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NOTICE

The concluding Part of Vol. XLI. Proceedings, containing Title-page and Index, will be issued shortly.

Proceedings of the American Academy of Arts and Sciences.

Vol. XLII. No. 1.—May, 1906.

CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.

New Series.—No. XXXII.

STUDIES IN THE EUPATORIEAE.

I. Revision of the Genus *Piqueria.*
II. Revision of the Genus *Ophryosporus.*
III. The Genus *Helogyne* and its Synonyms.
IV. Diagnoses and Synonymy of *Eupatorieae* and of certain other *Compositae* which have been classed with them.

By B. L. Robinson.

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UNIVERSITY, NEW SERIES, NO. XXXII.

STUDIES IN THE EUPATORIEAE.

By B. L. Robinson.

Presented March 14, 1906. Received February 21, 1906.

Introductory.

During the summer of 1905 the writer spent some weeks in visiting several of the larger herbaria of Europe in order to examine and photograph plants not hitherto authoritatively represented in the Gray Herbarium. In the course of this work considerable attention was given to the tropical American species of the great genus *Eupatorium* and several allied genera. As must be expected in all such large and difficult groups, which have not been subjected to recent revision, the study of the type-specimens of several hundred species has yielded much new information on the synonymy and proper classification of the group. The collections examined were: (1) the herbarium of the Museum of Natural History in Paris, where the herbaria of Jussieu, Lamarck, and Michaux were consulted, and special attention given to an admirably preserved and well-nigh complete set of the tropical American plants collected by Humboldt and Bonpland and critically described by Kunth in the Nova Genera et Species; (2) the rich private herbarium of the DeCandolle family in Geneva, including the invaluable Prodromus types; (3) the herbarium of the Imperial Museum of Natural History at Vienna, noteworthy among other ways by containing the fullest series available of the species of Jacquin and an excellent series of the plants of Pohl and species of Poeppig and Endlicher; (4) the herbarium of the Royal Botanical Museum at Berlin, remarkably rich in South American as well as in Old World types and in the study of *Eupatorieae* specially noteworthy by exhibiting to its fullest extent the recent critical work of Dr. Hieronymus; (5) Professor Urban's large and carefully selected West Indian herbarium; (6) the herbarium of the Botanical Museum of the University of Copenhagen, containing, together with much other material of interest, the extant types of Vahl; (7) the herbarium of the Linnean

Society of London, where special attention was given to the types of Linnaeus filius and of Sir James Edward Smith; (8) the herbarium of the Royal Botanical Gardens at Kew, noteworthy to the student of the *Eupatorieae* by exhibiting the very numerous Brazilian types of Gardner, Hooker & Arnott, Bentham, and Baker, as well as the Mexican work of Hemsley; (9) the herbarium of the British Museum of Natural History, including, among many other specimens of high interest, the plants of Clayton, Walter, and Philip Miller.

The writer would here express his sincere thanks to all those in charge of these herbaria for their cordial hospitality, uniform courtesy, and valuable aid during his researches. He is also indebted to Messrs. Oakes Ames and A. A. Eaton for several excellent photographs of type-specimens at Paris, to Mr. H. Hua for a critical comparison of a Peruvian *Piqueria* in the herbarium of Jussieu, to Miss Mary A. Day, Librarian of the Gray Herbarium, for bibliographical assistance, and to Mr. F. V. Coville and Dr. J. N. Rose for the loan of the material of *Piqueria* belonging to the United States National Museum.

About four hundred photographs of types were taken in the European herbaria, some important exchanges effected, and many notes and sketches prepared, which it is hoped may form an accurate basis for considerable work on the group concerned. In the present paper only a small part of the results of the summer's investigation can be presented, but as any complete or monographic treatment of so large a group must be delayed for a considerable time, it seems best to record such identities and synonymy as can be at once stated with definiteness, in order that certain traditional errors may not become more fixed by longer usage. The nomenclature adopted is that recommended by the international congress at Vienna.

I. Revision of the Genus Piqueria.

Piqueria is the typical genus of the *Piquerinae*, a small subtribe of the *Eupatorieae*. The *Piquerinae* are chiefly marked by their blunt anthers, which entirely lack the more or less expanded, oblong, or lanceolate prolongation of the connective, which is present almost without exception in other *Compositae*. In this subtribe the genus *Piqueria* is characterized by a complete absence or very rudimentary development of its pappus. Its natural affinities are obviously on one hand with *Ophryosporus*, which scarcely differs save in the presence of a well-developed setose pappus, and on the other hand with *Alomia* and *Ageratum*, which are habitally approached by the species of *Piqueria* belonging to the subgenus *Phalacraea*. Geographically *Piqueria* extends from the Sierras of northern Mexico through central and southern Mexico, Central America, and Andean South America to northern Chili. The only known exception to this range of the genus is the occurrence of *P. trinervia* Cav. in the mountains of Haiti, a station called to my attention by Professor Urban. The genus is capable of pretty clear division into four subgenera, of which *Erythradenia* is Mexican, *Eupiqueria* is of Mexico, Central America, and Haiti, *Phalacraea* of the moister Andean region from Colombia to Ecuador, and *Artemisioides* characteristic chiefly of the drier parts of the Andes of Peru and Chili.

Since the treatment of some eight species in DeCandolle's Prodromus in 1836 no effort has been made to revise the genus, although the number of its species has been considerably increased since that time. The following revision has been drawn up after personal examination of nearly all the specific types and of all the specimens of the genus found in several of the leading herbaria of Europe.

PIQUERIA Cav. (in memoriam Andreae Piquerii — hispanice, *Andrés Piquer* — medici hispanici et auctoris philosophici). — Capitula parva homogama 3–∞-flora, involucro ovoideo vel cylindrico vel campanulato, squamis saepius paucis subaequalibus laxe imbricatis vel subuniseriatis, receptaculo nudo plano vel leviter convexo. Corollae tubulosae albae vel caerulescentes, tubo proprio saepissime brevi piloso vel glanduloso-puberulo, faucibus saepe ampliatis, dentibus limbi ovato-deltoideis suberectis vel saepius ovati-oblongis et patentibus acutiusculis. Achaenia 5-angulata prismatica deorsum plus minusve angustata basi saepe oblique callosa apice rotundata calva vel disco annulari deciduo vel rarissime setis paucis brevibus coronata. Styli rami filiformi-clavellati longe exserti saepe valde recurvati. Antherae breviter oblongae, connectivo apice nec incrassato nec appendiculato. — Ic. iii. 18, t. 235 (1795); Usteri, N. Ann. xviii. 62 (1800); Pers. Syn. ii. 397 (1807); Rees, Cycl. xxvii. (1814); Cass. Bull. Soc. Philom. 1819, p. 127 (1819), et Dict. Sci. Nat. xli. 115 (1826); Less. Syn. 154 (1832); DC. Prod. v. 104 (1836); Endl. Gen. 366 (1838); Reichenb. Nom. 97 (1841); Jameson, Syn. Pl. Aequat. ii. 75 (1865); Benth. et Hook. f. Gen. ii. 238 (1873); Hemsl. Biol. Cent.-Am. Bot. ii. 77 (1881); Hoffm. in Engl. et Prantl, Nat. Pflanzenf. iv. Ab. 5, 133 (1890); Hook. f. et Jacks. Ind. Kew., ii. 544 (1894); Bailey et Scott in Bailey, Cycl. Hort. iii. 1357 (1901). *Phalacraea* DC. Prod. v. 105 (1836); Endl. Gen. 366 (1838). *Stevia* Hort. — Herbae annuae vel perennes vel frutices. Caulis erectus vel decumbens foliosus ramosus. Folia opposita vel alterna petiolata vel subsessilia integra vel saepius serrata vel dentata vel rarius angulata.

Species 19 generaliter bene distinctae quarum 5 Mexicanae sunt, una Mexicana et Centrali-Americana etiam Haitensis, reliquiae Andium montium Australi-Americae incolae.

Clavis subgenerum.

a. Corollae fauces tubulosi a tubo proprio non distincti. Folia alterna, glanduloso-punctata. . . . . Subg. I. ERYTHRADENIA.
a. Corollae fauces ampliati a tubo proprio bene distincti. Folia opposita vel alterna. *b.*
  b. Corollae fauces turbinati, brevissimi, dentibus limbi breviores; tubus proprius villosus vel rarissime glanduloso-puberulus. Subg. II. EUPIQUERIA.
  b. Corollae fauces campanulati vel cylindrici dentibus limbi longiores. Corolla externe praesertim tubo glanduloso-puberula. *c.*
    c. Capitula 4–5-flora. . . . . Subg. III. ARTEMISIOIDES.
    c. Capitula 15–∞-flora. . . . . Subg. IV. PHALACRAEA.

Subg. I. ERYTHRADENIA, subg. nov. Capitula circa 6-flora sessilia in panicula ampla pyramidata disposita. Corolla subcylindrica glanduloso-puberula sine faucibus distinctis; dentibus brevissimis. Folia alterna. — Species unica habitu distinctissima.

1. P. PYRAMIDALIS Robinson, caule tereti maculato 2–2.6 m. alto omnifariam puberulo-velutino; foliis alternis magnis longe petiolatis late ovatis vel suborbicularibus angulatis vel leviter lobatis crenato-dentatis 3-nerviis, basi rotundatis vel cordatis supra viridibus subtus griseis et glandulis parvis globosis numerosissimis brevissime stipitatis rubris munitis in nerviis et venis reticulatis velutino-tomentellis; panicula ampla folioso-bracteata, ramis ascendentibus; capitulis in glomerulis sessilibus ca. 6-floris; involucri squamis anguste oblongis dorso pubescentibus; corollis anguste tubulosis externe glanduloso-punctatis, dentibus brevissimis; achaeniis 5-angulatis, basin versus leviter angustatis parum obliquis, costis sursum hispidulis. — Proc. Am. Acad. xxxvi. 475 (1901). — MEXICO: in rupibus umbrosis montium supra Igualam, alt. 1250 m., *Pringle*, n. 8389 (hb. Gray, hb. U. S. Nat. Mus.).

Subg. II. EUPIQUERIA DC. Capitula parva 3–4-flora cymoso-corymbosa vel laxe paniculata, involucro anguste obovoideo vel subcylindrico, squamis 3–4 subaequalibus obovatis vel oblongis concavis tenuibus, apice obtusis vel rotundatis mucronatis eroso-subciliatis rarius dorso pubescentibus. Corollae tubus proprius extus pubescens, dentibus limbi oblongo-lanceolatis acutiusculis patentibus.— Prod. v. 104 (1836); Endl. Gen. 366 (1838); Hoffm. in Engl. & Prantl, Nat. Pflanzenf. iv, Ab. 5, 133 (1890) pro parte; nec Gardn.; nec Walp. — Vel herbae annuae vel perennes vel frutices. Folia opposita lanceolata vel ovata breviter petiolata.

Clavis specierum.

a. Involucri squamae dorso pubescentes. . . . . 2. *P. triflora.*
a. Involucri squamae dorso glabrae. *b.*
  b. Herba annua. Inflorescentia perlaxa, racemiformis. . . 3. *P. laxiflora.*
  b. Herbae perennes. Inflorescentia plus minusve corymbosa. Achaenia basi valde obliqua. *c.*
    c. Caulis omnifariam pilosus vel puberulus. . . . . . . 4. *P. pilosa.*
    c. Caulis bifariam solum puberulus. . . . . 5. *P. trinervia.*
  b. Frutices. Achaenia basi subrecta. . . . . 6. *P. serrata.*

2. P. TRIFLORA Hemsl., caule tereti vel supra subhexagono ubique pilosiusculo folioso 4 dm. alto plus minusve ramoso; foliis oppositis anguste lanceolatis basi perangustatis subsessilia caudato-attenuatis sed in apice vero obtusiusculis 3–6 cm. longis 5–9 mm. latis utrinque crispe puberulis remote serratis vel subintegris; inflorescentia laxe ramosa racemiformi; bracteolis linearibus; pedicellis filiformibus unilateraliter pilosis 3–7 mm. longis; capitulis numerosis obovoideis; involucri squamis 3 obovatis carinatis 3-nerviis scarioso-marginatis eroso-ciliatis apice rotundatis mucronatis, dorso hirsuto-pubescentibus; flosculis 3; corolla alba; tubo proprio extus lanato; faucibus quam dentes 5 oblongo-lanceolati brevioribus; achaeniis atro-fuscescentibus glabris 1.7 mm. longis. — Biol. Cent.-Am. Bot. ii. 77 (1881). — MEXICO: Cerro de Pinal, *Seemann*, n. 1478 (hb. Kew., hb. Gray).

3. P. LAXIFLORA Robinson & Seaton, herbacea tenuis laxe et copiose ramosa; radice annua; caule tereti viridi 4–5 dm. alto piloso; foliis oppositis, ovatis vel ovato-lanceolatis serratis 3–5-nerviis sparse pilosis basi cuneatis petiolatis apice obtusiusculis; inflorescentia trichotomo-furcata racemiformi; bracteolis subfiliformibus 2–4 mm. longis; pedicellis fere capillaribus 1 cm. longis flexuosis; capitulis obovoideis parvis 4-floris; involucri squamis viridibus obovatis mucronatis erosis tenuibus persistentibus; corollis albis; tubo perbrevi externe pubescenti; achaeniis atro-fuscescentibus 5-angulatis glabris lucidis. — Proc. Am. Acad. xxviii. 107 (1893). — MEXICO: in rupibus frigidulis convallium montanarum prope lacum Chapalam, *Pringle*, n. 4333 (hb. Gray, etc.); montibus prope Durango, *Pringle*, n. 10,067 (hb. Gray).

4. P. PILOSA HBK., herbacea perennis ramosa 5–13 dm. alta; caule subtereti omnifariam glanduloso-puberulo saepe purpurascenti folioso; foliis oppositis petiolatis ovatis serratis acuminatis 4–6 cm. longis 2–3 cm. latis utrinque puberulis; petiolo 4–10 mm. longo glanduloso-puberulo; inflorescentia trichotoma cymoso-corymbosa; capitulis parvis numerosis saepe congestis 4-floris; involucri squamis 4 ellipticis margine scariosis erosis apice rotundatis mucronatis dorso glabris; corollis albis, tubo brevi piloso-lanato, faucibus brevibus, dentibus

limbi 5 lanceolati-oblongis acutiusculis patentibus; achaeniis 5-angulatis glabris 2 mm. longis. — Nov. Gen. et Spec. iv. 153 (1820); Cass. Dict. Sci. Nat. xli. 116 (1826); DC. Prod. v. 104 (1836); Hemsl. Biol. Cent.-Am. Bot. ii. 77 (1881). *P. trinervia*, var. *pilosa* O. Kuntze, Rev. Gen. i. 355 (1891). *P. Pringlei* in sched. Pringlei pro parte, non Robinson & Seaton. — MEXICO (praecipue in montibus partis centralis rei publicae): *Humboldt et Bonpland*, n. 4342 (hb. Par.); *Alaman* (hb. DC.); Real del Monte, *Ehrenberg*, n. 481 (hb. Berol., hb. Gray); *Bates* (hb. Kew.); silvis deserti Vieja, *Bourgeau*, n. 828 (hb. Berol.); *Bourgeau*, n. 825 partim (hb. Par.); *Uhde*, nn. 414, 416 (hb. Berol.); Tacubaya, *Schaffner*, n. 300 (hb. Berol.); *Pringle*, nn. 3624, 4119, 4285 partim, 7930 (hb. Gray, etc.); Sierra de Pachucha, Hidalgo, *Rose*, n. 8867 (hb. U. S. Nat. Mus.).

Var. **Pringlei** (Robinson & Seaton), n. comb., caule et inflorescentia omnino eglandulosis pubescentibus; pilis brevibus albidis crispis. — *P. Pringlei* Robinson & Seaton, Proc. Am. Acad. xxviii 107 (1893). — MEXICO: saepe cum forma typica: pinetis convallis mexicanae, *Bourgeau*, n. 825 partim (hb. Gray, hb. Kew.); *Schmitz*, n. 398 (hb. Imp. Mus. Vindob.); *Pringle*, n. 4285 partim (hb. Gray); Sierra de las Cruces, alt. 3000 m., *Pringle*, n. 11,563 (hb. Gray).

5. P. TRINERVIA Cav., herbacea perennis erecta ramosa; radice fibrosa; caule tereti bifariam puberulo folioso 4–7 dm. alto; foliis oppositis lanceolatis vel anguste ovatis serratis subglabris 3 (–5)-nerviis crassiusculis basi cuneatis apice attenuatis; capitulis parvis saepius 4-floris laxe cymoso-corymbosis vel rarius in inflorescentia perlaxa racemiformi dispositis; involucri squamis ellipticis erosis margine tenuibus apice rotundatis mucronatis; corollis albis, tubo proprio brevi piloso, faucibus brevissimis, dentibus limbi 5 ovato-oblongis patentibus; achaeniis atrofuscis 5-angulatis basi oblique sigmoideo-callosis. — Ic. iii. 19, t. 235 (1794); Willd. Spec. iii. 1748 (1804); Pers. Syn. ii. 397 (1807); Jacq. f. Ecl. i. 70, t. 48; Bot. Mag. t. 2650 (1826); Cass. Dict. Sci. Nat. xli. 116 (1826); DC. Prod. v. 104 (1836); Hemsl. Biol. Cent.-Am. Bot. ii. (1881); Rose, Contrib. U. S. Nat. Herb. v. 231 (1899); Bailey, Cycl. Am. Hort. iii. 1357 (1901). *P. trinervis* J. E. Smith in Rees, Cycl. xxvii. n. 1 (1817). *P. ovata* G. Don in Loud. Hort. Brit. 337 (1830). *Ageratum febrifugum* Sess. ex DC. Prod. v. 104 (1836). *Stevia febrifuga* Moc. ex DC. l. c. *S. serrata* et *serratifolia* Hort. — MEXICO: vulgaris et late distributa praecipue in arvis et collinis ad 2900 m. alt. Exsiccatis visis: *Aschenborn*, nn. 201, 580; *Bates; Berlandier*, nn. 704, 1210, 1241; *Bilimek*, n. 578; *Botteri*, nn. 13, 391; *Bourgeau*, nn. 144, 149, 288, 1402; *Conzatti et González*, n. 1087; *Coulter*, n. 721; *Deam; Ehrenberg*, n. 480; *Galeotti*, nn. 2108, 2414,

2482; *Graham*, nn. 22, 23; *Halsted*, n. 27; *Harris; Humboldt et Bonpland*, nn. 4228, 4258, 4401; *Karwinski*, n. 104; *Kerber*, n. 328; *Lagasca*, n. 128; *Liebmann*, n. 110; *Nelson*, nn. 1720, 1928, 3176, 3463; *Palmer*, nn. 85, 313, 496½, 596; *Parry et Palmer*, nn. 85, 314; *Pringle*, nn. 241, 1748, 5686, 9053, 9951; *Purpus*, n. 55; *Rose*, n. 2767; *Sartorius; Schaffner*, n. 235; *Schiede*, n. 303; *Schmitz*, n. 73; *Schumann*, n. 62; *Seaton*, n. 276; *C. et E. Seler*, n. 1153; *C. L. Smith*, n. 1664; *L. C. Smith*, n. 291; *Uhde*, nn. 349, 412, 413, 448. GUATEMALA: *Heyde et Lux*, n. 3399 pl. exsic. J. D. Smithii (hb. Kew., hb. U. S. Nat. Mus.). COSTA RICA: *Cooper*, n. 5811 pl. exsic. J. D. Smithii (hb. Gray, hb. Kew.). HAITI: in montibus Furcy, *Picarda*, n. 1521 (hb. Urb.), forma capitulis paullo majoribus ad var. luxuriantem spectans.

NOTA. — Herba a horticultoribus sub nomine "Stevia" late culta et ob inflorescentia eleganter ramosa bene amata, a Mexicanis pro febrifuga, etiam a Cubensibus ut dicitur loco condimenti usa.

NOMINA VERNACULA: Empueshta, Hierba de San Nicolas, Hierba del tabardillo, Xoxonitzal, Xoxonitztac, Yoloxiltic; omnia fide cl. G. V. Alcocer.

Var. VARIEGATA Hort. ex Bailey, Cycl. Am. Hort. iii. 1358 (1901), est forma calidariarum foliis albo-marginatis.

Var. NANA Hort. ex Bailey, l. c., 1357, humilior 2–3.5 dm. alta foliosissima. — Forma ut videtur mexicana in calidariis saepe culta. Speciminia inculta sunt rara, e. g. MEXICO: *Méhédin*, 1864–5 (hb. Par.), Zacatecas, *Deam*, n. 141 (hb. Gray).

Var. LUXURIANS O. Kuntze, foliis ovatis argute serratis 4–9 cm. longis 2.3–4 cm. latis 5-nerviis glabriusculis basi rotundatis, petiolo ca. 1 cm. longo; capitulis quam ea formae typicae distincte majoribus; achaeniis 2.4 mm. longis. — Rev. Gen. i. 355 (1891). — COSTA RICA: silvis montanis in declivitatibus montis ignivomi Irazu, *C. Hoffmann*, n. 171 (hb. Gray); *Kuntze; Pittier*, n. 14,080 (hb. Gray).

6. P. SERRATA Gray, fruticosa ramosa primo aspectu glaberrima; caule tereti pallide viridi obsolete bifariam puberulo folioso; foliis oppositis ovati-oblongis acuminatis grosse arguteque serratis basi abrupte angustatis breviter petiolatis 7–9 cm. longis 2.5–4 cm. latis glaberrimis; capitulis cymoso-corymbosis numerosis 3-floris; involucri squamis elliptico-ovatis 3-nerviis apice rotundatis mucronatis margine ciliato-erosis dorso glabris; corollis albis, faucibus brevissimis, dentibus limbi oblongo-lanceolatis patentibus glabris, tubo proprio breviter subglanduloso-puberulo; achaeniis 5-angulatis glaberrimis annulo deciduo coronatis basi callosis parum obliquis. — Proc. Am. Acad. xv. 25 (1880). — MEXICO: in montibus Alvarez prope San Luis Potosi, *Parry et*

Palmer, n. 496 (hb. Gray, hb. U. S. Nat. Mus.), *Palmer*, n. 199 (hb. Gray).

Var. (?) ANGUSTIFOLIA Robinson & Greenman, foliis angustioribus lanceolatis obscure et remote serrato-crenatis basi cuneato-angustatis; capitulis eis formae typicae exacte similibus. — Am. Jour. Sci. l. 151 (1895). — MEXICO: in montibus Sierra de San Felipe, Oaxaca, alt. 2800–3300 m., *Pringle*, n. 4827 (hb. Gray, etc.), *Nelson*, n. 1049 (hb. Gray), *C. L. Smith*, n. 605 (hb. U. S. Nat. Mus.).

Subg. III. ARTEMISIOIDES DC. Capitula 3–4-flora, involucro subcylindrico, squamis saepius 4 subaequalibus. Corollae externe glanduloso-punctatae, glandulis minutis rubescentibus brevissime stipitatis; faucibus cylindricis vel subcylindricis tubo proprio subaequantibus, quam dentes limbi distincte longioribus. Achaenia deorsum plus minusve angustata parum obliqua. — Prod. v. 105 (1836). — Frutices ramosi andini. Folia opposita vel alterna rhomboidea vel lanceolata vel linearia basi cuneata petiolata.

Clavis specierum.

a. Folia alterna fasciculata. *b.*
  b. Capitula in panicula laxiuscula subrigida disposita. Involucri squamae obtusiusculae. . . . . 7. *P. galioides.*
  b. Capitula congesta. Involucri squamae breviter acuminatae vel attenuatae. Pappus e setis paucis brevissimis compositus vel nullus. *c.*
    c. Inflorescentia thyrsoidea. Involucri squamae lineari-lanceolatae. 8. *P. pinifolia.*
    c. Inflorescentia cymosa. Involucri squamae rhomboideo-oblongae acutatae vel breviter acuminatae. . . . . 9. *P. Cumingii.*
a. Folia opposita. *d.*
  d. Involucri squamae 3.5–5 mm. longae. *e.*
    e. Inflorescentia fulvo-tomentosa. Folia flabelliformi-ovata sinuato-dentata. . . . . 10. *P. pubescens.*
    e. Inflorescentia obscure glanduloso-puberula. Folia ovata serrato-dentata. *f.*
      f. Involucri squamae apice eroso-ciliatae. . . . . . 11. *P. Mathewsii.*
      f. Involucri squamae dorso granulatae nec erosae nec ciliatae. 12. *P. floribunda.*
  d. Involucri squamae 2–2.7 mm. longae. *g.*
    g. Capitula prope apices ramorum late patentium paniculae congesta. 13. *P. densiflora.*
    g. Capitula (numerosissima) spicato-racemosa in panicula subfastigiata folioso-bracteata disposita. *h.*
      h. Folia lanceolata dentata vel incisa. Capitula sessilia. 14. *P. peruviana.*
      h. Folia caulina linearia subintegra. Capitula breviter pedicellata. 15. *P. Hartwegi.*

7. P. GALIOIDES DC., fruticosa ramosa glabriuscula; foliis alternis fasciculatis linearibus opacis utrinque attenuatis acutis sessilibus integris subenerviis 1.5–2 cm. longis 2 mm. latis; panicula rigidiuscula patente ramosa pyramidali nudiuscula; capitulis haud congestis racemoso-spicatis subdivaricatis 3-floris; involucri ca. 4 mm. longi squamis subobtusis. — Prod. v. 105 (1836). — PERU: in Cordilleriis, *Haenke*, 1834 (hb. DC.). Species ut videtur bene distincta sed vix satis nota.

8. **P. pinifolia** (Phil.) Hieron. in herb., fruticosa ramosa 6–12 dm. alta, novellis plus minusve glutinosis; ramis arcuato-ascendentibus teretibus foliosissimis cortice griseo-flavido tectis; foliis in fasciculis alternis vel irregulariter sparseque dispositis suberectis anguste lanceolatis integerrimis vel utroque cum 2–3 dentibus brevibus patentibus subremotis instructis apice modice acutis basi attenuatis 1.5–3 cm. longis 2–6 mm. latis utrinque puberulis; thyrsis ovoideo-cylindricis 6–12 cm. longis 3–5 cm. crassis, ramulis rigidiusculis patentibus; capitulis in apice ramuli saepius 2–3 approximatis sessilibus 4–5-floris; involucri squamis 5 lineari-lanceolatis attenuatis subaequalibus laxis stramineis dorso rotundatis 1-nerviis 6 mm. longis glanduloso-puberulis margine involutis; corollis roseis tubulosis 3.5 mm. longis glanduloso-puberulis in fauces plus minusve ampliatis, dentibus limbi ovato-deltoideis patentibus; achaeniis 5-angulatis basi attenuatis puberulis summo saepius calvis rarius obsolete setuliferis. — *Stevia pinifolia* Phil. Ann. Mus. Nac. Chil. sec. 2 (botanica), 37 (1891); Reiche, Fl. de Chil. iii. 262 (1902). *Piqueria pinifolia* Hieron. in herb. Berol. — CHILI: Atacama, *Philippi* (hb. Berol.). PERU: In montibus Andinis supra Palcam, *d'Orbigny* (hb. Par.): Pachia, alt. 1200–1900 m., *Pearce*, Sept. 1862 (hb. Kew.). Specimen Pearcei est a cl. Benthamio (Gen. Pl. ii. 238) ad hoc genus sed sine nomine relatum.

9. **P. Cumingii,** n. sp., fruticosa ramosa, novellis vernicosis, ramis teretibus flavidis foliosissimis valde patentibus juventate pulverulento-puberulis; foliis fasciculatis anguste oblanceolatis obtusis vel obtusiusculis integerrimis vel cum dentibus unicis vel pluribus munitis 1–1.6 cm. longis 2–4 mm. latis basi attenuatis uninerviis glaberrimis plus minusve viscosis; inflorescentiis arcte congestis corymbosis, ramulis valde patentibus arcuato-ascendentibus; capitulis constipatis 5-floris; involucri squamis oblongis vel subrhomboideis acutis saepe vernicosis 5 mm. longis; corollis 3 mm. longis, tubo proprio quam fauces tubulosi breviori glanduloso-pulverulento; achaeniis pallidis substramineis prismaticis 5-angulatis 3 mm. longis basi attenuatis callosis in faciebus puberulis. — PERU MERIDIONALI et CHILI SEPTENTRIONALI: Cobija,

Iquiqui, et Arica, 1831, *H. Cuming*, n. 953 (hb. Kew.); Cobija, 1841, *Gaudichaud* (hb. Berol.).

10. P. PUBESCENS J. E. Sm., fruticosa oppositiramea 1–2 m. alta; caulibus teretibus a cortice griseo tectis; ramulis foliosis glanduloso-tomentellis; foliis oppositis, late rhomboideis, grosse arguteque inaequaliter dentatis basi cuneatis vel abrupte angustatis graciliter petiolatis 1.4–3.3 cm. longis 1–3.6 cm. latis sub lente utrinque papilloso-granulosis, petiolo fulvo-tomentoso 8–18 mm. longo; capitulis numerosis congestis corymbosis 4–5-floris; involucri squamis anguste oblongis attenuatis dorso glanduloso-puberulis; corollis albis vel (?) flavidis. — J. E. Smith in Rees, Cycl. xxvii. n. 2 (1814); DC. Prod. v. 105 (1836). *P. quinqueflora* Cass. Bull. Soc. Philom. 1819, p. 128 (1819), et Dict. Sci. Nat. xli. 116 (1826); DC. Prod. v. 105 (1836). — PERU: spec. typ. in hb. L. f. (hb. Linn. Soc.); *Dombey* (hb. Par.); *Mathews*, n. 946 (hb. Gray); Obrajillo, *Wilkes* (hb. Gray, hb. U. S. Nat. Mus.); Lima, *Cuming*, n. 1045 (hb. Brit. Mus.); ad pedem montis Amancaes prope urbem Lima, 21 Julio, 1876 (floribus albis), *André*, n. 4117 (hb. Gray); in rupibus prope Lima, 30 Nov. 1901, *Weberbauer* n. 9 (hb. Berol.).

11. **P. Mathewsii**, n. sp., suffruticosa oppositiramea; ramis tenuibus rubescentibus glaberrimis; foliis oppositis membranaceis ovatis crenato-serratis vel grosse-dentatis acuminatis graciliter petiolatis 3–5-nerviis utrinque viridibus glabris 4–6 cm. longis 2–3 cm. latis, petiolo 7–10 mm. longo; panicula laxe ramosa, ramis gracilibus teretibus puberulis valde patentibus vel arcuato-ascendentibus prope apicem capitula subcorymbosa ferentibus; capitulis 4–5-floris; involucri squamis breviter acuminatis vel saepius obtusiusculis ca. 5 mm. longis apice ciliolatis; corollis valde exsertis, tubo proprio gracili glanduloso-puberulo, faucibus cylindricis bene distinctis quam dentes limbi longioribus. — PERU: Purruchuca, *Mathews*, n. 1015 (hb. Kew.).

12. P. FLORIBUNDA DC., fruticosa oppositiramosa minutissime puberula vel subglabra; ramis patentibus griseo-fuscis foliosis; foliis oppositis ovatis serrato-dentatis acutis basi rotundatis vel abrupte angustatis graciliter petiolatis subglabris 1–2 cm. longis; panicula patente ramosa pyramidali 7–11 cm. lata; capitulis numerosis 4–6-floris; involucri anguste cylindrici squamis oblanceolati-oblongis vel -linearibus 3-costatis dorso convexis et glanduloso-puberulis acutis; corollis albis, tubo proprio gracili dense glanduloso-atomifero quam fauces cylindrici subglabri breviori, limbi dentibus deltoideis brevibus; achaeniis gracillimis prismaticis 5-angulatis basi gradatim attenuatis subrectis, angulis glanduloso-hispidulis. — Prod. v. 105 (1836); Phil. Cat. Pl. Vasc. Chil. 174 (1881). — PERU: in montibus Andinis, *Haenke*,

1834 (hb. DC.); Obrajillo, *Wilkes* (hb. Gray, hb. U. S. Nat. Mus.); in rupibus inter Matucana et Tambo, alt. 2370–2650 m., *Weberbauer*, n. 115 (hb. Berol.). CHILI: *Haenke* sec. DC. sed dubitative.

NOTA. — Habitatio originalis a DC. data "montanis Oronocensibus" possit forsan errore pro *montanis Huanoccensibus*.

13. P. DENSIFLORA Benth., fruticosa vel paene herbacea glabriuscula; ramis plus minusve flexuosis gracilibus teretibus, internodiis quam folia saepius longioribus; foliis oppositis membranaceis ovatis vel rhomboideis graciliter petiolatis acuminatis serratis 3-nerviis 5–6 cm. longis, petiolo ca. 1 cm. longo; foliis superioribus lanceolati-oblongis integris; panicula pyramidali oppositiramea, ramis patentissimis basi nudis prope apicem capituliferis puberulis; capitulis congestis 4–5-floris cylindricis; involucri squamis oblongo-linearibus 2.5 mm. longis glabriusculis apice subobtusis eroso-ciliatis; corollis valde exsertis 2.5 mm. longis, tubo gracili glanduloso-puberulo, faucibus campanulato-ampliatis glabris vel obsolete granulatis, dentibus limbi 5 anguste oblongis revolutis acutis; achaeniis glaberrimis 1.6 mm. longis nigris lucidis summo disco annulari coronatis. — Bot. Sulph. 110 (1845); Jameson, Syn. Pl. Aequat. ii. 75 (1865). — ECUADOR: Insula Puna prope Guayaquil, 1841, *Hinds*, n. 401 (hb. Kew.); in terra pingui agrorum fruticiferorum Insulae Punae 1–2 m. alta floribus albis, Sept. 1838, *Barclay*, nn. 412, 2426 (hb. Brit. Mus.); reg. trop., *Sodiro*, n. 3/4 (hb. Berol.). Specimen dubium ex herb. Thibaudii verosimiliter a Néeo lectum sed sine ullo indicio loci in hb. DC. invenitur.

NOTA. — Cl. Benthamius asseveravit in descriptione principali hujus speciei "antherae apice appendiculatae," sed dissectio a cl. Brittenio in herbario Musei Britannici benevolente permissa et ab auctore maxima cum cura facta flosculorum e specimine Barclayano antheras sine ulla dubitatione inappendiculatas exhibuit.

14. **P. peruviana** (Gmel.), n. comb., fruticosa 2–4 m. alta oppositiramea subglabra; foliis oppositis tenuibus rhomboideo-lanceolatis attenuatis inferioribus grosse arguteque serratis basi cuneatis 3-nerviis supra viridibus glaberrimis subtus vix pallidioribus in nerviis puberulis 6–10 cm. longis, petiolo 1.5 cm. longo, foliis superioribus multo minoribus vix serratis vel etiam integerrimis; panicula ramosissima pyramidali, ramis ascendentibus foliosis; capitulis parvis sessilibus basi unibracteolatis; bracteolis margine pubescentibus; involucri squamis 4 aequalibus anguste oblongis obtusis ciliatis basi calloso-incrassatis; flosculis saepius 4. — *Flaveria peruviana* [Juss.] Gmel. Syst. ii. 1269 (1791). *F.* sp. *Peruviana a Dombeyo data* Juss. Gen. 187 (1789). *F. spicata* J. E. Sm. in Rees, Cycl. xiv. n. 2 (1810). *Piqueria artemisioides* HBK. Nov.

Gen. et Spec. iv. 153 (1820); DC. Prod. v. 105 (1836); Loud. Hort. Brit. 337 (1830) ubi errore dicitur originem mexicanam habere. — PERU: Lima, *Dombey* (hb. Par., hb. Berol.); Lima et Peruvia septentrionali, *Cuming*, n. 1037 (hb. Kew.); *Mathews*, n. 413 (hb. Gray); *Besser* (hb. Berol.); *Wilkes* (hb. Gray); *Weberbauer*, nn. 8, 8a, 39, et 200 (hb. Berol.). ECUADOR: Alampi, *Humboldt et Bonpland*, n. 3229 (hb. Par.); in montibus Andium prope Alausi, Huataxi, etc., in fruticetis frequens, *Spruce*, n. 5965 (hb. Kew., hb. Par., hb. Vindob., hb. Gray), "floribus albis suaveolentibus."

15. **P. Hartwegi,** n. sp. fruticosa 9–12 dm. alta; ramis flexuosis teretibus juventate puberulis deinde glaberrimis a cortice griseo tectis; foliis oppositis saepius fasciculatis linearibus glabris obsolete crenulato-serratis subtus pallidioribus 4–5.5 cm. longis 4 mm. latis uninerviis et mediocriter reticulato-venulosis; capitulis numerosissimis 4-floris in panicula elongata ramosissima racemose dispositis, lateralibus breviter sed distincte pedicellatis, terminalibus plus minusve glomeratis sessilibus; corollis 1.7 mm. longis albis, tubo proprio puberulo fauces campanulato-ampliatos subaequanti, dentibus recurvis; styli ramis longe exsertis apice mediocriter incrassatis; achaeniis immaturis. — *P. artemisioides* Benth. Pl. Hartw. 136 (1844), non HBK. — PERU: El Catamayo, *Hartweg*, n. 762 (hb. Kew., hb. Par., hb. Vindob., hb. Berol.).

Subg. IV. PHALACRAEA Benth. et Hook. f. Capitula 15–∞ flora-corymbosa, involucro campanulato, squamis 7–∞. Corolla externe glanduloso-punctata, glandulis parvis rubescentibus brevissime stipitatis, tubo proprio brevi, faucibus campanulatis vel cylindricis quam dentes limbi longioribus. — Gen. ii. 238 (1873); Hoffm. in Engl. et Prantl, Nat. Pflanzenf. iv. Ab. 5, 133 (1895). *Phalacraea* DC. Prod. v. 105 (1838); Deless. Ic. iv. 3, t. 8 (1839); Regel, Gartenfl. iii. 388, t. 107 (1854). — Species andinae herbaceae vel suffruticentes pubescentes. Folia opposita ovata petiolata. Habitus *Agerati.*

Clavis specierum.

a. Corollae fauces campanulati. *b.*
  b. Achaenia glabra basi valde obliqua sigmoidea. . . . . 16. *P. Sodiroi.*
  b. Achaenia in costis sursum hispidula basi parum obliqua. 17. *P. callitricha.*
a. Corollae fauces cylindrici. *c.*
  c. Capitula ca. 18-flora. Involucri squamae ovatae obtusiusculae. 18. *P. latifolia.*
  c. Capitula ca. 100-flora. Involucri squamae lineari-lanceolatae, acuminatissimae. . . . . 19. *P. coelestina.*

16. P. Sodiroi Hieron., herbacea tenuis decumbens basi repens; caule flexuoso opposititrameo omnifariam hirsutulo, internodiis elongatis quam folia multo longioribus; foliis oppositis parvis 1–1.4 cm. longis deltoideo-ovatis crenato-serratis; capitulis paucis graciliter pedicellatis subglobosis ca. 38-floris; corollis externe glandulis rubris sparsissimis munitis, tubo proprio brevissimo basi pilis multicellularibus valde recurvatis vel reflexis hirsuto, faucibus campanulatis; achaeniis glaberrimis obovoideis basi valde obliquis. — Hieron. in Engl. Bot. Jahrb. xxix. 3 (1900). — Ecuador: Sarsarango, *Seemann* n. 705 (hb. Kew., hb. Gray); in umbrosis humidis juxta Quito, alt. 3000 m., floret Junio et Julio, *Jameson*, n. 205 (hb. Kew.), n. 307 (hb. Kew., hb. Par.) n. 760 (hb. Brit. Mus.); fruticetis superioribus regionis silvestris in declivitatibus occidentalibus Andium circa Mihuir, prov. Cuenca, alt. 2800–3300 m., floret in Oct., *Lehmann*, n. 5187 (hb. Berol.); inter virgulta in regione interandina, *Sodiro*, n. B/1 (hb. Berol.); Chilliquin, *Mathews*, n. 1401 (hb. Kew., hb. Brit. Mus.). Peru: Prov. Chachapoyas (hb. Kew.).

17. **P. callitricha**, n. sp., herbacea decumbens vel procumbens ramosa; caule elongato quadrangulari atropurpureo glabriusculo foliato; ramis ascendentibus; foliis oppositis ovato-deltoideis 3(–5)-nerviis petiolatis acutiusculis sed apice vero obtusis grosse crenato-dentatis 1–4 cm. longis 0.7–3 cm. latis supra atroviridibus pilis albis basi incrassatis scabriusculis subtus vix pallidioribus in nerviis venisque pubescentibus margine pilis conspicue septatis eleganter ciliatis; capitulis laxe cymoso-paniculatis graciliter pedicellatis ca. 15-floris; involucri squamis ca. 13 oblanceolatis acutiusculis carinatis in margine et carina ciliatis; corollis externe glanduloso-puberulis, tubo proprio brevissimo, faucibus campanulatis quam dentes limbi longioribus; achaeniis 5-angulatis obovoideo-prismaticis disco annulari coronatis basi subrectis. — Colombia: in summo monte Quendin, Maio 1846, *Purdie* (hb. Kew., hb. Gray); inter Boquia et Volconcito, *Holton* (hb. Gray).

18. P. latifolia (DC.) Gardn., herbacea perennis; caulibus decumbentibus subsimplicibus vel paucirameis foliatis omnifariam puberulis; foliis oppositis ovatis acutiusculis crenato-serratis trinerviis utrinque viridibus subtus in nerviis pubescentibus; inflorescentia cymoso-corymbosa trichotoma paucicapitata; capitulis ca. 18-floris; involucri campanulati squamis ca. 7 ovatis obtusiusculis 3-nerviis dorso glanduloso-puberulis; corollis externe glanduliferis, faucibus cylindricis tubo proprio multo longioribus, dentibus limbi brevibus patentibus; achaeniis obovoideis 5-angulatis basi attenuatis subrectis summo disco annulari coronatis costis sursum hispidulis. — Gardn. in Hook. Lond. Jour. Bot. vi. 430 (1847); Nicholson, Dict. Gard. iii. 148 (1886) pro

parte. *Phalacraea latifolia* DC. Prod. v. 106 (1836); Deless. Ic. iv. 3, t. 8 (excl. syn. *Ageratum latifolium* Cav.); Regel, Gartenfl. iii. 388. — PERU: prope Lima, *Née* (hb. DC.), *Haenke;* Cuzco, *Gay* (hb. Gray).

Var. **glabra** (DC.), n. comb., caule glabro; foliis paullo majoribus ovatis cordatis ad 7 cm. longis; pedicellis vix apice puberulis. — *Phalacraea latifolia β glabra* DC. Prod. v. 106 (1836). — PERU? *Née* (hb. DC.).

19. P. COELESTINA (Regel) Hieron., suffruticosa ramosissima 1 m. alta pubescens; foliis oppositis ovatis subcordatis crenato-dentatis; capitulis in cymis saepius 5-capitulatis longe pedunculatis dispositis ca. 100-floris; involucri squamis numerosis lineari-lanceolatis subtriseriatis acutissimis; corollis externe in tubo proprio glanduloso-puberulis, faucibus cylindricis quam dentes limbi longioribus. — Hieron. in Engl. Bot. Jahrb. xxix. 3 (1900). *Phalacraea coelestina* Regel, Gartenfl. iii. 388, t. 107 (1854). *P. latifolia* Nicholson, Dict. Gard. iii. 148 (1886) pro parte. — PERU: casualiter in terra cum speciebus orchidaceis et bromeliaceis a Warscewiczio lectis in horticulturam europaeam introducta. Species inquirenda ut videtur ab herbariis absens.

Species excludendae.

P. ageratoides (HBK.) Gardn. in Hook. Lond. Jour. Bot. vi. 430 (1847) est *Alomia ageratoides* HBK.

P. angustata Gardn. l. c. 432 est *Alomia angustata* Benth.

P. attenuata Gardn. l. c. 430 est *Gymnocoronis spilanthoides* DC.

P. cinerea Gardn. l. c. 422 est *Alomia cinerea* Benth.

P. eupatorioides Gardn. l. c. 431 est *Trichogonia salviaefolia* Gardn. var. *β calva* Baker.

P. Eupatorium Gardn. l. c. 430 est *Clibadium rotundifolium* DC.

P. fastigiata Gardn. l. c. 431 est *Alomia fastigiata* Benth.

P. foliosa Gardn. l. c. 432 est *Alomia foliosa* Benth. et Hook. f.

P. latifolia Gardn. l. c. 430 est *Ageratum conyzoides* L.

P. longipetiolata Sch. Bip. ex. Bak. in Mart. Fl. Bras. vi. pt. 2, 183 (1876) est *Gymnocoronis spilanthoides* DC.

P. myriadenia Sch. Bip. l. c. 192 est *Alomia myriadenia* (Sch. Bip.) Bak.

P. polyphylla Sch. Bip. l. c. 191 est *Alomia polyphylla* (Sch. Bip.) Bak.

P. subcordata Gardn. l. c. 430 est *Gymnocoronis spilanthoides* DC.

II. Revision of the Genus Ophryosporus.

Ophryosporus is a natural but not sharply delimitable group of South American *Eupatorieae Piquerinae*. It differs from *Piqueria* in the presence of a well developed barbellate or shortly plumose pappus of numerous capillary or rarely slightly thickened bristles. From *Eupatorium* it differs in the entire absence of the apical appendage of the anthers and usually may be distinguished furthermore by its subsimple involucre and rather conspicuously enlarged tips of the style branches. The boundary between the two genera has been variously drawn and must be regarded at best as a somewhat artificial line. To the writer it appears that if the genus *Ophryosporus* is to be maintained at all it must be restricted to those species in which the anthers are really destitute of any terminal appendage or trace of such a structure in any perceptible broadening or thickening of the connective at its summit. Close examination shows such rudimentary appendages in several species which recent authors have referred to *Ophryosporus*, plants which furthermore exhibit at least in some instances the more imbricated involucre and less enlarged style tips usual in *Eupatorium*. If these species were kept in *Ophryosporus* there would appear to be no single valid character and no combination of characters by which the two genera could be clearly divided. For this reason it seems best to refer these plants again to *Eupatorium*. *Ophryosporus* may be divided into two sections (scarcely of subgeneric rank) on the nature of the inflorescence and arrangement of the leaves. The typical section is Chilian, while the other and larger section has a wider distribution, occurring in Brazil, Argentina, Chili, Peru, Bolivia, and Ecuador. Most of the species of Brazil and Argentina are, so far as yet known, rather local. Those of the Andes, on the other hand, in some cases have a considerable north and south range. The writer has been able to examine the types or authentic specimens of all the species and varieties of the genus as here treated.

OPHRYOSPORUS Meyen. (Nomen ab ὀφρύς, *supercilium*, et σπορά, *semen*, derivatum, achaeniis in costis saepissime ciliatis.) — Capitula homogama parva numerosa paniculata vel thyrsoidea 3–12-flora plus minusve pedicellata; involucri anguste campanulati vel cylindrici, squamis subaequalibus 1–2-seriatim laxe vel vix imbricatis, disco nudo parvo leviter convexo. Corollae albae tubulosae sursum gradatim ampliatae vel in fauces distinctos dilatatae externe praesertim in tubo proprio et sub apicibus dentium glanduloso-puberulae vel atomiferae; limbi dentibus 5 brevibus triangulares patentibus. Filamenta gracilia glabra.

Antherae oblongae apice rotundatae vel truncatae vel retusae, connectivo angusto apice nullo modo expanso nec appendiculato. Achaenia 5-angulata prismatica vel saepius leviter deorsum angustata 5-costata saepius praesertim in costis hispidula vel glanduloso-ciliolata inter costis saepe glanduloso-puberula vel atomifera. Pappi setae 15–35 albae vel roseae barbellatae vel breviter plumosae quam corolla vix breviores. — Reise um die Erde, i. 402 (1834); DC. Prod. vii. 260 (1838); Walp. in Meyen, Beitr. zur Botan. 256 (1843); Benth. et Hook. f. Gen. ii. 239 (1873); Bak. in Mart. Fl. Bras. vi. pt. 2, 186 (1876) excl. spec. n. 1; Griseb. Abhandl. Gesellsch. Wiss. Goett. xxiv. 173 (1879); Hoffm. in Engl. et Prantl, Nat. Pflanzenf. iv. Ab. 5, 133 (1890); Löfgren, Commissão Geograph. e Geolog. de São Paulo, Boletim, xii. 139 (1897); Hieron. in Engl. Bot. Jahrb. xxii. 705 (1897); Reiche, Fl. de Chil. iii. 258 (1902). *Nothites* DC. Prod. v. 186 (1836) solum quoad spec. n. 5. *Pachychaeta* Sch. Bip. ex Bak. in Mart. Fl. Bras. vi. pt. 2, 186 (1876). — Frutices ramosi, foliis oppositis vel alternis saepius petiolatis rarius sessilibus triangulari-rhomboideis vel ovato-lanceolatis vel rarius linearibus dentatis vel serratis vel crenatis vel rarius integris.

Species 17 Americae australis incolae. Species aliae plurimae ab auctoribus adhuc relatae videntur ob involucri squamis valdius imbricatis et ob antheris plus minusve distincte (quamquam brevissime) apice appendiculatis melius ad *Eupatorium* referendae propterea quod his speciebus in *Ophryosporo* inclusis distinctio inter generibus omnino evanesceret.

Clavis sectionum.

Folia alterna parva, internodiis brevissimis. Panicula thyrsoidea.
Sect. I. EUOPHRYOSPORUS.
Folia opposita majora, internodiis bene evolutis. Capitula in paniculis amplioribus vel in cymis axillaribus disposita. . . . Sect. II. OPHRYOCHAETA.

Sect. I. EUOPHRYOSPORUS, sect. nov. Capitula 5–12-flora in thyrso angusto elongato folioso-bracteato disposita. Folia parva alterna triangulari-rhomboidea vel lanceolato-linearia saepius fasciculata, internodiis brevissimis.

Clavis specierum.

Capitula ca. 5-flora. Folia lanceolati-oblonga vel linearia subglabra.
1. *O. paradoxus.*
Capitula 7–12-flora. Folia rhomboidea vel anguste deltoidea tomentella.
2. *O. triangularis.*

1. O. PARADOXUS (Hook. et Arn.) Benth. et Hook. f., fruticosus ramosus glabriusculus; ramis teretibus foliosissimis erectis; foliis anguste lanceolati-oblongis vel linearibus utroque acutis 1.5–2.5 cm. longis te-

nuibus subsessilibus subglabris viridibus margine hinc inde dentibus argutis solitariis vel paucis instructis; panicula thyrsoidea ad 3 dm. longa, ramis ascendentibus foliosis subfastigiatis pluri- vel multi-capitulatis; capitulis numerosissimis 5-floris; involucri squamis oblongis apice rotundatis margine glanduloso-ciliolatis dorso sub-3-nerviis obscure glanduloso-puberulis; corollis 3.2 mm. longis, faucibus subcylindricis glabris, tubo proprio glanduloso-puberulo aequantibus; achaeniis in costis et inter costis glanduloso-puberulis; pappi setis ca. 20 subplumosis. — Gen. Pl. ii. 239 (1873) ex Hook. f. et Jacks. Ind. Kew. ii. 354 (1895). *O. triangularis* Reiche, Fl. de Chil. iii. 259 (1902), not Meyen. *Eupatorium paradoxum* Hook. et Arn. in Hook. Comp. Bot. Mag. i. 240 (1835). *Nothites baccharidea* DC. Prod. v. 187 (1836); Gay, Fl. Chil. iii. 476 (1847). *N. baccharoides* Meigen in Engl. Bot. Jahrb. xvii. 283 (1893). *Stevia polyphylla* DC. l. c. 123. *S. baccharoides* Meigen, l. c. — CHILI: in planitiebus incultis et locis saxosis prope marem vulgaris, *Gay*, n. 990 (hb. Gray); Valparaiso, *Cuming*, n. 337 (hb. Kew., hb. Brit. Mus., hb. Gray), *Bridges*, n. 52 (hb. Kew., hb. Brit. Mus., hb. Gray), *Gillies* (hb. Gray), *Bertero* (hb. Gray), *Philippi*, n. 406 (hb. Kew., hb. Brit. Mus.); Campana di Quillota, *Edmonston* (hb. Kew., hb. Gray), *Bertero*, n. 837 (hb. Gray, hb. Brit. Mus.), *Germain* (hb. Kew.).

2. O. TRIANGULARIS Meyen, fruticosus ramosus; ramis rectis vel saepius leviter arcuatis rigidiusculis foliosissmis teretibus tomentellis; foliis parvis saepe fasciculatis rhomboideis ca. 1 cm. longis crenato-dentatis vel lobulatis basi cuneatis utrinque tomentellis; thyrsis subcylindricis terminalibus 5–10 cm. longis 2–4 cm. crassis multicapitulatis; capitulis 7–12-floris; involucri squamis 6–10 aequalibus oblongis dorso enerviis tomentellis; corollis externe glanduloso-puberulis; faucibus paullo ampliatis tubo proprio longioribus. — Reise um die Erde, i. 402 (1834); Gay, Fl. Chil. iii. 481 (1847); Benth. et Hook. f. Gen. ii. 239 (1873); Phil. Cat. Pl. Vasc. Chil. 174 (1881). *Eupatorium decipiens* Hook. et Arn. in Hook. Comp. Bot. Mag. i. 240 (1835). *E. foliolosum* DC. Prod. v. 174 (1836). *Kuhnia multiramea* Turcz. Bull. Soc. Nat. Mosc. xxiv. pt. 1, 168 (1851). *E. Volckmanni* Phil. Anal. Univ. Chil. xviii. 51 (1861). *Ophryosporus foliolosus* Reiche, Fl. de Chil. iii. 259 (1902). — CHILI: in planitiebus desiccatis, Coquimbo, *Meyen*, *Gaudichaud*, *Macrae* (hb. Kew., hb. DC.), *Harvey* (hb. Gray); Vallenar, *Reed*, n. 47 (hb. Kew.); Concepcion, *Bridges*, n. 1412 (hb. Kew., hb. Brit. Mus., hb. Gray); Copiapo, *Gay* (hb. Gray); Cobija, *Gaudichaud* (hb. Gray); desertis Atacamae, *Morong*. n. 1149 (hb. Gray).

Nomen vulgatum: *rabo de zorra* (fide Gayii).

Sect. II. Ophryochaeta, sect. nov. Folia opposita, internodiis bene evolutis. Capitula in paniculis amplis vel in cymis axillaribus saepius laxe disposita.

Clavis specierum.

a. Squamae involucri saltim exteriores dorso puberulae vel pubescentes. *b.*

b. Achaenia glabra. Pubescentia involucri brevissima appressa. 3. *O. Charua.*

b. Achaenia pubescentia vel glandulifera vel saltim in angulis hispidula. Pubescentia involucri squamarum laxior. *c.*

c. Inflorescentiae axillares quam folia multo breviores. 4. *O. axilliflorus.*

c. Inflorescentiae corymboso-paniculatae terminales. *d.*

d. Flosculi minimi. Corollae ca. 2 mm. longae. *e.*

e. Folia ovata haud attenuata. *f.*

f. Inflorescentia laxa, capitulis graciliter pedicellatis. 5. *O. laxiflorus.*

f. Inflorescentia densa, capitulis subsessilibus. 6. *O. Regnellii.*

e. Folia ovato-lanceolata vel lanceolata conspicue attenuata. *g.*

g. Stigmata nigrescentia. Achaenia 1.8 mm. longa. Species Brasiliae et Argentinae. . . . . 7. *O. Freyreysii.*

g. Stigmata brunnea. Achaenia 1.5 mm. longa. Species andinae et Argentinae. . . . . 8. *O. piquerioides.*

d. Flosculi majores. Corollae 3.3–5.5 mm. longae. *h.*

h. Capitula 6–11-flora. Folia 1.6–4 cm. longa. 9. *O. origanoides.*

h. Capitula 4–5-flora. Folia ca. 7 cm. longa. 10. *O. venosissimus.*

a. Squamae involucri praeter marginem saepe eroso-ciliatam glabrae. *i.*

i. Folia graciliter quamquam saepe breviter petiolata. *j.*

j. Capitula saepissime 8–10-flora; involucri squamis ca. 5 mm. longis. 11. *O. macrodon.*

j. Capitula 4–5-flora; involucri squamis 2–3.6 mm. longis. *k.*

k. Involucri squamae lanceolati-oblongae vel ellipticae. *l.*

l. Achaenia gracillima 3 mm. longa. Species Argentinae. 12. *O. Lorentzii.*

l. Achaenia crassiora 2 mm. longa. Species bolivienses. *m.*

m. Cymae axillares compositae quam folia breviores. 13. *O. Cumingii.*

m. Cymae in corymbo convexo terminali aggregatae. 14. *O. Kuntzei.*

k. Involucri squamae plus minusve obovatae apice rotundatae. *n.*

n. Folia ovata subintegra. Species ecuadorensis. 15. *O. Sodiroi.*

n. Folia lanceolata distincte serrata. Species argentinensis. 16. *O. clavulatus.*

i. Folia sessilia. . . . . 17. *O. Pachychaeta.*

3. O. Charua (Griseb.) Hieron., fruticosus ramosus puberulus; ramis arcuato-ascendentibus; foliis oppositis petiolatis ovato-lanceolatis tenuibus acutis subgrosse serrato-dentatis margine utrinque cum dentibus 4–5 acutis instructis basi subcuneatis 2–3 cm. longis 1–1.5 cm. latis glabriusculis, petiolo ca. 8 mm. longo minutissime puberulo; cymis compositis ad axillas superiores glomeratis ca. 4–6 cm. diametro

rotundatis multicapitulatis; pedicellis puberulis gracilibus 2–5 mm. longis; capitulis 4-floris; involucri squamis 4 appresse puberulis anguste oblongis obtusis 3 mm. longis; corollis 3 mm. longis externe glanduloso-atomifero a tubo proprio brevi gracili in fauces longiores gradatim ampliatis; styli ramis brunneis longe exsertis apice modice incrassatis; achaeniis fusco-nigris maturitate glaberrimis; pappi setis ca. 30 barbellatis 2.5 mm. longis in specimine exsiccato sordide albis. — Hieron. in Engl. Bot. Jahrb. xxii. 705 (1897). *Mikania Charua*, Griseb. Abhandl. Gesellsch. Wiss. Goett. xxiv. 174 (1879). — ARGENTINA: Prov. de Catamarca, *Schickendantz*, n. 26 (hb. Berol.).

Nomen vulgatum: *charrua.*

4. O. AXILLIFLORUS (Griseb.) Hieron., suffruticosus; caulibus flexuosis teretibus striatis pilosiusculis; foliis oppositis membranaceis ovato-lanceolatis 3-nerviis argute serratis basi cuneatis apice acuminatis pilosiusculis; cymis axillaribus quam folia multo brevioribus; capitulis 5-floris; involucri squamis oblongis 3.5 mm. longis apice rotundatis dorso molliter pubescentibus margine ciliatis; corollis (immaturis) 4 mm. longis externe in tubo proprio glanduloso-puberulis in fauces leviter ampliatis, dentibus limbi 5 deltoideis; achaeniis 2 mm. longis in costis 5 sursum hispidulis disco crassiusculo depresso-hemisphaerico coronatis; pappi setis ca. 20 sursum barbellatis. — Hieron. in Engl. Bot. Jahrb. xxii. 706 (1897). *Eupatorium axilliflorum* Griseb. Abhandl. Gesellsch. Wiss. Goett. xix. 121 (1874). — ARGENTINA: Cordoba, prope Ascochinga, *Lorentz.*

5. O. LAXIFLORUS Bak., fruticosus ramosus erectus ad 1 m. altus; caule tereti glabrato a cortice brunneo tecto; ramulis dense fulvescenti-pubescentibus, pilis crispis; foliis oppositis ovatis breviter petiolatis grosse crenato-serratis a basi subintegra 3-nerviis obtusiusculis supra viridibus scabriusculis subtus paullo pallidioribus modice pubescentibus ca. 5 cm. longis ca. 2.7 cm. latis; inflorescentia laxe paniculata ampla; bracteis ellipticis vel linearibus, ramulis pedicellisque filiformibus, his ca. 4 mm. longis saepe arcuatis vel flexuosis; capitulis ca. 5-floris, squamis involucri 4–5 obovato-oblongis vel angustioribus 2.5 mm. longis apice rotundatis eroso-ciliatis dorso pubescentibus; corollis ca. 2.3 mm. longis, tubo proprio gracile subglabro; faucibus turbinato-campanulatis externe granulatis; achaeniis obovoideis 5-costatis in costis sparse hirsutis; pappi setis ca. 16 achaenio distincte longioribus firmiusculis barbellato-plumosis. — BRASILIA: in campis, Prov. Minas Geraes prope Caldas, *Regnell*, n. III. 709 (herb. Berol.).

6. O. REGNELLII (Sch. Bip.) Bak., fruticosus erectus ramosus tomen-

toso-pubescens 1–1.3 m. altus; ramis oppositis ascendentibus rectis vel leviter arcuatis foliatis, internodiis quam folia longioribus; foliis oppositis modice firmis ovatis obtusiusculis crenato-serratis 3-nerviis basi cuneatis supra scabriusculis subtus pallidioribus praecipue in nerviis molliter tomentosis 4–5 cm. longis 2–2.5 cm. latis, serraturis utrinque ca. 7; panicula ampla 3 dm. vel ultro longa folioso-bracteata, ramis cymiferis oppositis divergentibus tomentosis, cymis rotundatis; capitulis brevissime pedicellatis 5-floris; involucri squamis 4–6 oblongis 2.5 mm. longis erosociliatis nervoso-striatis dorso tomentosis apice rotundatis; corollis ca. 2.5 mm. longis, tubo proprio gracili puberulo, faucibus gradatim ampliatis tubum longitudine vix aequantibus subglabris, dentibus limbi 5 ovato-deltoideis patentibus; achaeniis 5-angulatis praecipue in costis valde hirsutis; pappi setis ca. 20 plumoso-barbatis. — Bak. in Mart. Fl. Bras. vi. pt. 2, 188, t. 53 (1876). *Eupatorium Regnellii* Sch. Bip. Linnaea, xxii. 572 (1849), xxx. 182 (1859), nomen solum. — BRASILIA: in planitiebus Prov. Minas Geraes prope Caldas, *Regnell*, n. I. 237 (hb. Berol.); prope Villam Francam, *Riedel*, n. 1018 (hb. Gray).

7. O. FREYREYSII (Dallm.) Bak., fruticosus oppositirameus; caulibus teretibus juventate tomentosis maturitate glabriusculis; ramis patentibus foliatis; foliis oppositis ovato-lanceolatis vel saepius lanceolatis attenuatis basi breviter cuneatis 3-nerviis leviter sed distincte serratis cum dentibus utrinque ca. 8 instructis supra scabriusculis vel glabriusculis subtus vix pallidioribus pubescentibus 5–8.5 cm. longis 1.5–3.5 cm. latis, petiolo supra sulcato tomentello 3–7 mm. longo; capitulis ca. 5 mm. longis saepissime 5-floris; involucri squamis obovatis apice rotundatis dorso tomentosis 3 mm. longis; corollis 2.4 mm. longis externe glanduloso-puberulis, faucibus gradatim ampliatis turbinato-campanulatis tubum proprium gracilem fere aequantibus, dentibus limbi 5 brevibus patentibus; styli ramis bene exsertis nigrescentibus apice incrassatis rotundatis; achaeniis nigris 1.8 mm. longis sursum praesertim in costis hirsutulis. — Bak. in Mart. Fl. Bras. vi. pt. 2, 188 (1876), as *Freyreissii*. *Eupatorium Freyreysii* Dallm. in Thunb. Decad. Nov. Pl. Bras. n. 19, et Flora, 1821, i. 332; DC. Prod. v. 183 (1836). *E. Freyresii* DC. Prod. v. 169 (1836) in syn. *E. Freyreissii* Bak. in Mart. Fl. Bras. vi. pt. 2, 188 (1876) in syn. *E. Freyreysi* Hook. f. & Jacks. Ind. Kew. i. 917 (1893). *E. Riedelianum* Gardn. in Hook. Lond. Jour. Bot. v. 478 (1846). *Mikania clavellata* DC. Prod. v. 192 (1836). — BRASILIA: Prov. Minas Geraes, *Freyreiss* (hb. Thunb.) fide Bakeri; in silvis Serro Frio, *Gardner*, n. 4851 (hb. Kew.); prope Marianna, *Vauthier*, n. 287 (hb. Kew., hb. Gray), *Gardner*, n. 4852 (hb. Kew., hb. DC.), sub hoc numero etiam *Symphyopappus polystachyus* (DC.) Bak. fide Bak. l. c. 368; prope Rio Janeiro et Ouro Preto,

Glaziou, n. 15,158 (hb. Kew.); *Riedel* (hb. Kew.); *Schuch*, n. 138 (hb. Vindob.).

8. O. PIQUERIOIDES (DC.) Benth., fruticosus erectus vel subscandens oppositirameus foliis et floribus precedenti simillimus; ramis juventate fulvescenti-tomentellis maturitate glabratis striatulis; foliis lanceolatis 3-nerviis serratis vel integerrimis breviter petiolatis 4–9 cm. longis, 1.2–3.3 cm. latis supra scabriusculis subtus in nerviis tomentosis vel pubescentibus; paniculis patente ramosissimis; capitulis numerosissimis 5–7-floris; involucri squamis oblongis vel elliptici-obovatis apice rotundatis erosis dorso laxe pubescentibus; corollis lutescenti- vel viridescenti-albis; styli ramis apice brevissime et modice incrassatis brunneis; achaeniis nigris 5-angulatis ca. 1.5 mm. longis pubescentibus. — Benth. ex Bak. in Mart. Fl. Bras. vi. pt. 2, 188 (1876). *Eupatorium piquerioides* DC. Prod. v. 175 (1836). *E. Tweedianum* Griseb. Abhandl. Gesellsch. Wiss. Goett. xxiv. 170 (1879), partim, non Hook. et Arn. *Mikania Mandonii* Sch. Bip. Linnaea, xxxiv. 536 (1865–6). *M. Mandoni* Bak. l. c. *Ophryosporus saltensis* Hieron. in Engl. Bot. Jahrb. xxii. 705 (1897). — PERUVIA: in montibus Guanoccensibus, *Haenke* (hb. DC.); Panahuanca, *Mathews*, n. 1122 (hb. Kew.); inter Palcam et Huacapistanam, Prov. Tarma, alt. 1700–2400 m., *Weberbauer*, n. 1776 (hb. Berol.). BOLIVIA: Yungas, *d'Orbigny*, n. 421 (hb. Vindob., hb. Gray); prope Soratam, Prov. Larecaja, regione temperata, alt. 2550 m., ad rivum in nemoribus, *Mandon*, n. 268 (hb. Kew., hb. Brit. Mus., hb. Gray); *Mandon*, n. 255 (hb. Kew., hb. Brit. Mus.); Calapampa, *Bang*, n. 2342 (hb. Gray), distrib. sub nomine "Willoughbya sp. n." CHILI: Santa Cruz, Aug. 1865, *Pearce*, (hb. Kew., hb. Brit. Mus.). ARGENTINA: Salta prope Yacone, *Lorentz et Hieronymus*, n. 536 (hb. Berol.).

NOTA. — *O. saltensis* Hieron. l. c. est forma ut videtur vix distincta foliis firmiusculis saepe conduplicatis verosimiliter e loco aridiori.

9. O. ORIGANOIDES (Meyen et Walp.) Hieron., suffruticosus oppositirameus; caulibus teretibus striatulis; foliis oppositis ovatis vel elliptico-lanceolatis acutis vel acuminatis haud longe attenuatis 1.5–3 cm. longis 8–10 mm. latis 3-nerviis petiolo 4–6 mm. longo gracili et nerviis tomentellis margine utrinque cum dentibus saepissime 4–7 instructis; involucri squamis uniseriatis 8–10 lanceolati-oblongis acutiusculis saltim juventate dorso pubescentibus; capitulis in corymbo composito dispositis 9–11-floris; corollis 5.5 mm. longis; achaeniis nigrescentibus 3.5 mm. longis. — Hieron. in Engl. Bot. Jahrb. xxii. 707 (1897), excl. pl. Rusbyi et syn. *Eupatorium eleutherantherum*. *E. origanoides* Meyen et Walp. Nov. Act. Acad. Caes.-Leopold. xix. Suppl. I. 257 (1843);

Walp. Rep. vi. 113 (1846); non HBK. — PERUVIA: in planitie circa Tacoram, alt. 4300–5200 m., *Meyen* (hb. Berol.).

NOTA. — *Eupatorium eleutherantherum* Rusby, Mem. Torr. Bot. Club, iii. pt. 3, 53 (1893), huc a cl. Hieronymo relatum exhibet antheras plus minusve appendiculatas saltim connectivo apice expanso, etiam folia longiora magis apice attenuata cum dentibus numerosioribus (7–11) utrinque instructa. Quapropter planta Rusbyi videtur verosimiliter distincta et melius ad *Eupatorium* referenda.

Var. (?) MICROCEPHALUS Hieron., habitu formae typicae similis; foliis magis attenuatis; capitulis minoribus 6–7-floris, squamis involucri obovalibus apice rotundatis ciliatis dorso laxe pubescentibus; corollis 4 mm. longis; achaeniis (immaturis) 2–2.5 mm. longis. — Hieron. in Engl. Bot. Jahrb. xxii. 708 (1897). — BOLIVIA: prope Cochabambam, ca. 4000 m. alt., *Kuntze* (hb. Berol.).

10. O. VENOSISSIMUS (Rusby) Robinson, fruticosus oppositirameus; ramis subangulatis ascendentibus foliatis; foliis oppositis lanceolatis ca. 7 cm. longis ca. 2 cm. latis serratis 3-nerviis attenuatis basi cuneatis utrinque viridibus sparse pubescentibus; corymbis rotundatis folioso-bracteatis; capitulis numerosis brevissime pedicellatis vel subsessilibus; involucri pallide viridis subcylindrici squamis anguste oblongis acutiusculis dorso rotundatis sparse laxeque pilosis ca. 3 mm. longis; flosculis 4–5; corollis albis vel ochroleucis 3–4 mm. longis, tubo proprio glanduloso-puberulo quam fauces glabri cylindrici breviori; achaeniis valde immaturis pubescentibus; pappi setis ca. 28 barbellatis tenuibus albis. — Proc. Am. Acad. xli. 271 (1905). *Eupatorium venosissimum*, Rusby, Mem. Torr. Bot. Club, vi. 57 (1896). — BOLIVIA: prope Cochabambam, *Bang*, n. 1113 (hb. Gray).

11. O. MACRODON Griseb., frutescens in parte superiori oppositirameus; ramis subteretibus dense fulvo-puberulis; foliis oppositis ovatis magnis grosse et duplice dentato-serratis acutis basi subrotundatis vel cuneatis membranaceis 3–5-nerviis ad 14 cm. longis ad 7 cm. latis supra scabris subtus pallidioribus pubescentibus, petiolo ca. 1.3 cm. longo; inflorescentiis terminalibus corymbosis convexis subcongestis; capitulis saepissime 8–10-floris pro genere maximis; flosculis (styli ramis elongatis exclusis) ca. 7 mm. longis; corollis ca. 5 mm. longis, tubo proprio puberulo quam fauces cylindrici glabri multo breviori. — Abhandl. Gesellsch. Wiss. Goett. xxiv. 173 (1879). — ARGENTINA: Nevado del Castillo, Prov. de Salta, *Lorentz et Hieronymus*, n. 156 (hb. Berol.).

12. O. LORENTZII Hieron., fruticosus oppositirameus 1.5–2 m. altus; ramis teretibus flexuosis ascendenti-patentibus striatulis; foliis oppositis ovato-lanceolatis attenuati-acuminatis argute serratis ca. 5–10 cm. longis 2–4 cm. latis membranaceis 3-nerviis petiolatis juventate utrinque sparse pubescentibus deinde glabratis; corymbis numerosis

lateralibus in axillis superioribus et terminalibus rotundatis quam folia brevioribus; capitulis 4-floris; involucri squamis 4 aequalibus linearibus acutiusculis 4 mm. longis; corollis 3.3 mm. longis fere a basi gradatim ampliatis in parte inferiori glanduloso-puberulis, dentibus limbi brevissimis erectis deltoideis; achaeniis in parte inferiori glabris in parte superiori sparse hispidulis; pappi setis ca. 23 albidis subplumosis 3 mm. longis. — Hieron. in Engl. Bot. Jahrb. xxii. 706 (1897). *Eupatorium laeve* Griseb. Abhandl. Gesellsch. Wiss. Goett. xxiv. 172 (1879), partim f. Hieron. l. c. — ARGENTINA: Cueste inter Yacone et Los Potreros, *Lorentz et Hieronymus*, nn. 333, 340 (hb. Berol.).

13. O. CUMINGII (Sch. Bip.) Benth., fruticosus glabriusculus; ramis flexuosis elongatis subteretibus laevibus striatulis medullosis, internodiis 5–12 cm. longis, ramulis gracillimis puberulis; foliis oppositis tenuibus lanceolatis longissime attenuatis acutissimis 3-nerviis argute serratis 3–10 cm. longis 7–20 mm. latis sparse in nerviis venulisque puberulis vel supra glaberrimis, petiolo 4–12 mm. longo gracili; cymis axillaribus quam folia subtendentia multo brevioribus in parti superiori caulium et ramorum aggregatis et paniculam foliosam formantibus, pedicellis filiformibus 1–3 mm. longis; capitulis 4-floris ca. 6 mm. altis; involucri squamis oblongis tenuibus stramineo-viridibus ciliolatis 3–4 mm. longis apice rotundatis dorso 2–3-nerviis; corollis albis 3.3 mm. longis, tubo proprio gracili externe glanduloso-puberulo fauces cylindrico-campanulatos sparsissime glanduliferos subaequanti, dentibus limbi 5 ovato-deltoideis plus minusve patentibus; achaeniis 5-angulatis 1.6 mm. longis prismaticis modice deorsum angustatis in costis sursum hispidis inter costis glaberrimis summa parte cupulo brevissimo coronatis; pappi setis capillaribus barbellatis. — Benth. ex Bak. in Mart. Fl. Bras. vi. pt. 2, 188 (1876); Hieron. in Engl. Bot. Jahrb. xxii. 705 (1897). *Mikania Cumingii* Sch. Bip. Bull. Soc. Bot. Fr. xii. 82 (1865) et Linnaea, xxxiv. 535 (1865–6), nomen nudum. — BOLIVIA: in nemoribus regionis temperatae 2700–2900 m. alt., inter Ladrilloni et Condurpata, Prov. Larecaja, *Mandon*, n. 264 (hb. Kew., hb. Gray); *Bridges*, 1847 (hb. Brit. Mus.); *Cuming*, n. 102 (f. Sch. Bip.).

14. O. KUNTZEI Hieron., fruticosus oppositirameus; ramis gracilibus patenti-ascendentibus juventate puberulis deinde glabratis usque ad inflorescentias terminales corymbosas foliatis; foliis oppositis lanceolatis vel ovato-lanceolatis acuminato-attenuatis serratis 3–7 cm. longis 1–2 cm. latis 3–5-nerviis utrinque sparse pubescentibus; corymbis multicapitulatis valde convexis, pedicellis ca. 1 mm. longis; capitulis 4-floris; squamis involucri 4 subaequalibus 2.6–3.3 mm. longis fere a basi gradatim expansis in parte inferiori glanduloso-puberulis; achaeniis nigris 5-angulatis cum vel absque angulis secondariis obscuris

2 mm. longis deorsum valde angustatis apice ab annulo albido coronatis; setis pappi ca. 25 sordido-albis. — Hieron. in Engl. Bot. Jahrb. xxii. 707 (1897). — BOLIVIA: Tunari australi, ca. 3000 m. alt., Apr.-Maio, 1892, *Kuntze* (hb. Berol.).

15. O. SODIROI Hieron., fruticosus verosimiliter scandens; ramis oppositis elongatis flexuosis puberulis, internodiis folia longitudine valde superantibus; foliis ovatis acutis petiolatis integris vel pauciserratis 3–4 cm. longis 1.7–2 cm. latis 3-nerviis tenuibus pellucide reticulato-venosis; capitulis sublaxe paniculatis 5–7-floris; pedicellis patentibus 2–3 mm. longis; involucri squamis 5–7 obovatis vel late spatulatis brunneis ciliatis apice rotundatis erosis dorso glabris ca. 2.8 mm. longis; corollis gradatim et modice apicem versus ampliatis ubique glanduloso-puberulis vel atomiferis; achaeniis (valde immaturis) brevissimis in angulis scabratis; pappi setis ca. 28 albidis inaequalibus barbellatis. — Hieron. in Engl. Bot. Jahrb. xxix. 3 (1900). — ECUADOR: in silvis subtropicis prope La Chima, *Sodiro*, n. 6/18 (hb. Berol.); Campamento Utañag in valle fluminis Chambo, alt. 3045 m., *Strübel*, n. 275 (hb. Berol.).

16. O. CLAVULATUS Griseb., fruticosus 2 m. altus superne puberulus laxe ramosus; ramis teretibus foliatis oppositis; foliis anguste ovato-lanceolatis longe acuminato-attenuatis 3-nerviis tenuibus serratis supra obscure puberulis subtus in nerviis pubescentibus 4–6 cm. longis 1.3–2.5 cm. latis; petiolis 6–11 mm. longis flexuosis planiusculis; paniculis ovalibus multicapitulatis, capitulis 4–5-floris; involucri squamis obovati-oblongis 2 mm. longis ciliolatis dorso glabris brunneis; corollis 2 mm. longis, tubo proprio gracili puberulo sursum in fauces subaequilongos gradatim expanso; achaeniis nigris 1.6 mm. longis 5-angulatis in angulis sursum hispidulis; pappi setis ca. 20 albidis subplumoso-barbatis. — Abhandl. Gesellsch. Wiss. Goett. xxiv. 173 (1879); Hieron. in Engl. Bot. Jahrb. xxii. 705 (1897). *Eupatorium clavulatum* Griseb. l. c. xix. 120 (1874). — ARGENTINA: Cuesta de Periquilla, *Lorentz*, n. 409; *Lorentz et Hieronymus*, n. 178 (hb. Berol.).

17. O. PACHYCHAETA Bak., suffruticosus erectus ramosus puberulus; ramulis angulatis viridibus foliatis flexuosis; foliis oppositis sessilibus lanceolatis firmiusculis 3-nerviis utrinque viridibus punctatis subglabris 2–3 cm. longis 4–7 mm. latis utrinque 1–3-angulato-dentatis acutis; corymbis terminalibus pyramidatis alternirameis 4–5 cm. diametro, bracteis subulatis brevissimis, ramis puberulis apicem versus 3–7-capituliferis; capitulis saepissime 4-floris; involucri squamis 3–5 oblongis vel anguste obovatis vel ellipticis acutiusculis 2.5 mm. longis; corollis glabriusculis 2.5 mm. longis, tubo proprio gracillimo fauces campanulatos subaequanti; achaeniis 1.2 mm. longis basin versus

attenuatis 5-angulatis fere glabris; pappi setis aequalibus crassiusculis haud barbatis salmoneis vel roseis attenuatis subpatentibus. — Bak. in Mart. Fl. Bras. vi. pt. 2, 187 (1876). *Pachychaeta eupatorioides* Sch. Bip. mss. fide Bak. l. c. — BRASILIA: in planitiebus Prov. Minas Geraes, *Lund; Claussen* (hb. Kew.); *Riedel*, n. 421; Rio Janeiro, *Glaziou*, n. 14,018 (hb. Kew., hb. Berol.), n. 14,019 (hb. Kew.).

Species reducendae vel excludendae.

O. Burchellii Bak. in Mart. Fl. Bras. vi. pt. 2, 187 (1876) est forma *Eupatorii tetranthii* Sch. Bip. quod videtur solum forma oppositifolia *E. dentati* Gardn.

O. Chilca (HBK.) Hieron. in Engl. Bot. Jahrb. xxii. 706 (1897) ob antheris distincte appendiculatis certe ex *Ophryosporo* excludendus est *E. Chilca* HBK.

O. Mandonii (Sch. Bip.) Benth. et Hook. f. ex Hook. f. et Jacks. Ind. Kew. ii. 354 (1895) est *O. piquerioides* (DC.) Benth.

O. ovatifolius (DC.) Benth. et Hook. f. ex Hemsl. Biol. Cent.-Am. Bot. ii. 79 (1881), ob antheris obscure appendiculatis et ob affinitate indubia cum aliis speciebus *Eupatorii* melius nunc ad hoc genus referendus, est vere ab *E. polybotryo* DC. Prod. v. 174 (1836) nullo modo distinctus.

O. solidaginoides (HBK.) Hieron. in Engl. Bot. Jahrb. xxix. 4 (1900) est eisdem rationibus *E. solidaginoides* HBK.

O. solidaginoides, var. *Bonplandianus* (Sch. Bip.) Hieron. l. c. = *Eupatorium solidaginoides*, var. **Bonplandianum** (Sch. Bip.), n. comb.

III. THE GENUS HELOGYNE AND ITS SYNONYMS.

Since its publication by Nuttall in 1841 the genus *Helogyne* has remained an obscure monotype known only from a single fragmentary specimen, collected in Peru and now preserved in the herbarium of the British Museum. The genus has been referred to the *Piquerinae* by all recent authors who have had occasion to classify it, but Bentham and Hooker, who first expressed this view as to its affinities, particularly state that they did not examine the anthers, which alone furnish the crucial character of the subtribe.

While at the herbarium of the British Museum, the author was kindly permitted to remove one of the very few flowers from a head and make a dissection. The anthers were found to be each supplied with a well-developed terminal appendage, showing clearly that the genus belongs not to the *Piquerinae* but to the *Ageratinae*. When transferred to the latter tribe, *Helogyne* is found to occupy the same

place as the later monotypic genus *Brachyandra* Phil., and there can be no doubt that these two plants are congeneric. They are both small-leaved glandular-pubescent xerophytic shrubs, with closely similar achenes and pappus, and both possess the peculiar narrowly tubular corollas with exceedingly short teeth and no expanded throat. The only difference between them which could possibly be regarded as of generic importance is that the involucre in *Helogyne* is about 2-seriate and of subequal bracts, while in *Brachyandra* it is about 3-seriate, the outer bracts being decidedly shorter. In view of the close correspondence in floral structure, achenes, leaf-arrangement, etc., this difference in the involucre, which finds frequent parallels within the limits of several other genera of the *Compositae*, seems by no means sufficient to warrant keeping these two genera separate. The later name proposed by Philippi must of course give way to the earlier one of Nuttall.

Leto Phil. is a third obscure Chilian xerophytic monotype of this affinity. Its generic relationship to *Brachyandra* was shrewdly surmised by Dr. O. Hoffmann (see Engl. & Prantl. Nat. Pflanzen. iv. Ab. 5, 334), notwithstanding a misleading statement in the original description to the effect that the corollas were irregular, which is not the case. More recently Reiche, Fl. de Chil. iii. 263 (1902), has formally transferred the single species to *Brachyandra*. While at Berlin, the writer had an opportunity to examine an authentic specimen of this plant (*Leto tenuifolius* Phil., *Brachyandra tenuifolia* Reiche), and failed to find even specific distinctions between it and the type of *Helogyne apaloidea* preserved at the British Museum. It is true that the specimen of *Leto* at Berlin shows some leaves much more deeply lobed than any on the specimen of *Helogyne* at the British Museum, but the latter consists only of a tip of a flowering branch on which the leaves, 3-toothed at the apex, correspond well with the uppermost leaves in the Berlin plant.

Still a fourth South American xerophytic monotype clearly belongs to the same group, namely, *Addisonia* Rusby, Bull. Torr. Bot. Club, xx. 432, t. 159 (1893). Like the three preceding, it is an erect viscid much-branched shrub with small narrow alternate glandular leaves, few-flowered heads, narrow subcylindric involucre, and disk free from pales. In common with them it has 5-angled achenes, slightly narrowed toward the base, and crowned with numerous purple-tinged barbellate setae. What is still more significant, it shares with them the peculiar very narrowly tubular corollas destitute of any distinct throat and provided with the same very short suberect teeth, of the same yellowish white color and exhibiting the same tendency to external granular puberulence. Like *Brachyandra*, *Addisonia* has an involucre with 3–4-seriate scales which are strongly imbricated and very unequal in length, but unlike any of the three other monotypes here discussed, *Addisonia* has its involucral scales arranged in four upright rows. This difference, if any, must be regarded as its claim to rank as a separate genus. It is a conspicuous characteristic, and at first sight might seem to be of considerable diagnostic importance. However, a second species, closely related to *Addisonia virgata* Rusby, has been collected in Peru by Weberbauer, and in it the scales are in five not always equidistant erect series, showing that the number of the series is not of generic significance. Furthermore, the tendency of closely imbricated, somewhat carinate involucral scales to assume more or less regularity in upright series is observable elsewhere in the *Compositae* in a way to cast much doubt upon the importance of the character as a sole basis for a generic separation. Thus, in the species of *Bigelovia*, of the *B. graveolens* group, an equally marked tendency of this sort is observable, but shows such inconstancy even in very nearly related forms, that it can scarcely be taken as a character of specific, not to mention generic, significance. In view, then, of the close correspondence of the four South American plants here discussed — a likeness which embraces, as we have seen, not merely habit, leaf-arrangement, etc., but all the more significant characters of flower and fruit — it seems best to unite them under the oldest name, both in order to show their obvious relationship and to avoid the adoption of a standard of generic classification solely on the basis of involucral differences, which would cause great difficulty and artificiality if applied to neighboring genera of the *Compositae*.

Attention may be called to the fact that all four of these plants maintain the chief distinction by which the genus *Brachyandra* has long been separated from the allied genus *Trichogonia*, namely, the very narrowly tubular corolla. The creation in botanical literature of these four successive genera for plants, which now appear to be of one generic type, is readily explained and to a great extent excused by the rarity of the plants concerned and by the natural misapprehensions which have arisen from mistakes in the original descriptions. Thus, the original *Helogyne*, founded on a small tip of a flowering branch, was thought by Nuttall to be probably an annual, and his description was likely to mislead the reader into supposing that the outer involucre was more foliaceous and the style-branches more expanded than is really the case. To this may be added the circumstance that Bentham long ago referred the genus to the *Piquerinae*, with which it has no close affinity. The original description of *Leto* states that the corolla

is irregular, a trait which, if it really were true, would place the genus in the *Mutisieae*. Finally *Addisonia* is described and figured as having corollas with campanulate throats. Under these circumstances, it is by no means remarkable that the real affinity of these plants has not been noticed until authentic specimens of each of them could be studied in relation to the others. The combined genus may be treated as follows:

HELOGYNE Nutt. (Nomen ab ἧλος, *clavis*, et γυνή, *mulier*, ob forma styli ramorum incrassatorum.) — Capitula parvula homogama 6–12-flora terminalia vel subcorymbosa vel spicato-racemosa; involucro subcylindrico vel anguste campanulato, squamis 2–4-seriatis imbricatis, receptaculo parvo nudo planiusculo. Corollae anguste tubulosae vel etiam angustissime fusiformes nullo modo in fauces ullos distinctos ampliatae externe glanduloso-puberulis vel pulverulis, limbi brevissime 5-dentati dentibus suberectis vel leviter patentibus. Antherae lineares inclusae apice obtuse appendiculatae basi integrae. Styli rami valde exserti plus minusve patentes valde sed gradatim incrassati flavi vel saepius nigrescentes. Achaenia 5-costata prismatica deorsum leviter angustata. Pappi setae subaequales tenues achaeniis subaequilongae vel superantes barbellatae vel subplumosae. — Trans. Am. Phil. Soc. n. s. vii. 449 (1841); Walp. Rep. vi. 107 (1846); Benth. et Hook. f. Gen. ii. 239 (1873); Hoffm. in Engl. et Prantl. Nat. Pflanzenf. iv. Ab. 5, 133 (1890). *Brachyandra* Phil. Fl. Atac. 34, t. 4, f. D (1860); Benth. et Hook. f., l. c. 244 (1873); Hoffm. l. c. 138 (1890); Reiche, Fl. de Chil. iii. 263 (1902). *Leto* Phil. Ann. Mus. Nac. Chil. sec. 2 (botanica), 33, t. 1, f. 3 (1891); Hoffm. l. c. 334 (1893). *Addisonia* Rusby, Bull. Torr. Bot. Club, xx. 432, t. 169 (1893). — Frutices ramosissimi foliosi glanduloso-puberuli. Folia alterna parva integra vel dentata vel lobato-subpinnatifida sessilia.

Species 4 regionis desiccatae Peruviae et Boliviae australis et rei publicae Chilensis borealis incolae.

Sect. I. EUHELOGYNE, sect. nov., involucri squamis subaequalibus ca. 2-seriatim imbricatis. — *Helogyne* Nutt. l. c. *Leto* Phil. l. c. — Species unica.

1. H. APALOIDEA Nutt., ramis arcuato-ascendentibus foliosis, internodiis brevibus; foliis parvis 8–15 mm. longis cuneato-oblanceolatis integerrimis vel saepius irregulariter pinnatim 2–3-lobatis vel supremis solum apice 2–3-dentatis glanduloso-puberulis amaris; capitulis 8–12-floris paucis erectis corymbosis, pedicellis gracilibus erectis vel arcuato-ascendentibus; involucri squamis subaequalibus anguste ellipticis vel

lanceolatis ca. 6–8 apice obtusiusculis vel rotundatis ca. 6 mm. longis pubescentibus; corollis verosimiliter flavescenti- vel viridiscenti-albis 4.5 mm. longis externe minute granulosis; achaeniis 2.7 mm. longis; pappi setis ca. 20 barbellatis 2 mm. longis. — Trans. Am. Phil. Soc. n. s. vii. 449 (1841). *Leto tenuifolius* Phil. Ann. Mus. Nac. Chil. sec. 2 (botanica), 34, t. 1, f. 3 (1891). *Brachyandra tenuifolia* Reiche, Fl. de Chil. iii. 263 (1902). — PERU: Arequipa, *Curson* (hb. Brit. Mus.). — CHILI: inter Sibayam et Chiapam in provincia Tarapacá, *F. Philippi* (specimen in hb. Berol. a cl. Reicheo missum).

Sect. II. BRACHYANDRA (Phil.), sect. nov., involucri squamis valde inaequalibus spiraliter 3–4-seriatim imbricatis. — *Brachyandra* Phil. Fl. Atac. 34, t. 4, f. D (1860). — Species unica desertorum Atacamae incola.

2. **H. macrogyne** (Phil.), n. comb., fruticosa ramosissima viscoso-tomentella 1 m. alta parte inferiori delapsu foliorum denudata cortice griseo tecta; ramis ascendentibus foliosissimis furcatis prope apicem capituliferis; capitulis 6–7 mm. altis saepe 4-floris breviter pedicellatis; involucri squamis subscariosis tenuibus pubescentibus appressis exterioribus brevissimis ovatis acutis, intermediis gradatim longioribus oblongo-ovatis acutis, intimis lineari-lanceolatis; corollis albis vel ex sicco purpurascentibus 5 mm. longis ubique externe resinoso-granulosis; antheris linearibus 1.8–2 mm. longis; achaeniis nigricantibus sursum atomiferis 3 mm. longis; pappi setis ca. 25 sordide albidis vel etiam purpurascentibus breviter plumoso-barbatis 4.5 mm. longis subinaequalibus. — *Brachyandra macrogyne* Phil. Fl. Atac. 34, t. 4, f. D (1860); Reiche, Fl. de Chil. iii. 263 (1902). — CHILI: Prov. Atacama versus Tilopozo, alt. 2300 m., *Philippi;* in desertis Atacamae, *Philippi*, n. 504 (hb. Berol.), Aug. 1864, *Pearce* (hb. Kew.).

Sect. III. ADDISONIA (Rusby), sect. nov., involucri squamis valde inaequalibus in seriebus 4–5 erectis imbricatis. — *Addisonia* Rusby, l. c. — Species 2 quarum una Boliviae australis altera Peruviae australis.

3. **H. virgata** (Rusby), n. comb., fruticosa ramosissima glanduloso-subviscosa ad 1 m. alta basi delapsu foliorum denudata, ramis subfastigiatis foliosissimis saepius supra mediam partem capituliferis; foliis ascendentibus vel appressis ad 11 mm. longis ad 1.3 mm. latis glaberrimis integris glanduloso-punctatis firmiusculis linearibus sessilibus subcarinatis 1-nerviis obtusis margine revolutis; capitulis spicato-racemosis breviter pedicellatis 4–5-floris; squamis involucri erecte 4-seriatim imbricatis subherbaceis acutissimis vel obtusiusculis et

argute mucronatis; corollis ocroleucis 5.5 mm. longis externe basi et prope apicem granuliferis; antheris 2 mm. longis linearibus; achaeniis obtuse 5-angulatis deorsum leviter angustatis ubique sed praesertim in angulis granulosis; pappi setis purpureis ca. 25 paullulo rigidiusculis barbellatis ca. 5 mm. longis. — *Addisonia virgata* Rusby, Bull. Torr. Bot. Club, xx. 432, t. 169 (1893). — Bolivia: Songo, Nov. 1890, *Bang*, n. 868 (hb. Gray, hb. Acad. Philad.) sub nomine *Chuquiragua* distributa.

4. **H. Weberbaueri,** n. sp., fruticosa ad 1 m. alta ramosissima in parte inferiori delapsu foliorum denudata cortice griseo tecta, ramis subfastigiatis flexuosis erectis foliosissimis supra mediam partem capituliferis; foliis linearibus crassiusculis 1 cm. longis 2 mm. latis prope marginem hispidulis glanduloso-punctatis; capitulis spicato-racemosis breviter pedicellatis ca. 5-floris; involucri squamis in seriebus 5 erectis imbricatis valde inaequalibus fere omnino brunnescenti-stramineis nec herbaceis carinatis apice attenuatis; corollis exacte tubulosis ocroleucis apice brevissime 5-dentatis externe solum versus basin granuliferis ca. 7 mm. longis; antheris linearibus 2.5 mm. longis; achaeniis prismatico-obpyramidatis ubique praesertim in angulis granuliferis 3 mm. longis; pappi setis subinaequalibus purpurascentibus rigidiusculis barbellatis ca. 7.5 mm. longis. — Peru: in arenosis subdesertis, Yura, alt. 2400 m., 31 Aug. 1902, *Weberbauer*, n. 1416 (hb. Berol.). Species praecedenti valde affinis differt foliis latioribus margine hispidulis, capitulis majoribus, involucro haud herbaceo, corollis longioribus basi tantum granuliferis.

IV. Diagnoses and Synonymy of Eupatorieae and of certain other Compositae which have been classed with them.

Apodocephala Bak. Jour. Linn. Soc. xxi. 417 (1885); S. Ell. ibid. xxix. 28 (1891); Hoffm. in Engl. & Prantl Nat. Pflanzenf. iv. Ab. 5, 134, 135 (1890), 388 (1894). Through the courtesy of Sir William Thiselton Dyer and Dr. Stapf. of the Royal Gardens at Kew the writer was permitted to make dissections of the flowers of both species of this problematic genus of Madagascar, which has been referred to the *Eupatorieae* by Mr. Baker and to the *Vernonieae* by Dr. Hoffmann. The anthers are clearly sagittate, the leaves alternate, the style-branches rather strongly recurved and acutish, and the involucre of a form and texture far more frequent in the *Vernonieae* than in the *Eupatorieae.* Indeed, all features observed seem to confirm fully the view of Dr. Hoffmann that the genus should be referred to the former tribe.

Alomia dubia, n. sp., herbacea erecta perennis sordide pubescens 4–7 dm. alta; radice lignescenti ramosa; caulibus 1 vel pluribus teretibus striatulis foliosissimis ad inflorescentiam corymbosam simplicissimis; foliis alternis oblanceolatis obtusiusculis crenato-serratis 2–3 cm. longis 4–10 mm. latis basi attenuatis subpetiolatis utrinque dense et sordide pubescentibus supra rugulosis subtus reticulatis; corymbis laxe ramosis 7–15 cm. latis supra modice planis, ramis ascendentibus saepius 3–5-capituliferis, bracteis linearibus 5–15 mm. longis; capitulis ca. 65-floris ca. 1 cm. diametro 8–10 mm. altis; involucri campanulati squamis subbiserialibus lanceolato-linearibus subaequalibus dorso puberulis nervosis apice attenuatis tomentellis; corollis roseis 4 mm. longis, tubo proprio gracili externe glanduloso-hirsutulo basi plus minusve expanso supra in fauces turbinato-campanulatos gradatim ampliato, limbo purpurascenti-tomentello; achaeniis nigris 5-angulatis glaberrimis basi mediocriter angustatis apice rotundatis ab annulo albido cartilagineo coronatis omnino calvis. — Brazil, presumably from the Prov. Goyaz, *Dr. A. Glaziou*, n. 21579 (hb. Kew.).

Although there can be no doubt that this species is technically an *Alomia* and must be referred to that genus, as the genera of this affinity are now divided, it must be confessed that *Alomia* looks suspiciously like an artificial aggregate of species which may well have had a very different origin. Its species are varied in habit, and approach on the one hand so close to *Ageratum*, and on the other to *Trichogonia*, that it may well be doubted whether they are not, at least in some cases, "*formae epapposae*" of these genera. The present species closely resembles in habit and many of its features *Trichogonia*. It should be noted that forms of at least two species of *Trichogonia* have been found in which part or all the achenes were entirely destitute of pappus. The species here described, however, is clearly distinct from any hitherto characterized species of either genus. *Trichogonia* with its plumose setiform pappus is certainly very distinct from *Ageratum* with a pappus of few distinct or somewhat connate scales, yet the *Alomiae*, which are entirely destitute of pappus or have only an annular rudiment in its place, show such transitions of habit, involucre, pubescence, etc., that they neither carry conviction as a distinct genus, nor are they capable of satisfactory grouping as pappusless forms of the pappus-bearing genera. The genus *Alomia* is as yet very poorly represented in herbaria, and until further material has been collected, it seems impracticable to revise the generic limits of the three genera here concerned.

HARTWRIGHTIA FLORIDANA Gray, Proc. Am. Acad. xxiii. 265 (1888). In characterizing this monotypic genus from Florida, Dr. Gray un-

fortunately described the anthers as exappendiculate. This has led to some misapprehension as to its affinities, and it has been placed in the *Piquerinae* by Hoffmann in Engl. & Prantl, Nat. Pflanzenf. Nachtr. zu iv. Ab. 5, 321 (1897), who seems to have overlooked Professor J. M. Holzinger's notes on the subject, Bull. Torr. Bot. Club, xx. 287, t. 160 (1893). Careful dissection shows that the anthers are each provided with an ovate deeply retuse membanaceous apical appendage. This trait throws the genus into the *Ageratinae*, where it should probably be placed nearest *Alomia*, a conclusion long ago reached by Professor Holzinger.

AGERATUM SCORPIOIDEUM Bak. in Mart. Fl. Bras. vi. pt. 2, 197 (1876). Mr. Baker's description of this species was drawn from Schomburgk's no. 353 in the herbarium of the Royal Gardens at Kew. He states that the plant is an erect doubtfully perennial herb. While at the Royal Botanical Museum of Berlin, the writer had an opportunity to examine an authentic specimen of *Caelestina repens* Sch. Bip., mentioned in Schomb. Fauna et Flora Guy. 1134 (1848), which proves identical with Baker's species. Schultz's name, although earlier, cannot be taken up, as it is accompanied by no description. It was founded on Schomburgk's no. 1188, collected on a moist savanna near the Canuku Mountains, British Guiana. The specimen is more complete than the one at Kew, and shows that the plant has a horizontal, evidently perennial, and slightly lignescent rhizome, which roots at the nodes and sends up erect subsimple stems.

Stevia simulans, n. sp., herbacea perennis fere a basi pauciramea erecta; caule et ramis simplicibus teretibus purpureis foliosissimis 3–3.5 dm. altis in summa parte glanduloso-puberulis; foliis linearibus crassiusculis firmiusculis 3–5-nerviis glabris 2.8–3.5 cm. longis 2–5 mm. latis utrinque glanduloso-punctatis obtusiusculis ascendentibus; capitulis paucis corymbosis saepius longe rigidiuscule pedicellatis ca. 1.3 cm. altis 13-floris; involucri squamis lanceolati-oblongis 9 mm. longis atropurpureis glanduloso-puberulis vix nervosis; corollis laete purpureis 7–9 mm. longis, tubo proprio externe glanduloso-puberulo modice et gradatim in fauces longiusculos ampliato, dentibus limbi anguste oblongis acutis patentibus vel recurvatis; styli ramis planis; achaeniis 5 mm. longis 10-costatis subteretibus puberulis; pappo duplici e squamis 5 brevibus hyalinis apice rotundatis et aristis 5 atropurpureis 5 mm. longis erectis scabriusculis composito. — On the mesa de la Sandia, Durango, Mexico, alt. 3050 m., 14 Oct., 1905, *C. G. Pringle*, n. 10,144 (type, in hb. Gray). This species possesses a very close habital similarity to *S. Pringlei* Wats., but may be readily distinguished by its copious glandular indumentum and especially by the presence of the aristate pappus, which is quite lacking in *S. Pringlei.*

Fleischmannia arguta, n. comb. *Eupatorium argutum* HBK. Nov. Gen. et Spec. iv. 121 (1820). *E. quinquesetum* Benth. ex Oerst. Vidensk. Meddel. 1852, p. 79. *Fleischmannia rhodostyla* Sch. Bip. Flora, xxxii. 417 (1850). The type of *Eupatorium argutum* HBK. is still extant at the Museum of Natural History in Paris. It is clearly just the plant which has long passed as *Fleischmannia rhodostyla* and its much earlier specific name must accordingly be taken up.

Trichocoronis sessilifolia, n. comb. *Argeratum sessilifolium* Schauer, Linnaea, xix. 715 (1847); Hemsl. Biol. Cent.-Am. Bot. ii. 83 (1881). *Trichocoronis Greggii* Gray, Pl. Wright. i. 89 (1852). The type of Schauer's *Ageratum sessilifolium* is Aschenborn's no. 4, of which there is a well preserved specimen in the Royal Botanical Museum in Berlin. The habitat is given as Mexico, but without more particular locality, and the species has remained obscure. On examination it proves to be identical with the species later described as *Trichocoronis Greggii* by Dr. Gray. Gregg's plant (no. 807 of his last Mexican collection) is said to have come from the region between Mazatlan and the City of Mexico. Fortunately Mr. Pringle has rediscovered the species, and the fuller data of his label give definite information of at least one station, namely, marshes of Atequiza in the state of Jalisco. Priority necessitates the transference of the original specific name.

EUPATORIOPSIS HOFFMANNIANA Hieron. in Engl. Bot. Jahrb. xviii. Beibl. 43, p. 46 (1893). This Brazilian monotype was placed by its author in the subtribe *Piquerinae.* Dissection, however, shows that the anthers have distinct apical appendages quite as well developed as in many of the *Ageratinae.* There can be no doubt that the true affinity of the genus is with *Trichocoronis* Gray, Pl. Fendl. 65 (1849), with which it agrees closely in general habit, in its opposite sessile leaves, its long pedicelled heads, campanulate involucre with subequal subherbaceous bracts, its short purplish corollas with capanulate throat, and in its abortive setiform pappus. In fact, almost its only claim to generic separation is in its broad obovate quasi two-winged achenes, those of *Trichocoronis* being narrow and prismatically 4–5-angled.

Dissothrix imbricata, n. comb. *Stevia imbricata* Gardn. in Hook. Lond. Jour. Bot. v. 458 (1846). *Dissothrix Gardneri* Gray in Hook. Jour. Bot. & Kew Misc. iii. 223 (1851); Bak. in Mart. Fl. Bras. vi. pt. 2, 272 (1876). Dr. Gray's specific name, coined at a time of greater nomenclatorial laxity, must, according to priority, give place to the original name given by Gardner.

Trichogonia rhadinocarpa, n. sp., suffrutescens ramosa; ramis subsimplicibus teretibus striatis viridibus puberulis foliosis; foliis alternis lanceolati-oblongis crenato-serratis ad apicem obtusiusculum angustatis basi rotundatis vel breviter abrupteque cuneatis petiolatis utrinque pubescentibus et glanduloso-atomiferis viridibus subtus vix pallidioribus membranaceis 3–5.5 cm. longis 1.3–2.5 cm. latis, petiolo 8–15 mm. longo subtomentoso; inflorescentia terminali corymbosa ca. 16-capitulata, bracteis filiformibus 3 mm. longis, pedicellis filiformibus plus minusve flexuosis ca. 7 mm. longis; capitulis ca. 18-floris 8–10 mm. altis; involucri turbinato-campanulati squamis linearibus vel anguste oblanceolatis attenuatis ca. 7 mm. longis subuniseriatis dorso leviter nervatis puberulis apice tomentosis purpurascentibus; corollis angustis, tubo proprio gracili glaberrimo, faucibus brevissimis, limbo purpureo-tomentoso; achaeniis 4 mm. longis nigris 5-angulatis basi longe attenuato-stipitatis angulis obsolete scabratis; pappi setis plumosis sordide albis 4 mm. longis basi brevissime connatis. — *T. podocarpa* Bak. in Mart. Fl. Bras. vi. pt. 2, 216 (1876) pro parte, non Sch. Bip. — Near Tovar, Venezuela, *Fendler*, n. 651 (hb. Gray); Mariara, Venezuela, 800 m. alt., Aug. 1899, *Preuss*, n. 1508 (hb. Berol.); Ocaña, Prov. Ocaña, Colombia, *Schlim*, n. 178 (hb. Kew.). The Fendler specimen was taken to Geneva and carefully compared with the type of *T. podocarpa* Sch. Bip. (*Kuhnia podocarpa* DC.) and it was found to be clearly a distinct species, differing in various ways but most strikingly in its densely tomentose corollas. In the type of *T. podocarpa* the corollas are covered on the outside by large scattered waxy atoms but are otherwise glabrous. From *T. campestris* Gardn., *T. rhadinocarpa* differs in its much broader leaves, longer and more tapering achenes, etc.

EUPATORIUM ALTISSIMUM L. Syst. ed. 12, 537 (1767). By the Index Kewensis, i. 915, this is referred to *E. ageratoides* L. f., but this is clearly a mistake. In the 12th edition of the Systema, p. 537, the description of *E. altissimum* is identical with the description of the same species in the first edition of the Species Plantarum, and it can be construed only as relating to the lanceolate-leaved plant which still very properly bears the name *E. altissimum.*

Eupatorium auriculatum Vahl, Symb. Bot. iii. 95, t. 72 (1794); DC. Prod. v. 174 (1836); Bak. in Mart. Fl. Bras. vi. pt. 2, 340 (1876); not Lam. This species, said by Vahl to come from Brazil, was fully described and clearly figured by him. It has, however, not been rediscovered since its description a century ago and has remained entirely obscure. Suspecting from Vahl's figure that the plant was not really a *Eupatorium* but a *Senecio*, the writer, with the aid of Dr. Greenman, who has a special knowledge of the latter genus, made some efforts to identify the species among the Brazilian *Senecioneae.* This search proved wholly unsuccessful, and accordingly a wish to examine the type of this problematic plant added no small incentive to a recent visit to Copenhagen, where many of Vahl's plants are preserved. The specimen of the plant in question was easily found and corresponded in all respects to Vahl's description and plate. It proved as anticipated a *Senecio*, but what was even more interesting, a faint but still quite legible label on the back of the sheet disclosed the fact that the specimen had not come from Brazil, but had been collected by Commerson on the Isle of Bourbon in the Indian Ocean. With this important clue, it has been easy to identify it positively with *Senecio penicillatus* (Cass.) Sch. Bip. The synonymy of the species is as follows: —

Eupatorium tomentosum Lam. Dict, ii. 410 (1786).
" *auriculatum* Vahl, Symb. iii. 95, t. 72 (1794).
Mikania tomentosa Willd. Spec. Pl. iii. 1744 (1804).
Cacalia penicillata Cass. Dict. xlviii. 460 (1827).
Senecio penicillatus Sch. Bip. Flora, xxviii. 499 (1845).
Senecio tomentosus Cordemoy, Fl. de l'Île de la Réunion, 543 (1895), not Michx.

Although the specific names *tomentosus* and *auriculatus* are both earlier than *penicillatus* they have already been employed for other valid species of *Senecio* and are accordingly not available for this plant, which should continue to pass as S. PENICILLATUS (Cass.) Sch. Bip.

Vahl seems to have been quite aware of the identity between his *Eupatorium auriculatum* and the earlier *E. tomentosum* of Lam., as he has indicated this upon his label. Although Lamarck's species was also founded on material collected by Commerson on the Isle of Bourbon, he appends to his description the note "on la trouve aussi dans le Brésil," having probably confused with the plant of the Indian Ocean some habitally similar species of South America. It was doubtless this circumstance which led Vahl to ascribe his *E. auriculatum* to Brazil, notwithstanding the fact that his type-sheet bears a note apparently in his own hand to the effect that the plant came from the Isle of Bourbon.

Eupatorium confertifolium Klatt, Abh. Naturf. Ges. Halle, xv. 324 (1881). This species does not differ essentially from E. VACCINIIFOLIUM Benth. Pl. Hartw. 200 (1845).

Eupatorium coperense Hieron. in Engl. Bot. Jahrb. xxi. 330 (1895). This species, examined and photographed at the Royal Botanical Museum at Berlin, appears identical with the earlier E. ANGUSTIFOLIUM

(HBK.) Spreng. Syst. iii. 415 (1826), the type of which was examined and photographed at the Museum of Natural History in Paris.

Eupatorium cremastum, n. sp., fruticosum 3–4 m. altum; ramis teretibus glabris fusco-brunneis laevibus; foliis magnis oppositis tenuibus 16 cm. longis 4.5–6.5 cm. latis utrinque viridibus sublucidis ovato-oblongis acuminatis serratis penninerviis supra glabris subtus in nerviis lanulosis basi breviter acuminatis; petiolo 3–3.5 cm. longo; internodiis ca. 3 cm. longis; inflorescentiis laxis lateralibus pendulis 5–8 cm. longis pauci- vel multi-capitulatis; pedicellis 2–2.5 cm. longis capillaribus flexuosis obscurissime puberulis; capitulis 1.3 cm. latis ca. 10-floris; involucro perlaxo, squamis linearibus attenuatis ca. 10 pilosulis viridibus subaequalibus ca. 5 mm. longis 3.5 mm. latis gracillimis nigris curvatis deorsum angustatis etiam apicem versus paullulo decrescentibus; corollis 4.5 mm. longis glabris albis, tubo proprio gracili quam fauces anguste cylindrici breviori; pappi setis ca. 18 laete albis corollam subaequantibus. — Crest of the Sierra Madre in Michoacan or Guerrero, Mexico, alt. 2200 m., 17 Feb. 1899, *Langlassé*, n. 893 (hb. Gray, hb. Berol.).

Eupatorium Cursonii, n. sp., ramis atropurpureis teretibus fusco-tomentellis, internodiis brevibus 8 mm. longis; foliis oppositis lineari-oblongis attenuatis supra bullato-reticulatis sparse puberulis subtus reticulato-venosis tomentosis 9 cm. longis 8 mm. latis crassiusculis margine leviter crenulatis valde revolutis, petiolo ca. 1 mm. longo; capitulis 2–3 in summis ramorum axillaribus permagnis ca. 80-floris, involucri squamis subaequalibus lanceolatis, exterioribus subcoriaceis 2 cm. longis 4 mm. latis striatulis dorso granulari-puberulis apice subfiliformi-attenuatis, interioribus angustioribus et tenuioribus; receptaculo paleifero saltem ad marginem, paleis 2 cm. longis ad apicem fere capillaceum attenuatis; corollis anguste tubulosis glaberrimis 1 cm. longis, dentibus 5 brevissimis patentibus atropurpureis; antheris linearibus apice valde appendiculatis basi integerrimis; styli ramis clavatis planiusculis 6 mm. longis atropurpureis; achaeniis linearibus 7 mm. longis 5-angulatis in costis minute scabratis; pappi setis ca. 50 fere 1 cm. longis, plurimis exteriorum brevioribus. — Collected at Arequipa, Peru, by Mr. Curson. The sole specimen of this clearly marked species is in the herbarium of the British Museum of Natural History. It had evidently been in the herbarium of Nuttall, who had recognized its novelty and assigned it a specific name under the genus *Campuloclinium.* Unfortunately Nuttall's manuscript name has already been used in *Eupatorium* and therefore cannot be now taken up. In habit and probably in its affinities *E. Cursonii* approaches *E. Ballii* Oliv., from which, however, it differs decidedly in its subequal involucral bracts.

Eupatorium gracilicaule Sch. Bip. in herb., fruticosum 3 m. altum oppositirameum; ramis teretibus glaberrimis gracilibus, internodiis ca. 5 cm. longis; foliis ovatis longe acuminatis basi rotundatis vel brevissime acuminatis 3-nerviis leviter serrato-dentatis plus minusve falcatis utrinque glabris subtus haud vel vix pallidioribus minute glanduloso-punctatis ad 8 cm. longis 3–4 cm. latis, venulis subtranslucentibus, petiolo gracili patenti 2–2.5 cm. longo; paniculis magnis 2–3 dm. diametro rotundato-corymbosis oppositirameis folioso-bracteatis multicapitulatis obscurissime puberulis vel granulosis, ramis laxe patentibus, pedicellis gracillimis saepe flexuosis minute bracteolatis; capitulis 6–8 mm. altis ca. 13-floris; involucri squamis laxissime imbricatis linearibus acutissimis dorso sordide puberulis, intimis ca. 3 mm. longis subaequalibus 6–9, exterioribus paucis (2–3) brevioribus angustissimis; achaeniis 5-angulatis gracilibus glabris fuscis deorsum levissime angustatis; corollis glabris 3 mm. longis verosimiliter albidis, tubo proprio gracili in fauces modice ampliatos gradatim expanso, dentibus limbi 5 brevibus patentibus; pappi setis ca. 20 tenuibus albidis aequalibus simplicissimis corollam fere aequantibus. — Tlacolula, Mexico, Dec. 1839, *Ehrenberg*, n. 11,711 (hb. Berol., hb. Gray). This species in habit somewhat suggests *E. stillingiaefolium* DC. and *E. Mendezii* DC., but differs from each of them in being nearly glabrous and in its much less imbricated involucre, etc.

Eupatorium hemipteropodum, n. sp., herbaceum robustum ad 3 m. altum; caule meduloso subtereti valde striato-costato glabrato in sicco stramineo-brunnescenti; foliis oppositis magnis tenuibus ovatis utrinque glabris et viridibus supra levissimis longe petiolatis, inferioribus ovatis subacuminatis 1.8–2.5 dm. longis 12–15 cm. latis argute grosseque dentatis basi acuminatis et breviter in petiolo decurrentibus, petiolo 4–6 cm. longo, foliis supremis deltoideo-ovatis 8–10 cm. longis 5–7 cm. latis subduplice crenato-serratis basi acuminatis, petiolo fere a media parte apicem versus alato; panicula ovoidea thyrsoidea oppositiramea ca. 7 cm. diametro aequialta multicapitulata; capitulis ca. 10-floris; involucri squamis stramineis in apice obtuso vel rotundato brunneis pluriseriatim imbricatis, interioribus lineari-oblongis, exterioribus gradatim brevioribus ovatis ciliolatis; corollis anguste tubulosis glabris dentibus limbi brevissimis; pappi setis sordide albis corollam subaequantibus; achaeniis valde immaturis in et etiam inter costis pubentibus. — *E. quadrangulare* Millsp. Field Columb. Mus. Bot. i. 324 (1896), not DC. *E. populifolium* Millsp. l. c., not HBK. *E. aromatisans* Millsp. l. c. iii. 92, with figs. (1904), not DC. — YUCATAN: Izamal (where, acc. to Millspaugh, l. c., in general cultivation), *Gaumer*, n. 552 (hb. Field Mus., hb. Gray); Merida, *Valdez*, n. 92 (hb. Field Mus., hb.

Gray). This perplexing plant has several times been referred to the present writer, who may well be responsible for some of the several names under which it has passed. Renewed study of it shows that it possesses distinctions which appear to separate it from any described species. From *E. quadrangulare* it differs in its essentially terete stem; from *E. populifolium* (a species which should henceforth bear the name *E. morifolium* Mill.), it may be readily distinguished by its essentially membranaceous leaves of different dentation, and by its pubescent achenes, as well as by its half-winged petioles. Its nearest affinity is doubtless, as pointed out by Dr. Millspaugh, with the West Indian *E. aromatisans* DC., yet it cannot be satisfactorily placed in that species, as the leaves are much thinner, the upper ones crenate-dentate, not serrate, and the petioles only partially winged instead of being winged clear to the base, as is the case in *E. aromatisans.*

Eupatorium Holwayanum, n. sp., herbaceum gracile erectum vel decumbens subglabrum 3–5 dm. altum; caulibus teretibus plus minusve purpurascentibus striatulis obscurissime puberulis foliosis gracilibus haud 2 mm. diametro; foliis ternis vel oppositis vel praecipue in parte superiore caulis alternis ovatis vel rhomboideis acutiusculis pauciserratis 1.5–2 cm. longis 7–10 mm. latis 3-nerviis utrinque glabris viridibus basi cuneatis brevissime petiolatis; panicula laxe ramosa basi plus minusve folioso-bracteata, ramis gracilibus purpureis ascendenti-divergentibus saepius 1–5-capituliferis; capitulis 9 mm. diametro ca. 60-floris; involucri squamis valde inaequalibus lineari-lanceolatis pluriseriatim imbricatis attenuatis acutissimis striatis viridibus vel purpurascentibus; corollis albis 3.3 mm. longis tubulosis, tubo proprio brevissimo, faucibus anguste tubulosis multo longioribus glabris, dentibus limbi 5 externe puberulis; achaeniis 5-angulatis fuscescentibus 1.7 mm. longis in angulis scabratis; pappi setis ca. 20 albis tenuissimis scabratis. — Dry banks, Sierra de San Felipe, alt. 2150 m., 17 Nov. 1894, *C. G. Pringle,* n. 5683 (hb. Gray), 14 Nov. 1903, *E. D. W. Holway,* n. 5418 (hb. Gray). This plant was at first referred to *E. trinervium* Sch. Bip., to which it has considerable similarity. It may be readily distinguished, however, by its much smaller leaves of an ovate or rhombic form, and by the lack of pubescence regularly found in *E. trinervium.*

Eupatorium leptophyllum DC. Prod. v. 176 (1836). This species has long been misunderstood and neglected. It was originally described from material collected about Savannah, Georgia, by Herbemont, and was regarded by Torrey and Gray (Fl. ii. 83) as merely a smoothish variety (their var. *glabrum,* l. c.) of *E. foeniculaceum* Willd. Dr. Chapman and Dr. Small in their floras of the Southern States do not mention the plant either as a species or variety. The material now available shows that *E. leptophyllum* was well grounded and is readily distinguishable from the related species. Among these, it most nearly approaches *E. capillifolium* (Lam.) Small (*E. foeniculaceum* Willd.), and has similar very fine filiform-linear entire, pinnate, or dissected leaves, but it differs in the long simple recurved-spreading secund-racemose branches of its inflorescence, the heads of *E. capillifolium* being borne in compound somewhat fastigiate leafy panicles. In *E. capillifolium*, furthermore, the involucral scales are green and oblong, rather abruptly pointed and but slightly scarious at the margins, while in *E. leptophyllum* they are inclined to be brown (in dried specimens) and have strongly contrasting white margins. In form they are linear-oblong or lance-linear and tend to be attenuate, often ending in a very sharp point. The simple recurved racemose branches of the inflorescence are 4–10 cm. long, and as DeCandolle remarks in the original description, the inflorescence suggests strongly that of some of the golden-rods. The following specimens of *E. leptophyllum* have been examined: — GEORGIA: near Savannah, *Herbemont* (hb. DC.). SOUTH CAROLINA: damp pine land, Santee Canal, September, *Ravenel* (hb. Gray). FLORIDA: without locality, *Leavenworth;* Braidentown, *Tracy*, n. 7099 (hb. Gray), distributed as *E. capillifolium.* The species shows some variation in its leaves. They are described as entire by DeCandolle, and this is true in the upper parts of the specimens at hand, but the lowermost leaves when shown are pinnately divided into filiform or narrowly linear segments. In a second specimen collected on the Santee Canal by Ravenel (October), the leaves are not only more dissected than in the others, but the segments, although still very narrow, are distinctly flat rather than filiform. The close correspondence of all these specimens, however, in the more essential characters of inflorescence, involucres, flowers, achenes, and pappus, confirm the belief that they are only individual or formal leaf-variations of *E. leptophyllum* DC.

Eupatorium loxense Klatt, Ann. k. k. Naturh. Hofmus. Wien, ix. 357 (1894). This species, founded on a plant collected by Jameson at Loxa, Ecuador, has been examined both as to its type preserved in the herbarium of the Imperial Natural History Museum at Vienna and the specimen in the herbarium of the late Dr. Klatt. It is clearly the staminate plant of a *Baccharis*, and so far as may be judged from the characters of B. BERBERIFOLIA HBK. Nov. Gen. et Spec. iv. 57 (1820), may well belong to that species.

Eupatorium menthaefolium Poepp. ex Spreng. Syst. iii. 412 (1826). Although this Cuban species is kept up as a *Eupatorium* by Hooker and Jackson in the Index Kewensis i. 118 (1893), it was long ago transferred to *Vernonia* by Lessing, Linnaea, iv. 268, a disposition fully confirmed by a recent examination of the type preserved in the herbarium of the Imperial Natural History Museum at Vienna. Unless some earlier specific name is found, the plant should certainly stand as VERNONIA MENTHAEFOLIA Less.

EUPATORIUM MORIFOLIUM Mill. Gard. Dict. ed. 8, no. 10 (1768). The type-specimen of this hitherto obscure species was recently examined at the herbarium of the British Museum of Natural History. It was collected at Vera Cruz, Mexico, by Dr. Houston, and proves to be identical with the plant long familiar under the later name of *E. populifolium* HBK. Nov. Gen. et Spec. iv. 111 (1820). *E. morifolium* Mill. seems to have been overlooked by Mr. Hemsley, as it is omitted from the Biologia Centrali-Americana, but it is as well characterized as most species of its date and must be reinstated for the well-known plant to which the name was clearly applied.

Eupatorium myosotifolium Jacq. Ic. Pl. Rar. iii. 15, t. 582 (1786–1793), & Coll. ii. 341 (1788). This species was described from cultivated specimens which had been raised from seeds supposed to come from tropical America. The plant is well figured by Jacquin, and it is clear, even from his illustration, that it does not belong to the genus *Eupatorium* nor even to the *Eupatorieae*, for the style-branches are short and decidedly recurved. An examination of an authentic specimen preserved in the herbarium of the Imperial Natural History Museum at Vienna fully confirmed the correctness of the plate and showed that the plant belongs to the baccharoid *Astereae*, probably to the genus *Conyza*.

Eupatorium nubigenoides, n. sp., inflorescentia excepta glaberrimum; ramis lignescentibus teretibus medulosis; foliis oppositis longe petiolatis tenuissimis ovati-oblongis vel late lanceolatis penninerviis serratis utrinque glaberrimis viridibus utroque acuminatis ca. 13 cm. longis ca. 5 cm. latis, dentibus parvis incurvis tenuiter cuspidatis, petiolo obcompresso nudo ca. 5 cm. longo; panicula oppositiramea ramosissima supra rotundata polycephala puberula vel tenuiter tomentella folioso-bracteata; capitulis pedicillatis ca. 10-floris 6 mm. altis; involucri campanulato-subcylindrici squamis albescentibus adpressis valde imbricatis ca. 4-seriatis multi-striatulis, exterioribus brevissimis ovatis acutiusculis ciliolatis intermediis ovato-oblongis apice rotundatis glabris gradatim longioribus, intimis similibus sed anguste oblongis obtusis; corollis albis anguste tubulosis 3 mm. longis glabris sine faucibus ullis distinctis, dentibus limbi deltoideis brevissimis patentibus; achaeniis 5-angulatis 1 mm. longis fuscis glabris ad basin leviter attenuatis; pappi subsparsi setis tenuissimis albis. — Pansamalá, Depart. Alta Verapaz, Guatemala, June, 1886, alt. 1150 m., *H. von Tuerckheim*, n. 928 (distrib. by J. D. Smith). This specimen was first sent out as *E. aromatisans* DC. Its identification was then changed at the mistaken suggestion of the writer to *E. nubigenum* Benth. Compared at the Royal Gardens of Kew with Hartweg's n. 587, the type of Bentham's *E. nubigenum*, this plant proved to be distinct. *E. nubigenum* has larger flowers, longer achenes, closely sessile heads, and strongly angled branches.

Eupatorium nutans HBK. Nov. Gen. et Spec. iv. 105 (1820). This name has been regarded as a somewhat doubtful synonym of *Brickellia secunda* Gray. The type, however, when examined at the herbarium of the Museum of Natural History in Paris, proved to be B. PENDULA Gray, Pl. Wright. i. 85 (1852). *Bulbostylis pendula* DC. Prod. v. 138 (1836). *Eupatorium pendulum* Schrad. Cat. Sem. Hort. Goett. 1830, acc. to DC. l. c. Although the specific name *nutans* is earlier than *pendulum*, it is no longer applicable in *Brickellia* owing to the existence of the valid homonym *B. nutans* Robinson & Greenman, Am. Jour. Sci. l. 152 (1895).

EUPATORIUM OVALIFLORUM Hook. & Arn. Bot. Beech. 297 (1840). This species was long known to the writer from the original description only, and from the characters there given could not be positively distinguished from the plant, also of western Mexico, later described as *E. Bertholdii* Sch. Bip. in Seem. Bot. Herald. 299 (1856). As others may encounter difficulty in separating these nearly related and habitally similar species, it may be well to record the differences, which were found on a comparison of authentic specimens in the herbarium of the Royal Gardens at Kew. In *E. ovaliflorum* the leaves are permanently tomentulose above as well as below and they are decidedly pale beneath; the involucres are ovoid and 3.8 mm. in thickness. In *E. Bertholdii*, on the other hand, the leaves are somewhat scabrous with scattered hairs above and green beneath; while the involucres are more narrowly ovoid, 2.3 mm. in thickness.

EUPATORIUM PALMERI Gray, Proc. Am. Acad. xxi. 383 (1886). From the typical form of this species, which has its leaves grayish-tomentulose beneath and its involucral scales finely pubescent, the following smoothish caudate-attenuate leaved plant would appear quite distinct were it not for the existence in western Mexico of some intermediate forms, which appear to show that the differences, although conspicuous, are by no means constant.

Var. **tonsum**, n. var., foliis ovato-lanceolatis longissime attenuatis utrinque glabris remote serrulatis vel integris ad 14 cm. longis ad

4.2 cm. latis; capitulis pyramidato-paniculatis ca. 5-floris; involucri squamis viridibus striatis acutissimis vix pubescentibus; antheris apice brevissime appendiculatis. — El Ocote, Michoacan or Guerrero, alt. 300 m., 10 November, 1898, *Langlassé*, n. 616 (hb. Gray, hb. Berol.). Said to be a shrub 1.5 m. high with grayish flowers.

Eupatorium pelotrophum, n. sp., fruticosum 1.5 m. altum; ramis gracilibus teretibus fuscis puberulis; internodiis ca. 5 cm. longis; foliis oppositis petiolatis ovatis longe acuminatis serratis basi rotundata et apice caudato-attenuato integerrimis a basi 3(–5)-nerviis 7–10 cm. longis 2–4 cm. latis firmiusculis utrinque viridibus scabriusculis, petiolo fusco-puberulo ca. 1 cm. longo; cymis axillaribus glomeruliformibus oppositis ca. 12-capitulatis 2–3 cm. diametro, pedicellis 1–5 mm. longis teretibus sordido-tomentellis, bracteolis minimis fuscis; involucris subcylindrici-campanulatis saepissime 4-floris, squamis ca. 9 valde inaequalibus appresse imbricatis lanceolato-linearibus obtusis brunneis dorso pilosiusculis, interioribus ca. 3 mm. longis; corollis albis ex involucro longe exsertis ca. 4 mm. longis glabris, tubo proprio gracili in fauces cylindricos subaequilongos leviter ampliato, dentibus limbi brevissimis suberectis; achaeniis (immaturis) fuscis obscure granulatis quam corollae multo brevioribus; pappi setis albis rectis ca. 20 sursum scabriusculis corollam aequantibus. — In clayey soil on the crest of the Sierra Madre in Michoacan or Guerrero, alt. 2300 m., 16 February, 1899, *Langlassé*, n. 880 (hb. Gray). This species is clearly related to *E. tubiflorum* Benth. and *E. areolare* DC., but may be readily distinguished by its peculiar globular axillary inflorescences and by the fact that the pappus is fully as long as the corolla.

EUPATORIUM PURPUREUM L., var **Bruneri** (Gray), n. comb., foliis oblongo-lanceolatis ternis vel quaternis vel quinis serratis firmiusculis subtus plus minusve cinerascenti-puberulis vel tomentellis albescente reticulato-venulosis; corymbis planiusculis vel modice convexis. — *E. Bruneri* Gray, Syn. Fl. i. pt. 2, 96 (1884); Coult. Man. Rocky Mt. Reg. 142 (1885). *E. atromontanum* Nelson, Bot. Gaz. xxxi. 400 (June, 1901). *E. Rydbergi* Britton, Man. 921 (Oct. 1901). — Dr. Gray's original characterization of *E. Bruneri* was drawn from a very poor specimen and is so misleading that it is by no means surprising that it has never been rightly understood. The leaves are not opposite, as described, but are in whorls of three. In pubescence, venation, toothing, and texture, they correspond accurately with the widely distributed western form of *E. purpureum*, which as stated above has received two subsequent specific names. The form, although fairly well marked in its extreme, appears to pass by easy transitions into the typical form of *E. purpureum* and would therefore seem best treated as a variety.

In range it extends from Iowa to British Columbia and southward in the Rocky Mountains to New Mexico.

Eupatorium rapunculoides, n. comb. *Stevia rapunculoides* DC. Prod. v. 124 (1836). *Eupatorium dasycarpum* Gray, Proc. Am. Acad. xxii. 420 (1887); Robinson, ibid. xxxvi. 478 (1901).

EUPATORIUM REMOTIFOLIUM DC. Prod. v. 165 (1836). This species is reduced without comment by Baker in Mart. Fl. Bras. vi. pt. 2, 205 (1876) to *E. Vitallae* DC., but the two species in question have not the smallest resemblance. It can only be inferred that Mr. Baker was misled through some transposition of labels or similar slip. The plants are by no means similar in habit, leaf texture, inflorescence, or involucre. The marked difference in the size of the heads is quite sufficient to show them distinct species. In *E. Vitalbae* the florets, including the mature achene, are 10–12 mm. long, while in *E. remotifolium* they are, when measured in the same manner, only 4–5 mm. long.

Eupatorium resinosum Poepp. [& Endl.] Nov. Gen. et Spec. iii. 54 (1845), not Torr. From an examination of the type-specimen of this obscure species, which is to be found in the herbarium of the Imperial Natural History Museum at Vienna, it is clear that it is identical with E. LAEVIGATUM Lam. Dict. ii. 408 (1786).

EUPATORIUM RUBRICAULE HBK. Nov. Gen. et Spec. iv. 124 (1820). Although excellently described by Kunth, this species appears never to have been recognized. Hemsley (Biol. Cent.-Am. Bot. ii. 100) refers no material to it beyond the original specimen collected by Humboldt & Bonpland, and no more recent collection of it seems to be on record. The type was found, however, to be exactly the plant for some years known as *E. amplifolium* Gray, Proc. Am. Acad. xv. 28 (1880), and well shown by the following exsiccatae: Parry & Palmer's no. 334 from San Luis Potosi, Pringle's no. 4272 from cool cañons near Pazcuaro, Michoacan, Palmer's no. 165 (coll. of 1902) from Alvarez, San Luis Potosi, and L. C. Smith's no. 858 from the mountains of Jayacatlan, alt. 2150 m., Oaxaca. Dr. Gray's later specific name must, of course, drop into synonymy. Why *E. rubricaule* HBK., which is a large-leaved perennial, should have been referred to *E. guadalupense* Spreng. by the Index Kewensis is not clear.

EUPATORIUM SAGITTATUM Gray, Pl. Wright. i. 88 (1852). This species has in general ovate-oblong sagittate or hastate leaves considerably longer than broad. In its wide deltoid leaves the following plant is strikingly different, although maintaining the essential characteristics of the species.

Var. **deltophyllum**, n. var., fruticosum oppositirameum habitu capitulis involucri squamis formae typicae simile; foliis multo latioribus deltoideis 3–4 cm. longis 2–4.4! cm. latis apice et angulis inferioribus transverse divaricatis acutis. — Near Culiacan, Sinaloa, Mexico, coll. *Schaffner* (type, in hb. Gray).

EUPATORIUM THYRSOIDEUM Moc. ex DC. Prod. v. 150 (1836). Some years ago the writer (Proc. Am. Acad. xxxvi. 484) stated the belief that this plant was identical with *E. quadrangulare* DC. l. c. However, subsequent study of more copious material and a re-examination of the type specimens in the Prodromus Herbarium at Geneva have shown conclusively that the two species are wholly distinct notwithstanding considerable habital resemblance. To *E. thyrsoideum*, which may be recognized by its roundish stem, more glomerate-thyrsoid inflorescence, and softer less stramineous involucral scales, the following specimens may be referred: Dr. Palmer's no. 1048 (coll. of 1890) from Manzanillo, and his no. 1162 (coll. of 1891) from Colima, W. G. Wright's no. 1355 from San Blas, Langlassé's no. 650, collected on granitic soil at Cajinicuilar, alt. 300 m., in Michoacan or Guerrero, Baker's no. 2302, Volcan El Viejo, Dept. Chinandega, Nicaragua, and Barclay's no. 2719 from Tiger Island, Gulf of Fonseca.

Eupatorium tolimense Hieron. in Engl. Bot. Jahrb. xix. 45 (1894). The writer, after examining the type of this species at the Royal Botanical Museum at Berlin and the type of E. PELLUCIDUM HBK. Nov. Gen. et Spec. iv. 108 (1820), which is preserved in the herbarium of the Museum of Natural History at Paris, is unable to find any differences of importance between them.

EUPATORIUM URTICAEFOLIUM Reichard, Syst. iii. 719 (1780). *Ageratum altissimum* L. Spec. Pl. ii. 839 (1753). *Eupatorium altissimum* Murr. Syst. Veg. ed. 13, 614 (1774), not L., nor Murr. l. c. 613. *E. ageratoides* L. f. Suppl. 355 (1781). *Kyrstenia altissima* Greene, Leafl. i. 8 (1903). A consistent application of the rules of priority necessitates the reviving of Reichard's name for our common and attractive North American *Eupatorium* which has long passed as *E. ageratoides.* Furthermore, in consequence of this revival of Reichard's *E. urticaefolium*, it is necessary to suppress the later *E. urticaefolium* L. f. Suppl. 354 (1781). This, however, is no great misfortune, for the species of Linnaeus filius has been in recent times entirely misinterpreted, the name having been applied by English and German botanists to a coarse annual, widely distributed in tropical America, having rather long pedicels and ovate-lanceolate to lance-linear leaves. This plant has borne many names, of which the earliest appears to be the hitherto obscure *E. pauciflorum* HBK. Nov. Gen. et Spec. iv. 120 (1820). Very different from this is the type of *E. urticaefolium* L. f., which is still extant in the herbarium of the Linnean Society of London. It is a Colombian plant, apparently perennial, with deltoid-ovate, strongly cordate leaves and contracted inflorescence, the pedicels being very short. It is not yet certain whether this plant has received a later name, but it is pretty close to *E. ballotaefolium* HBK. and of course has nothing whatever to do with the *E. urticifolium* of the Flora Brasiliensis or of recent writers on the flora of Argentina and Paraguay.

MIKANIA BADIERI DC. Prod. v. 194 (1896). This excellent species is well marked by its coriaceous glabrous essentially entire leaves, which are rather abruptly acuminate, ending in a caudate and often falcate tip. The petioles are furthermore much flattened and rather broad. The species, notwithstanding these distinctive characters, has been reduced by Grisebach (Veg. Karaib. 85) to *M. latifolia* J. E. Smith (Rees, Cyclop. xxiii. n. 8). Grisebach also implies that it is suspiciously close to *M. amara* Willd. However, the inflorescences in *M. Badieri* are pyramidal panicles in which the heads are subspicately arranged, while both in *M. amara* Willd. and in *M. latifolia* J. E. Smith (ex char.) the heads are borne in corymbs. In the herbarium of Lamarck there is a sheet of *M. Badieri* labelled in the hand of Lamarck himself "Eupatorium vincaefolium Lam. dict. no. 39 de Mr. Badier de la guadeloupe No 137." At first sight it would appear that this was an authentic type of *E. vincaefolium* Lam. and that his earlier specific name should take the place of DeCandolle's later *Badieri*. However, Lamarck did not cite the plant of Badier in his Dictionary, but on the other hand distinctly states that his *E. vincaefolium* was founded on South American material. Indeed, most of the characters are evidently drawn from Aublet's plate and description of *E. parviflorum*, which Lamarck reduces to a synonym of his own species. Although Badier's plant of Guadeloupe possesses some habital similarity to the South American one, the latter lacks the broad petioles and the peculiar acumination and is doubtless a distinct species. Obviously, it is to the South American plant that Lamarck's name *E. vincaefolium* was applied.

Mikania Houstoniana, n. comb. *Eupatorium Houstonianum* L. Spec. ii. 836 (1753). *E. Houstonis* L. Syst. ed. 10, 1204 (1759). *E. fruticosum* Mill. Dict. ed. 8, no. 6 (1768). *Mikania Houstonis* Willd. Spec. iii. 1742 (1804). The rule of priority of the specific name requires the restoration of the earlier adjectival form.

BRICKELLIA ATRACTYLOIDES Gray, Proc. Am. Acad. viii. 290 (1870). Of this species, *Coleosanthus venulosus* A. Nelson, Bot. Gaz. xxxvii. 262 (1904), is an exact synonym.

Brickellia microphylla Gray, Pl. Wright. i. 85 (1852); *Bulbostylis microphylla* Nutt. Trans. Am. Phil. Soc. n. s. vii. 286 (1841). To the synonymy of this species may be added *B. cedrosensis* Greene, Bull. Torr. Bot. Club, x. 86 (1883); *Coleosanthus cedrosensis* Greene, Erythea, i. 54 (1893).

Brickellia oblongifolia Nutt. Trans. Am. Phil. Soc. n. s. vii. 286 (1841). Of this species, *Coleosanthus humilis* Greene, Pittonia, iv. 124 (1900), appears to be a synonym.

Brickellia paniculata, n. comb. *Eupatorium paniculatum* Mill. Gard. Dict. ed. 8, no. 15 (1768). *E. Verae-Crucis* Steud. Nom. ed. 2, i. 609 (1840). *Ageratum paniculatum* Hort. and *Eriopappus paniculatus* Hort. ex Steud. l. c. *Eupatorium rigidum* Benth. Pl. Hartw. 88 (1841), not Sw. *Brickellia Hartwegi* Gray, Pl. Wright. i. 85 (1852). Miller's *Eupatorium paniculatum* seems never to have been studied or accurately identified. It is evident that Steudel's renaming of the species was purely a bibliographical change incident to his compilation, and involving no personal examination of the plant. Miller's type, collected at Vera Cruz, Mexico, by Dr. Houston, is still extant in the herbarium of the British Museum of Natural History, and proves to be the plant which has for some time been passing under the name of *Brickellia Hartwegi* Gray. There seems no reason why Miller's name should not be restored and transferred to the correct genus. Steudel's purpose in renaming *E. paniculatum* Mill. (1768) was doubtless that he might maintain the better known but much later homonym of Schrader (1832), which, however, has since proved a synonym of *E. microstemon* Cass. Link, Enum. ii. 306 (1822), erroneously united under the name *E. paniculatum* the very different plants of Miller and Schrader, belonging as we now see to two distinct genera. The unwarranted name *E. Verae-Crucis* still persists in some botanical gardens.

Cacalia asclepiadea L. f. Suppl. 352 (1781). *Senecio* (?) *asclepiadeus* DC. Prod. vi. 422 (1837). This doubtful species from Colombia seems never to have been studied or accurately placed. Fortunately the type-specimen is still extant in the herbarium of the Linnean Society of London. It proves to be identical with *Eupatorium angustifolium* Spreng. Syst. iii. 415 (1826); *Mikania angustifolia* HBK. Nov. Gen. et Spec. iv. 138 (1820). Although the Linnaean specific name is older, it cannot be applied in *Eupatorium* because of the existing homonym, *E. asclepiadeum* DC. Prod. v. 148 (1836). *Cacalia asclepiadea* L. f. should therefore drop into the synonymy of Eupatorium angustifolium (HBK.) Spreng. and the genus *Cacalia* (or *Senecio*) may thus be freed from a dubious species.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

ARCHITECTURAL ACOUSTICS.

I. *INTRODUCTION.*
II. *THE ACCURACY OF MUSICAL TASTE IN REGARD TO ARCHITECTURAL ACOUSTICS.*
III. *VARIATION IN REVERBERATION WITH VARIATION IN PITCH.*

BY WALLACE C. SABINE.

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ARCHITECTURAL ACOUSTICS.

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I. INTRODUCTION.

THE problem of architectural acoustics requires for its complete solution two distinct lines of investigation, one to determine quantitatively the physical conditions on which loudness, reverberation, resonance, and the allied phenomena depend, the other to determine the intensity which each of these should have, what conditions are best for the distinct audition of speech, and what effects are best for music in its various forms. One is a purely physical investigation, and its conclusions should be based and should be disputed only on scientific grounds; the other is a matter of judgment and taste, and its conclusions are weighty in proportion to the weight and unanimity of the authority in which they find their source. For this reason, these papers are in two series. The articles which appeared six years ago began the first, and the paper immediately following is the beginning of the second.

Of the first series of papers, which have to do with the purely physical side of the problem, only one paper has as yet been published. This contained a discussion of reverberation, complete as far as one note is concerned. There is on hand considerable material for a paper extending this discussion to cover the whole range of the musical scale, and therefore furnishing a basis for the discussion of what has sometimes been called the musical quality of an auditorium. There has also been collected a certain amount of data in regard to loudness, resonance, interference, echos, irregularities of air currents and temperature, and the transmission of sound through walls and partitions, — all of which will appear as soon as a complete presentation is possible in each case. Each problem has been taken up as it has been brought to the writer's attention by an architect in consultation either over plans or in regard to a completed building. This method is slow, but it has the advantage of making the work practical, and may be relied on to prevent the magnification to undue importance of scientifically interesting but practically subordinate points. On the other hand there is the danger that it may lead to a fragmentary presentation. An effort has been made to guard against this, and the effort for completeness is the reason for delay in the appearance of some of the papers. Sufficient progress has been made, however, to justify the assertion that the physical side of the problem is solvable, and that it should be possible ultimately to calculate in advance of construction all the acoustical qualities of an auditorium.

Thus far it is a legitimate problem in physics, and as such a reasonable one for the writer to undertake.

The second part of the problem, now being started, the question as to what constitutes good and what constitutes poor acoustics, what effects are desirable in an auditorium designed for speaking, and even more especially in one designed for music, is not a question in physics. It is therefore not one for which the writer is especially qualified, and would not be undertaken here were it not in the first place absolutely necessary in order to give effect to the rest of the work, and in the second place were it not the plan rather to gather and give expression to the judgment of others acknowledged as qualified to speak, than to give expression to the taste and judgment of one. It is thus the purpose to seek expert judgment in regard to acoustical effects, and if possible to present the results in a form available to architects. This will be slow and difficult work, and it is not at all certain that it will be possible to arrive, even ultimately, at a finished product. It is worth undertaking, however, if the job as a whole is worth undertaking, for without it the physical side of the investigation will lose much of its practical value. Thus it is of little value to be able to calculate in advance of construction and express in numerical measure the acoustical quality which any planned auditorium will have, unless one knows also in numerical measure the acoustical quality which is desired. On the other hand, if the owner and the architect can agree on the desired result, and if this is within the limits of possibility considering all the demands on the auditorium, of utility, architecture, and engineering, this result can be secured with certainty, — at least there need be no uncertainty as to whether it will or will not be attained in the completed building.

The papers following this introduction will be: "The Accuracy of Musical Taste in regard to Architectural Acoustics," and "Variation in Reverberation with Variation in Pitch."

II. The Accuracy of Musical Taste in regard to Architectural Acoustics.

Piano Music.

The experiments described in this paper were undertaken in order to determine the reverberation best suited to piano music in a music room of moderate size, but were so conducted as to give a measure of the accuracy of cultivated musical taste. The latter point is obviously fundamental to the whole investigation, for unless musical taste is precise, the problem, at least as far as it concerns the design of the auditorium for musical purposes, is indeterminate.

The first observations in regard to the precision of musical taste were obtained during the planning of the Boston Symphony Hall, Messrs. McKim, Mead, and White, Architects. Mr. Higginson, Mr. Gericke, the conductor of the orchestra, and others connected with the Building Committee expressed opinions in regard to a number of auditoriums. These buildings included the old Boston Music Hall, at that time the home of the orchestra, and the places visited by the orchestra in its winter trips, Sanders Theatre in Cambridge, Carnegie Hall in New York, the Academy of Music in Philadelphia, and the Music Hall in Baltimore, and in addition to these the Leipsig Gewandthaus. By invitation of Mr. Higginson, the writer accompanied the orchestra on one of its trips, made measurements of all the halls, and calculated their reverberation. The dimensions and the material of the Gewandthaus had been published, and from these data its reverberation also was calculated. The results of these measurements and calculations showed that the opinions expressed in regard to the several halls were entirely consistent with the physical facts. That is to say, the reverberation in those halls in which it was declared too great was in point of physical measurement greater than in halls in which it was pronounced too small. This consistency gave encouragement in the hope that the physical problem was real, and the end to be attained definite.

Much more elaborate data on the accuracy of musical taste were obtained four years later, 1902, in connection with the new building of the New England Conservatory of Music, Messrs. Wheelwright and Haven, Architects. The new building consists of a large auditorium surrounded on three sides by smaller rooms, which on the second and third floors are used for purposes of instruction. These smaller rooms, when first occupied, and used in an unfurnished or partially furnished condition, were found unsuitable acoustically, and the writer was consulted by Mr. Haven in regard to their final adjustment. In order to learn the acoustical condition which would accurately meet the requirements of those who were to use the rooms, an experiment was undertaken in which a number of rooms, chosen as typical, were varied rapidly in respect to reverberation by means of temporarily introduced absorbing material. Approval or disapproval of the acoustical quality of each room at each stage was expressed by a committee chosen by the Director of the Conservatory. At the close of these tests, the reverberation in the rooms was measured by the writer in an entirely independent manner as described in the paper on Reverberation (1900). The judges were Mr. George W. Chadwick, Director of the Conservatory, and Sig. Oresti Bimboni, Mr. William H. Dunham, Mr. George W. Proctor, and Mr. William L. Whitney, of the Faculty. The writer suggested and arranged the experiment and subsequently reduced the results to numerical measure, but expressed no opinion in regard to the quality of the rooms.

The merits of each room in its varied conditions were judged solely by listening to piano music by Mr. Proctor. The character of the musical compositions on which the judgment was based is a matter of interest in this connection, but this fact was not appreciated at the time and no record of the selections was made. It is only possible to say that several short fragments, varied in nature, were tried in each room.

As will be evident from the descriptions given below, the rooms were so differently furnished that no inference as to the reverberation could be drawn from appearances, and it is certain that the opinions were based solely on the quality of the room as heard in the piano music.

The five rooms chosen as typical were on the second floor of the building. The rooms were four meters high. Their volumes varied from 74 to 210 cubic meters. The walls and ceilings were finished in plaster on wire lath, and were neither papered nor painted. There was a piano in each room; in room 5 there were two. The amount of other furniture in the rooms varied greatly:

In room 1 there was a bare floor, and no furniture except the piano and piano stool.

Room 2 had rugs on the floor, chairs, a sofa with pillows, table, music racks, and a lamp.

Room 3 had a carpet, chairs, bookcases, and a large number of books, which, overflowing the bookcases, were stacked along the walls.

Room 4 had no carpet, but there were chairs and a small table.

Room 5 had a carpet, chairs, and shelia curtains.

Thus the rooms varied from an almost unfurnished to a reasonably furnished condition. In all cases the reverberation was too great.

The experiment was begun in room 1. There were, at the time, besides the writer, five gentlemen in the room, the absorbing effect of whose clothing, though small, nevertheless should be taken into account in an accurate calculation of the reverberation. Thirteen cushions from the seats in Sanders Theatre, whose absorbing power for sound had been determined in an earlier investigation, were brought into the room. Under these conditions the unanimous opinion was that the room, as tested by the piano, was lifeless. Two cushions were then removed from the room with a perceptible change for the better in the piano music. Three more cushions were removed, and the effect was much better. Two more were then taken out, leaving six cushions in the room, and the result met unanimous approval. It was suggested that two more be removed. This being done the reverberation was found to be too great. The agreement was then reached that the conditions produced by the presence of six cushions were the most nearly satisfactory.

The experiment was then continued in Mr. Dunham's room, number 2. Six gentlemen were present. Seven cushions were brought into the room. The music showed an insufficient reverberation. Two of the cushions were then taken out. The change was regarded as a distinct improvement, and the room was satisfactory.

In Mr. Whitney's room, number 3, twelve cushions, with which it was thought to overload the room, were found insufficient even with the presence in this case of seven gentleman. Three more cushions were brought in and the result declared satisfactory.

In the fourth room, five, eight, and ten cushions were tried before the conditions were regarded as satisfactory.

In Mr. Proctor's room, number 5, it was evident that the ten cushions which had been brought into the room had overloaded it. Two were removed, and afterwards three more, leaving only five, before a satisfactory condition was reached.

This completed the direct experiment with the piano.

The bringing into a room of any absorbing material, such as these cushions, affects its acoustical properties in several respects, but principally in respect to its reverberation. The prolongation of sound in a room after the cessation of its source, may be regarded either as a case of stored energy which is gradually suffering loss by transmission through and absorption by the walls and contained material, or it may be regarded as a process of rapid reflection from wall to wall with loss at each reflection. In either case it is called reverberation. It is sometimes called, mistakenly as has been explained, resonance. The reverberation may be expressed by the duration of audibility of the residual sound after the cessation of a source so adjusted as to produce an average of sound of some standard intensity over the whole room. The direct determination of this, under the varied conditions of this experiment, was impracticable, but, by measuring the duration of audibility of the residual sound after the cessation of a measured organ pipe in each room without any cushions, and knowing the coefficient of absorption of the cushions, it was possible to calculate accurately the reverberation at each stage in the test. It was impossible to make these measurements immediately after the above experiments, because, although the day was an especially quiet one, the noises from the street and railway traffic were seriously disturbing. Late the following night the conditions were more favorable, and a series of fairly good observations was obtained in each room. The cushions had been removed, so that the measurements were made on the rooms in their original condition, furnished as above described. The apparatus and method employed are described in full in a series of articles in the Engineering Record and American Architect for 1900. The results are given in the accompanying table.

The table is a record of the first of what, it is hoped, will be a series of such experiments extending to rooms of much larger dimensions and to other kinds of music. It may well be, in fact it is highly probable, that very much larger rooms would necessitate a different amount of reverberation, as also may other types of musical instruments or the voice. As an example of such investigations, as well as evidence of their need, it is here given in full. The following additional explanations may be made. The variation in volume of the rooms is only threefold, corresponding only to such music rooms as may be found in private houses. Over this range a perceptible variation in the required reverberation should not be expected. The third column in the table includes in the absorbing power of the room (ceiling, walls, furniture, etc.) the absorbing powers of the clothes of the writer, who was present not merely at all tests, but in the measurement of the reverberation the following night. From the next two columns, therefore, the writer and the effects of his clothing are omitted. The remarks in the last column are reduced to the form "reverberation too great," "too little," or "approved." The remarks at the time were not in this form, however. The room was pronounced "too resonant," "too much echo," "harsh," or "dull," "lifeless," "overloaded," expressions to which the forms adopted are equivalent.

Room Number.	Volume.	Absorbing Power of Room.	Gentlemen Present.	Absorbing Power of Clothing.	Number of Meters of Cushions.	Absorbing Power of Cushions.	Total Absorbing Power.	Reverberation in Seconds.	Remarks.
1	74	5.0	0	0	0	0	5.0	2.43	Reverberation too great.
		"	5	2.4	0	0	7.4	1.64	Reverberation too great.
		"	"	"	13	12.8	20.2	.60	Reverberation too little.
		"	"	"	11	10.1	17.5	.70	Better.
		"	"	"	8	7.3	14.7	.83	Better.
		"	"	"	6	5.5	12.9	.95	Condition approved.
		"	"	"	4	3.6	11.0	1.22	Reverberation too great.
2	91	6.3	0	0	0	0	6.3	2.39	Reverberation too great.
		"	6	2.9	0	0	9.2	1.95	Reverberation too great.
		"	"	"	7	6.4	15.6	.95	Reverberation too little.
		"	"	"	5	4.6	13.8	1.10	Condition approved.
3	210	14.0	0	0	0	0	14.0	2.46	Reverberation too great.
		"	7	3.4	0	0	17.4	2.00	Reverberation too great.
		"	"	"	12	11.0	28.4	1.21	Better.
		"	"	"	15	13.7	31.1	1.10	Condition approved.
4	133	8.3	0	0	0	0	8.3	2.65	Reverberation too great.
		"	7	3.4	0	0	11.7	1.87	Reverberation too great.
		"	"	"	6	5.5	17.2	1.26	Better.
		"	"	"	10	9.1	20.8	1.09	Condition approved.
5	96	7.0	0	0	0	0	7.0	2.24	Reverberation too great.
		"	4	1.9	0	0	8.9	1.76	Reverberation too great.
		"	"	"	10	9.1	18.0	.87	Reverberation too little.
		"	"	"	8	7.3	16.2	.98	Better.
		"	"	"	5	4.6	13.5	1.16	Condition approved.

If from the larger table the reverberation in each room, in its most approved condition, is separately tabulated, the following is obtained:

Rooms.	Reverberation.
1	.95
2	1.10
3	1.10
4	1.09
5	1.16
	1.08 mean.

The final result obtained, that the reverberation in a music room in order to secure the best effect with a piano should be 1.08, or in round numbers 1.1, is in itself of considerable practical value; but the five determinations, by their mutual agreement, give a numerical measure to the accuracy of musical taste which is of great interest. Thus the maximum departure from the mean is .13 seconds, and the average departure is .05 seconds. Five is rather a small number of observations on which to apply the theory of probabilities, but, assuming that it justifies such reasoning, the probable error is .02 seconds — surprisingly small.

A close inspection of the large table will bring out an interesting fact. The room in which the approved condition differed most from the mean was the first. In this room, and in this room only, was it suggested by the gentlemen present that the experiment should be carried further. This was done by removing two more cushions. The reverberation was then 1.22 seconds, and this was decided to be too much. The point to be observed is that 1.22 is further above the mean, 1.08, than .95 is below. Moreover, if one looks over the list in each room it will be seen that in every case the reverberation corresponding to the chosen condition came nearer to the mean than that of any other condition tried.

It is conceivable that had the rooms been alike in all respects and required the same amount of cushions to accomplish the same results, the experiment in one room might have prejudiced the experiment in the next. But the rooms being different in size and furnished so differently, an impression formed in one room as to the number of cushions necessary could only be misleading if depended on in the next. Thus the several rooms required 6, 5, 15, 10, and 5 cushions. It is further to be observed that in three of the rooms the final condition was reached in working from an overloaded condition, and in the other two rooms from the opposite condition — in the one case by taking cushions out, and in the other by bringing them in.

Before beginning the experiment no explanation was made of its nature, and no discussion was held as to the advantages and disadvantages of reverberation. The gentlemen present were asked to express their approval or disapproval of the room at each stage of the experiment, and the final decision seemed to be reached with perfectly free unanimity.

This surprising accuracy of musical taste is perhaps the explanation of the rarity with which it is entirely satisfied, particularly when the architectural designs are left to chance in this respect.

III. VARIATION IN REVERBERATION WITH VARIATION IN PITCH.

Six years ago there was published in the Engineering Record and the American Architect a series of papers on architectural acoustics intended as a beginning in the general subject. The particular phase of the subject under consideration was reverberation — the continuation of sound in a room after the source has ceased. It was there shown to depend on two things, — the volume of the room, and the absorbing character of the walls and of the material with which the room is filled. It was also mentioned that the reverberation depends in special cases on the shape of the room, but these special cases were not considered. The present paper also will not take up these special cases, but postpone their consideration, although a good deal of material along this line has now been collected. It is the object here to continue the earlier work rather narrowly along the original lines. The subject was then investigated solely with reference to sounds of one pitch, C_4 512 vibrations per second. It is the intention here to extend this over nearly the whole range of the musical scale, from C_1 64 to C_7 4096.

It can be shown readily that the various materials of which the walls of a room are constructed and the materials with which it is filled do not have the same absorbing power for all sounds regardless of pitch. Under such circumstances the previously published work with C_4 512 must be regarded as an illustration, as a part of a much larger problem — the most interesting part, it is true, because near the middle of the scale, but after all only a part. Thus a room may have great reverberation for sounds of low pitch and very little for sounds of high pitch, or exactly the reverse; or a room may have comparatively great reverberation for sounds both of high and of low pitch and very little for sounds near the middle of the scale. In other words, it is not putting it too strongly to say that a room may have very different quality in different registers, as different as does a musical instrument; or, if the room is to be used for speaking purposes, it may have different degrees of excellence or defect for a whisper and for the full rounded tones of the voice, different for a woman's voice and for a man's — facts more or less well recognized. Not to leave this as a vague generalization the following cases may be cited. Recently, in discussing the acoustics of the proposed cathedral of Southern California in Los Angeles with Mr. Maginnis, its architect, and the writer, Bishop Conaty touched on this point very clearly. After discussing the general subject with more than the usual insight and experience, possibly in part because Catholic churches and cathedrals have great reverberation, he added that he found it difficult to avoid pitching his voice to that note which the auditorium most prolongs notwithstanding the fact that he found this the worst pitch on which to speak. This brings out, perhaps more impressively because from practical experience instead of from theoretical considerations, the two truths that auditoriums have very different reverberation for different pitches, and that excessive reverberation is a great hindrance to clearness of enunciation. Another incident may also serve, that of a church near Boston in regard to which the writer has just been consulted. The present pastor, in describing the nature of its acoustical defects, stated that different speakers had different degrees of difficulty in making themselves heard; that he had no difficulty, having a rather high pitched voice; but that the candidate before him, with a louder but much lower voice, failed of the appointment because unable to make himself heard. Practical experience of the difference in reverberation with variation of pitch is not unusual, but the above cases are rather striking examples. Corresponding effects are not infrequently observed in halls devoted to music. Its observation here, however, is marked in the rather complicated general effect. The full discussion of this belongs to another series of papers, in which will be taken up the subject of the acoustical effects or conditions that are desirable for music and for speech. While this phase of the subject will not be discussed here at length, a little consideration of the data to be presented will show how pronounced these effects may be and how important in the general subject of architectural acoustics.

In order to show the full significance of this extension of the investigation in regard to reverberation, it is necessary to point out some features which in earlier papers were not especially emphasized. Primarily the investigation is concerned with the subject of reverberation, that is to say, with the subject of the continuation of a sound in a room after the source has ceased. The immediate effect of reverberation is that each note, if it be music, each syllable or part of a syllable, if it be speech, continues its sound for some time, and by its prolongation overlaps the succeeding notes or syllables, harmoniously or inharmoniously in music, and in speech always towards confusion. In the case of speech it is inconceivable that this prolongation of the sound, this reverberation, should have any other effect than that of confusion and injury to the clearness of the enunciation. In music, on the other hand, reverberation, unless in excess, has a distinct and positive advantage.

Perhaps this will be made more clear, or at least more easily realized and appreciated, if we take a concrete example. Given a room comparatively empty, with hard wall surfaces, for example plaster or tile, and having in it comparatively little furniture, the amount of reverberation for the sounds of about the middle register of the double-bass viol and for the sounds of the middle register of the violin will be very nearly though not exactly equal. If, however, we bring into the room a quantity of elastic felt cushions, sufficient, let us say, to accommodate a normal audience, the effect of these cushions, the audience being supposed absent, will be to diminish very much the reverberation both for the double-bass viol and for the violin, but will diminish them in very unequal amounts. The reverberation will now be twice as great for the double-bass as for the violin. If an audience comes into the room, filling up the seats, the reverberation will be reduced still further and in a still greater disproportion, so that with an audience entirely filling the room the reverberation for the violin will be less than one third that for the double-bass. When one considers that a difference of five per cent in reverberation is a matter for approval or disapproval on the part of musicians of critical taste, the importance of considering these facts is obvious.

This investigation, nominally in regard to reverberation, is in reality laying the foundation for other phases of the problem. It has as one of its necessary and immediate results a determination of the coefficient of absorption of sound of various materials. These coefficients of absorption, when once known, enable one not merely to calculate the prolongation of the sound, but also to calculate the average loudness of sustained tones. Thus it was shown in one of the earlier papers, though at that time no very great stress was laid on it, that the average loudness of a sound in a room is proportional inversely to the absorbing power of the material in the room. Therefore the data which are being presented, covering the whole range of the musical scale, enable one to calculate the loudness of different notes over that range, and make it possible to show what effect the room has on the piano or the orchestra in different parts of the register.

To illustrate this by the example above cited, if the double-bass and the violin produce the same loudness in the open air, in the bare room with hard walls both would be re-enforced about equally. The elastic felt brought into the room would decidedly diminish this re-enforcement for both instruments. It would, however, exert a much more pronounced effect in the way of diminishing the re-enforcement for the violin than for the double-bass. In fact, the balance will be so affected that it will require two violins to produce the same volume of sound as does one double-bass. The audience coming into the room will make it necessary to use three violins to a double-bass to secure the same balance as before.

Both cases cited above are only broadly illustrative. As a matter of fact the effect of the room and the effect of the audience in the room is perceptibly different at the two ends of the register of the violin and of the double-bass viol.

There is still a third effect, which must be considered to appreciate fully the practical significance of the results that are being presented. This is the effect on the quality of a sustained tone. Every musical tone is composed of a great number of partial tones, the predominating one being taken as the fundamental, and its pitch as the pitch of the sound. The other partial tones are regarded as giving quality or color to the fundamental. The musical quality of a tone depends on the relative intensities of the overtones. It has been customary, at least on the part of physicists, to regard the relative intensities of the overtones, which define the quality of the sound, as depending simply on the source from which the sound originates. Of course, primarily, this is true. Nevertheless, while the source defines the relative intensities of the issuing sounds, their actual intensities in the room depend not merely on that, but also, and to a surprising degree, on the room itself. Thus, for example, given an eight-foot organ pipe, if blown in an empty room, such as that described above, the overtones would be pronounced. If exactly the same pipe be blown with the same wind pressure in a room in which the seats have been covered with the elastic felt, the first upper partial will bear to the fundamental a ratio of intensity diminished over 40 per cent, the second upper partial a ratio to the fundamental diminished in the same per cent, the third upper partial a ratio diminished over 50 per cent, while the fourth upper partial will bear a ratio of intensity to the fundamental diminished about 60 per cent. Quality expressed numerically in this way probably does not convey a very vivid impression as to its real effect. It may signify more to say merely that the change in quality is very pronounced and noticeable, even to comparatively untrained ears. On the other hand, if one were to try the experiment with a six-inch instead of with an eight-foot organ pipe, the effect of bringing the elastic felt cushions into the room would be to increase the relative intensities of the overtones, and thus to diminish the purity of the tone.

All tones below that of a six-inch organ pipe will be purified by bringing into the room elastic felt. All tones above and including that pitch will be rendered less pure. The effect of an audience coming into a room is still different. Assuming that the audience has filled the room and so covered all the elastic felt cushions, the effect of the audience is to purify all tones up to violin C_4 512, and to have very little effect on all tones from that pitch upward. On very low tones the effect of the audience in the room is more pronounced. For example, again take C_1 64, the effect of the audience will be to diminish its first overtone about sixty per cent relative to the fundamental and its second overtone over seventy-five per cent.

The effect of the material used in the construction of a room, and the contained furniture, in altering the relative intensities of the fundamental and the overtones, is to improve or injure its quality according to circumstances. It may be, of course, that the tone desired is a very pure one, or it may be that what is wanted is a tone with pronounced upper partials. Take, for example, the "night horn" stop in a pipe organ. This is intended to have a very pure tone. The room in contributing to its purity would improve its quality. On the other hand, the mixture stop in a pipe organ is intended to have very pronounced overtones. In fact to this end not one but several pipes are sounded at once. The effect of the above room to emphasize the fundamental and to wipe out the overtones would be in opposition to the original design of the stop. To determine what balance is desirable must lie of course with the musicians. The only object of the present series of papers is to point out the fundamental facts, and that our conditions may be varied in order to attain any desired end. One great thing needed is that the judgment of the musical authorities should be gathered in an available form; but that is another problem, and the above bare outline is intended only to indicate the importance of extending the work to the whole range of the musical scale, — the work undertaken in the present paper.

The method pursued in these experiments is not very unlike that followed in the previous experiments with C_4 512. It differs in minor detail, but to explain these details would involve a great deal of repetition which the modifications in the method are not of sufficient importance to justify.

Broadly, the procedure consists first in the determination of the rate of emission of the sound of an organ pipe for each note to be investigated. This consists in determining the durations of audibility after the cessation of two sounds, one having four or more, but a known multiple, times the intensity of the other. From these results it is possible to determine the rate of emission by the pipes, each in terms of the minimum audibility for that particular tone. The apparatus used in this part of the experiment is shown in Figure 1. Four small organs were fixed at a minimum distance of five meters apart. It was necessary to place them at this great distance apart because, as already pointed out, if placed near each other the four sounded to-

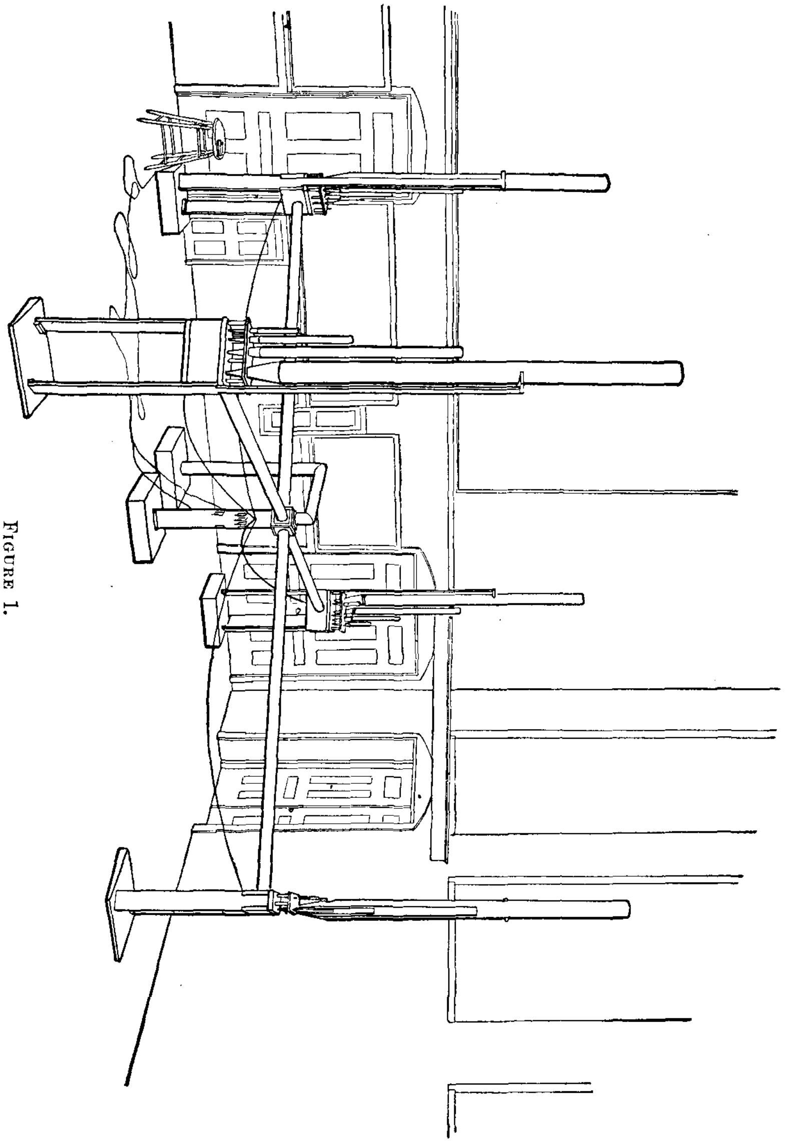

FIGURE 1.

gether do not emit four times the sound emitted by one. This wide separation was particularly necessary for the large pipes and the low tones; a very much less separation would have served the purpose in the case of the high tones.

From the point where the four tubes leading to the small organs meet, a supply pipe ran, as shown on the drawing, to an air reservoir in the room below. This was fed from an electrically driven blower at the far end of the building. The chronograph was in another room. The experiments with this apparatus, like the experiments heretofore recorded, were carried out at night between twelve and five o'clock.

The rate of emission of sound by the several pipes having been determined, the next work was the determination of the coefficients of absorption. The methods employed having already been sufficiently described, only results will be given.

In the very nature of the problem the most important data is the absorption coefficient of an audience, and the determination of this was the first task undertaken. By means of a lecture on one of the recent developments of physics, an audience was enveigled into attending, and at the end of the lecture requested to remain for the experiment. In this attempt the effort was made to determine the coefficients for the five octaves from C_2 128 to C_6 2048, including notes E and G in each octave. For several reasons the experiment was not a success. A threatening thunder storm made the audience a small one, and the sultriness of the atmosphere made open windows necessary, while the attempt to cover so many notes, thirteen in all, prolonged the experiment beyond the endurance of the audience. While this experiment failed, another the following summer was more successful. In the year that had elapsed the necessity of carrying the investigation further than the limits intended became evident, and now the experiment was carried from C_1 64 to C_7 4096, but including only the C notes, seven notes in all. Moreover, bearing in mind the experiences of the previous summer, it was recognized that even seven notes would come dangerously near overtaxing the patience of the audience. Inasmuch as the coefficient of absorption for C_4 512 had already been determined six years before in the investigations mentioned, the coefficient for this note was not redetermined. The experiment was therefore carried out for the lower three and the upper three notes of the seven. The audience, on the night of this experiment, was much larger than that which came the previous summer, the night was a more comfortable one, and it was possible to close the windows during the experiment. The conditions were thus fairly satisfactory. In order to get as much data as possible and in as short a time, there were nine observers stationed at different points in the room. These observers, whose kindness and skill it is a pleasure to acknowledge,

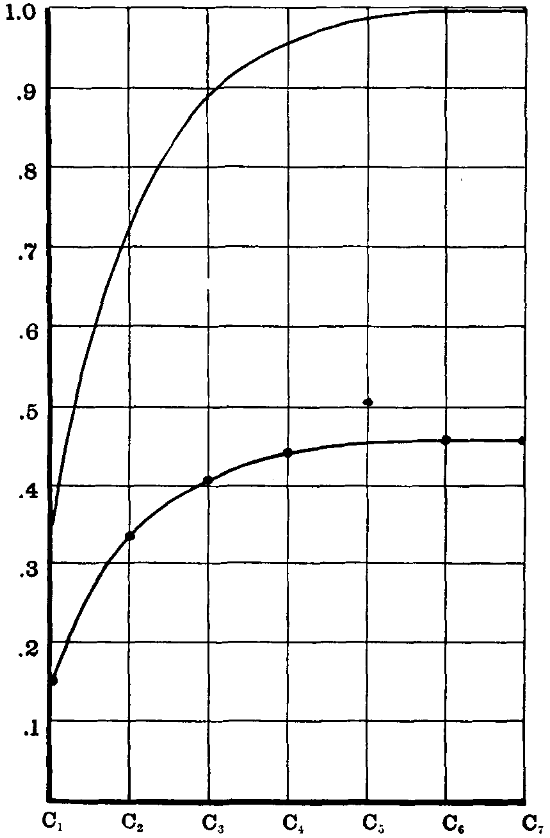

FIGURE 2.

The absorbing power of an audience for different notes. The lower curve represents the absorbing power of an audience per person. The upper curve represents the absorbing power of an audience per square meter as ordinarily seated. The vertical ordinates are expressed in terms of total absorption by a square meter of surface. For the upper curve the ordinates are thus the ordinary coefficients of absorption. The several notes are at octave intervals as follows, C_1 64, C_2 128, C_3 (middle C) 256, C_4 512, C_5 1024, C_6 2048, C_7 4096.

had prepared themselves by previous practice for this one experiment. As in the work of six years ago, the writer's key controlled the organ pipes and started the chronograph, the writer and the other observers each had a key which was connected with the chronograph to record the cessation of audibility of the sound. The results of the experiment are shown on the lower curve in Figure 2. This curve gives the coefficient of absorption per person. It is to be observed that one of the points falls clearly off the smooth curve drawn through the other points. The observations on which this point is based were, however, much disturbed by a street car passing not far from the building, and the departure of this observation from the curve does not indicate a real departure in the coefficient nor should it cast much doubt on the rest of the work, in view of the circumstances under which it was secured. Counteracting the perhaps bad impression which this point may give, it is a considerable satisfaction to note how accurately the point for C_4 512, determined six years before by a different set of observers, falls on the smooth curve through the remaining points. In the audience on which these observations were taken there were 77 women and 105 men. The courtesy of the audience in remaining for the experiment and the really remarkable silence which they maintained is gratefully acknowledged.

The curve above discussed is that for the average person in an audience. An interesting form in which to throw the results is to regard the audience as one side of a room. We may then look at it as an extended absorbing surface, and determine the coefficient per square meter. Worked out on this basis the absorption coefficient is indicated in the higher curve. It is merely the lower curve multiplied by a number which expresses the average number of people per square meter. It is interesting to note that the coefficient of absorption is about the same from C_4 512 up, indicating over that range nearly complete absorption. Below that point there is a very great falling off, down to C_1 64. The curve is such as to permit of an extrapolation indicative of even less absorption and consequently greater reverberation for the still lower notes. Without entering into an elaborate discussion of this curve, two points may be noted as particularly interesting. The first is the nearly complete absorption for the higher notes, a result which at first sight seems a little inconsistent with the results which will be shown later on in connection with the absorption by felt. The inconsistency, however, is only apparent. The greater absorption shown by an audience than that shown by thick felt arises from the fact that the surface of the audience is irregular and does not result in a single reflection, but probably, for a very large portion of the sound, of multiple reflection before it finally emerges. The physical conditions are such that they obviously do not admit of analytic expression, but the explanation of the great absorption by an extended audience surface is not difficult to understand. In addition to the above there is another partial explanation which contributes to the results. The felt forms a perfectly continuous medium, and therefore offers a comparatively rigid reflecting surface. The comparatively light, thin, and porous nature of the clothing of women, perhaps more than of men, contributes to the great absorption of the high notes.

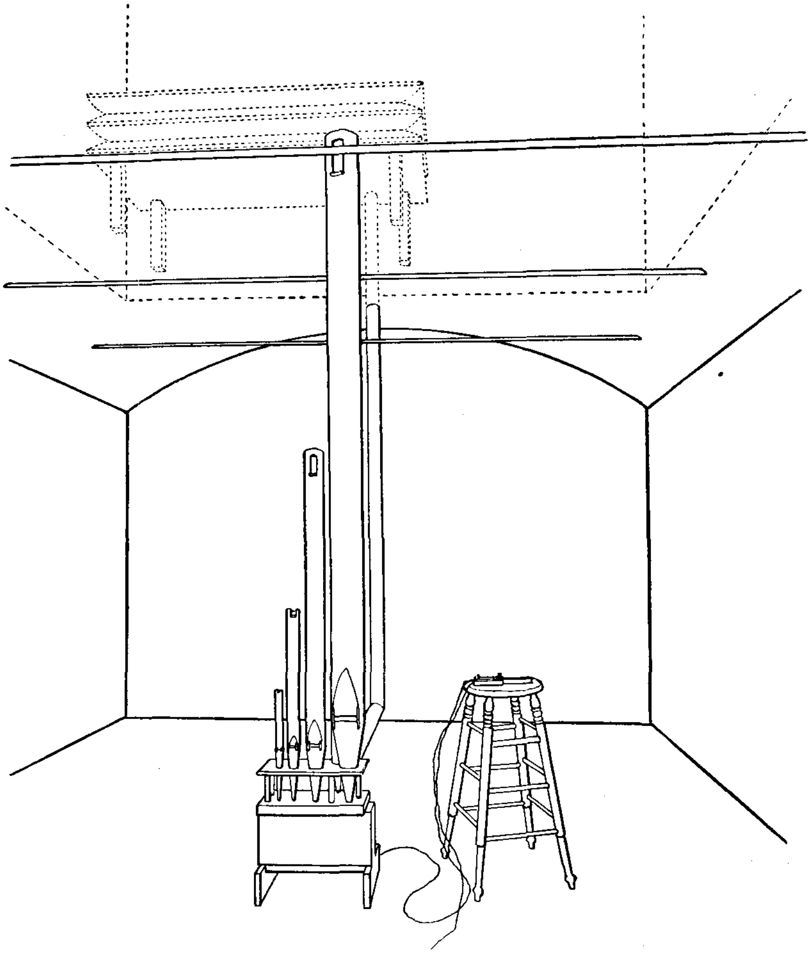

Figure 3.

The next experiment, taking them up chronologically, and perhaps next even from the standpoint of interest, was in regard to a brick wall surface. This experiment was carried out in the constant temperature room mentioned in the previous papers. The arrangement of apparatus is shown in Figure 3, where the air reservoir in the room

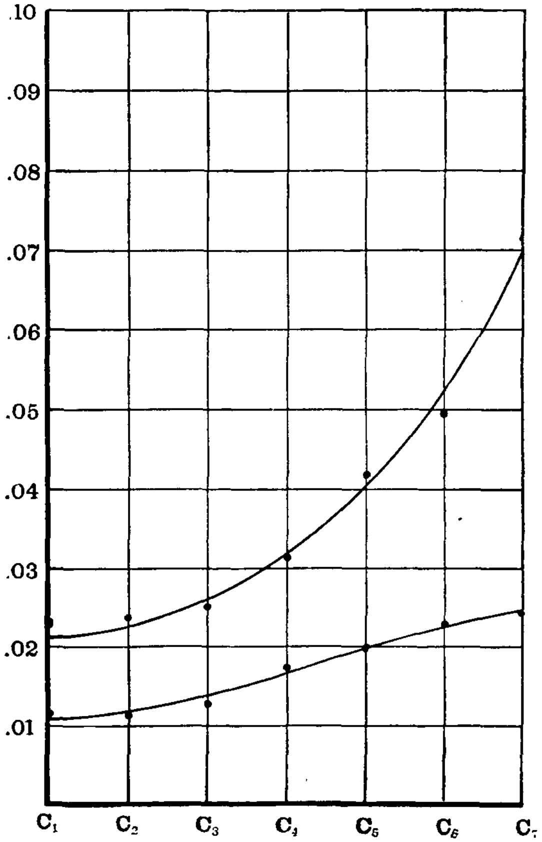

FIGURE 4.

The absorbing power of a 45 cm. thick brick wall. The upper curve represents the absorbing power of an unpainted brick surface. The bricks were hard but not glazed, and were set in cement. The lower curve represents the absorbing power of the same surface painted with two coats of oil paint. The difference between the two curves represents the absorption due to the porosity of the bricks. In small part, but probably only in small part, the difference is due to difference in superficial smoothness. C_3 (middle C) 256.

above is shown in dotted lines. In many respects the constant temperature room offered admirable conditions for the experiment. Its position in the centre of the building and its depth underground made it comparatively free from outside disturbing noises, — so much so that it was possible to experiment in this room in the earlier parts of the evening, although not, of course, when any one else was at work in the building. While it possesses these advantages, its arched ceiling, by placing it in the category of special cases, makes extra precaution necessary. Fortunately, at the beginning of the experiment the walls were unpainted. Under these conditions its coefficient of absorption for different notes was determined. It was then painted with an oil paint, two coats, and its coefficient of absorption redetermined. The two curves are shown in Figure 4. The upper curve is for the unpainted brick; the lower curve is that obtained after the walls were painted. The difference between the two curves would, if plotted alone, be the curve of absorption due to the porosity of the brick. It may seem, perhaps, that the paint in covering the bare brick wall made a smoother surface, and the difference between the two results might be due in part to less surface friction. Of course this is a factor, but that it is an exceedingly small factor will be shown later in the discussion of the results on the absorption of sound by other bodies. The absorption of the sound after the walls are painted is, of course, due to the yielding of the walls under the vibration, to the sound actually transmitted bodily by the walls, and to the absorption in the process of transmission. It is necessary to call attention to the fact that the vertical ordinates are here magnified tenfold over the ordinates shown in the last curve.

The next experiment was on the determination of the absorption of sound by wood sheathing. It is not an easy matter to find conditions suitable for this experiment. The room in which the absorption by wood sheathing was determined in the earlier experiments was not available for these. It was available then only because the building was new and empty. When these more elaborate experiments were under way the room had become occupied, and in a manner that did not admit of its being cleared. Quite a little searching in the neighborhood of Boston failed to discover an entirely suitable room. The best one available adjoined a night lunch room. The night lunch was bought out for a couple of nights, and the experiment was tried. The work of both nights was much disturbed. The traffic past the building did not stop until nearly two o'clock, and began again about four. The interest of those passing by on foot throughout the night, and the necessity of repeated explanations to the police, greatly interfered with the work.

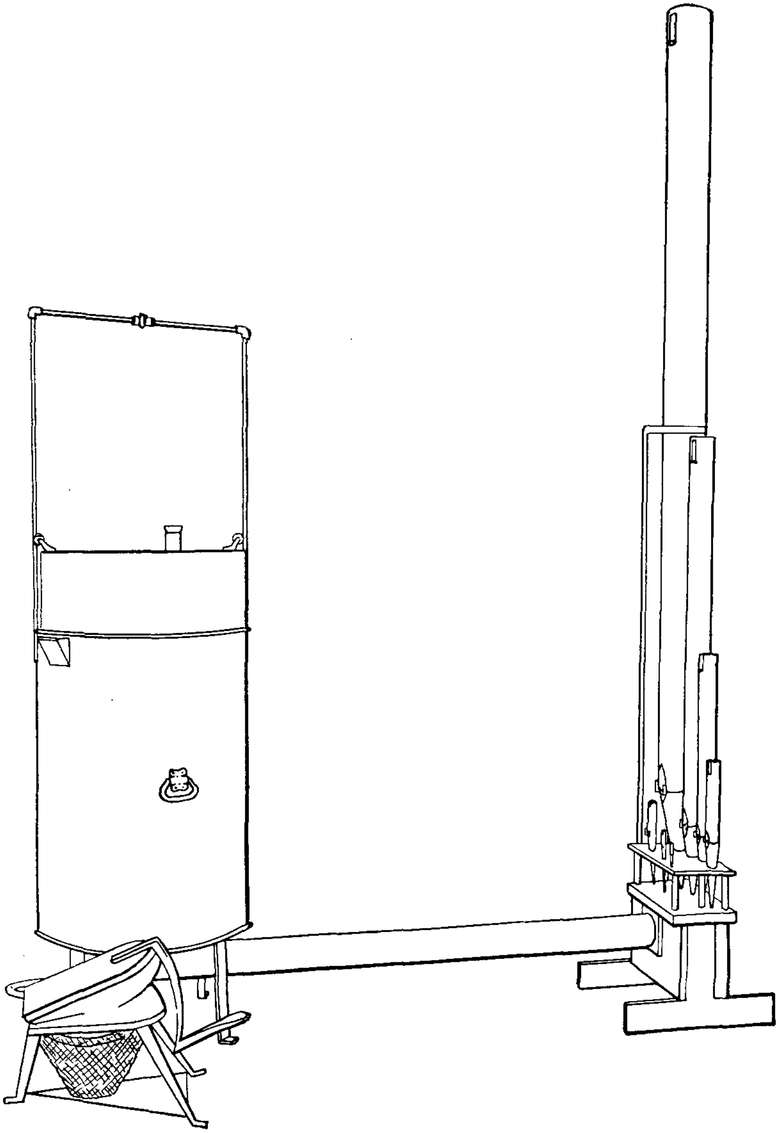

Figure 5.

This detailed statement of the conditions under which the experiment was tried is made by way of explanation of the irregularity of the observations recorded on the curve, and of the failure to carry this particular line of work further. The first night seven points were obtained for the seven notes C_1 64 to C_7 4096. This work was done by means

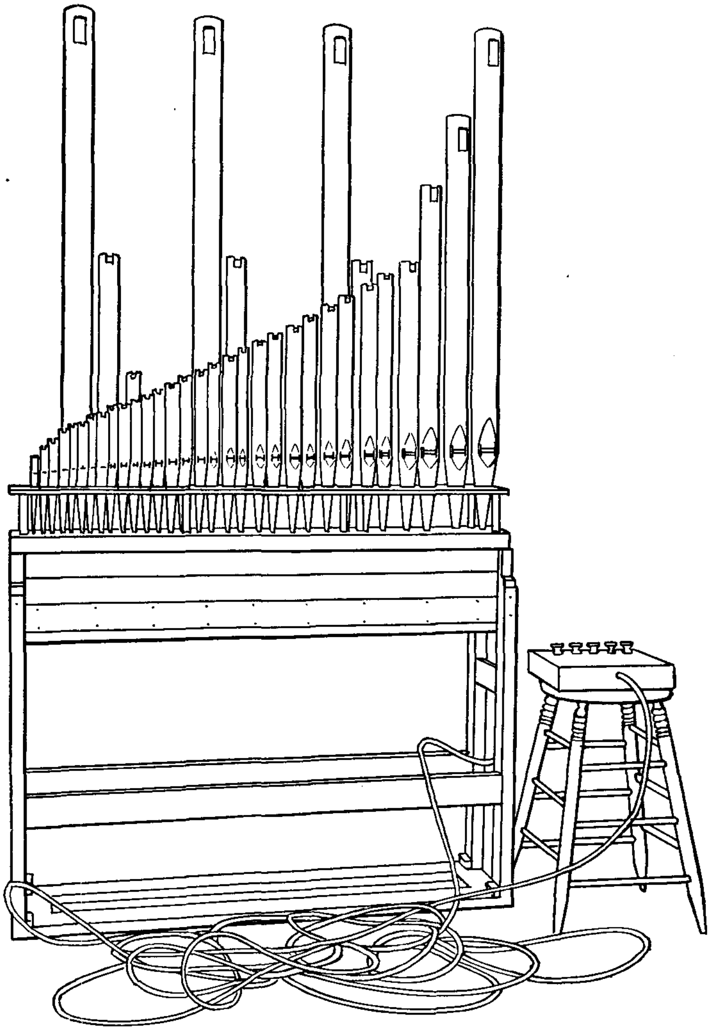

FIGURE 6.

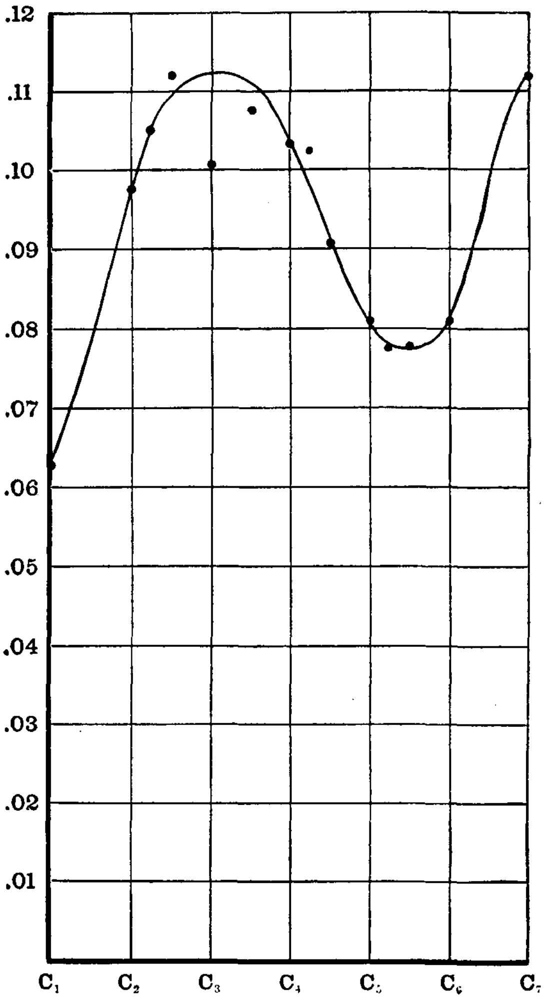

FIGURE 7.

The absorbing power of wood sheathing, two centimeters thick, North Carolina pine. The observations were made under very unsuitable conditions. The absorption is here due almost wholly to yielding of the sheathing as a whole, the surface being shellacked, smooth, and non-porous. The curve shows one point of resonance within the range tested, and the probability of another point of resonance above. It is not possible now to learn as much in regard to the framing and arrangement of the studding in the particular room tested as is desirable. C_3 (middle C) 256.

of a portable apparatus shown in Figure 5. The reduction of these results on the following day showed variations indicative of maxima and minima, which to be accurately located would require the determination of intermediate points. The experiment the following night was by means of the organ shown in Figure 6, and points were determined for the E and G notes in each octave between C_2 128 and C_6 2048. Other points would have been determined, but time did not permit. It is obvious that the intermediate points in the lower and in the higher octave were desirable, but no pipes were to be had on such short notice for this part of the range, and in their absence the data could not be obtained. In the diagram, Figure 7, the points lying on the vertical lines were determined the first night. The points lying between the vertical lines were determined the second night. The accuracy with which these points fall on a smooth curve is perhaps all that could be expected in view of the difficulty under which the observations were conducted and the limited time available. One point in particular falls far off from this curve, the point for C_3 256, by an amount which is, to say the least, serious, and which can be justified only by the conditions under which the work was done. The general trend of the curve seems, however, established beyond reasonable doubt. It is interesting to note that there is one point of maximum absorption, which is due to resonance between the walls and the sound, and that this point of maximum absorption lies in the lower part, though not in the lowest part, of the range of pitch tested. It would have been interesting to determine, had the time and facilities permitted, the shape of the curve beyond C_7 4096, and to see if it rises indefinitely, or shows, as is far more likely, a succession of maxima. The scale employed in this curve is the same as that employed in the diagram of the unpainted and painted wall surfaces. It may perhaps be noted in this connection that at the very least the absorption is four times that of painted brick walls.

The experiment was then directed to the determination of the absorption of sound by cushions, and for this purpose return was made to the constant temperature room. Working in the manner indicated in the earlier papers for substances which could be carried in and out of a room, the curves represented in Figure 8 were obtained. Curve 1 shows the absorption coefficient for the Sanders Theatre cushions, with which the whole investigation was begun ten years ago. These cushions were of a particularly open grade of packing, a sort of wiry grass or vegetable fiber. They were covered with canvas ticking, and that in turn with a very thin cloth covering. Curve 2 is for cushions borrowed from the Phillips Brooks House. They were of a high grade, filled with long curly hair, and covered with canvas ticking, which was in turn covered by a long nap plush. Curve 3 is for the cushions of Appleton Chapel, hair covered with a leatherette, and showing a sharper maximum and a more rapid diminution in absorption for the higher

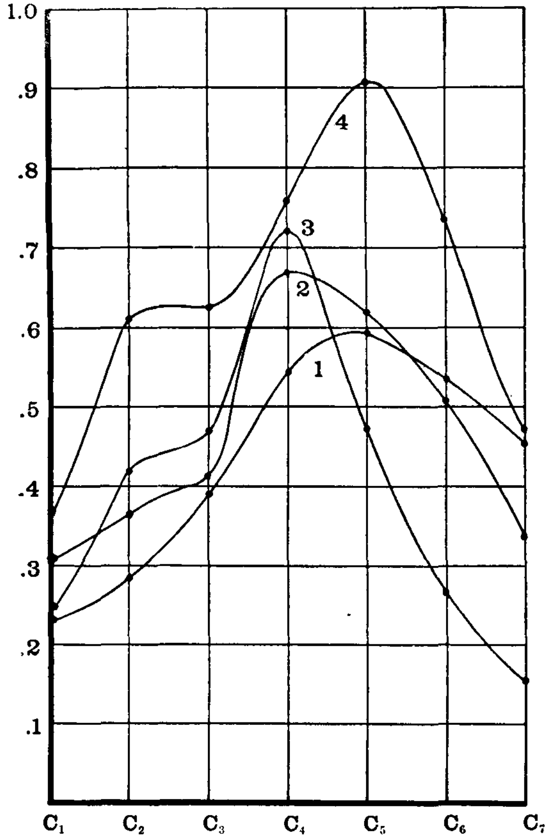

FIGURE 8.

The absorbing power of cushions. Curve 1 is for "Sanders Theatre" cushions of wiry vegetable fibre covered with canvas ticking and a thin cloth. Curve 2 is for "Brooks House" cushions of long hair covered with the same kind of ticking and plush. Curve 3 is for "Appleton Chapel" cushions of hair covered with ticking and a thin leatherette. Curve 4 is for the elastic felt cushions of commerce of elastic cotton covered with ticking and short nap plush. The absorbing power is per square meter of surface. C_3 (middle C) 256.

frequencies, as would be expected under such conditions. Curve 4 is probably the most interesting, because for more standard commercial conditions. It is the curve for elastic felt cushions as made by Sperry and Beale. It is to be observed that all four curves fall off for the higher frequencies, all show a maximum located within an octave, and three of the curves show a curious hump in the second octave. This break in the curve is a genuine phenomenon, as it was tested time after time. It is perhaps due to a secondary resonance, and it is to be observed that it is the more pronounced in those curves that have the sharper resonance in their principal maxima.

Observations were then obtained on unupholstered chairs and settees. The result for chairs is shown in Figure 10. This curve gives the absorption coefficient per single chair. The effect was surprisingly small; in fact, when the floor of the constant temperature room was entirely covered with the chairs spaced at usual seating distances, the effect on the reverberation in the room was exceedingly slight. The fact that it was so slight and the consequent difficulty in measuring the coefficient is a partial explanation of the variation of the results as indicated in the figure. Nevertheless it is probable that the variations there indicated have some real basis, for a repetition of the work showed the points again falling above and below the line as in the first experiment. The amount that these fell above and below the line was difficult to determine, and the number of points along the curve were too few to justify attempting to follow their values by the line. In fact the line is drawn on the diagram merely to indicate in a general way the fact that the coefficient of absorption is nearly the same over the whole range. A varying resonance phenomenon was unquestionably present, but so small as to be negligible; and in fact the whole absorption by the chairs is an exceedingly small factor. The chair was of ash, and its type is shown in the accompanying sketch, Figure 9.

The results of the observations on settees is shown in Figure 11. Those plotted are the coefficients per single seat, there being four seats to the settee. The settees were placed at the customary distance. Here again the principal interest attaches to the fact that the coefficient of absorption is so exceedingly small that the total effect on the reverberation is hardly noticeable. Here also the plotted results do not fall on the line drawn, and the departure is due probably to some slight resonance. The magnitude of the departure, however, could not be determined with accuracy because of the small magnitude of the total absorption coefficient. For these reasons and because the number of points was insufficient, no attempt was made to draw the curve through the plotted points, but merely to indicate a plotted tendency. The

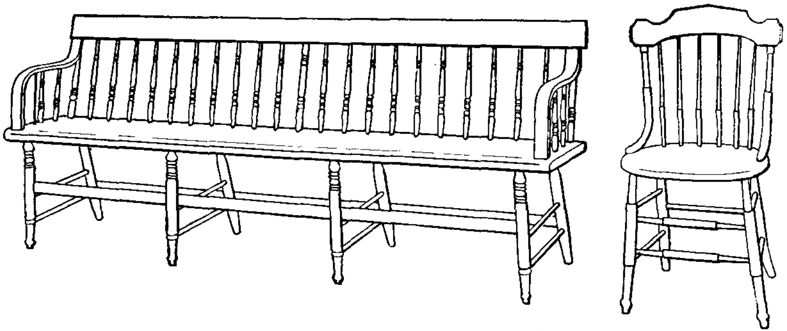

FIGURE 9.

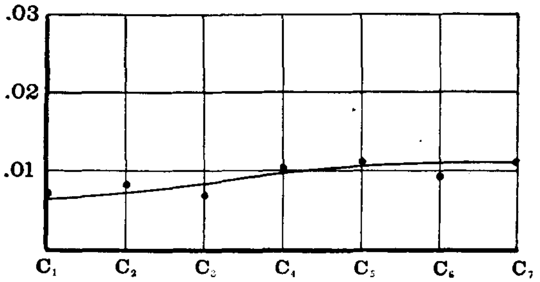

FIGURE 10.

The absorbing power of ash chairs shown in Figure 9.

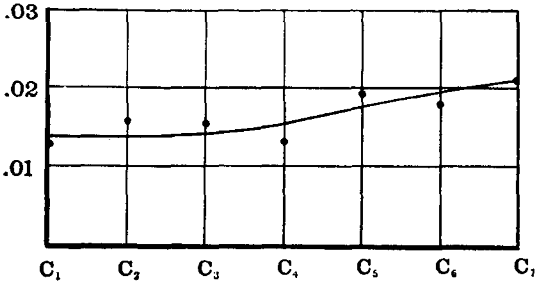

FIGURE 11.

The absorbing power of ash settees shown in Figure 9. The absorption is per single seat, the settee as shown seating five.

settees were of ash, and their general style is shown in the accompanying sketch.

An investigation was then begun in regard to the nature of the process of absorption of sound. The material chosen for this work was a very durable grade of felt, which, as the manufacturers claimed, was all wool. Even a casual examination of its texture makes it difficult to believe that it is all wool. It has, however, the advantage of being porous, flexible, and very durable. Almost constant handling for several years has apparently not greatly changed its consistency. It is to be noted that this felt is not that mentioned in the papers of six years ago. That felt was of lime-treated cow's hair, the kind used in packing steam pipes. It was very much cheaper in price, but stood little handling before disintegrating. The felt employed in these experiments comes in sheets of various thicknesses, the thickness here employed being about 1.1 cm.

The coefficient of absorption of a single layer of felt was measured for the notes from C_1 64 to C_7 4096 at octave intervals. The experiment was repeated for two layers, one on top of the other, then for three, and so on up to six thicknesses of felt. Because the greater thicknesses presented an area on the edge not inconsiderable in comparison with the surface, the felt was surrounded by a narrow wood frame. Under such circumstances it was safe to assume that the absorption was entirely by the upper surface of the felt. The experiment was repeated a great many times, first measuring the coefficient of absorption for one thickness for all frequencies, and then checking the work by conducting experiments in the other order; that is, measuring the absorption by one, two, three, etc., thicknesses, for each frequency. The mean of all observations is shown in Figure 12 and Figure 13. In Figure 12 the variations in pitch are plotted as abscissas, as in previous diagrams, whereas in Figure 13 the thicknesses are taken as abscissas. The special object of the second method will appear later, but a general object of adopting this method of plotting is as follows:

If we consider Figure 12, for example, the drawing of the line through any one set of points should be made not merely to best fit those points, but should be drawn having in mind the fact that it, as a curve, is one of a family of curves, and that it should be drawn not merely as a best curve through its own points, but as best fits the whole set. For example, in Figure 12 the curve for four thicknesses would not have been drawn as there shown if drawn simply with reference to its own points. It would have been drawn directly through the points for C_1 64 and C_2 128. Similarly the curve for five thicknesses would have been drawn a little nearer the point for C_2 128, and above instead of below the point for C_1 64. Considering, however, the whole family of curves and recognizing that each point is not without some error, the

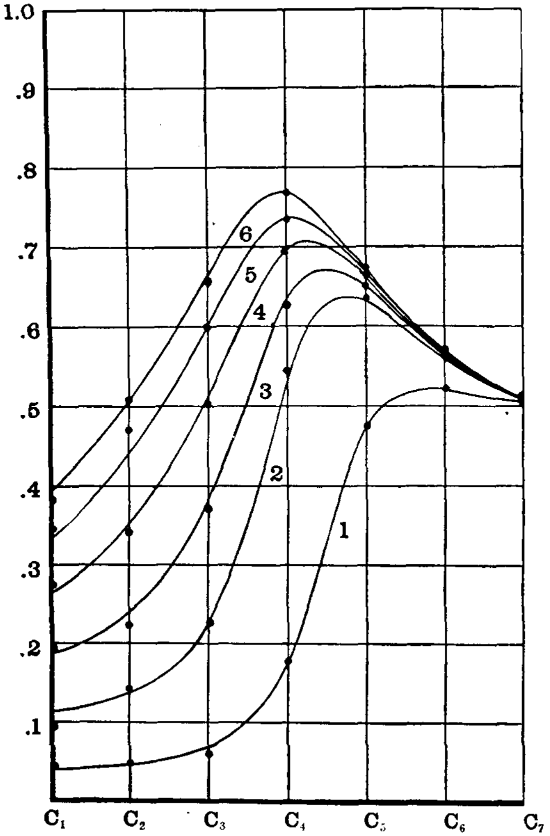

FIGURE 12.

The absorbing power of felt of different thicknesses. Each piece of felt was 1.1 cm. in thickness. Curve 1 is for a single thickness, curve 2 for two thicknesses placed one on top of the other, etc. As shown by these curves, the absorption is in part by penetration into the pores of the felt, in part by a yielding of the mass as a whole. Resonance in the latter process is clearly shown by a maximum shifting to lower and lower pitch with increase in thickness of the felt. C_3 (middle C) 256.

curves as drawn are more nearly correct. The best method of reconciling the several curves to each other is to plot two diagrams, one in which the variations in pitch are taken as abscissa and one in which the variations in thickness of felt are taken as abscissas; then draw through the points the best fitting curves and average the corresponding ordinates taken from the curves thus drawn; and with these average ordinates redraw both families of curves. The points shown on the diagram are of course the original results obtained experimentally. In general they fall pretty close to the curves, although at times, as in the points noted, they fall rather far to one side.

The following will serve to present the points of particular interest revealed by the family of curves in Figure 12, where the absorption by the several thicknesses is plotted against pitch for abscissas. It is to be observed that a single thickness scarcely absorbs the sound from the eight, four, and two foot organ pipes, C_1 64, C_2 128, and C_3 256, and that its absorption increases rapidly for the next two octaves, after which it remains a constant. Two thicknesses absorb more — about twice as much — for the lower notes, the curve rising more rapidly, passing through a maximum between C_4 512 and C_5 1024, and then falling off for the higher notes. The same is true for greater thicknesses. All curves show a maximum, each succeeding one corresponding to a little lower note. The maximum for six thicknesses coincides pretty closely to C_4 512. The absorption of the sound by felt may be ascribed to three causes, — porosity of structure, compression of the felt as a whole, and friction on the surface. The presence of the maximum must be ascribed to the second of these causes, the compression of the felt as a whole. As to the third of these three causes, it is best to consult the curves of the next figure.

The following facts are rendered particularly evident by the curves of Figure 13. For the tones emitted by the eight-foot organ pipe, C_1 64, the absorption of the sound is very nearly proportional to the thickness of the felt over the range tested, six thicknesses, 6.6 cm. The curves for notes of increasing pitch show increasing value for the coefficients of absorption. They all show that were the thickness of the felt sufficiently great, a limit would be approached, — a fact, of course, self-evident, — but for C_5 1024 this thickness was reached within the range experimented on; and of course the same is true for all higher notes, C_6 2048 and C_7 4096. The higher the note, the less the thickness of felt necessary to produce a maximum effect. The curves of C_1 64, C_2 128, C_3 256, and C_4 512, if extended backward, would pass nearly through the origin. This indicates that for at least notes of so low a pitch the absorption of sound would be zero, or nearly zero, for zero thickness. Since zero thickness would leave surface effects, the argument leads to the conclusion that surface friction as an agent in the absorption of sound is of small importance. The curves plotted

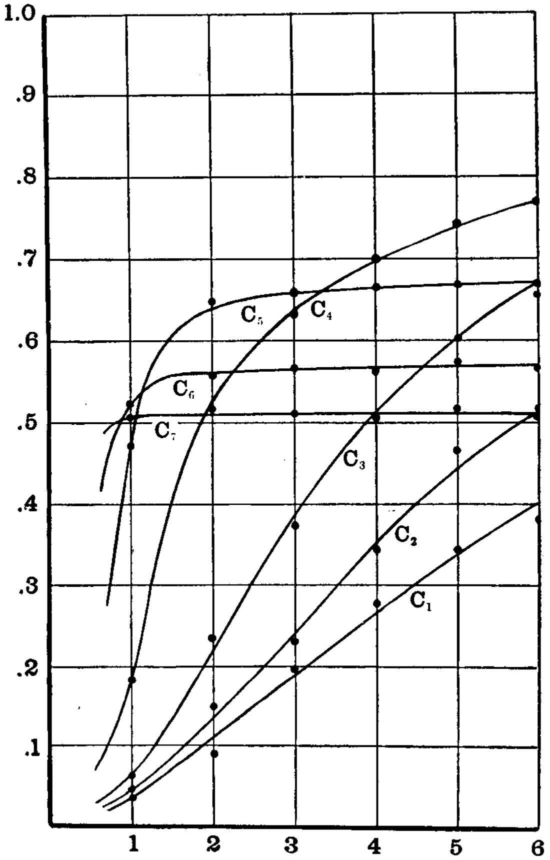

FIGURE 13.

The absorbing power of felt of different thicknesses. The data, Figure 12, is here plotted in a slightly different manner, — horizontally on plotted increasing thickness, — and the curves are for notes of different frequency at octave intervals in pitch. Thus plotted the curves show the necessary thickness of felt for practically maximum efficiency in absorbing sound of different pitch. These curves also show that for the lowest three notes surface friction is negligible, at least in comparison with the other factors. For the high notes one thickness of felt was too great for the curves to be conclusive in regard to this point. C_3 (middle C) 256.

do not give any evidence in this respect in regard to the higher notes, C_5 1024, C_6 2048, and C_7 4096.

It is of course evident that the above data do not by any means cover all the ground that should be covered. It is highly desirable that data should be accessible for glass surfaces, for glazed tile surfaces, for plastered and unplastered porous tile, for plaster on wood lath and plaster on wire lath, for rugs and carpets; but even with these data collected the job would be by no means completed. What is wanted is not merely the measurement of existing material and wall surfaces, but an investigation of all the possibilities. A concrete case will perhaps illustrate this. If the wall surface is to be of wood, there enter the questions as to what would be the effect of varying the material, — how ash differs from oak, and oak from walnut or pine or whitewood; what is the effect of variations in thickness; what the effect of panelling; what is the effect of the spacing of the furring on which the wood sheathing is fastened. If the wall is to be plaster on lath, there arises the question as to the difference between wood lath and wire lath, between the mortar that was formerly used and the wall of to-day, which is made of hard and impervious plaster. What is the effect of variations in thickness of the plaster? What is the effect of painting the plaster in oil or in water colors? What is the effect of the depth of the air space behind the plaster? The recent efforts at fireproof construction have resulted in the use of harder and harder wall surfaces, and great reverberation in the room, and in many cases in poorer acoustics. Is it possible to devise a material which shall satisfy the conditions as to fireproof qualities and yet retain the excellence of some of the older but not fireproof rooms? Or, if one turns to the interior furnishings, what type of chair is best, what form of cushions, or what form of upholstery? There are many forms of auditorium chairs and settees, and all these should be investigated if one proposes to apply exact calculation to the problem. These are some of the questions that have arisen. A little data have been obtained looking toward the answer to some of them. The difficulty in the way of the prosecution of such work is greater, however, than appears at first sight, the particular difficulties being of opportunity and of expense. It is difficult, for example, to find rooms whose walls are in large measure of glass, especially when one bears in mind that the room must be empty, that its other wall surfaces must be of a substance fully investigated, and that it must be in a location admitting of quiet work. Or, to investigate the effect of the different kinds of plaster and of the different methods of plastering, it is necessary to have a room, preferably an underground room, which can be lined and relined. The constant temperature room which is now available for the experiments is not a room suitable to that particular investigation, and for best results a special room should be constructed. Moreover, the expense of plastering and replastering a room — and this process, to arrive at anything like a general solution of the problem, would have to be done a great many times — would be very great, and is at the present moment prohibitive. A little data along some of these lines have been secured, but not at all in final form. The work in the past has been largely of an analytical nature. Could the investigation take the form of constructive research, and lead to new methods and greater possibilities, it would be taking its more interesting form.

The above discussion has been solely with reference to the determination of the coefficient of absorption of sound. It is now proposed to discuss the question of the application of these coefficients to the calculation of reverberation. In the first series of papers, reverberation was defined with reference to C_4 512 as the continuation of the sound in a room after the source had ceased, the initial intensity of the sound being one million times minimum audible intensity. It is debatable whether or not this definition should be extended without alteration to reverberation for other notes than C_4 512. There is a good deal to be said both for and against its retention. The whole, however, hinges on the outcome of a physiological or psychological inquiry not yet in such shape as to lead to a final decision. The question is therefore held in abeyance, and for the time the definition is retained.

Retaining the definition, the reverberation for any pitch can be calculated by the formula

$$T = \frac{K\,V}{a},$$

where V is the volume of the room, K is a constant depending on the initial intensity, and a is the total absorbing power of the walls and the contained material. K and V are the same for all pitch frequencies. K is .164 for an initial intensity 10^6 times minimum audible intensity. The only factor that varies with the pitch is a, which can be determined from the data given above.

In illustration, the curves in the accompanying Figure 14 give the reverberation in the large lecture room of the Jefferson Physical Laboratory. The upper curve defines the reverberation in the room when entirely empty; the lower curve defines this reverberation in the same room with an audience two thirds filling the room. The upper curve represents a condition which would be entirely impractical for speaking purposes; the lower curve represents a fairly satisfactory condition.

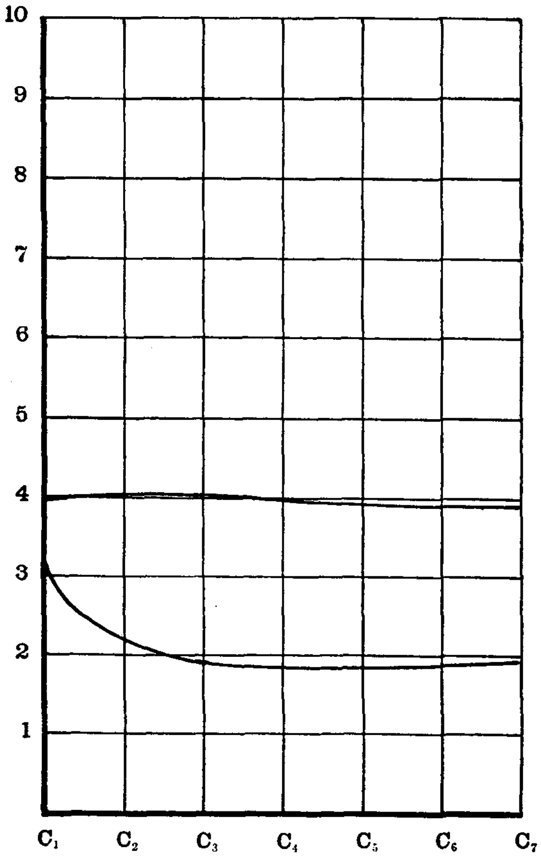

FIGURE 14.

Curves expressing the reverberation in the large lecture room of the Jefferson Physical Laboratory with (lower curve) and without (upper curve) an audience. These curves express in seconds the duration of the residual sound in the room after the cessation of sources producing intensities 10^6 times minimum audible intensity for each note. The upper curve describes acoustical conditions which are very unsatisfactory, as the hall is to be used for speaking purposes. The lower curve describes acoustically satisfactory conditions. C_3 (middle C) 256.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

ON THE PERMEABILITY AND THE RETENTIVENESS OF A MASS OF FINE IRON PARTICLES.

By B. Osgood Peirce.

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ON THE PERMEABILITY AND THE RETENTIVENESS OF A MASS OF FINE IRON PARTICLES.

BY B. OSGOOD PEIRCE.

Presented February 14, 1906. Received March 13, 1906.

IN a familiar lecture-room experiment, a mass of iron filings, filling a straight glass tube to a length of thirty or forty times its diameter, is forced to become a rather weak "bar magnet" by subjecting it to a strong exciting field in a solenoid; and then, by rearranging the particles, it is made to lose its magnetic moment almost completely, although, if, after the filings have been poured out of the tube, a few of them be examined under a microscope of moderate power, it is usually easy to see that most of the elongated particles retain some magnetism for a good while. A number of persons[1] have studied the magnetic properties of masses of iron filings or of chemically deposited "iron dust," as well as of mixtures of iron particles in various proportions with non-magnetic powders of various kinds. Concise statements of the results of experiments on the subject are to be found in the papers of Maurain and Trenkle.

I have lately had occasion to measure the permeability and the retentiveness of each of several masses of very fine cast-iron particles, or dust, made by a fine cutting end-mill in a milling-machine, and, as testing the effect upon their magnetic properties of subjecting the particles to a "hardening" process, the results seem to have some slight interest.

The material to be examined was tamped solidly into a glass tube almost exactly one centimeter in diameter, until a column was formed fifty diameters long. After its ends had been closed by corks, the comparatively short tube was put into the middle of a long solenoid S, the axis of which was horizontal and perpendicular to the meridian. The connections of the apparatus are shown schematically in Figure 1.

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[1] Töpler, Pogg. Ann. d. Phys., 1877; v. Waltenhofen, Wied. Ann. d. Phys., 1870; Jamin, Comptes rendus, 1875; Börnstein, Pogg. Ann. d. Phys., 1875; Auerbach, Wied. Ann. d. Phys., 1880; Baur, Wied. Ann. d. Phys., 1880; Maurain, Éclairage, Élect., 1903; Trenkle, Erlangen. Sitzungsberichte, 1905; Wied. Ann. d. Phys., 1906.

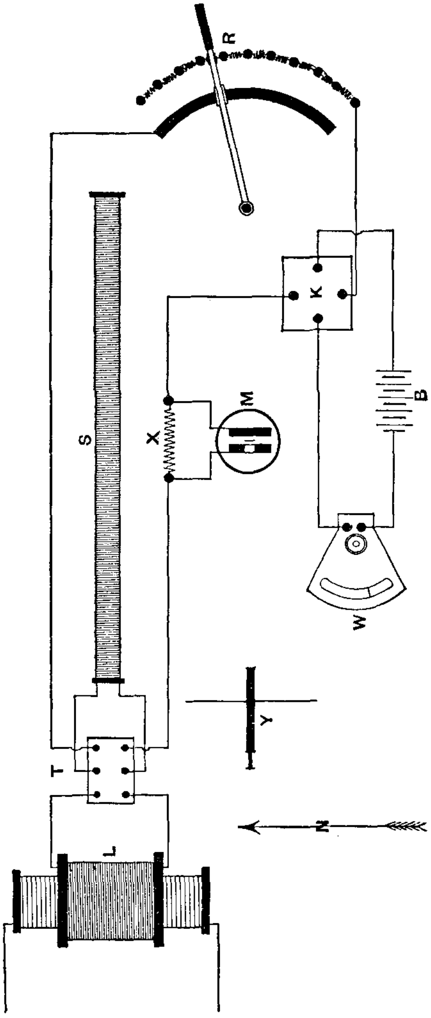

FIGURE 1.

After the tube (Q) had been placed in S, the double switch T was closed to the left, so as to connect S with the secondary (L) of a transformer the primary of which was attached to the alternating street circuit. The secondary coil was so suspended with counterbalancing weights in a tall frame that it could be moved at will in the direction of its axis to a distance of several feet away from the primary, and thus a great number of alternations of a current gradually decreasing in intensity could be sent through S to demagnetize the specimen in it.

M represents a magnetometer in the form of a mirror galvanometer placed in Gauss's B Position with respect to Q; the galvanometer was so shunted by an adjustable resistance X that the effect on the galvanometer needle of the partial current through the coils of the instrument almost exactly compensated for the effect of the whole current through the solenoid S when empty. R is an adjustable rheostat of 200 ohms total resistance, designed to carry currents of some intensity, K is a commutator, and W a milliamperemeter furnished with a set of four shunts. When the switch T was closed to the right, it was possible, by manipulating the rheostat arm and the commutator K', to put Q through any desired hysteresis cycle in the usual manner. The current came from a battery of storage cells, any number of which could be used at pleasure. A current of 1 ampere in the solenoid gave rise to a field of 54.8 gausses in the space within it. The field about M's needle had to be artificially strengthened to suit the circumstances, and a piece of soft Bessemer steel rod of almost exactly the same dimensions as the column of filings was used to determine the sensitiveness of the apparatus at any time. By means of a coil of 20 turns of extremely fine insulated copper wire wound directly on this piece at its centre and connected with a carefully standardized ballistic galvanometer, the induction flux through the centre of the rod could be found and the corresponding deflection of the magnetometer needle determined.

The work was undertaken in order to compare the magnetic properties of masses of iron particles, as they came from the milling-machine, with those of masses of particles from the same lot which had undergone the treatment used in hardening iron castings for permanent magnets. These "filings" were prepared by Mr. G. W. Thompson, the mechanician of the Jefferson Laboratory, who has had much experience in the process. A completely closed iron crucible with thin walls, containing a mass of the particles to be treated, was heated white hot under a power blast in a gas furnace, and then suddenly chilled in an acid bath. After this experience, during which the particles had been carefully excluded from the air, they had a somewhat altered color and lustre, but under a microscope of low power showed very little difference from the untreated particles; at best all such particles cut by machine tools from iron castings are most irregular in form, and are so much seamed by deep furrows and pits as to look like clinkers from a furnace. All the particles were kept quite free from oil or dirt, and the surfaces of the "hardened" ones were only very slightly tarnished, but it was not possible to pack quite so large a mass of the material into a given space after the treatment as before. This might have arisen from changes of shape, but it is a suspicious fact that the induction flux through the centre of a column (of given dimensions) of the filings under a given excitation was almost exactly the same whether the filings had been hardened or not. Two uniform glass tubes from the same piece and practically of the same dimensions were filled respectively with 103 grams of untreated particles and 96 grams of the others, and each was put several times through a hysteresis cycle, using about 250 gausses as the intensity of the maximum exciting field. After this 22 simultaneous determinations of H and B were made in each half-cycle, and both cycles were carefully plotted on such a scale that each was about 40 centimeters long. It appeared that the maximum values of the induction were almost identical, and that at no point were the plotted curves so much as 1 millimeter apart. In filling the same glass tube a number of times in succession from the same lot of filings, it was of course impossible to pack exactly the same mass into the same space twice; but the hysteresis diagrams, many of which were obtained by Mr. J. Coulson and myself, were always of the same shape, and the intensity of magnetization due to a given exciting field seemed to be strictly proportional to the mass. The density of the untreated filings was only about four tenths of that of massive cast iron.

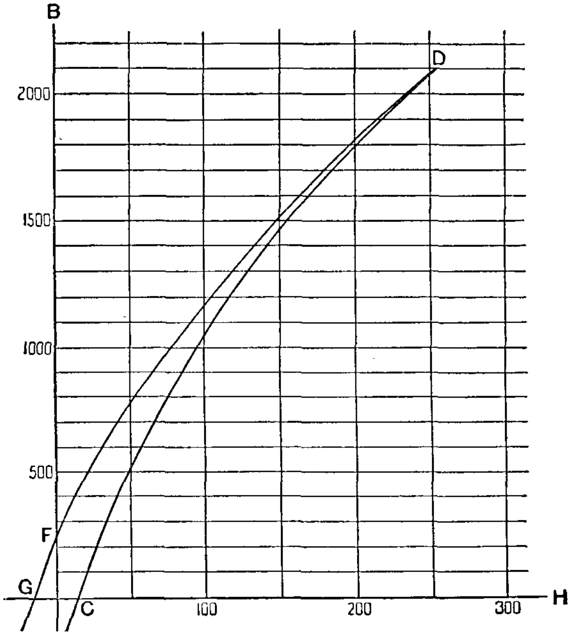

FIGURE 2.

The demagnetizing effect of the ends in a rod of solid iron only fifty diameters long would of course be very serious, but in the present case, due to the relatively small value of I for a given value of H, it is far less important. In order to determine from my observations the permeability at the centre of an endless column of material like mine, I have used the value of N computed by Du Bois[2] from the experimental results of Ewing, Tanakadaté, and himself.

Figure 2 shows half of a hysteresis diagram obtained by shearing very slightly a diagram obtained by Mr. J. Coulson and me from a column of untreated particles 50 centimeters long and 1 centimeter in diameter, which weighed just over 100 grams. This diagram has all the characteristics of the many others for which I have the materials.

As the figure shows, B was almost exactly 2100 for an exciting field of 255 gausses, and this corresponds to a value for I of only about 147. When the external field was removed, the remanent value of I was about 20.8; the coercive force, however, was comparatively large, being about 16. It is evident that the exciting field would need to be very strong to magnetize the column of particles approximately to saturation. The subject of the saturization of masses of iron filings has been discussed at length by Maurain and by Trenkle.

THE JEFFERSON LABORATORY,
HARVARD COLLEGE.

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[2] Ewing, The Philosophical Transactions; Tanakadaté, The Philosophical Magazine, 1888; Du Bois, Annalen der Physik., 1892; "The Magnetic Circuit," chapter vi. For a criticism on this process, see Trenkle, l. c.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD COLLEGE.

ON THE LENGTH OF THE TIME OF CONTACT IN THE CASE OF A QUICK TAP ON A TELEGRAPH KEY.

By B. Osgood Peirce.

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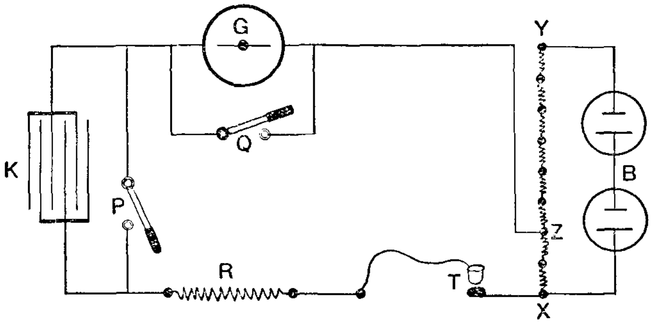
Presented February 14, 1906. Received March 13, 1906.

In the use of a sensitive mirror, needle, ballistic galvanometer, in cases where a moving-coil galvanometer is objectionable, and the familiar sliding-coil-on-a-long-magnet arresting apparatus unsuitable, it is sometimes desirable to control the needle by currents of short duration, sent through either a coil of the galvanometer or an auxiliary coil, in one direction or the other, by aid of a pair of reversing keys. The current through the controlling circuit during a comparatively long depression of the key must be strong enough to check quickly a large swing of the needle, and yet the operator must be able to bring the needle practically to rest when its whole range, as measured by the indication on its scale, is only a small fraction of a millimeter. To do this quickly and surely requires some skill in making short key contacts, as every person who has tried it knows, and different observers find it well to use different strengths of current in the controlling circuit. In order to be able to plan wisely some apparatus, I have lately found it desirable to learn about how short a contact the average observer can make, either with a telegraph key or with a thimble on the finger, and, through the kindness of a number of my colleagues and friends, I have been able to get the results given below.

The measurements were made according to a well-known principle, by determining either the fractional part of the whole charge of a given loaded condenser lost during the key contact by short circuit through a given resistance, or the fractional part of a full charge gained by an originally empty condenser, through a given resistance, when a fixed electromotive force was applied during the time of closure of the key. It is well known that with proper choice of the apparatus this procedure is susceptible of great accuracy, and that it furnishes satisfactory means of determining the velocity of a rifle ball, or the rate of propagation of an elastic wave in a bar of metal, or the time of contact of a hammer and an anvil. Some time ago I had to measure with some care an interval of time of about one five thousandth of a second, and to test the apparatus, used a good number of different combinations of condensers and resistances in the circuit, with the result that the determinations agreed within the very small uncertainty caused by the fact that it was difficult to be sure that the suspended system of the ballistic galvanometer used was originally exactly at rest.

The arrangement of the apparatus for measuring the time of closure of the key is shown schematically in the subjoined diagram. *K* represents a divided mica condenser of about 28.8 microfarads capacity; *G*, a ballistic mirror galvanometer; *R*, a large known resistance; *T* is the point of contact of the key with its base, or of the thimble with its block; *P* and *Q* are keys mounted on a large block of parrafine for discharging the condenser and short-circuiting the galvanometer after every throw. In order to make the potential difference across the open key at all times small, the ends of the condenser circuit were attached to two points (*X*, *Z*) in the closed high resistance current of one or two cells, so chosen as to make the galvanometer throw, corresponding to a complete loading of the condenser, a convenient number of centimeters; the final adjustment of this throw to a predetermined value was made by moving the scale (along a track made for the purpose) away from or nearer to the galvanometer mirror.



It is evident that if k is the capacity in farads of the condenser, r the whole resistance in ohms of the condenser branch circuit, and $(u + v)$, of which v is the resistance of the plain conductor between X and Z, in the resistance of the battery circuit, then, when P and Q

are open, the charge in coulombs in the originally empty condenser, t seconds after the key, T, is closed, is

$$Q_t = \frac{e\,k\,v}{u+v}\left(1 - e^{\frac{-(u+v)t}{k(uv+ur+vr)}}\right),$$

$$\text{and} \quad Q_\infty = \frac{e\,v\,k}{u+v},$$

$$\text{so that} \quad t = \frac{k\,(u\,v + u\,r + v\,r)}{u+v} \cdot \log_e\left(\frac{Q_\infty}{Q_\infty - Q_t}\right).$$

The values of the natural logarithms which enter into this equation for several values of the ratio of Q_t to Q_∞ are as follows:

Q_t/Q_∞.	$\log_e[(Q_\infty - Q_t)/Q_\infty]$.	Q_t/Q_∞.	$\log_e[(Q_\infty - Q_t)/Q_\infty]$.
0.05	0.05130	0.40	0.51083
0.10	0.10537	0.45	0.59784
0.15	0.16252	0.50	0.69315
0.20	0.22315	0.55	0.79851
0.25	0.28769	0.60	0.91629
0.30	0.35668	0.65	1.04983
0.35	0.43079	0.70	1.20398

For the long scale distances actually used, the throw of the galvanometer caused by a discharge of electricity through its coil was, as measured on the scale of the instrument, sensibly proportional, up to a reading of say 25 centimeters, to the quantity of the discharge. The galvanometer and the resistances u and v were usually so adjusted that a complete load (Q_∞) of the condenser, when sent through the coil, caused a throw of exactly 20 centimeters. After k, u, v, and r had been fixed, the coefficient of the logarithm remained constant; if, for example, its mean was 0.05, a throw of 50 millimeters would be caused by closing the key for about 0.0144 seconds.

In order to make sure that the apparatus had been properly set up, I made use of a body falling from rest through a measured space to close the gap T for a short interval of calculable length. A heavy weight sliding on a stout vertical brass rod carried with it an insulated frame which held securely a printer's lead, which formed one terminal of the gap T. Projecting about a millimeter from a hole in the vertical rod, near the lower end, was a piece of sharpened drill rod which formed the other terminal of T, and ploughed a clean furrow in the lead as the weight, in falling, carried the lead by. In this way sure contact across the gap could be made for a time interval, which could be varied between limits, the length of which could be computed with some accuracy. The agreement between the results obtained by this device and those calculated from the electrical constants was always so close as to lie within the uncertainty of the zero of the galvanometer.

Using the apparatus just described, a large number of trials were made by about twenty different persons to determine how short a contact each could surely make with a telegraph key or with a thimble on a plate. If a telegraph key is furnished with a very stiff spring it is possible for any one, after a little practice, to make short contacts, not more than a thousandth of a second long, by *striking* the key with the hand; the observations recorded below were all made with a key which had no spring, and which had to be lifted, after the contact, by hand. It generally appeared that in the course of a long series of trials a person made two or three contacts of abnormal length (sometimes five or ten times as long as the mean of the others), but it did not seem well to leave them out of account; I have, therefore, given

TABLE I.

Observer.	Average time of contact of key as obtained from all the trials.	Average time of contact of key as obtained from the shorter half of the observations.	Observer.	Average time of contact of key as obtained from all the trials.	Average time of contact of key as obtained from the shorter half of the observations.
B	0.0329	0.0223	L	0.0203	0.0130
C	0.0278	0.0185	M	0.0303	0.0257
D	0.0368	0.0292	N	0.0166	0.0095
E	0.0223	0.0141	O	0.0175	0.0112
F	0.0383	0.0305	P	0.0107	0.0050
G	0.0606	0.0525	Q	0.0277	0.0112
H	0.0239	0.0166	R	0.0416	0.0362
I	0.0871	0.0788	S	0.0318	0.0284
J	0.0210	0.0147	T	0.0350	0.0280
K	0.0239	0.0178			

the average length of all the contacts, and also the average length of the half of the whole number which were shortest. I am indebted to Mr. John Coulson for help in making the observations.

The recorded averages are given in ten thousandths of a second, but of course the average of two long sets of trials made by the same person would not agree exactly. There is, however, in most cases, a striking agreement between observations made at different times by one observer, and except in the cases of two persons whose records at the beginning were not very good, practice did not shorten the time appreciably. In my own case the shortest average that I have been able to get differs from my longest by about one sixth part of the latter. The records of thimble contacts made by two persons were 0.0036 and 0.0083 seconds in November, and 0.0037 and 0.0083 seconds in January.

TABLE II.

Observer.	Hand.	Average time of contact of thimble and plate as obtained from all the trials.	Average time of contact of thimble and plate as obtained from the shorter half of the observations.
A	Right	0.0077	0.0037
B	Right	0.0060	0.0028
C	Right	0.0083	0.0046
C	Left	0.0061	0.0037
D	Right	0.0126	0.0066
E	Right	0.0077	0.0035
E	Left	0.0062	0.0038
F	Right	0.0034	0.0027
G	Right	0.0217	0.0077
H	Right	0.0053	0.0032
I	Right	0.0303	0.0083
J	Right	0.0110	0.0045
J	Left	0.0136	0.0086
K	Right	0.0170	0.0062
L	Right	0.0139	0.0078
M	Right	0.0246	0.0035
N	Right	0.0082	0.0036
O	Right	0.0032	0.0022
P	Right	0.0037	0.0032
P	Left	0.0110	0.0028
T	Right	0.0064	0.0051

Table I shows results obtained with the telegraph key, and, though the range is large, it appears that, when the key must be lifted away from its seat after the contact by the observer, *the average person can surely make a contact as short as a thirtieth of a second*, and some can always do better than this. Table II shows records of fair contacts obtained with a thimble and block. While *the lengths of the shorter contacts of the average person seem to be rather less* — when made in this manner — *than one two hundredth of a second*, three of the persons (F, O, P) who made trial of the apparatus could surely make contacts the average lengths of which were only about one three hundredth of a second.

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE. E. L. MARK, DIRECTOR. — No. 179.

SOME STAGES IN THE SPERMATOGENESIS OF THE HONEY BEE.

By E. L. Mark and Manton Copeland.

With a Plate.

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SOME STAGES IN THE SPERMATOGENESIS OF THE HONEY BEE.

By E. L. Mark and Manton Copeland.

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In 1903 F. Meves published a short account of spermatogenesis in the honey bee in which it was held that the process was remarkably unlike that of other animals, and simulated in an interesting way the maturation of the female sexual products.

As is well known, an important parallelism between the formation of polar cells in the maturing egg and the last two cell divisions leading to the formation of spermatozoa was established for Ascaris by O. Hertwig, and has since been shown to be a general condition of maturation throughout the animal kingdom. But while in the sexual cells of the female maturation results in the formation of one functional and three (or two) non-functional elements, in the male the usual outcome is four elements, the spermatozoa, all of which are functional.

In the honey bee Meves showed that the maturation divisions of the primary spermatocytes resulted, as in the case of the primary ovocyte generally, in the production of a single functional cell, inasmuch as there are produced from the primary spermatocyte in succession two "Richtungskörper." However, this fundamental difference was noted: whereas in the formation of polar cells during the maturation of the primary ovocyte there are produced two nucleated "Richtungskörper," in the spermatogenesis of the honey bee only the second of the corresponding bodies is nucleated, the first one being composed exclusively of cytoplasm.

At the time Meves published these observations we had already begun the study of the germinal cells in the male honey bee, and having now arrived at somewhat different conclusions from those set forth by him, will give here a preliminary account of our results thus far, the intention being to publish later a more detailed and comprehensive paper on the subject.

In the final generation of spermatogonia in the honey bee there is at the apex of each conical cell a spheroidal, nearly homogeneous body, which represents the remnants of the interzonal filaments of the preceding cell division. These bodies are stained black in iron haematoxylin, and on being washed out assume a characteristic yellowish gray color. Since they are admittedly the metamorphosed remnants of filamentous structures first named by Mark (**'81**, pp. 198, 539) interzonal filaments, we shall henceforth speak of them as the *interzonal bodies*. The interzonal body is identical with the "Zellkoppel" of Paulmier (**'99**, p. 228), to which Prowazek (**:01**, p. 201) has given the name "Spindelrestkörper." It is to be noted that the term "Zellkoppel" was used by Zimmermann (**'91**, p. 189), who introduced the name into cytology, with a somewhat different meaning from that employed by Paulmier. This is in itself a reason for applying another name — interzonal body — to this structure.

At the end of the growth period, which follows the last spermatogonial division, the cells have increased greatly in size and have become in general spherical, a form which is more or less modified by the mutual pressure of the closely packed elements. At this stage (Figure 1) the interzonal body (x) is clearly visible, and is in contact with the cell membrane. Meves shows it in his Figure 1, but makes mention of it neither in his explanation of the figure nor in the accompanying text.

The first evidence of spermatocyte division is seen when the centrosome, which lies in contact with the cell membrane, divides and the two daughter centrosomes move apart along the periphery of the cell. The centrosomes during their migration appear to exert a marked influence on the form of the cell, which exhibits two more or less conspicuous elevations, the apex in each case being occupied by one of the centrosomes. Figure 2 represents a fairly early stage in the migration of the centrosomes and shows clearly the marked change in the form of the cell due to their presence. The distance between the two centrosomes increases until these ultimately arrive at opposite poles of the cell (Figure 3). Up to this time each centrosome seems to have exercised nearly the same amount of influence as the other in modifying the form of the cell; but from this time forward the influence of one is seen to predominate over that of the other, until at length (Figure 4) one end of the cell is drawn out into a long, slender, slightly tapering, finger-like process, at the tip of which is located the centrosome. This centrosome will be designated as the proximal one, the other as the distal centrosome.

The choice of these designations rests on a later condition in the arrangement of the spermatocytes. These become grouped in the cyst in a radial manner with their interzonal bodies clustered at the centre, very much as the spermatogonia are arranged in their follicles. This constant orientation of the cells in relation to the cluster as a whole makes it possible to use without ambiguity the terms proximal and distal.

The nucleus, which hitherto has been near the centre of the spermatocyte, now changes its position, migrating to the distal end of the cell; it generally comes to lie close to the corresponding centrosome. Coarse fibres, which stain deeply in iron haematoxylin, now make their appearance, extending from the distal centrosome around the nucleus to the region of the proximal centrosome. At this time the finger-like process terminating in the proximal centrosome may be reduced to a very narrow cytoplasmic stalk (Figure 4), so that it is difficult to observe whether the fibres at this stage actually reach the centrosome or not. It is, however, probable that they do, as they may be traced for some distance into the base of the finger-like projection.

Changes, which we shall not attempt to describe here in detail, have meantime taken place within the nucleus. The chromatin passes through a spireme stage and gives rise — apparently by segmentation and a concentration of the segments — to sixteen double, dumb-bell shaped chromosomes (Figure 5), which lie in rather scattered positions. The bivalent structure of the chromosomes becomes less conspicuous when, a little later, the nucleus enters on the metaphase.

At about this stage the nucleus elongates, first in the direction of the distal end of the cell (Figure 7), and later at the opposite end, until finally it becomes spindle shaped, extending more or less completely from pole to pole (Figure 8). At the time of nuclear elongation a few thick, deeply staining intranuclear-spindle fibres are seen connecting the chromosomes with the distal centrosome, and later, as the nucleus increases in length, similar fibres also run to the proximal pole of the cell. These fibres are clearly separate from one another in the region of the chromosomes, but as they approach the centrosomes they seem to coalesce, until it is impossible to make out separate elements. In the cell represented in Figure 8, a bundle of fibres is traceable from the chromosomes to the distal centrosome, and on the opposite side of the chromosomes a bundle may be followed for some distance into the base of the finger-like process which terminates in the proximal centrosome. In other cases the nucleus, although much attenuated at the poles, has been clearly seen to stretch from one centrosome to the other, the chromosomes being midway between the two centrosomes. I find nothing to corroborate the condition shown by Meves in his Figure 2, where the elongated nucleus with its contained spindle figure is entirely isolated from the two centrosomes of the cell. Apparently he has overlooked the fibrous prolongations which connect the ends of the spindle with the centrosomes. The coarse, granular, extranuclear fibres already referred to are still to be seen occupying the greater part of the cell, extending in somewhat irregular sinuous courses outside the nucleus from pole to pole.

The mitotic figure here described is evidently that of the first spermatocyte division; but now a peculiar phenomenon occurs. Instead of a division of the chromosomes taking place, as would be expected, the figure does not for the present progress beyond the beginning of the metaphase. The spindle fibres either break down, or, what is more likely, become closely pressed together into a single bundle; but the chromosomes maintain their form for some time. A cross section through the middle of the spindle at this stage (Figure 6) shows the chromosomes still surrounded by a nuclear membrane, and likewise the cut ends of the cytoplasmic fibres lying outside that membrane.

The interzonal body persists throughout the period of the nuclear changes last described without any apparent alteration in form or size, but toward the end of this period it has a position at, or very close to, the proximal end of the cell, and soon afterward it is found directly beneath the proximal centrosome. Eventually it occupies the base of the finger-like process of cytoplasm (Figure 9) previously described. There is some variation in the sequence of phenomena during this period. As early as the stage shown in Figure 7, the centrosome and the interzonal body are sometimes seen closely applied to each other, while in other cases at a much later stage in the nuclear metamorphosis (Figure 9) a portion of the finger-like process may still be seen between the two. At length the interzonal body occupies the place of the finger-like process of cytoplasm and protrudes from the main body of the cell (Figure 10), the centrosome continuing to occupy its former position at the proximal end of the projection. At this stage the interzonal body is a spherical object supported on a cylindrical process of the cell, and resembles superficially the first polar cell formed during the maturation of eggs; but it differs from the polar cell in obvious particulars. Not only is it destitute of visible chromatic substance and the usual remnants of interzonal filaments, but it exhibits from the beginning a fairly well-defined outline on the side toward the body of the cell. Except for the persisting centrosome, its substance is homogeneous and slightly more refractive than the neighboring cytoplasm.

During the migration of the interzonal body into the finger-like cell process, other portions of the cell undergo conspicuous changes. The extranuclear fibres which, in the earlier stages, extended from the distal centrosome around the nucleus and the interzonal body to the proximal centrosome, have now increased in number, and crowd upon the nucleus. The latter continues to change in form and size; it becomes considerably flattened and much smaller. The chromosomes are crowded together so that they often appear (Figure 10) as a single deeply staining mass, generally much nearer the proximal than the distal end of the cell, and sometimes actually in the projection of the cell close to the interzonal body. Careful decolorizing at this stage reveals the chromosomes in the form of dyads, with possibly an actual separation of the dyads into their components. This perhaps represents what Meves describes as the breaking up of the chromosomes into granules.

Soon after this the cylindrical process of cytoplasm connecting the interzonal body with the main portion of the cell becomes elongated and more attenuated. Eventually it assumes a slender neck-like form, and then by rupturing, effects a complete detachment of the interzonal body from the spermatocyte (Figure 11). The region of greatest attenuation and final rupture is such that a small, though possibly variable, amount of undifferentiated cytoplasm is cut off with the interzonal body. The rôle of this body in spermatogenesis is now finished; apparently it gradually diminishes in size (Figure 15), and ultimately disappears entirely.

The extranuclear fibres, which were well developed as far back as the stage represented in Figure 4, are possibly concerned in some way with the elimination of the interzonal body; they are at no time connected with any chromatin, and yet they are as large and stain as deeply as the fibres within the nucleus. They persist and are to be recognized in the cell throughout the succeeding division.

At about the time of the detachment of the interzonal body the nuclear membrane disappears and the spindle presents a form resembling that which usually characterizes such a structure. The chromosomes are now seen to be in the metaphase (Figure 11), having returned to the condition which they exhibited immediately before the elimination of the interzonal body.

The history of the nucleus during the elimination of the interzonal body is hard to trace and difficult to interpret. Meves states that, after the formation of a spindle figure inside the persisting nuclear membrane, with sixteen chromosomes irregularly arranged at the equator, there is no further progress; that, on the contrary, the chromosomes remain clustered together in one place, and finally break up into granules; and that the spindle fibres are converted into a network. While we agree with the main point implied in this description, — viz., that the chromosomes do not separate into two groups representative of two daughter nuclei, — our observations lead us to think that the nuclear matter does not pass through a true reticular stage. For the chromosomes retain more or less distinctly their individuality, and the spindle fibres do not seem to take on the reticulate condition, but rather, to become confluent into a smaller number of strands. Whatever may be the method of its formation, however, the spindle and its chromosomes present almost exactly the same conditions immediately after the elimination of the interzonal body that they did just before that event. We have found, then, in our material no evidence of a typical prophase leading up to the formation of the spindle which is connected with the production of the second (the nucleated) bud.

During the period intervening between the formation of the first and second buds the centrosomes sometimes present a condition that may have a bearing on the interpretation of the process we are dealing with. Either the distal or the proximal centrosome, or rarely both, becomes elongated in a plane perpendicular to the axis of the spindle (Figures 10, 11), and in some instances a constriction divides it more or less completely into two bodies.

When the interzonal body has been detached, the spindle figure again assumes the condition characteristic of a nucleus in the metaphase; its cross section becomes greater, and the chromosomes, sixteen in number, immediately undergo division. The small daughter chromosomes, of the same size as each of the components of a dyad previously described, migrate to the poles of the spindle, leaving stretched between them interzonal filaments, which stain in iron haematoxylin like the spindle fibres. Nearly up to the stage represented by Figure 12, nothing suggests an unequal division of the cell; but as soon as the chromosomes arrive at the poles of the spindle the proximal end of the cell elongates and the chromosomes appear as a deeply stained mass at the apex of a second finger-like process of cytoplasm. The chromosomes at the opposite pole become surrounded by a membrane, thus giving rise to a small typical nucleus. Between these two nuclear bodies are seen the interzonal filaments together with remnants of the more peripheral fibres already described in connection with the account of the elimination of the interzonal body. The finger-like process at the proximal end of the cell with the nuclear mass at its tip continues to elongate (Figure 14), and eventually its terminal part, containing the chromatin, is detached in the form of a very small cell, — much smaller, indeed, than the interzonal body previously set free. We have called this a cell, though it is difficult to determine how much, if any, undifferentiated cytoplasm is cut off with the nuclear matter. This, however, is a difficulty which is often encountered in studying the polar cells formed during the maturation of eggs, and does not in our opinion preclude the use of this designation. When fully separated from its sister cell, this small cell assumes a spherical form and increases in size. The nucleus of the larger cell, the spermatid, likewise increases in size and becomes to all appearances the counterpart of the detached polar cell. Figure 15 represents the two cells at this stage, and it also shows the previously eliminated interzonal body, now much diminished in size. The chromatin in both nuclear bodies becomes variously aggregated, but assumes in general a peripheral position; the metamorphosis of the larger cell — the spermatid — now begins.

Meves in his description of the first "Richtungskörper" is evidently dealing with what we have called the interzonal body, the peculiar history of which has been briefly described. If this so-called bud were in truth made up simply of undifferentiated cytoplasm, as Meves leads us to believe, perhaps one would be justified in considering it a rudimentary spermatocyte of the second order, even though it contained no chromatin. But if it is composed of the remnants of the interzonal filaments, which have become metamorphosed into a definite body within the cell, and is surrounded by only the slightest amount of undifferentiated cytoplasm, if any, it is difficult to find any sufficient basis for homologizing it with the first spermatocyte division of typical spermatogenesis. Moreover, such a casting forth of spermatogonial spindle remnants has already been observed in cases where four functional spermatids are formed in the normal way, and where, consequently, this elimination of interzonal matter can have nothing to do with the first spermatocyte division. Paulmier (**'98**, p. 228) has described such conditions in Anasa tristis in the following terms: "At the tip of the resting cell [spermatogonium] is a structureless mass which is formed from the remains of the intermediate spindle fibres [interzonal filaments] of the preceding division." Further on he adds, "As a group of chromosomes approaches the end of its movement the cell loses its conical shape and becomes more cylindrical, and the Zell-Koppel [interzonal body] of the preceding generation is cast off, remaining for a time as a small isolated mass, which ultimately disappears." Blackman (**:05**, p. 37) likewise describes a somewhat similar phenomenon in the male germ cells of Scolopendra, where the spindle remnants of both the first and the second spermatocyte divisions are eliminated during a rotation of one daughter cell upon the other.

It seems possible, therefore, that this globule, composed principally of the substance of the interzonal filaments, is not comparable with the spermatocyte of ordinary spermatogenesis, and therefore not with a polar body in ovogenesis.

The smaller of the two bodies resulting from the real mitosis which follows the elimination of the interzonal body bears a striking resemblance, it must be admitted, to the polar cell of eggs, and it is unquestionably the result of a true, though modified, cell division. This phenomenon seems, however, never to have been seen in the spermatogenesis of any other animal. There is here an equal division of chromatin accompanied by a very unequal division of the cytoplasm, precisely as in the formation of the polar cell in eggs. The small cell *apparently begins to undergo a metamorphosis parallel to that of its* larger sister cell, as Meves has maintained. Although Meves believes that the small cell eventually degenerates, positive evidence of this has not yet been produced. If it is to be interpreted as the homologue of a polar cell, the question at once arises, Which of the two polar cells usually produced does it represent? From the standpoint of Meves, the first body might be looked upon as a partially abortive attempt to produce the equivalent of the first polar cell, and the second body might then be regarded as in some sense the equivalent of the second polar cell; but even were his view about the first (interzonal) body correct, the division of nuclear substance accomplished by the formation of the second bud would more strongly point to this, rather than the first, as being the homologue of the first polar cell. That view is, perhaps, strengthened by the facts here presented, which tend to show that the body first produced has nothing whatever to do with cell division, and that in the occasional doubling of the centrosome (Figures 10, 11) some progress is made toward a second cell division, which, if completed, would result in the formation of either two spermatids or a second true polar cell. However, it must be admitted that, on the assumption that the interzonal body has nothing to do with an attempted cell division, the peculiar changes passed through by the chromatin preceding and during the period of the elimination of that body remain unexplained and without a parallel.

On the other hand, the beginning metamorphosis of the small nucleated body (second bud) suggests that it is the equivalent of a spermatid rather than a spermatocyte of the first order, and this view is strongly supported by the statement of Meves that in Vespa germanica the cell division corresponding to this results in the production of two spermatids of equal size.

BIBLIOGRAPHY.

Blackman, M. W.

:05. The Spermatogenesis of Myriapoda. III. The Spermatogenesis of Scolopendra heros. Bull. Mus. Comp. Zoöl. Harvard Coll., Vol. 48, No. 1, pp. 1–138, 9 pl., 9 fig.

Mark, E. L.

'81. Maturation, Fecundation, and Segmentation of Limax campestris, Binney. Bull. Mus. Comp. Zoöl. Harvard Coll., Vol. 6, pp. 173–625, 5 pl.

Meves, F.

:03. Ueber "Richtungskörperbildung" im Hoden von Hymenopteren. Anat. Anz., Bd. 24, pp. 29–32, 8 Fig.

Paulmier, F. C.

'99. The Spermatogenesis of Anasa tristis. Jour. Morph., Vol. 15, Suppl., pp. 223–272, 2 pl., 6 fig.

Prowazek, S.

:01. Spermatologische Studien. Arb. Zool. Inst. Wien, Bd. 13, pp. 197–236, Taf. 11 u. 12, 2 fig.

Zimmermann, K. W.

'91. Über den Kernteilungsmodus bei der Spermatogenese von Helix pomatia. Verh. Anat. Gesell., Bd. 5, pp. 187–193.

EXPLANATION OF PLATE.

All figures were drawn with the aid of a camera lucida and are magnified 2073 diameters.

FIGURE 1. Primary spermatocyte, resting stage.

FIGURE 2. Primary spermatocyte; the two centrosomes moving apart.

FIGURE 3. Primary spermatocyte; the centrosomes have arrived at opposite poles of cell.

FIGURE 4. Proximal finger-like process of the cell, terminating in the proximal centrosome; from the distal centrosome extend prominent extranuclear fibres.

FIGURE 5. Nucleus of primary spermatocyte, showing sixteen dyads.

FIGURE 6. Cross section of a primary spermatocyte in the same stage as that of Figure 9.

FIGURE 7. Elongation of the nucleus toward the distal centrosome.

FIGURE 8. The nucleus exhibits a spindle-shaped elongation, and intranuclear spindle fibres are established.

FIGURE 9. The interzonal body has migrated to the base of the finger-like process.

FIGURE 10. The interzonal body in the course of being constricted off from the cell.

FIGURE 11. The interzonal body is detached, and there is a typical spindle in the beginning of the metaphase.

FIGURE 12. Late anaphase.

FIGURE 13. Finger-like process surmounted by chromatin of polar cell.

FIGURE 14. Chromatin of finger-like process nearly detached from the process.

FIGURE 15. Spermatid, interzonal body reduced in size, and polar cell.

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Problem in General and Results in General.

This is a preliminary paper on an investigation whose object is threefold.

It is known that at pressures lower than about 1 cm. of mercury the resistance which a solid body meets in passing through a gas is noticeably smaller than it is when the pressure is that of an atmosphere. This is because the phenomenon of slip presents itself at the low pressures. The first object of the investigation is to study fully the relation between the friction of a gas on a solid body and the pressure in the gas at those pressures where slip is appreciable.

Again, it is known that, if a given volume of gas is divided into two portions by means of a partition one side of which is kept hotter than the other, and if a small opening of any form be made in the partition, the gas will flow from the cold side to the hot side of the partition until there is an excess of pressure in the chamber contiguous to the latter. This excess of pressure depends on the difference of temperature of the two sides of the partition and on the mean pressure.

The second object is to examine in a particular simple case the relation between the force on the partition (called the transpiration force), resulting from this flow of gas towards the hot side of the partition, and the mean pressure in the gas.

The third object of the investigation is to examine the feasibility of measuring the pressure in a gas when either of these relations is known without having recourse to the McLeod gauge. That this third object is by no means the least significant will appear when the evidence as to whether the McLeod gauge is reliable for the measurement of small pressures is adduced.

The progress made in the solution of these problems up to the present may be summed up as follows:

(1) The great difficulty in devising and carrying out methods of constructing the pieces of apparatus required has been overcome. This, which was by no means the least difficulty encountered, can be appreciated only when one knows the form of the apparatus required, and the treatment which the apparatus must receive in order that all traces of moisture may be removed from all the solid surfaces exposed to the interior of the containing vessel.

(2) The friction has been measured at pressures ranging from that of an atmosphere to 0.00024 mm. of mercury, the small pressures being measured by the McLeod gauge. Figure 6, in which ordinates are proportional to friction and abscissas proportional to pressure, shows how the friction diminishes as the pressure diminishes. The curve, whose error is certainly less than one per cent (if the McLeod gauge can be considered trustworthy over the range of pressures indicated), is very regular; and a discussion of the numerical results in that part of the curve which corresponds to pressures above 0.1 mm. of mercury shows that the law

$$\left[\frac{L-K}{l-K}-1\right]p=c,$$

first deduced by Kundt and Warburg for pressures down to 0.6 mm., holds very well down to about 0.1 mm. Beyond this point, however, the theoretical curve, i. e., the curve obtained by calculating p from the above relation after the constants have been determined from observations at comparatively large pressures, does not coincide with the actual curve.

(3) The force due to transpiration has been measured over a range of pressure varying from 1.42 mm. to 0.0093 mm.

(4) Figure 7 (the error in the curve may very well be five per cent), in which the ordinates are proportional to the transpiration force and abscissas to pressure, shows that, for pressures below that for which the force is a maximum, the relation between the force and the pressure rapidly approaches a relation of mere proportionality, or at worst a proportionality disturbed by a constant term. In other words, the curve becomes a straight line passing very nearly through the origin.

PROBLEM IN DETAIL.

Introduction.

Aside from the interest which attaches itself to an investigation of the law relating viscosity to pressure at low pressures, the advantage of being able to measure gas pressure by measuring the friction of the gas on a pendulum is apparent. By this method the state of the gas remains unchanged during the time the measurement is in progress, while by the method of the McLeod gauge the gas must be compressed in order that the pressure may be measured. It is, however, beyond doubt very important that in establishing a method of gas pressure measurement by friction, or otherwise, a second method should if possible be obtained to serve as a check upon the one which we may consider most convenient and therefore the most desirable. Though the investigation of the law relating pressure in a gas to transpiration force when the transpiration space is of special form is interesting and important for its own sake, yet additional interest and importance is imparted to it when the possibility of making its results serve as the desired check upon the proposed friction method of measuring gas pressure is considered.

Thus, in so far as the object of the present investigation is to test or replace the McLeod gauge, the function of what has been called the transpiration apparatus is to serve as a check upon the results of pressure measurement obtained by the viscosity apparatus. As will appear in the sequel, the problem to be solved with the transpiration apparatus is that of investigating the law of gas action on a radiometer vane of special form surrounded by a containing vessel whose parts are symmetrically placed with respect to the vane.

That at least another attempt should be made to decide the question as to how much reliance can be placed on the McLeod gauge for measurement of pressures below 0.1 cm. is very forcibly brought home to one who examines the conflicting evidence of high authority.

The attempts to investigate the behavior of the gauge when the vacua are high, have as a rule been made, not with the avowed intention of testing the gauge, but with the object of testing the validity of Boyle's law at various pressures, any departure from which for any gas would at once limit the use of the gauge for that particular substance. The method used was to enclose a certain quantity of gas, measure accurately its volume and its pressure, calculate $p\,v$; change this volume, and therefore pressure; measure again, and so on. The various values of $p\,v$ thus obtained should be the same, that Boyle's law may be obeyed, and hence that the principle of the gauge may be sound.

Siljeström[1] found departures from the law in the case of air and oxygen. Amagat[2] found the law obeyed, while Bohr[3] found an anomaly in the case of oxygen at 0.7 mm. pressure. In the curve representing $p\,v$ plotted against p there is a sudden drop at this pressure. Batelli's[4] experience corresponds well with that of Bohr. At 0.7 mm. with oxygen there is departure, while with air the departure is at pressures between 2 mm. and 5 mm. Carbon dioxide departs from the law, while hydrogen is found to obey it from one atmosphere to 0.002 mm. Baly and Ramsay[5] have pronounced the gauge worthless for oxygen, while the results given for hydrogen are said to be reliable.

Lord Rayleigh,[6] working with values of p ranging from 1.5 mm. of mercury to 0.01 mm., found no evidence of any anomaly with oxygen, nitrogen, or hydrogen; while at pressures ranging from 75 mm. to 150 mm. he found the law fully obeyed.

There is, then, conflicting evidence, with much in favor of irregularity.

Although with the gauge which Rayleigh employed in his investigation one can measure, with an accuracy of about five per cent, pressures of 0.01 mm. of mercury, yet for pressures which in many cases must be measured, this method is not sensitive enough, and then one is forced back upon the use of the McLeod gauge, and that in a region where we know nothing of its action.

We are indebted to Sutherland[7] for the suggestion that the lowest pressures may possibly be measured by measuring the friction of the gas on a pendulum at pressures where the phenomenon of slip can be detected experimentally. Having had some experience in determining the absolute value of viscosity in gases, I have undertaken to investigate the whole subject.

As has been stated at the beginning of this paper, the problem involves the solution of two others, which will now be discussed in detail.

The Friction Problem.

When a gas flows over a solid surface at a uniform rate, or a solid surface is made to move through a gas with uniform velocity, there is brought into play a resistance to the motion due to friction. If, for example, we have two parallel planes placed in a mass of gas at, say, unit distance apart, and, while keeping one fixed, we cause the other to move in a certain direction in its own plane, a certain force must be continually applied in the direction of motion in order that a uniform velocity may be maintained. That is, during the motion there is a certain tangential stress exerted by the gas on the solid which opposes the motion of the solid. If the layer of gas which is in contact with the moving plane moves with the same velocity with which the solid moves, the tangential stress between the gas and the solid, that is, the resistance experienced by the solid, must depend upon the force required to cause one layer of the gas to move with a certain velocity relative to the next layer. Experiment shows that the relative velocity of contiguous layers is a measure of the tangential force. If, however, the solid and the gas in contact with it do not move together, then, for the same velocity of the moving solid, the velocity of a layer of gas relative to its contiguous layer is less than before, and hence the friction between them is less, and therefore the stress at the solid is less. In the case considered, when the moving plane and the layer of gas next to it move together the force per square centimeter of the plane necessary to keep up a uniform velocity of one centimeter per second in the given direction is the coefficient of viscosity or internal friction. If the moving plane and the layer of gas next to it have not the same velocity, the gas is said to slip on the solid. The force per unit area of the plane which must be applied in a given direction in the plane to maintain a uniform relative velocity between the plane and the layer of gas is proportional to this relative velocity. When the relative velocity is one centimeter per second the force required is the coefficient of external friction.

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[1] Pogg. Ann., **151**, 1874.

[2] Ann. de Chim. et Phys., **28**, 1883.

[3] Wied. Ann., **27**, 1886.

[4] Phys. Zeits., **3**, 1901.

[5] Phil. Mag., **38**, 1894.

[6] Phil. Trans., **196**, 1901.

[7] Phil. Mag., [5], **43**, 1897.

Maxwell showed that the internal friction of a gas is constant for pressures varying from atmospheric to one sixtieth of atmospheric pressure. In his paper[8] he defines the coefficient of slip to be the ratio μ/σ, where μ is the coefficient of viscosity of the gas, and σ the coefficient of external friction. The coefficient of slip is then a quantity which decreases as σ increases and μ decreases, and which increases as σ decreases and μ increases. In order that there should be no slipping, σ must be infinitely great compared with μ. As this is probably never the case, there is some slip under all circumstances. Maxwell also showed that when the conditions are such that slip must be considered, the resistance to the moving solid in the foregoing discussion is the same as it would be were the fixed surface removed a distance 2β farther from the moving surface where β is the coefficient of slip.

A consideration of Figure 1 will probably help to make the motion of the gas between the planes better understood in the case where slip must be considered as well as in the case where it is negligible. Let A B represent the section of the fixed plane just considered by the plane of the paper. Likewise, let C D represent the similar section of the moving plane. Let the moving plane have a velocity of one centimeter per second in the direction C D, and let the distance between the planes be one centimeter. If the layer of gas next to the solid moves with the same velocity with which the solid moves, the line of particles represented by X Y at the beginning of any second will, if X O equals X Y, be represented by X O at the end of this second. That the particles will at the end of the second lie in the line X O is seen by considering the forces acting on any layer of the gas between the planes and parallel to them. The layer above the one in question exerts a tangential force on the latter which tends to move it in the direction C D, but the layer below it holds it back with a force just equal to this. Otherwise there would be a net constant force acting on the layer parallel to C D, which would produce an acceleration. Clearly there can be no acceleration of the layer while the moving plane maintains its uniform velocity. Moreover, since action and reaction are equal, the layer which is thus holding back the layer above it is being dragged forward with the same force. It in turn exerts the same forward force on the next succeeding layer, and so on. It thus appears that the force exerted by the layer of gas next to the fixed solid, A B, in the direction A B, is just the same and in the same direction as that exerted by any layer of gas upon that layer just beneath it; and also that the force with which the fixed solid resists the motion of the gas is the same as the force with which any layer resists the motion of the layer above it. Clearly, a similar statement may be

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[8] Sci. Papers, **2**, 1.

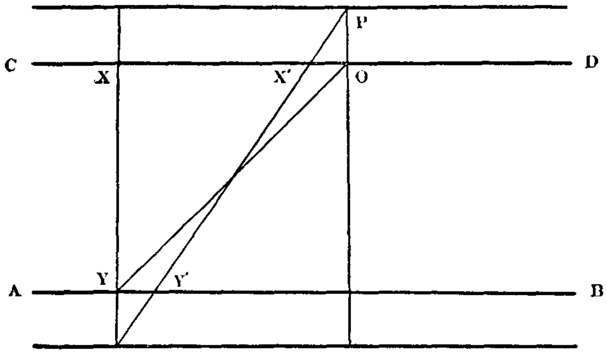

FIGURE 1.

made regarding the resistance which the gas in contact with the solid, C D, offers to the motion of C D. Under these circumstances, in any second, any one layer must move with respect to the next layer below it exactly the same distance which the one above it moves with respect to the layer in question. It follows, then, that the line of particles, X Y, at the beginning of a second will become the line X O at the end of that second.

If, however, the solid and the gas in contact with it have different velocities, that is, if the gas slips on the solid, the line X′ Y′ will represent the position of the line X Y at the end of the second. It is clear from the figure that if we remove the planes farther from each other by the distance O P plus Y P′, that is, twice O P, and assume that there is no slipping at the surface of the solids, then, since the relative positions of the successive layers are unaltered, the friction between contiguous layers of gas, and therefore the tangential stress on the solids will be the same as it is where there is slipping and where the planes are at the original distance X Y from each other.

That the distance O P is, as Maxwell showed, equal to μ/σ, is readily seen. For, if we assume that the surfaces of the fixed and moving solids are alike, σ will be the same for both. If, also, we call the distance Y Y′, or X′ O, x, the tangential stress at either solid must be σx, since x is the velocity of C D relative to the gas in contact with it, and also the velocity of the gas in contact with A B, relative to A B. But the stress between consecutive layers is, since the distance between the planes is one centimeter, equal to $\mu\,(1-2\,x)$, and we have seen that this is the same as the stress at the solid. We have, then,

$$\sigma x = \mu\,(1 - 2\,x)$$

or,

$$x = \frac{\mu}{2\,\mu + \sigma}$$

or,

$$\sigma x = \frac{\sigma\,\mu}{2\,\mu + \sigma}$$

$$= \mu\left(\frac{1}{1 + 2\,\mu/\sigma}\right)$$

= the force at the solid.

But the form of this result shows that this is the force which must be applied to the solid, C D, to maintain a velocity of one centimeter per second if the distance from the fixed solid is $(2\,\mu/\sigma + 1)$ and there is no slipping. If there is no slipping, then, in order that the stress at the solid should be the same as when there is slipping, namely σx, the distance between the planes must be increased by $2\ \mu/\sigma$, that is, twice the coefficient of slip as defined by Maxwell. In the figure, then, the distance O P is equal to μ/σ.

The investigation of the law of friction on a body at pressures smaller than those at which Maxwell worked was taken up by Kundt and Warburg,[9] but without adequate means of measuring pressure. They verified Maxwell's law, and concluded, both on theoretical grounds and from their experimental data, that over a certain range of pressure the coefficient of slip varies inversely as the density of the gas, and therefore, so long as Boyle's law holds, it varies inversely as the pressure. Sutherland[10] reached the same conclusion by a different method. If this is true and μ is constant, then σ, as defined above, must be proportional to the pressure. When the density of the gas has been so far reduced that the molecules seldom collide in passing from one solid surface to the other, then the friction is largely superficial or external, and one would expect that at less densities the resistance offered to the movement of the solid body would be well-nigh proportional to the pressure.[11]

Maxwell's viscosity apparatus consisted, essentially, of a disk of glass suspended with its plane horizontal and between two other larger fixed parallel disks of glass. The middle disk performed oscillations in its own plane. When the distance D, between the fixed and moving surfaces is small, Maxwell's formula for this apparatus becomes

$$\lambda - K = \frac{c\,\mu}{D} \tag{I}$$

Where λ is the total logarithmic decrement,

K is that due to the friction in the suspending fibre, and is a constant.

μ is the coefficient of viscosity of the gas.

D, the distance between fixed and moving surfaces,

c, a proportionality factor.

This is the formula only when slipping is not taken into account. When slipping must be considered, as has been said above, the fixed and moving surfaces may be considered removed from each other by a distance $2\ \beta$, where β is the coefficient of slip.

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[9] Wied. Ann, **158**, 1876.

[10] Phil. Mag., [5], **43**, 1897.

[11] Kundt and Warburg, Wied. Ann., **158**, 1876.

In that case equation (I) becomes,

$$l - K = \frac{c\,\mu}{D + 2\beta} \qquad \text{(II)}$$

where

l is the total decrement at a lower gas pressure, and the other quantities have the same significance as before.

Now, on dividing (I) by (II),

$$\frac{\lambda - K}{l - K} = \frac{D + 2\beta}{D}$$

$$= 1 + \frac{C}{p}$$

since β is inversely proportional to p. This equation may be written in the form

$$\left(\frac{\lambda - K}{l - K} - 1\right) p = C. \qquad \text{(III)}$$

This equation is deduced by Sutherland; and it is also of the form of that used by Kundt and Warburg in their investigation to verify their theoretical conclusion that the coefficient of slip varies inversely as the pressure. As has been said, however, they were unable to measure the lower pressures, and so the law was not submitted to a test at very low pressures.

The relation between pressure and logarithmic decrement expressed here is the one utilized by Sutherland[12] to determine p after the constants in the foregoing equation have been determined. The data for this purpose he found in Crookes' paper on 'Viscosity of Gases at very High Exhaustions.' There the logarithmic decrement of a vane of mica, suspended with its plane vertical, and performing oscillations about a vertical diameter, is given for pressures ranging from that of an atmosphere to 0.02 millionths of an atmosphere. The pressure was measured by a McLeod gauge. Sutherland's formula applied to these results gave very good agreement from about 0.2 mm. to about 0.01 mm. in the pressure as measured by the gauge, and the pressure as calculated from the formula. Below 0.01 mm. the agreement was not good.

Stokes[13] has shown that, in this apparatus, the coefficient of viscosity is not proportional to the logarithmic decrement, and therefore, as

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[12] Phil. Mag., [5], **43**, 1897.

[13] Note added to Crookes' paper.

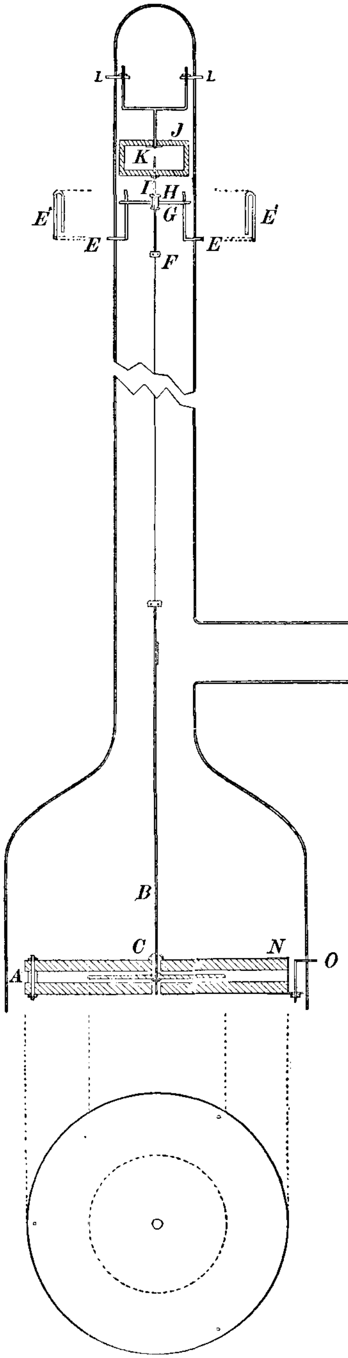

FIG. 2

Sutherland has pointed out, a fairer test of the correctness of (III) might be made with a viscosity apparatus like that for which the formula was deduced.

This suggestion has been followed in the construction of the instrument whose description follows.

Viscosity Apparatus — Description. — The viscosity apparatus used in these experiments consists of two circular glass plates 8.2 cm. in diameter and 0.3 cm. thick (A, Figure 2). They are pierced by a hole in the centre 0.3 cm. in diameter, and by three holes near the edge, equally spaced, and 0.15 cm. in diameter. Through the latter are passed small threaded platinum bolts, provided with nuts, for the purpose of binding the plates together. In order that the latter may be fixed at a certain distance apart, they are separated by three short glass tubes, through the bore of each of which passes one of the aforesaid bolts. These glass tubes are ground to the same length, 0.35 cm.

These are the two fixed plates in Maxwell's apparatus, while between them, concentric with them, and parallel to them, is suspended the vibrating disk, which consists of a plate of glass 4.4 cm. in diameter and 0.09 cm. thick. It has a hole in the centre about 0.075 cm. in diameter, through which passes the closely fitting supporting wire, B, Figure 2. The latter is provided with a hard-soldered shoulder, against which the plate is clamped by means of the nut below. Platinum foil washers are placed between the glass and the nut. On the supporting wire at C are fixed three feet, long enough so that, if the swinging plate were lowered, they would reach the upper surface of the upper plate just before the suspended plate could touch the lower plate. This is a measure of safety, as will be seen when the process of making and suspending is described. At the proper height for the window, D, is fastened a mirror of platinized glass. It is supported on the wire by thin platinum foil, which is fused to the wire, and then folded around the edges of the mirror. At the end of the wire is a clamp, consisting of two flat pieces of platinum 1.25 mm. thick, 3 mm. wide, and 6 mm. long. One is hard-soldered to the wire so that one surface is in the axis of the wire. The other is fastened upon this by two screws. The fibre is fastened at the top in the similar clamp, F, at the end of the wire, I. The nut, H, into which the wire, I, is screwed, carries a cross-bar, G, the ends of which are engaged by the platinum wire hooks, E and E, shown detached at E′ and E′. There is also a check-nut, K, on the wire, I, above H, which can be screwed down upon H to hold the clamp and cross-bar in any desired relative position. The upper end of the wire, I, screws into a nut of the same material as itself on the soft iron armature, J, which is supported by the swivel-head, K. The supporting cross-bar is itself fastened to the two vertical threaded pieces passing through the supports, L L, and furnished with nuts for support and adjustment.

As the telescope and scale were to be used in observing, and as the whole apparatus was to be heated, a plane parallel window, D, which would bear a temperature of 300° C., must be secured. It was decided to close in the end of a glass tube, and grind the inside and outside flat and parallel to each other, and then seal the short horizontal tube at the proper place to the long vertical one. By using a large lump of optical glass, and using the oxygen flame, it was found possible to close the end with glass of fairly uniform density. In ordinary glass tubes, the streaks in the glass cause difficulty, and besides, almost invariably a small sort of pit is formed at the centre in the process of sealing. At this point, the density of the glass is different from that of the rounded part, thus giving an irregular lens effect. The method of grinding is to use a ring tool with which to make a groove around the inside just as large as the size of the tube will allow. The groove allows clearance for the emery when the process of grinding flat is undertaken. A thoroughly satisfactory window is obtained in this way.[14]

After the glass vessel has been hermetically sealed, the soft iron armature, J, can be turned by means of a magnet fixed suitably outside. The magnet is placed on a circular platform of brass surrounding the tube so that the poles of the magnet control the armature within. The platform is supported on the top of the box containing the glass part of the apparatus. The platform has a circular groove in it which a circular brass ring fastened to the lower side of the magnet exactly fits, so that the magnet may be slid around without altering its position with respect to the tube.

To put this apparatus together required some care in handling, and no little skill in glass-blowing.[15] The large cylinder of glass was blown about 40 cm. long, and when the three supporting wires, one of which is shown at O, had been sealed in, the lower end of the cylinder was opened. To the upper end the narrower tube was joined, and the short tube bearing the window, D, was then sealed on at the proper place. The next step was to fasten the fibre so that it might be heated to a temperature of, say, 300° C., without danger of slipping or breaking. Various attempts were made. For example, the end of the fibre was platinized, and electroplated with copper to the supporting wire, — a very troublesome operation. It was found, moreover, that when heat was applied a break occurred where the quartz came in contact with the metal, so that this method was abandoned. Carbon cement was also tried, but discarded owing to the uncertainty whether all of the volatile substances which it contains were driven off. Clamping was resorted to, and the simple form of clamp shown in the figure adopted. With the clamp, the danger of snapping the fibre just at the edge of the metal is considerable. To minimize this, platinum foil was wrapped around the part of the fibre to be placed in the clamp. The upper large plate was now pushed on to the wire, B, and the small disk clamped to the wire in the manner already described. The lower plate was then placed on a support, the glass separating pieces placed temporarily upon it, and the disk suspended in position between the plates. A short range horizontal telescope was then focussed on the edge of the suspended disk, so that, when the latter was made to revolve, one could tell very readily if the wire, B, had been fixed at right angles to the disk. In order to remedy any want of perpendicularity the upper plate and the disk were lifted together, and the wire carefully bent at a point close to the disk. After some trials it was found possible to adjust, so that the maximum difference in height of the two ends of a diameter of the disk was less than 0.07 mm.[16] The width of the disk was 44 mm. The large plates were then fastened together as described above, and the three clamps, one of which is shown at N, were placed so as to fit the supporting wires, one of which is seen at O.

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[14] This work was done by Mr. Lundin of the Alvan Clark Optical Company, Cambridge.

[15] Mr. Oelling, the glass worker of the Jefferson Physical Laboratory, has done the glass-blowing with great care and efficiency.

Suspending cords were now placed temporarily around the glass separating tubes and the plates raised by them, in an inverted position, allowing the armature and its belongings to be suspended by the fibre. The whole was then carefully lowered into the enclosing vessel, which had been placed temporarily bottom upward. When the plates were in position on the supporting wires, the various wires of the suspended part were put in place by means of a tool made for the purpose, the various nuts turned on, and the whole turned into its natural position. The disk was raised by turning the armature, J, and the plates then made parallel to it. This adjustment was made by turning the supporting nuts on the wires O, O, O. Finally, the check-nut, K, being loose, the upper clamp was turned in the cylindrical nut, H, until the mirror faced the window, D. The check-nut was then tightened, and the suspended part lowered so as to rest on the tripod at C. The instrument could now be handled with only moderate care. The upper and lower ends were then sealed off, and it remained only to join this to the other parts of the apparatus.

The Transpiration Problem.

The first experiments on Thermal Transpiration were made by Feddersen,[17] but the full investigation of the phenomenon is due to Professor O. Reynolds.[18] He showed that, if the two surfaces of a plate of porous material which divides a mass of gas into two separate portions are kept at different temperatures, the gas will force itself through the channels in the material from the cold surface to the hot until a certain difference of pressure is reached. In the case of air, he found it possible to establish, in this way, as much as 6 mm. of mercury difference of pressure between the two sides of the plate, where the mean pressure in the gas was 760 mm. His experiments established the fundamental law that the difference of pressure reached is increased by increasing the difference of temperature between the surfaces of the porous plate, and that, given a certain difference of temperature, the amount of the gas transpiring, or the height to which the difference of pressure will grow, is governed by the relation between the mean free path of the molecules and the diameter of the conducting space. When, at a given pressure, the diameter of this space is diminished, the difference of pressure attainable is increased. Since this is true, it is readily seen that, when the transpiration spaces are large, it is only necessary to reduce the density of the gas to get the same results as those obtained at greater densities with spaces of capillary dimensions.

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[16] Stokes has shown that a small inaccuracy in this adjustment involves a rather large error in the measurement of the resistance encountered by the moving disk. If the layer of air between the latter and the fixed disk is wedge-shaped, considerable energy is used up in crowding the gas between the fixed and moving disks. The adjustment attained here is sufficiently accurate to avoid this difficulty.

[17] Pogg, **148**, 302 (1873).

[18] Phil. Trans., **170**, 1880.

Sutherland[19] renews the theoretical discussion of this phenomenon, and by a different method arrives at substantially the same result. His result appears in an expression relating the difference of pressure to the mean pressure, the coefficient of viscosity, the diameter of the transpiration tube, and the mean velocity of the molecules. He then considers the case of a circular vane of badly conducting material, placed in a circular space, which it fits rather closely, and, assuming that in the transpiration space thus formed the temperature of the gas is controlled by that of the vane and the surrounding annulus, he deduces an expression for the force which will tend to push the vane out of the plane of the annulus, when one side of the vane and the corresponding side of the annulus are heated. His result is expressed thus:

$$F = \frac{c}{A\,p + B + 1/p} \qquad \text{(IV)}$$

where F is the force on the vane in arbitrary measure, and p the mean pressure expressed in, say, millimeters of mercury.

Sutherland has submitted his equation, with a considerable degree of success, to the experimental test, using data accumulated by Crookes in his work, on viscosity of gases at high exhaustions. Crookes measured not only the logarithmic decrement of the vane of mica mentioned above, but also the angle through which the vane was deflected when the light from a candle was made to fall on the blackened half of one of its surfaces. By measuring F and the pressure for three different pressures, the constants in (IV) can be determined, and then, by measuring F, p can be calculated for any other degree of exhaustion and the result compared with the value of p obtained by the McLeod gauge. If (IV) is the proper relation between F and p, the values of the latter obtained from (IV) and from the gauge should be the same over any range of pressures where the results obtained by the gauge are reliable.

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[19] Phil. Mag., **42**, 1896.

In Sutherland's hands, this formula has stood the test fairly well, but it must be added that the experimental conditions from which the data used by him were obtained did not well conform to the mathematical conditions assumed in the deduction of the above formula. Still, the results obtained by him indicate the direction in which to look for the full solution of the problem.

Transpiration Apparatus — Description. — Figure 3 is a horizontal section of the instrument through the centre of the suspended vane. The essential parts of it are a cylindrical glass vessel, A, Figure 3, about 7.5 cm. in diameter, in which is fixed an annulus of mica 1.25 cm. wide, the plane of the annulus being a cross section of the vessel. The vane, suspended in a manner to be described later, will pass through the opening of the annulus, leaving about 0.75 mm. clearance. The vane is 4.7 cm. in diameter, and is less than 0.1 mm. in thickness. The vane is clamped between a shoulder and nut at B, Figure 3. The supporting wire, C, is in turn fastened to another, D, by collar and check-nut, U and V. D bears a mirror, E, and F and G are counterpoises. At K is a short metal tube, shown in vertical section at H, Figure 3*a*. The wire, I, Figure 3*a*, is furnished with shoulder and check-nut, seen at L and J, and just fits the tube, H. The fastening of the fibre is the same here as in the viscosity apparatus.

In Figure 4 is shown the arrangement by means of which the suspended system can be raised or lowered without orienting, and oriented without raising or lowering. A is a soft iron armature, D a supporting swivel-head, the rod of which presses through guide-pieces, E and F, and terminates in a second supporting swivel-head, G. To the vertical rod is fixed a cross-bar, H, furnished with vertical guide-pieces, I I, which pass through the wire loops, J J. To the armature, A, the fibre clamp is fixed. The clamp is kept in the centre of the tube by means of the wire, K, which has a loop through which the clamp wire, L, just passes. M is another soft iron armature supported, as shown, on a screw passing through a nut, N, which, in order to avoid complications from inequalities of expansion, is made of the same material as the screw.

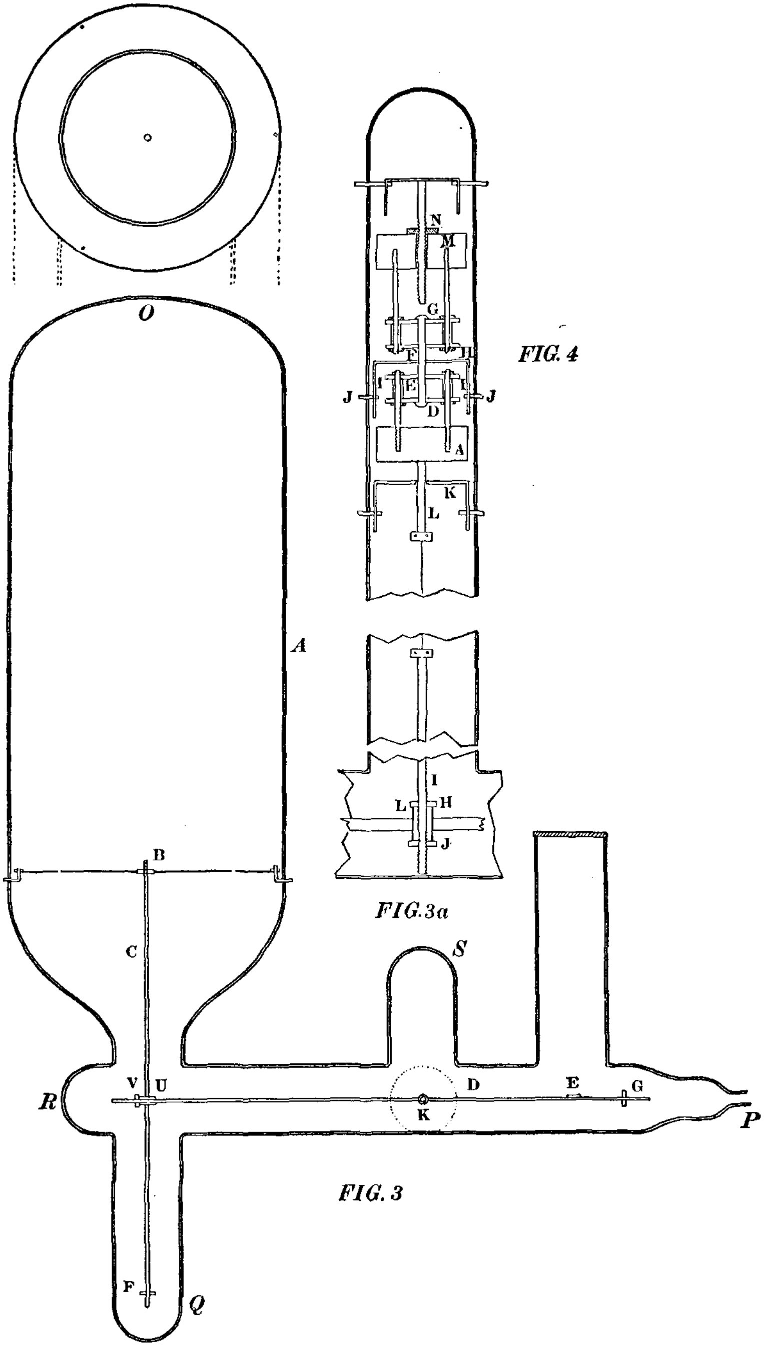
N
M
G
F
H
J
I
E
I
J
D
A
K
L
I
L
H
J
O
A
B
C
S
R
V
U
D
E
G
K
P
F
Q

FIG. 4

FIG. 3a

FIG. 3

By turning M alone, the whole suspended system is raised or lowered; while by turning A alone, a twist is given to the fibre while the height of the system is unchanged. Magnets, similar to the one used for turning the armature in the viscosity apparatus, and mounted so that they are entirely free from the glass part of the apparatus, are used to turn the armatures A and M.

Openings were left at O, P, Q, R, S, and at a point directly beneath the suspending fibre, until the annulus and vane with its supporting wire, C, were put in place, the wire D, bearing mirror and counterpoise, screwed into the collar U, and the adjusting apparatus bearing the fibre and the wire I placed in position. When J had been tightened, the whole suspension was raised by turning M. This operation was carried out in order to determine whether the wire D had been screwed just far enough into the collar U so that, when the system was raised, the plane of the vane might be parallel to that of the annulus. To remedy any want of parallelism, the suspended system was lowered again, and the collar U turned with respect to D. After going through the process of adjusting many times, the vane was finally placed in the proper adjustment, which was maintained by means of the nut, V.

In order to avoid breaking the fibre, during the process of closing the various openings in the apparatus, the suspension was lowered to resting loops not shown in the figure. All of the openings mentioned above were now closed save P, to which the connecting tubes shown in Figure 5 were joined.

The McLeod Gauges. — Two McLeod gauges of the form shown at A, Figure 5, were used. They have different factors, one of them being suited for the measurement of comparatively high pressures, while the other, the highest factor of which is about 69,000, is suited for the highest vacua. At B B are air-traps to prevent small bubbles of air, which may have adhered to the glass tube below, from slowly rising and destroying the vacuum. As it was necessary to keep the mercury in the gauge for some time and to keep it clean, the ordinary method of raising the mercury by raising a reservoir of it attached by rubber tubing to the barometer tube of the gauge, was replaced by the method which Figure 5 will explain. Here are two reservoirs C and D, attached to their respective gauges. The entrance to each is fitted with a stopcock, and the single leading tube has a three-way cock at E. F is an air-tight reservoir, S a pipe leading to the water tap, and T a waste tap. Water rises through S and forces air from the top of F against the mercury surfaces in C and D.

Arrangement of the Apparatus as a whole. — Figure 5 shows how these three pieces of apparatus are connected. It will be seen that the viscosity apparatus and the transpiration apparatus are joined by the glass tube L, and that a common tube leads from them to the pump by way of a tube, M, containing granular silver, and one, N, containing sulphur which has been fused and then powdered. The sulphur is intended to prevent mercury vapor from passing from the parts of the apparatus in which there is mercury to the other parts. The silver is to absorb the sulphur vapor. As a means of testing the purity of the air used, a spectrum tube, not shown in Figure 5, is inserted between the tube containing silver and the main part of the apparatus. The figure shows how the McLeod gauges are connected to the other parts of the apparatus. The tube, P, leads to the pump. The connection between the viscosity apparatus and the transpiration instrument is a long spiral tube, so that the former may be rotated through a considerable angle without disturbing the latter.

The piece marked A′ is for the purpose of admitting dry air or other gases, and consists of about 2.5 m. of 2.5 cm. tubing containing phosphoric anhydride, and 1.25 m. containing chloride of calcium. The gas is admitted through a tube, G, whose end passes under the flared out end of the barometer tube, H. The bottle, I, contains mercury forming a seal. The drying tubes may be connected to a bellows, if air is to be experimented with, or to a gas generator. The gas rises in bubbles through the mercury to the bulb, J. The small bent tube K is to prevent the mercury from being driven through the apparatus when a bubble of gas rises.

The mercury pump has no stop-cocks. It has one mercury-sealed valve. The auxiliary is a mechanical pump which can reduce the pressure to two or three millimeters.

The viscosity apparatus, the transpiration apparatus, and the gauges, are placed on an iron support inside of an electric oven. The first and second pieces are each placed in a double walled sheet iron box, whose wall space is packed with asbestos. The fronts of these boxes are removable, as is also the front of the oven. The purpose of the oven is to provide means of heating the whole apparatus to a high temperature to insure drying and to free the inside of the glass from the carbon dioxide which invariably adheres at ordinary temperatures and pressures.

When the different parts of the apparatus were set in position and sealed together, and before the suspensions had been raised, or any pumping had been done, the front of the oven was closed and heat applied so as to maintain the temperature at 200° C. during a whole day. There was no cracking of the glass, so that one was assured that the joints had all been well annealed.

Drying Process.

This process consisted in first pumping out the air until a pressure of a few millimeters of mercury was reached, then forcing in through the tube G a large quantity of dry air. This was pumped out again, and more dry air then forced in. This process was repeated many times. After this preliminary drying, and when the pressure was low, the front of the oven was closed and a current turned on sufficient to raise the temperature to 200° C. This temperature was maintained for several hours, and during this time the pump was kept going, and for about one hour the connecting tubes were kept hot by a Bunsen flame.

Perhaps it may be well to state here, that in the effort to distinguish between a real leak in the apparatus and what will produce a similar effect, viz., the slow "evaporation" of the gas from the inner surface of the vessel, it was found expedient to remove the sulphur tube and silver tube. Under these conditions the pumping can be done much more quickly. When the heating and pumping process above described had been resorted to, and after the oven had been allowed to cool down to room temperature, it was found that a vacuum of about 0.0003 mm., as indicated by the gauge, had been reached. When, however, the apparatus was simply allowed to stand the pressure greatly diminished, and a condition was finally reached when a slight rise in the temperature of the room would cause the pressure to increase greatly, much more than could be accounted for by the temperature coefficient. Then again a small decrease in temperature, say four or five degrees, would cause a corresponding anomalous decrease in the pressure. In one case where the temperature of the room reached 14° C. and remained there for over a day, the pressure was less than one third of what it was when the temperature had reached 18° C. for the first time after the oven had been allowed to cool down. This is mentioned merely to show how important a part the temperature of the glass plays in determining what the vacuum in the apparatus at any time is.[20]

After the complete drying process the spectrum showed that the moisture had been removed, and the vessel was then filled with dry air.

The suspensions were then raised to the proper height by turning the magnet which controls the armature, A, Figure 2, and M, Figure 4. A cathetometer was used to determine when the swinging disk in the viscosity apparatus was midway between the other two. As the mica vane could not be well viewed with the cathetometer, its proper adjustment was judged by its symmetry of position with respect to the surrounding ring of mica.

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[20] After it had been proved that there was no leak in the apparatus the sulphur and silver tubes were replaced.

Method of Experiment.

Viscosity Apparatus. — This instrument is held in position at the top by a snugly fitting collar fixed in the top of the enclosing box. It is fixed at the bottom, as shown in Figure 5, so that by giving the arm, L, a slight, slow angular movement, the disk can be set rotating without giving it a serious pendulous motion. This is especially true at the higher pressures, but more care in starting is necessary when the gas becomes rarer. About three quarters of an hour's training is always given to the fibre before observations begin. After allowing everything to become steady, the position of rest for every swing is observed, and when the damping is rapid about seven or eight swings is all one can get for any one start given to the disk. Sufficient accuracy is obtained by using the results got from starting the disk four times. This applies only to the work at the higher pressures; as will be seen, the plan is changed when the density diminishes, for then many more arcs can be obtained before the amplitude of swing becomes too small.

In getting the mean decrement, the logarithms of the fifth, sixth, seventh, and eighth arcs are taken respectively from the first, second, third, and fourth, and the result in each case divided by four. At the lower pressures, where many more arcs can be obtained in a series, and where as a consequence the error in observing any one is increased, the eleventh is taken from the first, the twelfth from the second, and so on, and the results divided each by ten. The decrement used is then got by taking the mean of some sixteen such decrements.

Transpiration Apparatus. The source of light used for the purpose of illuminating the blackened face of the mica vane and annulus was a twenty-five candle power incandescent lamp, suited to a voltage of forty. The current used was that from a storage battery.

The zero position of this instrument is that position of the armature, M, Figure 4, which allows the vane to hang in the plane of the annulus, when there is no irradiation of any part of the apparatus. To read this position, there is a pointer attached to the magnet which controls the armature. The pointer coincides with a radius of a graduated circle placed on the upper magnet platform, concentric with the tube in which the fibre is. Great care is taken in setting to turn the magnet in one direction so that any movement which the magnet, and therefore the pointer, may make, which is not made by the armature, may be eliminated. It was found, however, that with an ordinary permanent magnet this elimination was not satisfactory, so that a strong electro-magnet was finally used. This proved quite satisfactory, and certainly the error here is not so great as that arising from the innate difficulty of setting the vane to the proper position. To determine when the vane is in the proper plane a telescope is focussed on a scale reflected in the mirror, E, and the point on the scale which corresponds to the proper position of the vane is noted. This, of course, remains the same so long as the disposition of the apparatus is unchanged.

From what has been said previously on the relation between the force on the vane and the size of the transpiration space, it follows that for any given pressure the force on the vane in this instrument will be greatest when the vane and annulus are in the same plane, for then the annular space is least. The object is to measure the force when this disposition is secured. Since the blackened surface is turned towards the heat source, when this surface is irradiated the vane recedes, because pressure grows on the hot side. By turning the magnet, enough torsion is given to the fibre to bring the vane forward again into its zero position. If too much torsion is given, and the vane is thrown in the least past the annulus, then it continues to swing forward, because the force on it diminishes as its distance from the annulus increases. A glance through the telescope shows when this has happened, and in this case the magnet must be turned back and another trial be made. Much practice is required that one may set the vane, even with moderate success. The difference between the readings on the circular scale when the vane is and when it is not irradiated is the torsion necessary. As is allowable, the force on the vane is assumed to be proportional to the angle of torsion when the latter is not large.

In this maximum effect we have a method of finding the zero position without actually looking to see that the vane and annulus are co-planar. The method consists in illuminating the vane, setting to the maximum point, and noting the mark indicated by the cross-hair of the telescope. This mark indicates the zero position. It is really best to determine the two balancing points for any angle of torsion, one when the vane is behind the annulus and the other when it is in front. As the torsion on the fibre is increased the places of balance, indicated by the cross-hair of the telescope, approach each other, and the mean of these locates with sufficient accuracy the maximum point.

So long as the shutter over the window of the containing box is closed, the zero position should remain unchanged. With ordinary jacketing, it was found that this was not the case. Indeed, in some cases there was as much variation as 25°. This could be due to one or all of three causes, viz.: convection currents, unequal heating of the bulb or indeed any change of temperature of the apparatus, or to small charges of electricity on the vane or containing vessel.

To guard against the first of these, as has been said, the apparatus was enclosed in a double-walled box with an asbestos interspace. But since the McLeod gauge, which, of course, must be left uncovered, and is connected with this instrument through various obstructed tubes, responds quickly to any change of temperature in the room, any change in this temperature will cause a flow of gas either towards the transpiration instrument or away from it. To overcome this difficulty, a house whose walls are of asbestos and double, with an air interspace, was built surrounding the apparatus. In it were placed four electric heaters, the current through which can be made and broken by a relay worked by a battery circuit, in which is a thermostatic strip. In this way the temperature of the whole space — about 24 cubic meters — enclosed can be maintained very nearly constant. The temperature was kept near 22° C. day and night, so that one might begin work at any time and know that the apparatus was not far from a uniform temperature.

The second cause, that of temperature change in the instrument itself, proved the most serious. Before the constant temperature room was resorted to, invariably during the day with rising temperature there was an advance in the zero position, i. e., the apparatus acted as if the vane were illuminated, while during the night, as the temperature fell, the contrary was the case. The blackened surface would, of course, absorb heat which was being conducted in through the walls of the containing vessel faster than the clear surface, and a transpiration would begin which would have the same direction as that arising from the illumination of the vane. When the rise of temperature ceased, then the two surfaces would gradually reach the same temperature, and in the absence of cause for transpiration the vane could be put into its zero position without using force. Only by enclosing the bulb of the apparatus in a pretty thick (1.5 mm.) brass shell (the object of which is to distribute quickly the heat which enters from without) and then packing the box full of cotton wool, leaving only a channel to the mirror, E, closing the door of the box, and then packing it all around with wool, was it possible, even with the help of the constant temperature room, to remove serious disturbances arising from this cause. It was also necessary to prevent, as much as possible, light from shining on any part of the apparatus, as this would produce unequal heating. The light from the telescope lamp passed through a water window, and was allowed to shine only when observations were being made.

The trouble arising from electrical charges can readily be distinguished from those just mentioned, for, with it, the vane tends to take up a fixed position, and force is required to change that position in either direction. The force on the vane arising from unequal heating is unidirectional. Trouble from electrical charges showed itself most distressingly after an attempt to wrap the bulb in tin foil for the purpose of minimizing outside heat effects. The friction between the metal and the glass produced a very pronounced charge. It was noticed also that after filling the vessel with air in the manner already described the suspension, or rather the whole apparatus, had become so charged that its attraction for a pith-ball could easily be detected. A rather weak radioactive substance was placed within the brass shield as near as possible to the vane, and after several days the charge had disappeared. During the time when the box was being packed with the wool the brass shield was kept to earth, and it has remained so connected.

By attending carefully to all of these points it was found possible to control the vane so that after the current was turned on in the lamp and before the shutter was opened there was no great change in the zero reading.

Since the object is to establish a relation between pressure and the difference of pressure maintained between the ends of a transpiration space, and since any surface along which there is a variation of temperature must give rise to transpiration, it is clear that if the entire bulb of the instrument is illuminated, there will be various causes contributing to the total transpiration pressure, because the different parts of the surface of the bulb will not be equally heated. Now Sutherland has developed equation (IV) on the assumption that the only transpiration space contributing to the effect is annular in form. This is the simplest form of space to deal with analytically, and the attempt to establish the desired relation is likely to prove more successful with such a space than with an irregularly shaped surface unequally heated. In the form of the apparatus finally adopted the bulb is of a somewhat different form from that shown in Figure 3. The mica ring is placed against the conical posterior part of the glass vessel, so that the whole of the vane and ring, together with that narrow part of the glass in contact with the ring, may be irradiated. Thus the gas will transpire past not only the inner edge of the ring, but also the outer edge, and yet the whole bulb is not heated. There are then really two concentric annuli, and no space by which difference of pressure can be effaced except through the transpiration spaces themselves.

Before packing the box with wool, the lamp was placed at its proper distance, about 80 cm., and in such a position that the light from it fell perpendicularly upon the vane. Then a bright metal diaphragm was placed on the glass opening in the door of the box just large enough so that the base of the cone of light from the lamp would cover the desired area. The only part of the bulb illuminated, other than the mere ring contiguous to the outer edge of the mica ring, was, then, that immediately within the box and opposite the hole in the diaphragm, so that the heated portion of the bulb was at least 16 cm. away from the vane. The spot of glass irradiated was only about 4 cm. in diameter. To minimize the heating of this by absorption, the light was made to pass through about 2.5 cm. of glass and about 7 cm. of water. The water was kept running from one bottle to another through a water window placed between the lamp and the apparatus. The temperature of the water suffered no perceptible change in the course of an experiment which lasted some three hours.

Perhaps the greatest difficulty experienced in the mere handling of this apparatus is that caused by the great inertia of the suspended system. It is easily seen that the least jar given to the apparatus in turning the control magnet may give to the system a momentum which is easily many times greater than the force which it is desired to measure. To make conditions as favorable as possible, the platform on which the magnet was placed was finally fastened so that it and the magnet were entirely free from the glass vessel and from the containing box; but the more or less jerky motion of the armature as it moves under the influence of the magnet still gives the same trouble, though in a much less degree. This difficulty is greatly increased at low pressure; for when the density is small, the viscosity of the air is so small that the vane, though large, is a very inefficient damper, and so when the suspended system suffers a slight disturbance it moves freely until it strikes one side of the containing vessel; then it rebounds, strikes the opposite side, and so on. It is true that the suspended system might have been made lighter than it is, but, at best, it must be rather heavy, for the form which it must assume in order that it may be put together is such as to preclude its being made as light as it should be for convenience.

It is clear also that the equilibrating torsion given to the fibre will depend upon how long the light is allowed to shine on the vane, because the vane, after it is heated, becomes a radiator, and the surrounding vessel is heated by radiation from the vane. This increase in the force goes on for some hours, and in that time there is a chance for very inconvenient changes in the temperature of the vessel and vane due to extraneous sources of heat, e. g. the different distribution of the heat in the room when the lamp is lighted, and when the experimenter is in the room. The source of light was enclosed in a double-walled asbestos chimney, which communicated at the top with a flue, but the sides of this chimney became heated in time. It was found best to set the illuminated vane as accurately as possible (a process taking from a half to three quarters of an hour), and then close the shutter and set again. The difference is taken as the torsion necessary. The results given below were obtained in this way. It was found that the zero position of the vane, i. e., the position of equilibrium after the shutter had been closed, did not change as much as 5° in an hour. At this point, however, an error is introduced which tends to make the result too large. The maximum angle of torsion obtained was 250°, so that the error here considered is not very serious for such an angle of torsion; but as the pressure is diminished the angle diminishes, and at pressures where the angle of torsion has fallen to, say, 25°, this error becomes serious.

McLeod Gauges. — In reading the gauges, special attention was paid to these points:

First, the mercury, when lowered, was never allowed to sink much below the point of connection between the volume tube and the pressure tube. The reason for taking this precaution is that the narrow bore of the latter offers considerable resistance to the passage of gas into it, and thus when the mercury is raised after having been allowed to sink well down, more than the right quantity of gas is thrown up into the bulb, and the pressure will read too high.[21] Still further to ensure accuracy on this point, the mercury is made to rise very slowly indeed until it has passed the entrance to the pressure tube.

Second, during the ascent of the mercury the bulb and tubes are tapped repeatedly; and in setting on any mark, care is taken to tap both tubes until one is certain that the mercury has really settled down.

Third, in the measurement of each pressure the mercury is raised twice, and, for each time, generally one measurement is taken at each of two marks on the volume tube of the gauge. The cathetometer is used to learn when the setting is right, and to measure the mercury column.

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[21] Ramsay and Baly mention the necessity of taking this precaution. Phil. Mag., [v], **38**, 1894.

Of course, the nearer the top of the volume tube one works, the more serious the error made in setting becomes. Indeed, it is better not to use a mark which is less than, say, 2 cm. from the top.

Results.

The following table gives the results obtained so far. In the first column is given the gas pressure as measured by the McLeod gauge. The pressure is expressed in millimeters of mercury. The gas employed in this case was air. In the second column the values of the logarithmic decrement in the viscosity apparatus corresponding to these pressures are given, and in the third the amount of torsion given to the fibre of the transpiration instrument at the various pressures to balance the force due to gas action on the vane. The significance of the fourth and fifth columns will appear later.

The table shows how slowly the logarithmic decrement decreases at first as p is diminished. That there is an appreciable diminution in l even when the pressure is great, e. g. 20 cm., is due to the fact that the distance between the fixed and moving surfaces in the apparatus is very small, and therefore the coefficient of slip, β, is comparable with it. To put the matter in another way, l is large because of the small distance between the plates, so that small relative changes in it can be perceived. Yet the table shows that at a pressure of 5.8 cm. the resistance to the moving disk is only one half per cent smaller than it is at atmospheric pressure.

With falling pressure the rate of decrease of l increases, and for pressures less than 0.01 mm. l and p become approximately proportional. At such pressures the mean free path of the molecules of air is over half a centimeter in length, and is therefore several times the distance between the fixed and moving disks, so that the friction is largely superficial. Figure 6 shows the relation between l and p over the range from where $p = 0.530$ mm. to where $p = 0.00085$ mm. A unit on the axis of abscissas represents a pressure of 0.01 mm., and a unit on the axis of ordinates represents $l = 0.00333$.

The resistance which the disk meets is due to at least two causes, and we shall see that there is some evidence that there is a third. There is first the friction of the air on the disk, and second, the friction in the suspending fibre. The former diminishes with decreasing density of air, but the latter is a constant provided that the temperature of the fibre is maintained constant, as was the case in these experiments. The change in the temperature of the apparatus from day to day was rarely as much as 0°.5 C., and was generally much less than this during the course of one experiment.

The proportionality already referred to would evidently be more exact if the constant part of the resistance, i. e., that due to the imperfect elasticity of the fibre itself, could be determined and taken from the total resistance. An effort has been made to determine what is the resistance due to the air in the vessel alone. The method of procedure was to pump out the air to a certain low pressure, measure the decrement, then pump to a still lower pressure, measure the decrement again, and so on. If the exhaustion be carried far enough, a limiting value for l should be approached. This did, indeed, happen, but the limiting value seemed larger than one would have expected it to be; for it would seem that the friction in such a suspending fibre should be exceedingly small. The following results show how the decrement and the pressure diminished as the pumping proceeded. Pump began at 10.30 A.M., when the pressure was about 0.001 mm.

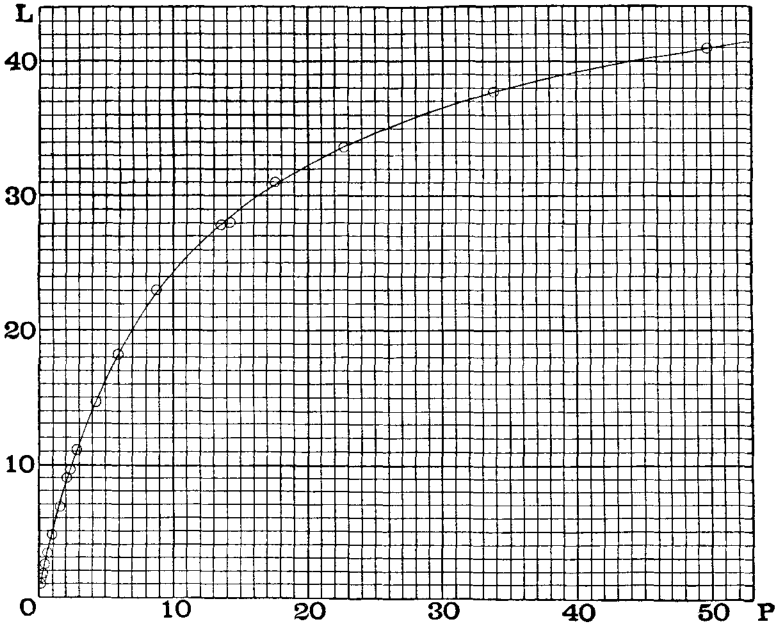

FIGURE 6. The curve shows the relation between pressure and logarithmic decrement over the range of pressure, $p = 0.530$ mm. to $p = 0.00085$ mm. The unit of pressure is 0.01 mm. The unit of logarithmic decrement is 0.00333.

TABLE I.

p	l	R	C	p, cal. from l.	p, cal. from R.
mm.					
761.5	0.1655				
514.1	0.1652				
200.8	0.1650				
58.4	0.1648				
1.42	0.1533	62°			
0.824	0.1461	122°	0.1112		
0.498	0.1364	170°	0.1076		
0.339	0.1258	190°	0.1085		
0.226	0.1121	202°	0.1096		
0.176	0.1032		0.1082		
0.142	0.0938		0.1108		
0.135	0.0931	248°	0.1072		
0.0880	0.0765	240°	0.1050	0.0913	
0.0590	0.0607	210°	0.1051	0.0611	
0.0425	0.0491	170°	0.1048	0.0442	
0.0284	0.0370	143°	0.1040	0.0297	0.033, or 0.41
0.0232	0.0324	115°	0.1012	0.0249	0.025, or 0.54
0.0211	0.0300	115°	0.1017	0.0226	0.025, or 0.54
0.0148	0.0228	70°	0.1011	0.0159	0.015, or 0.93
0.00928	0.01580	50°	0.1002	0.0101	0.010, or 1.32
0.00595	0.01113		0.1000	0.00649	
0.00410	0.00837		0.1002	0.00446	
0.00261	0.00607		0.1007	0.00282	
0.00177	0.00450		0.1113	0.00173	
0.00186	0.00478		0.1053	0.00193	
0.00085	0.00332		0.0999	0.00093	
0.00024	0.00229		0.1119	0.00023	

Decrement at 11.30 A.M. was 0.00234
" " 12.30 P.M. " 0.00234
" " 4.45 P.M. " 0.00228

In this time there had been about four hours of continuous pumping. According to the gauge the pressure at the final stage was 0.00019 mm. It is possible, however, that the pressure was not completely equalized through the apparatus, although the time allowed to elapse between successive strokes of the pump was sufficient to insure that the equalization at this time must have been nearly complete. At 9.30 P.M. the pressure had become 0.00024 mm. and the decrement at the same time was 0.00229. No pumping had been done since the last measurements were made at 4.45 P.M. At 12.10 P.M. on the next day the pressure was 0.00027 mm., while the decrement was 0.00236. The limiting value of the decrement seems to be not much less than 0.00230.

It is probable that this limiting value of l is due not only to the friction in the fibre, but also to the friction of mercury vapor on the disk. It was noticed that in the spectrum, after the current had been allowed to run for some time, the mercury lines gradually appeared, as if there were some slight deposit of mercury on the walls of the tube which slowly evaporated under the heat due to the current. This would indicate the possibility of a like deposit on the inner surface of the other parts of the apparatus, for it will be remembered that the spectrum tube is placed between the tube of silver and the main part of the apparatus.

The fact that the limiting value of l obtained is larger than had been anticipated would be explained by the presence of mercury vapor. Its presence would add a third factor to the resistance experienced by the disk, and one which would diminish certainly while pumping was in progress, but which would increase again as evaporation of the mercury, or the diffusion of its vapor to the viscosity apparatus from the other parts of the apparatus, proceeded.

Proceeding for the moment on the assumption that the decrement due to the air is proportional to the air pressure in the apparatus where the pressure is small, we can deduce the value of that part of the decrement due to friction in the fibre and the friction of the mercury vapor combined. For if m represents this constant part of the decrement we have,

$$p_1 : (l_1 - m) :: p_2 : (l_2 - m)$$

Also, for corresponding values for p and l, we find from Table I,

$$p_1 = 0.00177 \text{ mm.} \qquad l_1 = 0.00450$$
$$p_2 = 0.00024 \text{ mm.} \qquad l_2 = 0.00229$$

Using these values, we find for m the value 0.00194. If now this quantity m be inserted for K in the equation relating p and l, as given above, viz.:

$$\left(\frac{\lambda - K}{l - K} - 1\right) p = C$$

C can be determined for the different pressures, or C having been determined for the higher pressures, for example for those between 1 mm. and 0.1 mm., the values of p corresponding to the lower values of l can be solved for. The value of λ used in this process was 0.1655, and the value of C used was the mean of the first seven values given in the fourth column of Table I. The fourth and fifth columns of Table I give these results.

A comparison of the numbers given in the fifth column with those in the first show that the values of p deduced from the observed values of l are in general greater than the observed values of p.

It must be admitted at once that the foregoing method of getting the value of K is a very imperfect one, and it is inserted only tentatively. It is now proposed to remove the sulphur and silver tubes and place a vessel containing liquid air so as to surround a portion of the tubes connecting the pump with the apparatus, so that not only may all vapor be removed, but also that the very highest possible vacuum may be reached. The decrement will then be measured. This should give the value of the part of the decrement due to friction in the fibre. The liquid air will then be replaced by liquid carbon dioxide, which will remove the vapor but not the gas to be experimented with. It is hoped that this method of procedure will settle the only point that seems to remain in doubt in this part of the investigation. The full discussion of the law relating l and p is reserved until this step has been taken.

With regard to the results given in Table I for the transpiration instrument it must be stated that the smaller numbers in the third column may quite easily have an error of ten per cent. Figure 7 shows the results graphically. In this figure, a unit on the axis of abscissas corresponds to 0.01 mm. of pressure, while on the other axis a unit represents 10° of torsion. That portion of the curve which corresponds to pressures below those for which the torsion is a maximum, approaches a straight line, and it is apparently a line which passes very nearly through the origin. It is perhaps allowable to assume that it does go through the origin; for the force here, unlike the friction in the other apparatus, depends entirely on the gaseous contents of the vessel. There can be no force when these have all been removed. If there is any vapor present, however, there will still remain some force when the air has been pumped out. As we have seen, there are indications that mercury vapor is present, but in small quantity, and at the lowest pressure at which the transpiration apparatus was used the torsion necessary to balance the force on the vane, were the contents of the vessel the vapor alone, could not be more than 5°. This is readily seen by considering the torsion at these small pressures to be proportional to the pressure.

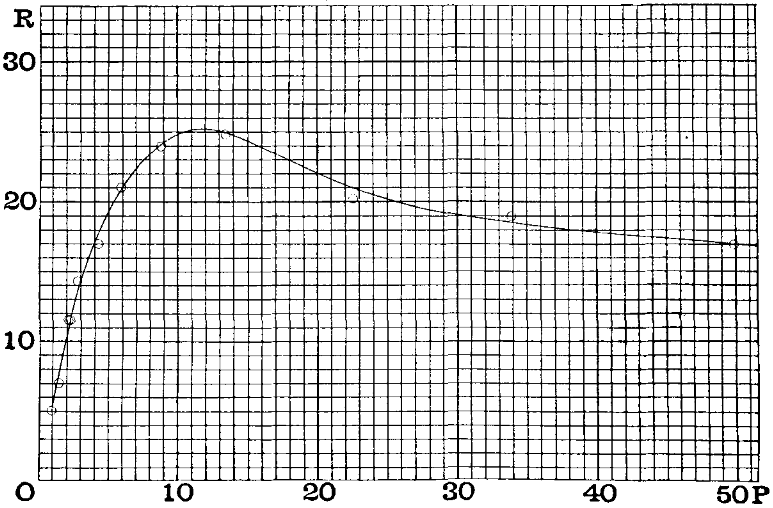

FIGURE 7. The curve shows the relation between pressure and force due to gas action on a circular vane over the range of pressure, $p = 0.498$ mm. to $p = 0.0093$ mm. The unit of pressure is 0.01 mm. The unit on the axis of ordinates represents an angle of torsion of 10° given to the fibre supporting the vane. This torsion is proportional to the force due to gas action on the vane.

If we confine our attention to the part of the curve from the origin up to the maximum point, and use the larger values of the torsion from which to determine the constants in Sutherland's equation, viz.:

$$F = \frac{C}{Ap + B + 1/p} \qquad \text{(IV)}$$

we get $A = 0.00187$, $B = 0.0152$, and $C = 25.67$. In the computations 0.005 mm. has been taken as the unit of pressure and 1° as the unit of torsion. If now we insert these numbers in the above equation and solve for the values of p, which correspond to the various values of the torsion given in the third column of the table, the results given in the last column are obtained. The form of the equation shows that there are two values of p for any particular value of the torsion. If we differentiate (IV) with respect to p to get the value of p, for which the torsion is a maximum, we get $p^2 = 1/A$, or $p = 23.1$. With the millimeter as the unit this is the same as $p = 0.116$ mm. The value of the torsion for this value of p is 254°. The curve indicates a maximum where $p = 0.120$ mm., and its value is 252°. The further discussion of equation (IV) is reserved until more and more accurate data have been collected.

With the object of rendering the instrument more easily handled, and thus making it susceptible of greater accuracy, it is proposed now to modify the form of the suspended system in order to reduce its moment of inertia. When this has been done the results from this instrument should be quite as reliable as those from the other, and with the experience which has been gained both in constructing and handling the apparatus it seems quite certain that the desired object will soon be accomplished.

Grateful acknowledgment is made to Professor Trowbridge for placing at my disposal all the resources of the laboratory, including the services of a mechanician[22] and of a glass-blower.[23] Without the assistance rendered by them the apparatus could not have been constructed. To Professor Hall, who first called my attention to the problem, I am indebted for much advice and encouragement.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
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ON THE CONDITIONS TO BE SATISFIED IF THE SUMS OF THE CORRESPONDING MEMBERS OF TWO PAIRS OF ORTHOGONAL FUNCTIONS OF TWO VARIABLES ARE TO BE THEMSELVES ORTHOGONAL.

BY B. OSGOOD PEIRCE.

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ON THE CONDITIONS TO BE SATISFIED IF THE SUMS OF THE CORRESPONDING MEMBERS OF TWO PAIRS OF ORTHOGONAL FUNCTIONS OF TWO VARIABLES ARE TO BE THEMSELVES ORTHOGONAL.

By B. Osgood Peirce.

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If $\phi_1(x, y)$, $\phi_2(x, y)$ are the potential functions due to two columnar distributions of matter the lines of which are perpendicular to the xy plane, and if $\psi_1(x, y)$, $\psi_2(x, y)$ are conjugate to ϕ_1 and ϕ_2, respectively, the families of curves obtained by equating ψ_1 and ψ_2 to parameters, are lines of force of the two distributions. Moreover, $\phi_1 + \phi_2$ is the potential function due to a combination of the two distributions, and the function $\psi_1 + \psi_2$ equated to a parameter gives the corresponding lines of force. The fact that if (ϕ_1, ψ_1) are any pair of conjugate functions and (ϕ_2, ψ_2) any other such pair, the functions $(a\phi_1 + b\phi_2, a\psi_1 + b\psi_2)$ are also conjugate — with similar facts for other classes of functions — lies at the foundation of the graphical methods so successfully used by Maxwell[1] and by others in drawing equipotential lines, and lines of force or flow, due to combinations of simple elements. If (ϕ_1, ψ_1) are merely a pair of orthogonal functions and (ϕ_2, ψ_2) another such pair, it is generally not true that $(\phi_1 + \phi_2, \psi_1 + \psi_2)$ are an orthogonal pair: thus (x, y), $(x^2 + y^2, y/x)$ are pairs of orthogonal functions, but $x + x^2 + y^2$, $y + y/x$ are not orthogonal.

In certain classes of physical problems one encounters potential functions which are not themselves harmonic and the lines of which are not possible lines of any harmonic function, and it is often desirable in cases where the analytical processes become too complex, to determine graphically the forms of lines of force or flow due to a combination of two simple elements. This note discusses briefly the conditions under which the ordinary method of procedure is possible.

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[1] Maxwell, Treatise on Electricity and Magnetism, Vol. I, Ch. VII. Minchin, Uniplanar Kinematics, § 112. See also P. W. Bridgman, The electrostatic field surrounding two special columnar elements, These Proceedings, **41**, 28.

Let (α, β) and (λ, μ) be two pairs of orthogonal functions of the two variables (x, y), so that

$$\frac{\partial \alpha}{\partial x} \cdot \frac{\partial \beta}{\partial x} + \frac{\partial \alpha}{\partial y} \cdot \frac{\partial \beta}{\partial y} = 0, \tag{1}$$

$$\frac{\partial \lambda}{\partial x} \cdot \frac{\partial \mu}{\partial x} + \frac{\partial \lambda}{\partial y} \cdot \frac{\partial \mu}{\partial y} = 0\,; \tag{2}$$

then if $(\alpha + \lambda, \beta + \mu)$ are to form an orthogonal pair, the equation

$$\left(\frac{\partial \alpha}{\partial x} + \frac{\partial \lambda}{\partial x}\right)\left(\frac{\partial \beta}{\partial x} + \frac{\partial \mu}{\partial x}\right) + \left(\frac{\partial \alpha}{\partial y} + \frac{\partial \lambda}{\partial y}\right)\left(\frac{\partial \beta}{\partial y} + \frac{\partial \mu}{\partial y}\right) = 0 \tag{3}$$

must be identically satisfied. Since (1) and (2) are true, (3) takes the form

$$\left(\frac{\partial \lambda}{\partial x} \cdot \frac{\partial \beta}{\partial x} + \frac{\partial \lambda}{\partial y} \cdot \frac{\partial \beta}{\partial y}\right) + \left(\frac{\partial \alpha}{\partial x} \cdot \frac{\partial \mu}{\partial x} + \frac{\partial \alpha}{\partial y} \cdot \frac{\partial \mu}{\partial y}\right) = 0. \tag{4}$$

If h_α, h_β, h_λ, h_μ represent the values of the gradients of α, β, λ, μ, and if the angle at any point between the directions in which λ and β increase most rapidly be denoted by $[\lambda, \beta]$, (4) becomes

$$h_\lambda \cdot h_\beta \cdot \cos\,[\lambda, \beta] + h_\alpha \cdot h_\mu \cdot \cos\,[\alpha, \mu] = 0. \tag{5}$$

Whatever the sequence of the directions of the gradient vectors might be, the two angles which appear in (5) would be either equal or supplementary, and their cosines would be equal in absolute value, but the gradients themselves are intrinsically positive and the sequences must therefore be such that

$$h_\lambda / h_\mu = h_\alpha / h_\beta\,. \tag{6}$$

Suppose that in the case of two given pairs of orthogonal functions $(\alpha, \beta,)$ (λ, μ), the necessary condition (6) is satisfied, and that the value of the gradient ratio, h_α / h_β, obtained from the given values of α and β, is the function Ω of x and y; then

$$\left(\frac{\partial \alpha}{\partial x}\right)^2 + \left(\frac{\partial \alpha}{\partial y}\right)^2 = \Omega^2\left(\frac{\partial \beta}{\partial x}\right)^2 + \Omega^2\left(\frac{\partial \beta}{\partial y}\right)^2, \tag{7}$$

and if, for $(\partial \alpha)^2/(\partial y)^2$ in this equation, we substitute the value

$$\left(\frac{\partial \alpha}{\partial x}\right)^2 \left(\frac{\partial \beta}{\partial x}\right)^2 \Big/ \left(\frac{\partial \beta}{\partial y}\right)^2 \tag{8}$$

obtained from (1), it appears, since the gradient of a real function cannot vanish, that

$$\frac{\partial \alpha}{\partial x} = \pm\, \Omega \left(\frac{\partial \beta}{\partial y}\right). \tag{9}$$

If for $(\partial \alpha)^2/(\partial x)^2$ in (7), we substitute its equivalent

$$\left(\frac{\partial \alpha}{\partial y}\right)^2 \left(\frac{\partial \beta}{\partial y}\right)^2 \Big/ \left(\frac{\partial \beta}{\partial x}\right)^2 \tag{10}$$

derived from (1), we shall learn that

$$\frac{\partial \alpha}{\partial y} = \mp\, \Omega \left(\frac{\partial \beta}{\partial x}\right). \tag{11}$$

Either the upper signs or the lower signs must be used in (9) and (11).

If now we treat the equation

$$h_\lambda^{\,2} = \Omega^2 h_\mu^{\,2} \tag{12}$$

in a similar way we shall obtain the equations

$$\frac{\partial \lambda}{\partial x} = \pm\, \Omega \left(\frac{\partial \mu}{\partial y}\right), \tag{13}$$

$$\frac{\partial \lambda}{\partial y} = \mp\, \Omega \left(\frac{\partial \mu}{\partial x}\right), \tag{14}$$

and, so far as the relation (12) is concerned, we may use either the upper signs or the lower signs, but if (4) is to be identically satisfied, the *same* sign must be used in (9) and (13) and the sign opposite to this in (11) and (14). Equation (6), then, together with the proper choice of sequence of directions for the gradient vectors which corresponds to the convention with regard to signs just made, will ensure the orthogonality of $\alpha + \lambda$, $\beta + \mu$. For practical purposes, however, it is well to approach the problem from another side.

If (α, β) and (λ, μ) are given pairs of orthogonal functions, and if we denote the given scalar point functions obtained by dividing $\partial\beta/\partial x$ by $\partial\alpha/\partial y$, and by dividing $\partial\mu/\partial x$ by $\partial\lambda/\partial y$, by ζ and η, the equations (1) and (2) can be written in the forms

$$\frac{\partial\beta}{\partial x}\Big/\frac{\partial\alpha}{\partial y} = -\frac{\partial\beta}{\partial y}\Big/\frac{\partial\alpha}{\partial x} = \zeta, \tag{15}$$

and

$$\frac{\partial\mu}{\partial x}\Big/\frac{\partial\lambda}{\partial y} = -\frac{\partial\mu}{\partial y}\Big/\frac{\partial\lambda}{\partial x} = \eta, \tag{16}$$

or

$$\frac{\partial\beta}{\partial x} = \zeta\cdot\frac{\partial\alpha}{\partial y}, \qquad \frac{\partial\beta}{\partial y} = -\zeta\cdot\frac{\partial\alpha}{\partial x}, \tag{17}$$

and

$$\frac{\partial\mu}{\partial x} = \eta\cdot\frac{\partial\lambda}{\partial y}, \qquad \frac{\partial\mu}{\partial y} = -\eta\cdot\frac{\partial\lambda}{\partial x}. \tag{18}$$

If the values of the derivatives of β and μ given in (17) and (18) be substituted in (4) this equation becomes

$$(\zeta - \eta)\left(\frac{\partial\lambda}{\partial x}\cdot\frac{\partial\alpha}{\partial y} - \frac{\partial\alpha}{\partial x}\cdot\frac{\partial\lambda}{\partial y}\right) = 0, \tag{19}$$

and if $(\alpha + \lambda, \beta + \mu)$ are to be orthogonal, α and λ must be such as to satisfy it. If λ were expressible as a function of α, and μ as a function of β, the second factor would vanish, but this case is of no practical interest and (19) demands in general that ζ and η shall be identical, so that

$$\frac{\partial \beta}{\partial x} = \zeta \cdot \frac{\partial \alpha}{\partial y}, \qquad \frac{\partial \beta}{\partial y} = - \zeta \cdot \frac{\partial \alpha}{\partial x}, \tag{17}$$

and

$$\frac{\partial \mu}{\partial x} = \zeta \cdot \frac{\partial \lambda}{\partial y}, \qquad \frac{\partial \mu}{\partial y} - \zeta \cdot = \frac{\partial \lambda}{\partial x}. \tag{20}$$

If in these equations the arbitrary function ζ is made equal to unity, the conditions degenerate into the familiar definitions of any two pairs of conjugate functions.

In order that a single function (β) may exist the partial derivatives of which with respect to x and y shall be equal, respectively, to

$$\zeta \cdot \frac{\partial \alpha}{\partial y} \text{ and } - \zeta \cdot \frac{\partial \alpha}{\partial x},$$

it is necessary and it is sufficient that α and ζ should satisfy the condition

$$\frac{\partial}{\partial y}\left(\zeta \cdot \frac{\partial \alpha}{\partial y}\right) + \frac{\partial}{\partial x}\left(\zeta \cdot \frac{\partial \alpha}{\partial x}\right) = 0, \tag{21}$$

or

$$\frac{\partial \zeta}{\partial x} \cdot \frac{\partial \alpha}{\partial x} + \frac{\partial \zeta}{\partial y} \cdot \frac{\partial \alpha}{\partial y} + \zeta \cdot \nabla^2 (\alpha) = 0. \tag{22}$$

In order that μ may exist, ζ and λ must satisfy the equation

$$\frac{\partial \zeta}{\partial x} \cdot \frac{\partial \lambda}{\partial x} + \frac{\partial \zeta}{\partial y} \cdot \frac{\partial \lambda}{\partial y} + \zeta \cdot \nabla^2 (\lambda) = 0. \tag{23}$$

If $\log_e \zeta$ be represented by ϖ, the last two equations take the forms

$$\frac{\partial \varpi}{\partial x} \cdot \frac{\partial \alpha}{\partial x} + \frac{\partial \varpi}{\partial y} \cdot \frac{\partial \alpha}{\partial y} + \nabla^2 (\alpha) = 0, \tag{24}$$

$$\frac{\partial \varpi}{\partial x} \cdot \frac{\partial \lambda}{\partial x} + \frac{\partial \varpi}{\partial y} \cdot \frac{\partial \lambda}{\partial y} + \nabla^2 (\lambda) = 0, \tag{25}$$

and, if each of these be differentiated with respect to x and with respect to y, ϖ may be eliminated from the resulting equations and a necessary condition for α and λ obtained, which may be stated in the form of the determinantal equation —

$$\begin{vmatrix} \dfrac{\partial \alpha}{\partial x} & \dfrac{\partial \alpha}{\partial y} & 0 & 0 & 0 & \nabla^2(\alpha) \\ \dfrac{\partial \lambda}{\partial x} & \dfrac{\partial \lambda}{\partial y} & 0 & 0 & 0 & \nabla^2(\lambda) \\ \dfrac{\partial^2 \alpha}{\partial x^2} & \dfrac{\partial^2 \alpha}{\partial x \cdot \partial y} & \dfrac{\partial \alpha}{\partial x} & \dfrac{\partial \alpha}{\partial y} & 0 & \dfrac{\partial}{\partial x}\Big(\nabla^2(\alpha)\Big) \\ \dfrac{\partial^2 \lambda}{\partial x^2} & \dfrac{\partial^2 \lambda}{\partial x \cdot \partial y} & \dfrac{\partial \lambda}{\partial x} & \dfrac{\partial \lambda}{\partial y} & 0 & \dfrac{\partial}{\partial x}\Big(\nabla^2(\lambda)\Big) \\ \dfrac{\partial^2 \alpha}{\partial x \cdot \partial y} & \dfrac{\partial^2 \alpha}{\partial y^2} & 0 & \dfrac{\partial \alpha}{\partial x} & \dfrac{\partial \alpha}{\partial y} & \dfrac{\partial}{\partial y}\cdot\Big(\nabla^2(\alpha)\Big) \\ \dfrac{\partial^2 \lambda}{\partial x \cdot \partial y} & \dfrac{\partial^2 \lambda}{\partial y^2} & 0 & \dfrac{\partial \lambda}{\partial x} & \dfrac{\partial \lambda}{\partial y} & \dfrac{\partial}{\partial y}\cdot\Big(\nabla^2(\lambda)\Big) \end{vmatrix} = 0. \qquad (26)$$

If α and λ happen to be harmonic, the elements of the last column vanish and the equation is satisfied, as it should be.

It is possible to factor the determinant, after it has been reduced, and if

$$L \equiv \frac{\partial^2 \alpha}{\partial x^2} \cdot \frac{\partial \lambda}{\partial x} - \frac{\partial^2 \lambda}{\partial x^2} \cdot \frac{\partial \alpha}{\partial x} + \frac{\partial^2 \alpha}{\partial x \cdot \partial y} \cdot \frac{\partial \lambda}{\partial y} - \frac{\partial^2 \lambda}{\partial x \cdot \partial y} \cdot \frac{\partial \alpha}{\partial y},$$

$$M \equiv \frac{\partial^2 \alpha}{\partial x \cdot \partial y} \cdot \frac{\partial \lambda}{\partial x} - \frac{\partial^2 \lambda}{\partial x \cdot \partial y} \cdot \frac{\partial \alpha}{\partial x} + \frac{\partial^2 \alpha}{\partial y^2} \cdot \frac{\partial \lambda}{\partial y} - \frac{\partial^2 \lambda}{\partial y^2} \cdot \frac{\partial \alpha}{\partial y}, \qquad (27)$$

$$N \equiv \frac{\partial}{\partial x}\Big(\nabla^2(\alpha)\Big) \cdot \frac{\partial \lambda}{\partial x} - \frac{\partial}{\partial x}\Big(\nabla^2(\lambda)\Big) \cdot \frac{\partial \alpha}{\partial x} + \frac{\partial}{\partial y}\Big(\nabla^2(\alpha)\Big) \cdot \frac{\partial \lambda}{\partial y} - \frac{\partial}{\partial y}\Big(\nabla^2(\lambda)\Big) \cdot \frac{\partial \alpha}{\partial y},$$

the condition of (26) demands either that α and λ satisfy the equation

$$\begin{vmatrix} \dfrac{\partial \alpha}{\partial x} & \dfrac{\partial \alpha}{\partial y} & \nabla^2(\alpha) \\ \dfrac{\partial \lambda}{\partial x} & \dfrac{\partial \lambda}{\partial y} & \nabla^2(\lambda) \\ L & M & N \end{vmatrix} = 0, \qquad (28)$$

or else that α and λ have the same level curves: this last case, as being uninteresting, may be left out of account.

Sometimes (28) is more convenient than the unexpanded form of the same condition which follows immediately if we solve (24) and (25) for $\partial\varpi/\partial x$ and $\partial\varpi/\partial y$,

$$\frac{\partial\varpi}{\partial x} = \frac{\dfrac{\partial\alpha}{\partial y}\cdot\nabla^2(\lambda) - \dfrac{\partial\lambda}{\partial y}\cdot\nabla^2(\alpha)}{\dfrac{\partial\alpha}{\partial x}\cdot\dfrac{\partial\lambda}{\partial y} - \dfrac{\partial\alpha}{\partial y}\cdot\dfrac{\partial\lambda}{\partial x}}, \tag{29}$$

$$\frac{\partial\varpi}{\partial y} = \frac{\dfrac{\partial\lambda}{\partial x}\cdot\nabla^2(\alpha) - \dfrac{\partial\alpha}{\partial x}\cdot\nabla^2(\lambda)}{\dfrac{\partial\alpha}{\partial x}\cdot\dfrac{\partial\lambda}{\partial y} - \dfrac{\partial\alpha}{\partial y}\cdot\dfrac{\partial\lambda}{\partial x}}, \tag{30}$$

and equate the derivative with respect to y of the second member of the first equation to the derivative with respect to x of the second member of the other. This process yields the relation,

$$\frac{\partial}{\partial y}\left[\frac{\dfrac{\partial\alpha}{\partial y}\cdot\nabla^2(\lambda) - \dfrac{\partial\lambda}{\partial y}\cdot\nabla^2(\alpha)}{\dfrac{\partial\alpha}{\partial x}\cdot\dfrac{\partial\lambda}{\partial y} - \dfrac{\partial\alpha}{\partial y}\cdot\dfrac{\partial\lambda}{\partial x}}\right] = \frac{\partial}{\partial x}\left[\frac{\dfrac{\partial\lambda}{\partial x}\cdot\nabla^2(\alpha) - \dfrac{\partial\alpha}{\partial x}\cdot\nabla^2(\lambda)}{\dfrac{\partial\alpha}{\partial x}\cdot\dfrac{\partial\lambda}{\partial y} - \dfrac{\partial\alpha}{\partial y}\cdot\dfrac{\partial\lambda}{\partial x}}\right] \tag{31}$$

and it is possible to check the fact that (28) and (31) are equivalent by a straightforward but somewhat laborious comparison of the two.

If α and λ satisfy (31), a function ζ exists which satisfies (22) and (23), functions β and μ exist which satisfy (17) and (20), and (α, β), (λ, μ), $(\alpha + \lambda, \beta + \mu)$ are orthogonal pairs of functions.

If, for instance, both α and λ represent values in the xy plane of harmonic space functions (V, W) the level surfaces of which are surfaces of revolution about the x axis, so that

$$\frac{1}{y}\cdot\frac{\partial}{\partial y}\left(y\cdot\frac{\partial V}{\partial y}\right) + \frac{\partial^2 V}{\partial x^2} = 0 \tag{32}$$

with a similar equation for W,

$$\nabla^2(\alpha) = -\frac{1}{y}\cdot\frac{\partial\alpha}{\partial y}, \quad \nabla^2(\lambda) = -\frac{1}{y}\cdot\frac{\partial\lambda}{\partial y}, \tag{33}$$

equation (31) is satisfied and

$$\frac{\partial \varpi}{\partial x} = 0, \quad \frac{\partial \varpi}{\partial y} = \frac{1}{y}, \quad \zeta = c\,y. \tag{34}$$

In this case, if we put $c = 1$, β and μ are the Stokes functions corresponding to α and λ. If the level surfaces of the harmonic space functions, V and W, are surfaces of revolution about two different straight lines in the xy plane, the functions α and λ which represent the values of V and W in this plane do not in general satisfy (31).

Graphical superposition of the lines of force in the xy plane due to an infinitely long, homogeneous cylinder of revolution parallel to the axis, and to a homogeneous sphere with centre in the plane, will not in general yield the lines of force in the xy plane due to a combination of the two masses.

If α and λ are harmonic, any linear function (but no other than a linear function) of α is harmonic, and any two linear functions of α and λ satisfy (31). There generally exist, however, non-linear functions of α and λ which, although they are not harmonic, satisfy the condition. The functions $(x^2 - y^2)$, $(x^2 + y^2)^n$, the second of which is not harmonic, obey (31), as do the harmonic pair $(x^2 - y^2)$, $\log(x^2 + y^2)$.

As a simple example of the fact that a harmonic function and a function which is not even isothermal may satisfy the condition (31), we may consider $(2\,y^2 - x^2)$ and $(y^2 - x^2)$.

The non-isothermal functions $x^2 - a\,y^2$, $y^2 - a\,x^2$, which are solutions of the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} - \frac{\partial V}{x \cdot \partial x} - \frac{\partial V}{y \cdot \partial y} = 0, \tag{35}$$

evidently satisfy the equation (31).

If α and λ are any two solutions of the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + f'(x)\,\frac{\partial V}{\partial x} = 0, \tag{36}$$

where $f(x)$ is any given function of x, the condition (31) is satisfied and $\varpi = f(x)$.

If α and λ satisfy the equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + a\,\frac{\partial V}{\partial x} + b\;\frac{\partial V}{\partial y} = 0, \tag{37}$$

ϖ is of the form $a\,x + b\,y + c$.

In general α and λ must both be solutions of an equation of the form

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + P\cdot\frac{\partial V}{\partial x} + Q\cdot\frac{\partial V}{\partial y} = 0, \tag{38}$$

where P and Q are any functions of x and y such that $\partial P / \partial y = \partial Q / \partial x$.

The question whether if (α, β) and (λ, μ) are orthogonal pairs and $(\alpha + \lambda, \beta + \mu)$ is not an orthogonal pair, it is possible to find a function (B) of β, and a function (M) of μ such that $(\alpha + \lambda, B + M)$ shall form an orthogonal pair, has already been answered; for α and λ must satisfy (31) in any case, and if they do this, ϖ may be determined from (29) and (30) and B and M from (17) and (20).

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ON THE CORRECTION FOR THE EFFECT OF THE COUNTER ELECTROMOTIVE FORCE INDUCED IN A MOVING COIL GALVANOMETER WHEN THE INSTRUMENT IS USED BALLISTICALLY.

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Presented March 14, 1906. Received April 12, 1906.

At the present time, when on account of their many good qualities ballistic galvanometers of the moving coil type are in such general use, and when some of them fall into the hands of persons who are not acquainted with their vagaries, it seems desirable to emphasize the fact that the counter electromotive force induced in the coil as it swings in the field of its own permanent magnet is often large, and that in certain classes of absolute measurements the result might be in error by many per cent if the effect of this electromotive force were not allowed for. Of course, in comparing condenser charges, mirror, ballistic d'Arsonval galvanometers are usually entirely satisfactory, and, within wide limits, the "throws" of the coil, as measured on the scale of the instrument, are nearly proportional to the quantities of electricity sent impulsively through the coil. If, however, a moving coil galvanometer, the resistance of which is known, be connected in series with a resistance r, wound non-inductively, and an earth inductor, permanently mounted at a given station, it will usually be found that if r be varied and if no account be taken of the electromotive force induced in the galvanometer coil, throws will be obtained from a given movement of the earth inductor, the relative values of which can only be explained by assigning to the resistance of the galvanometer a value much greater than the correct one. The amount of this difference between the apparent resistance of the galvanometer and its real resistance depends, it is clear, upon many things, such as the whole effective area of the turns of the coil, the strength of the field about the coil, the stiffness of the gimp by which the coil is suspended, the relative resistances of the galvanometer and the rest of the circuit, the moment of inertia of the coil, and the magnitude of the coefficients of the air and electromagnetic damping. Two galvanometers which have the same ballistic sensitiveness to a given discharge from a condenser, and equal resistances, may, of course, act very differently so far as the effect of the induced counter electromotive force is concerned when placed in inductive circuits which outside of themselves are alike. It is useless, therefore, to attempt to predict with any great accuracy the magnitude of the disturbance due to the induced current in any particular instance, but it may be worth while to give briefly the general results of observations in some actual cases, by way of illustration.

(A) A certain d'Arsonval ballistic galvanometer, such that a condenser discharge of one microcoulomb sent through it caused a throw corresponding to a scale reading of about 11.0 cms., was connected in series with a Du Bois and Rubens "Panzer Galvanometer" (P) and with one of two coils of fine insulated wire which formed single layers, the one on a wooden core 2.38 cms. in diameter, the other upon a rod of soft iron 0.94 cms. in diameter. Either coil could be placed within a solenoid of 22.3 turns per centimeter through which a small steady current from a storage cell could be sent, and reversed at pleasure by aid of a commutator in the circuit. By means of a clamp, operated from without the case of the d'Arsonval galvanometer, its coil could be held firmly fixed in such a position that the flux through it of the field of its permanent magnet was practically zero, and it could be released again at will. A heavy suspended system was used in P in order to make the sensitiveness about the same as that of the other instrument when the time of swing was a little shorter than that of the moving coil. The whole resistance of the galvanometer circuit was about 2000 ohms. The slightest movement of the coil of the d'Arsonval galvanometer was sufficient to set the needle of the other instrument in motion also, and the reversal of a given current in the solenoid always caused a larger throw in P when the moving coil was clamped than when it was free. In a typical case the throw of P's needle as measured on the scale of the instrument was 83.5 mm. when the suspended coil was held still, but only 69.9 mm. when it was released. Each of these numbers is the mean of a number of closely accordant readings. Whether the strength of the current in the solenoid was made so small that the throws caused by reversing it could just be read, or so large that they could hardly be measured on the scale, the ratio of the indications when the suspended coil was fixed and when it was free was almost exactly the same; but it was easy to magnify the effect of the electromotive force of the swinging coil by increasing the current in the solenoid and decreasing the resistance of P so that the throws could be determined. For the purpose of this experiment it was allowable to shunt P, and when a one ninth shunt was used, the "throw" caused by the reversal of a certain current in the solenoid was 73.9 mms. or 45.0 mms. according as the suspended coil was clamped or free.

(B) A certain d'Arsonval ballistic galvanometer the total resistance of which at the room temperature was almost exactly 123.7 B. A. units, was connected in series with a rheostat and an earth inductor of 11.7 B. A. units resistance, the frame of which was fixed at a certain station. The moving coil of the instrument was furnished with a horizontal wire which projected about 23 mms. beyond the coil on each side and carried on each end a brass weight. The object of this device was to increase materially the moment of inertia of the suspended system and thus to make the time of swing larger without increasing the already inconveniently great sensitiveness of the galvanometer. When the inductor ring, which lay in a vertical plane perpendicular to the meridian, was reversed, the galvanometer throw, as measured on the scale at a distance of 120 cms. from the mirror, was 9.41 cms., 5.51 cms., or 3.95 cms., according as the resistance of the rheostat in B. A. units was 0, 100, or 200: if no account were taken of the counter electromotive force induced in the suspended coil, these figures would point to a circuit resistance of something more than 141 units instead of 135.4 units. In the condition just described the periodic time of the suspended system was 34.5 seconds: when that time was reduced to 12.4 seconds by the removal of the weights attached to the horizontal wire, the throw caused by reversing the inductor ring was 12.98, 7.72, or 5.49 according as the rheostat resistance was 100, 300, or 500; these figures point to an apparent resistance of 193 units in the circuit or 42 per cent more than the real resistance.

(C) A certain extremely sensitive d'Arsonval ballistic galvanometer, the coil of which was about twice as long as is usual in such instruments, was connected in series with a rheostat and a "magneto-inductor," i. e., a short coil of very fine insulated wire wound on a spool which could be slipped between near, fixed stops on a long magnetized rod. The permanent magnet of the galvanometer was built up of a large number of thin horseshoe plates hardened separately. The resistance of the galvanometer current when the rheostat resistance was zero was about 8691 at room temperature. When the exploring coil was slipped from one stop to the other, the throw of the galvanometer, as measured on the scale, was 67.0, 58.1, 51.3, 48.3, or 40.05, according as the resistance of the rheostat was 0, 2000, 4000, 5000, or 8500. Of course it is not possible to compute the apparent resistance of the circuit with any great accuracy from these numbers, any one of which might possibly be in error by 0.5% of its own value and thus introduce a much greater fractional error into the result; but it is interesting to find that if we compare the first number with the others in succession, we get for the apparent resistance of the circuit about 13060, 13070, 12915, 12990, and these numbers point to a value 50% above the real value. The resistance of the "magneto-inductor" alone was 40.3.

(D) When another similar ballistic galvanometer of the same general type was substituted for the one mentioned in (C) so that the resistance of the whole circuit was 4270, Mr. John Coulson obtained throws of 116.0, 100.1, 88.6, 72.5, or 65.8, according as the rheostat resistance was 0, 1000, 2000, 4000, or 5000. If we compare the first of these numbers with each of the others in succession, we get as values of the apparent resistances of the circuit, 6295, 6470, 6665, 6550, and these indicate a value about 50% above the true one.

(E) A d'Arsonval galvanometer such that the resistances of the coil, suspending gimp, and connections amounted in all to only 4.8, was connected in series with a rheostat and the "magneto-inductor" already mentioned. The coefficient of air damping in this galvanometer was considerable, and the field due to the permanent magnet extremely weak. The total resistance of the circuit was 45.1, plus the rheostat resistance. By fastening the stops which limit the journeys of the sliding coil on the inductor rod near together at the centre of the rod or further apart near an end, the electromotive force induced in the sliding coil as it was moved quickly from one stop to the other could be varied within wide limits. Three different positions of the stop were used, and in each case a large number of observations were taken and the mean used, as in all the results here given. For the first position the throw of the galvanometer corresponded to a scale reading of 77.8 mm. or 36.9 mm., according as the rheostat resistance was 0 or 50; these numbers point to a circuit resistance of 45.2, which is almost exactly the true value. The readings for the second position were 129.3 and 61.4, and the calculated resistance was again 45.2. The readings for the third position were 170.4 and 81.7, and the calculated resistance was 46.0 units. The results obtained in this extreme case illustrate what one may expect if the field is extremely weak and the number of turns in the coil small.

(F) Another d'Arsonval galvanometer of very low coil resistance but furnished with much finer gimp so that its total resistance was about 21, and with a strong permanent magnet, was treated in much the same way as the galvanometer mentioned in (E). Two positions of its stops on the inductor rod were used, and in each of them throws were observed when the rheostat resistance was 0, 50, and 70; the total resistance of the galvanometer circuit outside the rheostat was 60.8. For the first position the throws were comparatively large (196.3, 112.2, and 95.7), and the apparent resistance of the galvanometer circuit was 66.6 besides the rheostat resistance. For the other setting of the stops, the throws were much smaller (42.1, 24.6; 20.95), and the apparent circuit resistance 70.2 ohms. In some galvanometers the apparent resistance, when calculated from large throws as measured on the calibrated scale, will be larger than when deduced from small throws. In other galvanometers the opposite rule holds.

(G) A galvanometer of the same design as the one mentioned in (F), but with many more turns in the coil and a resistance of 730 units, was tested with the "magneto-inductor." This fine instrument, which is furnished with a strong permanent magnet, has been used in making a great number of measurements of permeability, and the very great difference between the apparent and the real resistance (770.3) of the circuit outside of the rheostat, illustrates strikingly the danger of neglecting the correction for the counter electromotive force induced in the coil during a swing. For one setting of the stops on the inductor coil, the readings caused by the galvanometer throw were 66.2 or 39.7, according as the rheostat resistance was 0 or 1000; these numbers point to a circuit resistance of 1498 units. For the second setting of the stops, the readings were 88.2 and 52.9, and these correspond to an apparent circuit resistance of 1499 units. The apparent resistance is in this case nearly double the real resistance. For wide ranges of throws the apparent resistance of any circuit in which this galvanometer may be placed remains almost exactly constant, and the instrument is wholly trustworthy when absolute measurements are to be made, if the apparent resistance instead of the true resistance be used. The apparent resistance of this galvanometer is naturally not quite the same in all circuits; when the electromotive force to be induced in a coil attached to it is larger, it is of course necessary to introduce a large extra resistance into the circuit, and this seems to reduce very slightly the difference between the apparent and the real resistance of the circuit; but so far as my experience goes, it has always been about 700 units. A condenser discharge of one microcoulomb sent through this galvanometer gives a scale reading of about 100 mm.

(H) A very useful instrument for hysteresis work on iron cores of electro-magnets is the microamperemeter of Paul, which has the form of an ordinary portable commercial amperemeter, and is of moderate cost.

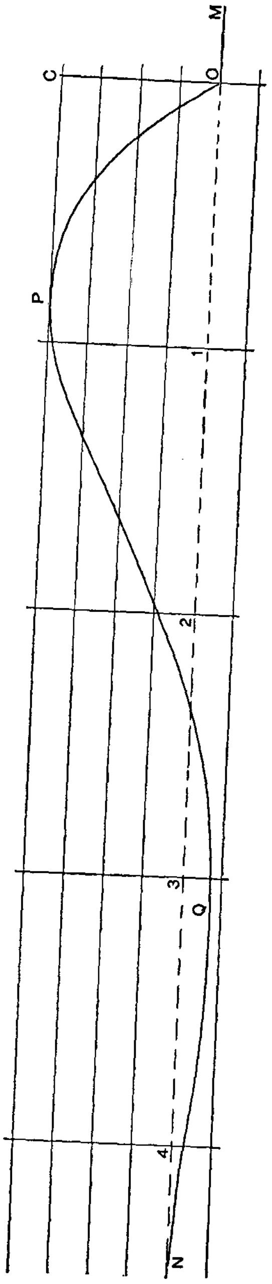
M
O
C
P
1
2
3
Q
4
N

FIGURE 1.

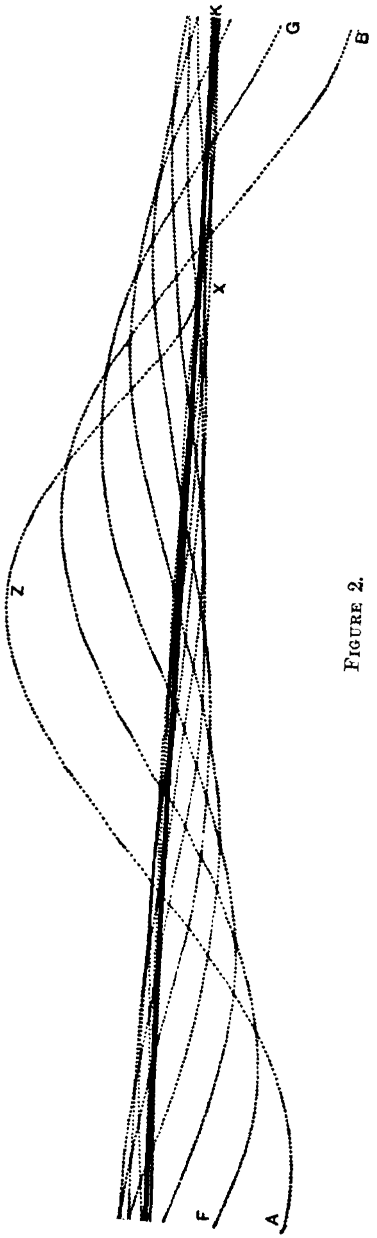

FIGURE 2.

The coil hangs from a single jewel, and the galvanometer is much more sensitive than the "millivoltmeters" usually found in American laboratories. The calibration curve for ballistic purposes is straight and the scale practically one of equal parts; one of these instruments which I have used has a resistance of about 60 ohms, and one microcoulomb sent impulsively through the coil causes a throw of 2 scale divisions. This galvanometer was connected with a rheostat and the low resistance secondary of a small induction coil, the primary of which could be opened and shut by means of a commutator; a rheostat in this primary circuit made it possible to change at pleasure the amount of electricity set in motion in the secondary circuit. The real resistance of the galvanometer circuit was about 74.9, and the apparent resistance for all discharges up to 20 microcoulombs seemed to be somewhere about 147. I took seventeen sets of observations with different quantities, and got some results as low as 143.5 and others as high as 151. The results all lay on a curious wavy line which seemed to mark some idiosyncracy of this particular instrument.

(I) The galvanometers used in (A) to (G) were all of the compact coil form without the inside soft iron core of the original d'Arsonval instrument. Galvanometers of this latter type, however, have their own advantages, as is well known, and are of course in very common use in commercial amperemeters and voltmeters and in laboratory mirror instruments of moderate sensitiveness for use in bridge work and in measuring ballistically condenser charges. As an extremely satisfactory representative of this class I chose one of the Leeds and Northrup Company's "Type H Galvanometers," of which there are several in the Jefferson Laboratory. The calibration curve of this instrument for throws due to condenser discharges is nearly straight, for quantities up to about 4 microcoulombs; the throw due to 1 microcoulomb being about 3.2 cm., whatever the resistance, up to 240,000 ohms, in the discharge circuit. A battery the electromotive force of which was about 8 volts was made to charge — through a resistance large enough (2000 ohms) to prevent possible polarization — a set of standard condensers, which were afterwards discharged through the galvanometer, and the throws corresponding to capacities of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.00 microfarads, were about 2.75, 5.49, 8.21, 10.99, 13.82, 16.71, 19.70, 22.80, 26.02, 29.37. The "magneto-inductor" and rheostat were attached to this galvanometer, and, with the stops on the inductor rod in certain fixed positions, the throws of the moving coil as measured on the scale were 15.81, 13.33, 9.76, 8.19, or 7.05, according as the rheostat resistance was 0, 300, 1000, 1500, or 2000. The first of these numbers in connection with each of the others gives the four values 1613, 1613, 1612, 1610 for the apparent resistance of the galvanometer circuit. These results, which agree wonderfully with each other considering the slight uncertainty in the readings, differ widely from the true resistance of the circuit, which was 531. If, when the rheostat resistance was zero, the galvanometer coil had been held still, the whole quantity of electricity set in motion in the circuit when the inductor coil was moved between its stops would have been more than three times the actual flux (rather less than 5 microcoulombs) when the coil was allowed to swing. This fact illustrates the magnitude of the error which might be made in a permeability measurement if the true resistance of the circuit instead of the apparent resistance were used. As is evident from the numbers given above, the apparent resistance is nearly independent of the actual resistance in the circuit when the impulse is constant and comes from a given source, but when the character of the impulse is changed the apparent resistance changes slightly also sometimes, and in a manner to be determined for each instrument by experiment. For a certain impulse which caused a throw of 8.52 cm., as indicated on the scale of the galvanometer just mentioned, the apparent resistance of the circuit was about 1704.

In such ballistic magnetic measurements as allow more than one observation at each stage of the experiment, it is possible, by alternately introducing and taking out of the circuit a known resistance of suitable value wound non-inductively, to determine the apparent resistance of the galvanometer circuit for each impulse. In other cases it is feasible to put into the circuit once for all such a large resistance, say 200,000 ohms, that changes in the apparent resistance of the circuit and changes in the inductive impulses become relatively unimportant. So far as my experience goes, a given instrument, as was to be expected, holds its apparent resistance for a given impulse unchanged for an indefinite time.

If a condenser discharge be sent through a d'Arsonval galvanometer which has no electromagnetic damping other than that which accompanies currents in its own coil, the form of the curve which shows the position of the swinging coil as a function of the time is hardly affected at all by the introduction of a large inductance or a resistance of several thousand ohms into the discharging circuit; but if the condenser be displaced by a magneto-inductor of small resistance, the damping is much increased and the shape of the curve is much altered. Figure 1, in which the horizontal divisions represent seconds, shows a careful copy (M O P Q N) on a small scale of a large photographic record of the excursion of the coil of a certain d'Arsonval galvanometer of short period, when a very sudden impulse from a magneto-inductor was sent through it; the curve starts very abruptly at O. Figure 2 is a photographic reduction, on the same scale as that of the other figure, of the record, just as it was removed from the drum, of an excursion of the same galvanometer when a condenser discharge was sent through it. The straight line through K represents the zero position of the coil; the curve begins at X, but not so abruptly as before. When the paper was on the drum, the points A, B and the points F, G fell together. It will be noticed that the time of the first swing of the coil on one side of its zero is nearly the same, about 2.34 seconds, in both diagrams.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

A SIMPLE DEVICE FOR MEASURING THE DEFLECTIONS OF A MIRROR GALVANOMETER.

By B. Osgood Peirce.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD COLLEGE.

A SIMPLE DEVICE FOR MEASURING THE DEFLECTIONS OF A MIRROR GALVANOMETER.

By B. Osgood Peirce.

Presented April 11, 1906. Received May 25, 1906.

For a good many years we have used in the Jefferson Laboratory, to determine the deflections of mirror galvanometers and other similar instruments, a simple form of tubeless telescope, which, at first intended for students only, has proved in its way very convenient, and has been adopted for some standard fixed instruments, because it is not easily put out of adjustment after it has once been properly set up, and because the scale image is so large and clear that a prolonged use of the device does not involve the eye strain which most reading telescopes would cause.

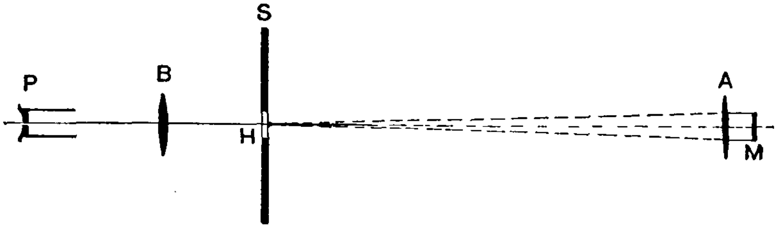

Figure A.

In front of the plane mirror (M) of the galvanometer, instead of the usual cover glass, is placed (Figure A) a convex lens (A) of focal length equal to the desired scale distance. This lens need not be achromatic ; a spectacle lens is quite good enough. At a distance in front of A equal to its focal length is placed a horizontal scale (S) mounted on a thin strip of wood at least twice as wide as the scale itself. Through the middle of this strip, above the scale, is bored a round hole (H) rather more than 20 millimeters in diameter ; just behind the scale is stretched a fine vertical wire or silk fibre (W) to serve as a cross-hair. Behind H and at a distance suited to its focal length, and to the eye of the observer, is placed another spectacle lens (B) to serve as an eyepiece. The centre of H should lie approximately in the common axis of A and B. A peephole (P) on this same line is placed at such a distance behind B, — which may conveniently have a focal length of 12 or 15 centimeters — that B's aperture shall appear *wholly filled* with a large, clear, uncolored image of the scale, with the wire W running vertically across it. The proper distance between A and S is determined by making the parallax of W across the lines of the scale, when the eye is moved sidewise across the peephole, to disappear: when this distance has once been found, the scale is firmly clamped in position. The distances between S and B and between B and P may be adjusted by every person to please himself, though it is usually possible to fasten P securely in a suitable position and to move B only to fit the eyes of different observers. It is important to notice that if the distance between the eye-lens B and the peephole P is not properly chosen, only a small round portion of the image of the scale will be seen, not nearly large enough to fill the aperture of B, and this will appear to move about in a distressing manner, when the eye at the peephole is moved slightly. When the simple adjustments have been properly made, however, the eye need not be pressed close to the peephole, for the deflections of the galvanometer can be accurately and easily read whether the eye is one inch or four or five inches behind P. It is not necessary to close the unused eye, for the field should be extremely bright and clear; the observer is, of course, looking at a life-size image of the scale at a distance PH from the peephole, and it is easy to get any desirable magnification of this image by a proper choice of the eye-lens. Ordinary astigmatism on the part of the observer's eye can be partially corrected by a suitable spectacle lens at B. One slight drawback to the use of this reading device, where space is limited, is the fact that the eye will generally need to be 25 or 30 centimeters behind the scale. When it is desirable to do so, the eye-lens, B, may be mounted in a tube which slips over the tube which carries the peephole.

Figure 1 shows one form of the arrangement just described, mounted, for students' use, with a mirror amperemeter on a wooden frame. Several adjustments are here provided for, so that different instruments may be used on this frame. Figure 2 shows the scale and eyepiece as mounted on a brass rod, by Mr. J. Coulson, and Figure 3 represents a convenient way of using the device with a wall galvanometer.

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CONTRIBUTIONS FROM THE CRYPTOGAMIC LABORATORY OF HARVARD UNIVERSITY. — LXIII.

ON THE CYTOLOGY OF THE ENTOMOPHTHORACEAE.

By Lincoln Ware Riddle.

With Three Plates.

CONTRIBUTIONS FROM THE CRYPTOGAMIC LABORATORY OF HARVARD UNIVERSITY.—LXIII.

ON THE CYTOLOGY OF THE ENTOMOPHTHORACEAE.

By Lincoln Ware Riddle.

Presented by W. G. Farlow, May 9, 1906. Received May 14, 1906.

Introduction.

The Entomophthoraceae constitute a small, and, in some ways, peculiar family of Phycomycetes, including, according to Schroeter's arrangement in Engler & Prantl, seven genera. Of these seven genera, we have more or less knowledge of the nuclear conditions in four. In Conidiobolus (Brefeld, '**84**) we know only that the hyphae are non-septate in the early stages, but in the older stages break up into short sections (hyphal bodies) which are multi-nucleate. Basidiobolus (Eidam, '**87**) is perhaps the best known of the genera from a cytological point of view, chiefly on account of the work of Fairchild ('**97**), although the papers of Raciborski ('**96**), Lowenthal (:**03**), and Woycicki (:**04**) are also to be mentioned. The hyphae are here septate, dividing the fungus into cells, which are typically uni-nucleate; the conidia also possess single nuclei, derived directly without division from the parent-cells. The zygospore is formed by the fusion of the nuclei of two adjacent cells, this process being preceded, however, by the production of two beaks on the fusing cells, with a simultaneous division of the two gamete-nuclei so that a daughter-nucleus is cut off in each beak, while the other two daughter-nuclei enter the young zygospore and sooner or later fuse. Raciborski ('**96**^{b}) and Fairchild ('**97**) have both described the division of the nucleus of Basidiobolus by a mitotic method.

The two remaining genera, upon which a certain amount of cytological work has been done, are Empusa and Entomophthora, all the species of which are parasitic on the larvae, pupae, or imagines of various insects, and resemble one another in their general life-history. For the general morphology of these genera reference may be made to the account given by Thaxter ('**88**) in his monograph of the North

American species. It was at Professor Thaxter's suggestion that the present study of the cytology of certain species of these two genera was taken up, in the hope of adding to our somewhat imperfect knowledge of this, and especially, of ascertaining the relation of the conditions involved in the formation of the resting-spores to the conditions in other Phycomycetes.

The first mention of the nucleus of Empusa is found in a paper by Maupas, published in 1879. By staining with picrocarmine, he was able to differentiate the nuclei, which, if seen by earlier observers, had been considered to be vacuoles, and to show that the hyphae of Empusa contain a large number of "petits nuclei, d'un diamètre d'environ 4 microns." The next paper was that of Vuillemin (**'87**), who worked with material of *Entomophthora gleospora*, which was stained in haematoxylin but was not sectioned; he describes the relatively large nuclei (10 microns in diameter) regularly spaced in the hyphae, one nucleus migrating without division into each conidium. Vuillemin made out the presence of chromatin contents in the nucleus, but found no nucleolus. In 1899 appeared a preliminary note, soon followed by the complete paper, by Cavara, describing the conditions in the hyphae and conidia of *Empusa Muscae* and *Entomophthora Delpiniana*, including also an incomplete account of the azygospores of the latter. Cavara's paper was marked by an important technical advance, inasmuch as he used microtome-sections with the most favorable stains. The following year Vuillemin (**:00**) published another paper on *Entomophthora gleospora*, describing the behavior of the nuclei in the formation of the azygospores, and including a theoretical discussion of certain points concerning the relationships of the genera of the Entomophthoraceae. These last two papers will be discussed in some detail in connection with the results here presented on the cytological processes in several species of Empusa and Entomophthora.

MATERIAL AND METHODS

In his "Monograph of the Entomophthoreae of the U. S." Thaxter (**'88**) combined the species usually separated into Empusa and Entomophthora in one genus under the name Empusa, considering that the simple or divided character of the conidiophores, and the presence or absence of rhizoids were features too inconstant to allow the separation of the species into two genera. Since, however, he has not been followed in this procedure by other writers, and since, further, Cavara (**99**^{a}) has pointed out the importance of the fact that in the species put in Empusa the conidia are typically multi-nucleate, while in those species put in Entomophthora, they are typically uni-nucleate, it seems best to consider them as two distinct genera.

The results given in this paper are based upon a study of preparations from the following species.

1. *Empusa Grylli*, Nowakowski, on Hyphantria larvae (the "Fall Web-worm"), and on the larvae of Spilosoma.

2. *Entomophthora Americana*, Thaxter, on the imagines of Musca sp. and various other large flies.

3. Also an undescribed form closely related to *E. Americana*, found by Professor Thaxter in various localities in New England, and called in this paper *Entomophthora* "*x*." The form "*x*" resembles *E. Americana* in hosts and in conidia, but differs from it in the zygote, the epispore of which is smooth in *Americana*, but is furnished with bullate processes in *Entomophthora* "*x*." (See Figure 16.)

4. *Entomophthora echinospora*, Thaxter, on the imagines of Sapromyza (a small yellow-winged fly).

5. *Entomophthora Geometralis*, Thaxter, on the imagines of a geometrid-moth.

6. *Entomophthora rhizospora*, Thaxter, on caddis-flies.

7. *Entomophthora Fresenii*, Nowakowski, on a species of Aphis.

Most of the material was fixed in 0.75 % chrom-acetic acid, which gave satisfactory results, except for the more mature stages of the resting-spores, when it became necessary to use hot sublimate-acetic. Flemming's weaker solution was also tried, but seemed to be no improvement over the chrom-acetic mixture. It is worthy of notice, however, that material gave excellent preparations after being left in Flemming's for some weeks (cf. Chamberlain, :**01**, p. 27); the figures in Plate I illustrating mitosis were drawn from such material. Small pieces of the hosts infected with the fungus were imbedded in paraffin by the usual procedure, and sections were cut either three or six microns in thickness.

In staining, safranin-gentian-violet gave decidedly the best results. The use of orange G in connection with these stains seemed to have no added advantage. Heidenhain's iron-alum-haematoxylin was also tried, but was a decided failure, as the stain showed too great affinity for the general cell-contents. Unless otherwise stated the preparations from which the plates were drawn were fixed in chrom-acetic and stained in safranin-gentian-violet.

ENTOMOPHTHORA.

Nucleus.

The nucleus of *E. Americana* may be taken as typical of the genus. The resting-nucleus varies in diameter from five to ten microns, with an average of six. A successful stain gives a beautiful differentiation of the nucleus and shows its structure in considerable detail (Figure 1). The nuclear membrane is sharply defined, taking a much darker stain than the rather faint gray-blue of the cytoplasm. Within the nucleus there is a relatively small nucleolus, which stains red with the safranin-gentian-violet combination. As a rule, the nucleolus is single, but in some cases two or three may occur, when they are correspondingly smaller (Figure 13), while in other cases the nucleolus may be altogether absent (Figure 16). Around the nucleolus and occupying a large part of the nuclear cavity occurs the chromatin, in the form of densely packed granules. These usually appear evenly granular, but at times this even appearance is much less marked, and the chromatin may occur in the form of a fine meshed network (Figure 12). The chromatin in the resting-nucleus stains deeply with the gentian-violet.

The structure of this nucleus appears to be quite typical, and certainly offers no support to the statement of Cavara ('99) that "the structure of these nuclei is quite singular, and perhaps has no counterpart in those of any other plants, with the exception of the Saccharomycetes . . . the nuclei of which recall very strongly those of the Entomophthoreae." A comparison of the account of the nucleus of the yeast given by Wager ('98), which is quoted by Cavara, with the material of the present study fails to suggest the slightest similarity in the two cases. Cavara further says, "It appears that the process by which multiplication of the nuclei is effected must be ascribed to division by fragmentation or to a modification of this which is transitional to mitotic division." The present study shows that the nucleus of Entomophthora divides by a well-developed process of mitosis.

The first indication of the approach of nuclear-division is to be seen in the gradual loss of the dense, evenly granular condition of the nucleus. The chromatin becomes less densely distributed, while the granules are seen to be fewer and larger. This is followed by the gathering of more and more of these granules in the central region of the nucleus, which then shows a dense central mass of large chromatin-granules surrounded by a clear area where a few scattered granules occur, and linin-fibres are plainly visible (Figures 23, 24). Meanwhile a change in staining-reaction has taken place, the granules retaining more and more of the safranin, so that the larger granules show a decided red. These granules then become aggregated into a definite number of spherical chromosomes (Figures 2–6). The organization of the chromosomes thus appears due to the direct aggregation of the granules, no spireme stage having been observed.

During this process of aggregation the nucleolus has taken up a position near the centre of the nucleus; but with the formation of the chromosomes the nucleolus is no longer distinguishable, since it is identical in size, shape, and staining, with the chromosomes. There is also no difference in behavior during the later stages of division. It must therefore be concluded that the so-called nucleolus of Entomophthora is in reality a chromatin-nucleolus, or karyosome. This karyosome does not contain, however, all the chromatin-content of the nucleus, as has been described by Wolfe (:**04**) for Nemalion, and by Mitzkewitsch ('**98**) for Spirogyra; on the contrary, it is equivalent to a single chromosome. The total number of chromosomes in the nucleus of *E. Americana* (including this "nucleolar chromosome") appears to be eight. This was seen clearly in many cases, and is illustrated by Figure 6 and by the lower daughter-nucleus in Figure 8. We have apparently in this species a condition where one of the chromosomes persists during the resting-stage of the nucleus as a chromatin-nucleolus. The writer has been unable to find anything comparable to this described for plants, but among animals Montgomery ('**98**) has described a similar condition in the spermatogenesis of Pentatoma.

The organization of the spindle in Entomophthora shows interesting conditions. It is always intranuclear and develops in the following manner. As the chromatin-granules draw toward the central region of the nucleus, they leave evident in the outer clear zone, numerous linin-fibres radiating in all directions from the central aggregation of chromatin (Figure 4) and running to the wall of the nucleus, which persists through all stages up to the telophase. It is highly probable that the chromatin-granules were, in the resting-period, distributed along these linin-fibres, which become evident, however, only when the granules draw toward the centre. The linin-fibres would thus be attached to the chromosomes by their proximal ends. The fibres, at first radiating in all directions, tend to separate into two groups, which gradually migrate toward the respective poles of the now somewhat drawn-out nucleus (Figure 5). These fibres form the spindle of the mitotic figure, which is thus derived wholly from the linin, and is therefore strictly intranuclear in origin and position (Figure 7). Intranuclear spindles have been described for a large number of plants (see Davis, :04, Am. Naturalist, vol. xxxviii, p. 435, and literature there cited). Stevens (:03) has described the origin of the spindle from the linin in Synchytrium, but his material was apparently less favorable for the exact determination of the process. In Phyllactinia, Harper (:05) has shown that the spindle-fibres of the mitotic figure are identical with the linin-fibres of the resting-nucleus, but the method of spindle-formation differs from the present case, since the process is controlled by centrosomes in Phyllactinia. From the method of origin in Entomophthora, it is clear that the spindle-fibres are identical with the linin-fibres and are attached to the chromosomes permanently. In this process of spindle-formation there is no place for centrosomes, and no structures resembling centrosomes were seen in any case.

During this migration of the spindle-fibres the chromosomes are grouped in a more or less definite equatorial plate (Figure 5). The typical metaphase with the equatorial plate and the bipolar spindle, if it occurs in Entomophthora, must be of very short duration, as it is the only phase that has not been seen in examining a large number of sections showing various stages of mitosis. The nearest approach to the metaphase figure is the condition shown in Figure 5. The absence of the typical metaphase may perhaps be explained if we assume that the chromosomes split and begin to separate before the spindle becomes definitely bipolar. The next older stage of mitosis seen in the preparations studied, is one where the chromosomes have divided and the daughter-chromosomes are beginning to draw apart. The spindle, occupying the entire nuclear-cavity, elongates at this time, and the daughter-chromosomes are clearly half the size of those in the equatorial plate; this stage (Figure 7) is unmistakable and was met with repeatedly. As the daughter-chromosomes reach the poles the wall of the nucleus becomes more rounded at these points. The chromosomes remain very clear up to a late stage, and eight could often be counted.

Two methods occur in the reconstruction of the daughter-nuclei. In the first method, a vacuole appears between the two daughter-nuclei, and the spindle-fibres in this region break down, leaving only a few granules (Figure 9). The old wall of the mother-nucleus persists up to this time, connecting the two daughter-nuclei. This old wall forms the walls of the daughter-nuclei, except on the side toward the vacuole, where new walls are laid down, probably by the plasma-membrane, bordering the vacuole (cf. Lawson, :03). The portion of the old wall connecting the daughter-nuclei now disappears and the latter draw apart freely. The second method is much less common than the first, but was met with several times. The portion of the wall of the mother-nucleus and the spindle-fibres connecting the daughter-nuclei draw apart (Figure 10), giving a very characteristic dumbbell-shaped appearance, similar to that figured by Stevens (: **01**) in Albugo. The entire wall of the daughter-nuclei in this case would be formed from the wall of the mother-nucleus. Whichever method is followed, the daughter-chromosomes remain more or less visible until after the wall is completed, when they fragment and become redistributed in the form of granules. Owing to the smaller size of the nuclei and to the density of their contents and to their strong affinity for stains at this time, it was found impossible to follow the process in detail. But it is not improbable that the process is the reverse of that followed in the organization of the chromosomes, in which case the fragmentation would be accompanied by a redistribution of the chromatin over the persistent linin-fibres.

To sum up: the process of mitosis in Entomophthora shows a chromatin-nucleolus; an organization of chromosomes by the aggregation of granules, without the intervention of a spireme-stage; the formation of a typical, intranuclear, bipolar spindle, without centrosomes, directly from the linin-fibres, which retain their connection with the chrosomomes throughout mitosis; and, in the daughter-nuclei, a redistribution of the chromosomes in a granular form over these fibres.[1]

Conidia.

The conidia of the following species have been examined and found to agree in the essential details: *E. Americana*, *E. Geometralis*, *E. echinospora*, *E. rhizospora*, *E. Fresenii*. As an illustration of the development and structure of the conidia in these species, the process may be described as seen in *E. Geometralis*.

About the time of the death of the host the hyphae break up into short multi-nucleate portions known as hyphal bodies (Thaxter, '**88**). The young hyphal body is a thin-walled structure, the cytoplasm of which shows a more or less definite structure, which Cavara ('99) has considered to be reticulate, but which, in the preparations studied, is distinctly alveolar. This difference in appearance may be due to different nutritive conditions, or more probably to difference in fixation. The nuclei are relatively few, and show, as a rule, regular spacing. Each hyphal body sends up a long hypha to the surface of the body of the host. Into this hypha pass a varying number of nuclei. In the conidiophore, as this hypha may be called, the nuclei attain their greatest size (Figure 19), owing, in all probability, to the active metabolism necessary in connection with the formation of the conidia. The nuclei in the conidiophore, as in the hyphal body, show a strikingly regular spacing, and in the older portions a septate condition is occasionally to be seen, these septa cutting off mostly bi-nucleate cells (Figure 17). At the top of each branch of the conidiophore the cytoplasm becomes much denser, showing an upward pressure and condensation at that point. Under this pressure a small protuberance soon appears, forming the young conidium, into which passes the dense cytoplasm (compare Empusa, Figure 24). A single nucleus enters the young conidium, becoming smaller and denser during its passage. The conidium is then cut off. There is never any nuclear-division during this process, and the single nucleus of the conidium comes directly from the conidiophore. The mature conidium (Figure 18) is a comparatively thin-walled structure, densely filled with very fine-meshed cytoplasm and with a single much condensed nucleus.

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[1] Since the completion of this work, two papers have appeared by E. W. Olive, under the general title, "Cytological Studies on the Entomophthoreae" (Botanical Gazette, **41**, 192–205 and 229–258, with pls. 14–16). The first paper presents a discussion of certain points in the life-history of various species of Empusa, in particular a new species, *E. Sciarae*, with a general confirmation of the results of previous workers in regard to the details of conidium formation. The second paper discusses cell- and nuclear-division. Olive finds that cell-division takes place through the entrance of a ring-shaped cleavage-furrow. The account of nuclear-division is in striking contrast to the conditions seen in the course of the present investigations, so much so that a close comparison is impossible. Olive describes what he calls a "primitive mitosis" resembling that of the lower Protozoa, a process controlled by "intranuclear centrosomes" and without definitely organized chromosomes.

Zygospores.

The formation of the zygospores has been studied by the writer in *E. Americana*, and the results there obtained have been confirmed by a further study of *E. echinospora* and *E. Geometralis.* There has been hitherto no account of the cytological processes involved in the formation of sexual resting-spores in any of the genera of the Entomophthoraceae, with the exception of Basidiobolus, in which the conditions have been generally recognized as peculiar and difficult to bring into relation with the processes known in the other Phycomycetes.

Entomophthora Americana shows two methods of zygospore formation. In the first case two hyphal bodies fuse at a point near their tips (Figure 12); the young zygospore then buds out at this point of fusion, in a manner similar to that seen in Piptocephalis, among the Mucorales. In the second case, the fusion of the two hyphal bodies is distinctly lateral (Figure 11), forming what may be compared to an H with a very short bar. The young zygospore in this case buds out from one of the gametes, at a point usually far removed from the point of fusion. This condition is comparable to that seen in the case of *Syncephalis nodosa* (compare description by Thaxter, '97).

In both of these methods, however, the essential cytological conditions are the same. The two hyphal bodies fuse, the wall breaking down between them, and the cytoplasm becoming completely continuous, so that no line of fusion is discernible. The wall near the point of fusion, in the first case, or the wall of one of the hyphal bodies at some other point, in the second case, then swells, and this swelling, under the pressure of inflowing protoplasm, continues until a definite ampulla is formed. The wall of this ampulla is only slightly thicker than that of the hyphal body from which it is derived, and forms, as will presently be shown, the epispore of the zygote. Into this ampulla pass all the nuclei of both the fusing hyphal bodies (see Figures 11–13). It appears also that most, if not all, of the cytoplasm also passes into the young zygospore. Figure 11 shows the much vacuolated cytoplasm of the hyphal bodies; and Figures 13 and 14 show young zygospores attached to empty hyphal bodies.

The fusing hyphal bodies are accordingly multi-nucleate structures, the entire nuclear and cytoplasmic contents of which pass into the young zygospore. The hyphal bodies are therefore strictly gametes, and since they are multi-nucleate, are certainly to be considered coenogametes. Since the coenogamete is a type of sexual organ which has been shown by Davis (:00), Stevens (:01), and others to be characteristic of the Phycomycetes, the establishment of its occurrence in the Entomophthoraceae is a matter of great importance.

After the entire contents of the two gametes have passed into the young zygospore, the latter is cut off by a cross-wall. The exact method of the formation of this wall has not been determined, but stages, such as are shown in Figure 14, prove that the zygospore is not abjuncted by a process of constriction, as is the case with the conidiospore. At this time the zygote is a spherical body surrounded by a single thin wall. The cytoplasm is alveolar, and the relatively few (probably not more than eighteen) large nuclei are irregularly placed, with no apparent definiteness of arrangement. No fusion of these nuclei has taken place up to this period of formation.

The next process of interest in the development of the zygote is the formation of the thick endospore wall. As has been mentioned above, the epispore is derived directly from the wall of the hyphal body. The endospore, however, does not appear until after the zygote is cut off by a cross-wall and the old walls of the hyphal bodies have disappeared. The first sign of the formation of the endospore is the marked increase of vacuolation in a zone just inside the epispore (Figure 15). On the inner side, this clearer zone is marked off by a densely granular region. The conditions at this period suggest strongly the periplasm found in so many Phycomycetes. This appearance, however, may be accidental and without special significance. From a study of such stages as are illustrated by Figure 15, it is evident that the granular cytoplasm is being gradually transformed into the clear material of the mature endospore (Figure 16). The endospore in Entomophthora thus appears to be formed, not as a secretion of the outer plasma-membrane, but by a direct transformation of the outer zone of cytoplasm, this transformation evidently involving more than the plasma-membrane itself, and possibly representing a primitive type of periplasm.

After this formation of the endospore, the zygote enters into a resting period which lasts until the following summer. Sections cut from material fixed three months after formation show that no fusion of the nuclei has taken place up to that time (Figure 16). Whether or not fusion of the nuclei in pairs takes place at the time of germination, as we might expect, it has been impossible to determine, owing to the difficulty of germinating the zygospores under artificial conditions. That the process of zygospore formation thus given for *E. Americana* is confirmed in full by *E. echinospora* may be seen by comparing Figures 11 to 16 with Figures 20 to 22.

It will be seen from the foregoing description that the processes connected with the formation of the sexual resting-spores in Entomophthora are closely comparable with the corresponding processes in the Mucorales, in so far as they have been made known by the work of Gruber (:**01**) on Sporodinia. Entomophthora has, in common with Sporodinia, coenogametes (multi-nucleate sexual cells) which are of a simple type, inasmuch as there is no differentiation of the contents and all of the nuclei function as gamete-nuclei.

EMPUSA.

Material of *E. Grylli* on two different hosts furnished the preparations upon which was based the study of this genus. Under *E. Grylli*, Now., Thaxter ('**88**) included as a synonym *E. Aulicae*, Reichhardt (in Bail, '**69**) first described on *Euprepia aulica*, but also occurring on other hosts, and especially in the United States, on the larvae of Spilosoma. Several authors (Cohn, '**75**; Schröter, '**86**; Lindau, '**97**) have kept this as a distinct species. The writer, in the course of the present study, has seen no such differences as should be considered of specific rank, and therefore prefers to follow Thaxter and consider both of the above forms as true *Empusa Grylli*, Nowakowski. The results here given are based on material on the Spilosoma larvae; but these have been confirmed in every particular by material from the larvae of Hyphantria.

Nucleus.

The nuclei of Empusa are smaller (average diameter 4 microns), more numerous in each hyphal body than in Entomophthora, and show no regularity of spacing (Figure 23). The smaller size of the nucleus makes it a less favorable object for study; so far as has been observed, however, the structure of the nucleus of Empusa agrees perfectly with that of Entomophthora. Although caterpillars killed by *E. Grylli* were fixed at various hours of the day and night, in various conditions of moisture, and at longer and shorter periods after death, no nuclear-divisions were seen in any case. It is therefore impossible to state whether or not the details of this process also agree with those in Entomophthora, as we should expect from the close relationship of the two genera.

Conidia.

The general process of the formation of the conidia in Empusa (Figures 24 and 25) is similar to that already described for Entomophthora. There are a few differences, however, the most important of which is the fact that the conidia of Empusa are multi-nucleate structures, and the conidiophores are unbranched.

Some time before the death of the host the hyphae break up into hyphal bodies in the usual manner; and, at the time of death, each hyphal body sends up a conidiophore. In this case, however, the conidiophores and consequently the conidia are much larger than in Entomophthora, conditions correlated with the absence of branching in the conidiophore, and with the large amount of nuclear substance in the multi-nucleate conidium. The nuclei in the conidiophore are very numerous, and show no evidence of regular spacing (Figure 24). There is the usual accumulation of dense cytoplasm in the tip of the conidiophore as it swells to form the conidium. It will be recalled that in Entomophthora a single nucleus passes into the young conidium at this time. In the case of Empusa, a large number of nuclei (about twelve to fifteen) pass in and the conidium is cut off. No nuclear-division takes place during this process of conidium-formation.

After the discharge of the conidium, the nuclei left in the conidiophore degenerate, certain stages of this degeneration presenting a misleading resemblance to the prophase of nuclear-division. The nuclei can, however, be followed through all the stages of the process, ending as vacuoles containing a few oil-like globules.

The centre of the mature conidium is pretty constantly occupied by a single, large, well-defined vacuole around which the numerous, much condensed nuclei are most often clustered (Figure 25).

The conidia of *Empusa Grylli* are, then, multi-nucleate structures and corroborate the statement of Cavara ('99ª), based on his examination of *E. Muscae*, that a multinucleate conidium is characteristic of the genus Empusa, while, as the same author shows in *Entomophthora Delpiniana*, and as has been shown above for a number of other species of the genus, Entomophthora has constantly a uni-nucleate conidium. The conidia of Empusa are also to be compared to those of Albugo and other genera of the Peronosporales, where, in all forms which have been examined (Ruhland, :03), the conidia are multi-nucleate. But the absence of a nuclear-division preceding conidium-formation in Empusa is in marked contrast to that of Albugo, a difference to be explained by the fact that, while in Albugo a series of conidia are formed, in Empusa but a single conidium is formed from each conidiophore.

Azygospores.

Just as the hyphal bodies and conidia of Empusa are multi-nucleate structures, so also we find multi-nucleate conditions in the azygospores. This multi-nucleate condition appears from the first formation stages, and these nuclei undergo neither divisions nor fusions. As these observations are not in accord with those of previous workers on the azygospores of Entomophthora, namely Cavara ('99) on *E. Delpiniana* and of Vuillemin (:00) on *E. gleospora*, it will be well to examine their results before describing the development of the azygospores in Empusa.

In *Entomophthora Delpiniana*, a species in which the hyphae do not break up into hyphal bodies, Cavara observed two sorts of structures which he called azygospores. First, a thick-walled, intercalary organ with granulated cytoplasm which shows clear reticulations, the joints of the meshes being occupied by "chromophilous granules"; this organ contains no definite morphological nucleus, but Cavara regards the "chromophilous granules" as a diffused nucleus. Such an intercalary organ appears not to have been seen by any other workers. Certainly in the present study no structures have been seen resembling Cavara's description, so that an interpretation of these curious conditions is here out of the question. Secondly, and more commonly, Cavara finds an organ of acrogenous formation; a hypha swells at the end into an ampulla; into this ampulla passes a single nucleus; a cross-wall is then formed cutting off the spore, which is, therefore, at this time, uni-nucleate; this single nucleus divides to four, six, eight, and perhaps more, since Cavara was unable to follow the process in its further development.

Vuillemin (:00) states that the early development of the azygospores of *Entomophthora gleospora* is the same as in the acrogenous azygospores described for *E. Delpiniana*. There is an ampulla into which passes a single nucleus, which after the formation of a cross-wall, divides to four, six, eight, and then to sixteen. It appears, however, that this number is by no means constant, as Vuillemin records finding twelve, fourteen, fifteen, seventeen, or eighteen, which he supposes to be due to the failure of some of the nuclei to divide, or to a greater number of divisions than usual. After the nuclei have reached this number, there occurs successive fusion in pairs till the number is reduced to two; the azygospore rests in this condition for some time, but these two also ultimately fuse so that the mature azygospore is once more uni-nucleate.

Vuillemin gives no figures for any of these numerous nuclear-divisions and nuclear-fusions. His evidence seems to be based upon the different numbers of nuclei in different cases and in their smaller size when the number is greater. The number of the nuclei, however, according to the results to be described presently, is closely dependent upon the number in the hyphal body from which the azygospore is formed. And the difference in size, apart from individual variations, within restricted limits, which seem to be quite common (cf. nuclei in Figure 26), might easily be explained by the maintenance of the equilibrium of nucleo-cytoplasmic relations, the importance of which has been shown by the recent work of Gerassimow (:01, :02, :04), Hertwig (:03), and others. As Vuillemin seems to consider the conditions described to be typical for the family, it will be well to compare the account given above with results obtained from a study of the azygospores of *Empusa Grylli*.

After the hyphae of the fungus have broken up into hyphal bodies, the formation of azygospores takes place. At some point on the hyphal body, not necessarily at one end, a small protuberance appears, into which there is a flow of protoplasm from the hyphal body; the protoplasm stains more densely in this region and shows pressure. On account of this pressure the protuberance continues to expand into a rounded ampulla, the wall of which is at first only slightly thicker than that of the hyphal body (Figure 26). Into this ampulla gradually passes all of the cytoplasmic contents of the hyphal body, together with all the nuclei, which the hyphal body originally contained. Figure 26 shows a stage where this process is under way; and Figure 27, a stage where it is completed. It will be seen here that the young azygospore is a multi-nucleate structure; and the attached empty hyphal body together with the absence of a cross-wall proves that this multi-nucleate condition is brought about, not by the successive divisions of a single primordial nucleus, but rather by the inclusion in the azygospore of the entire number of nuclei which were present in the hyphal body. As this number is, within certain limits, somewhat variable, the number of nuclei in the azygospore is correspondingly variable.

The next stage in the formation of the azygospore is marked by the formation of a cross-wall, cutting off the recently formed resting-spore; this being immediately followed by the dissolution of the walls of the empty hyphal body leaving the rounded azygospore, which at first often shows the position of the cross-wall (Figure 28). The cytoplasm is reticulated in structure, the meshes being comparatively small, and clearly granular; the wall is still only slightly thicker than that of the hyphal body, but the thickening process begins immediately.

An additional piece of evidence against the occurrence of nuclear-division within the azygospore is the fact that all of the stages described above, from the first sign of a small protuberance up to the fully formed spore, may be found on a single slide. Preparations of material in which the spores had been formed from one to several days were also studied. Yet in no case was there the slightest evidence of nuclear-divisions. Further, the number shows no striking variation, although it is often difficult to follow a spore through all the successive sections in which it should appear; and, likewise, there is no marked diminution in the size of the nuclei, such as would take place during nuclear-division (as is shown by comparing the mother-nucleus with the daughter-nuclei in Figures 1 to 10).

Finally, preparations were made from azygospores, three months old at the time of fixation. A median section of one of these is shown in Figure 29. The thinness of the wall in this case is anomalous and difficult to explain, since the normal procedure is for the wall to thicken immediately after the formation of the cross-wall, the mature azygospore having a thick wall with epispore and endospore differentiated similar to that described for Entomophthora. The appearance of this spore, fixed three months after formation (Figure 29), will be seen to be clearly similar to that of the newly-formed spore (Figure 28). The spore has increased in size somewhat, but with a corresponding increase in the vacuolation of the cytoplasm, these vacuoles being filled, in the living material, with oil-globules. The nuclear conditions are essentially identical with those in the young azygospore, and show clearly that no nuclear-divisions, and, likewise, no nuclear-fusions, have taken place up to this time.

When it is recalled that in *Entomophthora gleospora*, which, as has been mentioned, Vuillemin considers typical, the young azygospore, after the formation of the cross-wall, is uni-nucleate, and that the nuclear-divisions and fusions described take place immediately, it will be seen that here, in *Empusa Grylli*, we have a very different type of resting-spore formation — a resting-spore multi-nucleate from the beginning and showing neither nuclear-divisions nor nuclear-fusions.

The evidence seems to indicate that the azygospore of Empusa is more in the nature of a chlamydospore; and the occasional substitution of an encysted hyphal body, such as is shown in Figure 30, as a means of tiding over the winter period, is in favor of this view.

Conclusions.

In conclusion it will be well to consider what light the observations here set forth may throw on the interrelationship of the genera of the Entomophthoraceae, and the relationship of this family to other Phycomycetes. In comparing the nuclear conditions in the three genera, which have been studied from the cytological point of view, namely Basidiobolus, Entomophthora, and Empusa, two possible lines of development are offered for consideration. First, it is possible that Basidiobolus, with its uni-nucleate cells, may be a primitive type, from which the development may have proceeded through the conditions seen in the hyphal bodies of Entomophthora, with their several nuclei regularly spaced, to the conditions in Empusa, where the hyphal bodies have a large number of irregularly distributed nuclei. Or, secondly, it is possible that Empusa is the primitive type of a series showing progressive nuclear reduction, which reaches its highest expression in Basidiobolus.

The first of these two views is supported by Vuillemin in his paper of 1900. He says: "Au point de vue cytologique, l'*E. gleospora* (ainsi que l'*E. Delpiniana*) forme un trait d'union entre les Basidiobolus et les Empusa; il nous montre par quelle gradation la structure cellulaire s'est dans la série des Entomophthorées pour passer à la structure multi-nucléée. Cette dernière apparaît alors comme un dérivé phylogénétique de la première : c'est ce qui m'a depuis longtemps suggéré l'idée de la nommer une structure apocytique. Ceci posé, nous interpreterons les vues de Raciborski en disant que la famille des Entomophthorées chevauche sur les Archimycètes et les Phycomycètes, qu'elle prend ses racines dans le premier groupe au niveau des Basidiobolus, franchit la frontière au niveau des *Entomophthora gleospora* et *Delpiniana* pour s'epanouir en pleins Phycomycètes au niveau des Empusa. Les détails publiés par Cavara sur la structure de l'*Empusa Muscae*, détails que j'ai pleinement vérifiés, montrent clairement ce dernier stade de l'évolution apocytique des appareils végétatif et conidien des Entomophthorées." Vuillemin thus makes Basidiobolus the primitive form for the family. But such a primitive form should show some points of relationship to the forms from which it has itself been derived. These points further should be shown in connection with the sexual processes, the period of the life-history, which is generally agreed to be of the greatest phylogenetic importance. In connection with his views Vuillemin offers no special form to which Basidiobolus might be directly related.

Such a line of evolution, as Vuillemin suggests, has been discussed by Davis (:**30**) in connection with his consideration of the origin and evolution of the coenogamete. Davis says: "It is conceivable that a uni-nucleate sexual element might become multi-nucleate perhaps through such an increase in the protoplasmic content that more than one nucleus would be required to control satisfactorily its activities. This possibility has absolutely no evidence in its support. There is no series of forms whose sexual cells pass from a uni-nucleate condition to a multi-nucleate. There are no indications that such an evolutionary process has ever taken place among plants."

So much for negative evidence against such a process as Vuillemin supposes to have occurred. Let us now see what positive evidence there is for the second of the two views stated above. The most interesting piece of such evidence, because the most complete, is that of the Albugo series established by Stevens (:**01**) (and also quoted by Davis (l. c.)). Here the progressive increase in the development of the physiologically important coenocentrum, and the corresponding decrease in the now functionless receptive papilla indicate clearly the line of evolutionary advance. While along with this, we have a progressive reduction in the nuclei, starting with *Albugo Bliti* with many functional gamete-nuclei, through *A. Tragopogonis* with many potential but only a few functional, and ending with *A. candida*, with its single functional gamete-nucleus. This view may now be applied to the conditions in the three genera of the Entomophthoraceae.

In Empusa, the originally non-septate hyphae early break up into short segments, the hyphal bodies, which contain large numbers of small nuclei distributed irregularly through the plasm. The conidia are multi-nucleate. A large number of nuclei enter the apogamous resting-spore. In Entomophthora the hyphae usually break up into hyphal bodies, but as has been shown above in the case of *E. Geometralis* and as Cavara has shown in *E. Delpiniana*, the formation of hyphal bodies may not take place, in which case the hyphae often become septate. These septa cut off cells containing relatively few, regularly spaced nuclei which from their larger size have evidently an increase of efficiency. The tendency toward nuclear reduction has extended to the conidium resulting in a uni-nucleate condition. The resting-spores are formed by the fusion of coenogametes, with relatively few nuclei, — these coenogametes being of a simple type, where all of the nuclear material enters the zygote. In Basidiobolus the hyphae are regularly septate and do not normally form hyphal bodies. It has been shown, however, by Raciborski (**'96^a**) that under certain culture conditions such a formation of hyphal bodies may be induced. The tendency to nuclear reduction has here reached its highest expression, resulting in uni-nucleate cells. Accordingly the zygote is formed by the fusion of two uni-nucleate cells, which must, however, from a phylogenetic view-point, be considered coenogametes. The production of the peculiar beak-cells is clearly a late specialization and makes these coenogametes of the second of Davis's types, namely, where only a portion of the nuclear material is functional.

If the evolutionary line thus indicated be the true one, it is reasonable to expect that Empusa, as the primitive form, will show a possible derivation from other Phycomycetes. The close agreement of the methods of zygospore-formation, both as to general morphology and as to cytological conditions, between Entomophthora and the Mucorales, which has been pointed out above, indicates relationship in that direction. If we consider such a form as Sporodinia, we find hyphae with very numerous small nuclei; a sporangium forming a number of uni-nucleate spores; and zygospores formed by the fusion of primitive coenogametes. Apogamy in this species is of frequent occurrence; and it is reasonable to assume that the azygospores thus formed are multi-nucleate. Comparing with this the conditions in Empusa, we see at once a possible line of derivation. The hyphal bodies, into which, on account of the peculiar endophytic habit, the hyphae break, contain many small nuclei, which are, however, less numerous and larger than those of Sporodinia. Thaxter (**'88**) has already shown that the "conidium" of Empusa might be regarded as a sporangium. It is clear therefore that, just as the coenogamete is a gametangium into which has been extended the coenocytic habit and which accordingly functions as a unit, so the "conidium" of Empusa is a sporangium into which has been extended the coenocytic habit and which likewise functions as a unit. Apogamy has become constant in Empusa, resulting in the formation of multi-nucleate azygospores. The close relationship of Empusa and Sporodinia seems undeniable.

We may now sum up the line of development indicated in this paper.

I. Sporodinia: Hyphae non-septate, with very numerous very small nuclei. Non-sexual reproduction by a sporangium producing numerous uni-nucleate spores. Resting-spore formed by the fusion of primitive coenogametes with very numerous nuclei. Apogamy frequent.

II. Empusa: Hyphae breaking into hyphal bodies with numerous small nuclei irregularly distributed. Non-sexual reproduction by a multi-nucleate "conidium" which is morphologically a coenocytic sporangium. Apogamy constant.

III. Entomophthora: Hyphae either breaking into hyphal bodies after passing through a septate condition or else permanently septate, with few large nuclei regularly spaced. Tendency toward nuclear reduction extended to the non-sexual reproduction resulting in a uni-nucleate coenosporangium. Resting-spores formed by the fusion of primitive coenogametes with few nuclei. Apogamy occasional.

IV. Basidiobolus: Hyphae septate, breaking into hyphal bodies only under abnormal conditions, with a single nucleus in each cell. Non-sexual reproduction as in Entomophthora. Resting-spores formed by the fusion of coenogametes of a specialized type, which through the extension of the tendency to nuclear-reduction have become uni-nucleate. Apogamy absent.

These three genera of Entomophthoraceae thus form a specialized line, derived from a Mucor-like ancestry, and reaching a high stage in the development of certain well-defined evolutionary tendencies.

Summary.

Material of the following species was studied during the present investigation: *Entomophthora Americana*, *E.* "*x*," *E. Geometralis*, *E. echinospora*, *E. rhizospora*, *E. Fresenii*, and *Empusa Grylli*.

The nucleus of Entomophthora shows a well-developed structure. There is a relatively small chromatin-nucleolus surrounded by a more or less dense zone of chromatin granules. The division of the nucleus is by a well-developed process of mitosis.

During mitosis, spherical chromosomes are organized by the direct aggregation of the chromatin-granules, no spireme stage being present. In *E. Americana*, eight chromosomes were counted with reasonable certainty.

The linin-fibres freed by this process of aggregation separate into two groups which migrate to the respective poles of the nucleus, forming a typical, intranuclear, bipolar spindle, without centrosomes.

The conidia of Empusa are multi-nucleate, and of Entomophthora, uni-nucleate. No nuclear-division is present in the process of conidium-formation.

The zygospores of Entomophthora are formed by the fusion of multinucleate hyphal bodies, which are coenogametes of a primitive type, since all of the nuclear material is retained in the zygote. No fusion of the nuclei takes place during the first three months after formation. Whether or not such fusion in pairs takes place at the time of germination it has been impossible to determine, owing to the difficulty of germinating the zygospores under artificial conditions.

The epispore of the zygote is derived directly from the walls of the hyphal bodies. The endospore is formed by the direct transformation of an outer zone of the cytoplasm of the zygote.

In *Empusa Grylli* the azygospores are formed by the budding out of a hyphal body. The entire nuclear and cytoplasmic contents of the hyphal body passes into the azygospore, which is therefore always multi-nucleate. No division and no fusion of these takes place.

The cytological evidence suggests the derivation of the Entomophthoraceae from a Mucor-like ancestry, with the extension of the coenocytic habit to the non-sexual methods of reproduction, and with the progressive reduction in the number of nuclei, with the increasing efficiency of the survivors, a process beginning in Empusa, carried further in Entomophthora, and reaching its highest expression in Basidiobolus.

A preliminary notice of the results of the present study appeared in Rhodora (Vol. 8, p. 67) on March 27, 1906.

The writer takes this opportunity to make a grateful acknowledgment of his indebtedness to Professor Roland Thaxter, under whom the work was begun, for invaluable assistance in the collection of material, and for much advice; and to Professor W. G. Farlow, under whom the work was finished, for help in the preparation of this paper.

BIBLIOGRAPHY.

Bail, Th.

'69. Ueber Pilzepizootien der forstverheerenden Raupen. Schriften d. naturforsch. Gesel. Danzig, Vol. 2, pt. 2, pp. 1–25, pl. 1.

Brefeld, O.

'84. Conidiobolus utriculosus und C. minor n. spp. Untersuchungen aus dem Gesammtgebiete Mykologie, Pt. 6, p. 35.

Cavara, F.

'99^a. I nuclei delle Entomophthoreae in ordine alla filogenesi di queste piante. Bull. Soc. bot. italiana (1899), pp. 55–60.

'99^b. Osservazioni citologiche sulle Entomophthoreae. Nuovo Giornale botanico italiano, Vol. 6, pp. 411–466, pl. 4, 5.

Chamberlain, C. J.
:01. Methods in Plant Histology. Chicago, Ill. 1901.

Cohn, F.
'75. Ueber eine neue Pilzkrankeit der Erdraupen. Beiträge z. Biol. d. Pflanzen, Vol. 1, Pt. 1, pp. 58–84, pl. 45.

Davis, B. M.
:00. The Fertilization of Albugo candida. Bot. Gazette, Vol. 29, pp. 297–311, pl. 22.
:03. Oogenesis in Saprolegnia. Bot. Gazette, Vol. 35, pp. 233 and 320, pl. 9, 10.
:04. Studies on the Plant Cell. Amer. Naturalist, Vols. 38, 39.

Eidam, E.
'87. Basidiobolus, eine neue Gattung der Entomophthoraceen. Beiträge z. Biol. d. Pflanzen, Vol. 4, pp. 181–251, pl. 9–12.

Fairchild, D. G.
'97. Ueber Kerntheilung und Befruchtung bei Basidiobolus. Jahr. wiss. Bot., Vol. 30, pp. 285–296, pl. 13, 14.

Gerassimow, J. J.
:01. Ueber den Einfluss des Kerns auf das Wachstum der Zelle. Bull. Soc. Imp. Nat., Vol. 15, pp. 185–220, pl. A, B.
:02. Die Abhängigkeit der Grösse der Zelle von der Menge der Kernmasse. Zeitschr. für allg. Physiol., Vol. 1, pp. 220–258.
:04. Ueber die Grösse der Zellkerns. Beih. zum Bot. Centr., Vol. 18, p. 43.

Gruber, E.
:01. Ueber das Verhalten der Zellkerne in den Zygospore von Sporodinia grandis. Berichte d. Deutsch Bot. Gesell., Vol. 19, pp. 51–55, pl. 2.

Harper, R. A.
:05. Sexual Reproduction and the Organization of the Nucleus in certain Mildews. Carnegie Inst., No. 37, pp. 1–87, pl. 1–7.

Hertwig, R.
:03. Ueber Korrelation von Zell und Kerngrösse, usw. Biol. Centr., Vol. 23, pp. 49 and 108.

Lawson, A. A.
:03. On the Relationship of the Nuclear Membrane to the Protoplast. Bot. Gazette, Vol. 35, pp. 305–317, pl. 15.

Lindau, G.
'97. Zur Entwickelung von Empusa Aulicae, Reich. Hedwigia, Vol. 36, pp. 291–296.

Lowenthal, W.
:03. Beiträge zur Kenntniss des Basidiobolus lacertae, Eidam. Archiv. f. Protistenkunde, Vol. 2, pp. 369–420, pl. 10, 11.

Maupas, E.
'79. Sur quelques protorganismes animaux et vegetaux multinuclées. Compte Rendus, Vol. 89, pp. 250–253.

Mitzkewitsch, L.
'98. Ueber die Kerntheilung bei Spirogyra. Flora, Vol. 85, pp. 81–85, pl. 5.

Montgomery, T. H.
'98. Spermatogenesis in Pentatoma up to the formation of the Spermatid. Zool. Jahrb. (Abth. Anat.), Vol. 12, pp. 1–82, pl. 1–5.

Raciborski, M.
'96a. Ueber den Einfluss äusserer Bedingungen auf die Wachstumsweise der Basidiobolus ranarum. Flora, Vol. 82, pp. 107–132.
'96b. Mykologische Studien: Karyokinese bei Basidiobolus; usw. Anzeiger der Akad. Wissenchaften in Krakaw (1896), pp. 377–386, pl. 1.

Riddle, L. W.
:06. Contributions to the Cytology of the Entomophthoraceae: Preliminary Communication. Rhodora, Vol. 8, pp. 67, 68.

Ruhland, W.
:03. Studien ueber die Befruchtung der Albugo Lepigoni und einiger Peronosporeen. Jahr. wiss. Bot., Vol. 39, pp. 135–166, pl. 2, 3.

Schroeter, J.
'86. Entomophthorei. In Krytogamen-Flora von Schlesien, Vol. 3, p. 217.

Stevens, F. L.
:01. Gametogenesis and Fertilization in Albugo. Bot. Gazette, Vol. 32, pp. 77, 157, and 286, pl. 1–4.

Stevens, F. L., and Stevens, A. C.
:03. Mitosis in the Primary Nucleus in Synchytrium decipiens. Bot. Gazette, Vol. 35, pp. 405–415, pl. 16, 17.

Thaxter, R.
'88. The Entomophthoreae of the United States. Memoirs Boston Soc. Nat. Hist., Vol. 4, pp. 133–199, pl. 14–21.

Vuillemin, P.
'87. Études biologiques sur les Champignons. Bull. Soc. Sci. Nancy (1887), pp. 1–126, pl. 1–6.
:00. Développement des Azygospores chez les Entomophthorées. Comptes Rendus, Vol. 29, pp. 670–684, pl. 6.

Wager, H.
'98. The Nucleus of the Yeast Plant. Annals of Bot., Vol. 12, pp. 499–537, pl. 29, 30.

Wolfe, J. J.
:04. Cytological Studies on Nemalion. Annals of Bot., Vol. 18, pp. 607–630, pl. 40, 41.

Woycicki, Z.
:04. Einige neue Beiträge zur Entwicklungs-geschichte von Basidobolus ranarum, Eidam. Flora, Vol. 93, pp. 87–97, pl. 4.

EXPLANATION OF PLATES.

All the figures were drawn at stage-level with the aid of an Abbé camera, and with a Zeiss apochromatic 2 mm. objective and compensating ocular 4 (= × 500), 6 (= × 750), or 8 (= × 1000).

PLATE I.

Entomophthora Americana.

Figures 1–10. Mitosis (× 1000).

FIGURE 1. Resting-nucleus.

FIGURE 2. Prophase. Beginning of chromatin aggregation.

FIGURE 3. Prophase. Later stage of aggregation.

FIGURE 4. Prophase. Formation of the chromosomes, exposing the radiating linin-fibres.

FIGURE 5. Metaphase. Linin-fibres migrating to the poles.

FIGURE 6. Late prophase, showing eight chromosomes.

FIGURE 7. Early anaphase, showing typical spindle, with the daughter-chromosomes being drawn toward the poles.

FIGURE 8. Late anaphase. The daughter-chromosomes have reached the poles; the lower pole shows eight chromosomes.

FIGURE 9. Telophase. Reconstruction of the daughter-nuclei, which are separated by a vacuole.

FIGURE 10. Telophase. Separation of the daughter-nuclei by constriction.

Figures 11–14. Zygospore-formation (× 750).

FIGURE 11. Lateral fusion of the gametes, with budding of the spore from one of the gametes, the several nuclei of the hyphal bodies passing into the young zygospore (form "*x*").

FIGURE 12. Fusion of the gametes by their tips, with budding of the spore at the point of union.

FIGURE 13. Young zygospore before the formation of the cross-wall, with an attached empty hyphal body.

FIGURE 14. The same after the formation of the cross-wall. Figures 13 and 14 show the multi-nucleate condition.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

A REVISION OF THE ATOMIC WEIGHT OF BROMINE.

By Gregory Paul Baxter.

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By Gregory Paul Baxter.

Presented by T. W. Richards. Received May 28, 1906.

In numerous investigations in this laboratory upon the atomic weights of certain metals, in which metallic bromides were first titrated against the purest silver, and then the precipitated silver bromide was collected and weighed, the relation between the silver used in the titrations and the silver bromide obtained has yielded data from which the atomic weight of bromine may be calculated. Furthermore, in all these investigations, as a check upon the purity of the silver and bromine employed, silver bromide was synthesized directly from weighed quantities of silver and an excess of ammonium bromide or hydrobromic acid. Many of these results have already been collected and discussed by Richards,[1] nevertheless they are cited in the following table together with a few more recent determinations.

From the first of these ratios the atomic weight of bromine, referred to silver 107.930, is found to be 79.956, and from the second 79.955.

Very recently, in experiments in which silver iodide was heated first in a current of air and bromine until the iodine was completely displaced, and then in a current of chlorine to displace the bromine, the ratio of silver bromide to silver chloride was determined in six cases. From the results of these experiments the atomic weight of bromine was calculated to be 79.953,[2] if the atomic weight of chlorine is assumed to be 35.473.[3]

These values for bromine are in close agreement with those of Stas.[4] In his experiments weighed quantities of pure silver and bromine were

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[1] Proc. Amer. Phil. Soc., **43**, 119 (1904).

[2] Baxter, These Proceedings, **41**, 82 (1905).

[3] Richards and Wells, *Publications of the Carnegie Institution*, No. 28 (1905).

[4] Œuvres Completes, **1**, 603.

	Bromide analyzed.	Number of Experiments.	Analyst.	Reference.	Ratio $\frac{Ag}{AgBr}$
	Indirect Determinations.				
1	$BaBr_2$	Last seven	Richards	Proc. Amer. Acad., **28**, 28	57.444
2	$SrBr_2$	Seven	"	Ibid., **30**, 389	57.444
3	$ZnBr_2$	One	"	Ibid , **31**, 178	57.445
4	$NiBr_2$	Seven	Cushman	Ibid., **33**, 111	57.444
5	$CoBr_2$	Last five	Baxter	Ibid., **33**, 127	57.446
6	UBr_4	Three	Merigold	Ibid., **37**, 393	57.447
7	CsBr	Three	Archibald	Ibid., **38**, 466	57.444
8	$FeBr_2$	Two	Baxter	Ibid., **39**, 252	57.443
9	$CdBr_2$	Eight	Hines	Jour. Amer. Chem. Soc. **28**, 783	57.444
10	$MnBr_2$	Thirteen	"	Not yet published	57.444
	Average, weighted according to the number of determinations . .				57.4443
	Direct Determinations.				
11	HBr	Two	Richards	Proc. Amer. Acad., **28**, 17, 18	57.445
12	NH_4Br	One	"	Ibid., **30**, 380	57.446
13	HBr	Two	"	Ibid., **31**, 165	57.444
14	NH_4Br	One	Cushman	Ibid., **33**, 106	57.445
15	NH_4Br	One	Baxter	Ibid., **33**, 122	57.444
16	NH_4Br	Two	"	Ibid., **34**, 353	57.447
17	NH_4Br	Three	"	Ibid., **39**, 250	57.444
18	NH_4Br	One	Hines	Not yet published	57.443
	Average, weighted according to the number of determinations . .				57.4447

first titrated against each other, and then the precipitate of silver bromide was collected and weighed. Of the four results by the first method, one should be rejected according to his own statements, since the bromine was not thoroughly dried. The remaining three, 79.959, 79.961, and 79.960, give as an average 79.960. From the weight of silver bromide four values were obtained, 79.950, 79.952, 79.955, and 79.957, with an average of 79.954.

Marignac[5] also determined the ratio of silver to silver bromide, with somewhat lower results, — 79.959, 79.941, and 79.952; average, 79.950.

Scott,[6] in his analyses of ammonium bromide, obtained six values for the same ratio, varying between 79.936 and 79.948, with an average of 79.943. One of his results is here rejected, since the silver used in this experiment was known to be impure.

Dumas[7] by heating silver bromide in chlorine found the values 80.06, 79.89, and 79.96.

In computing the atomic weight of bromine from these data, great weight is always given to Stas's determinations, the value 79.955 being usually assumed as the most probable one for the constant in question. Certainly, as pointed out by Richards,[8] the true value must lie between 79.95 and 79.96. Clarke calculates the value 79.949 as the weighted average of the different investigations previous to Scott's.[9]

Considerable uncertainty exists as to the purity of the materials employed in much of the foregoing work. Richards and Wells[10] have already exhaustively investigated the various methods of preparing pure silver, and have found that while it is a comparatively simple matter to free this substance from metallic impurities, the absence of gaseous impurities is by no means so easy to secure. Oxygen may be eliminated best by fusion in an atmosphere of pure hydrogen gas,[11] or by prolonged fusion in a vacuum, while a lime boat was found to be the most suitable support for the silver during fusion.

In most of the experiments cited on page 202, one of the final steps in the purification of the silver was fusion of electrolytic crystals on lime, in many cases in a vacuum, but without especial care to prolong the fusion. Silver prepared in this way was found by Richards and Wells to contain traces of oxygen, derived from silver nitrate occluded by the electrolytic crystals. In cases 8, 9, 10, 17, and 18, however, the silver was fused in hydrogen. Richards and Wells showed also that Stas's silver contained at least one one hundredth of a per cent of impurity, since it yielded one one hundredth of a per cent less silver chloride than their purest silver.[12] Scott's silver in three cases was merely heated, not fused, in hydrogen, and in two of the others was fused before a blowpipe on calcic phosphate. In one experiment only the metal was fused on lime. No details are given as to the purification of the silver used by Marignac.

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[5] Œuvres Completes, **1**, 81.

[6] Jour. Chem. Soc. Trans., **79**, 147 (1901).

[7] Ann. Chem. Pharm., **113**, 20 (1860).

[8] Proc. Amer. Phil. Soc., **43**, 119 (1904).

[9] A Recalculation of Atomic Weights, Smith. Misc. Coll., 1897.

[10] Publications of the Carnegie Institution, No. 28, 16.

[11] Baxter, These Proceedings, **39**, 249 (1903).

Bromine also may be freed from impurities only with some difficulty. Experience in this laboratory has shown that chlorine may be eliminated most conveniently by distilling or precipitating the bromine from solution in a bromide. One such distillation is sufficient to remove chlorine completely only when the substance is initially comparatively pure. If, however, the process is repeated by converting a portion of the partially purified product into a bromide, and dissolving the remainder of the bromine in this comparatively pure bromide, the chlorine is eliminated so completely that further repetitions of this process have no apparent effect.[13] The removal of iodine may be easily effected by converting the bromine into hydrobromic acid or a soluble bromide, and boiling the solution with a small quantity of free bromine. Here again it is well to repeat the process several times, since the reaction between free bromine and the iodine ion, like that between free chlorine and the bromine ion, is undoubtedly incomplete.

The greater part of the experiments cited on page 202 were made with bromine which had been purified with due observance of these precautions. Of the other investigators, Stas seems to have been the only one to use sufficient pains to secure purity of the bromine. Stas removed iodine by shaking potassium bromide several times with free bromine and carbon disulphide, and in the course of the prolonged purification distilled the bromine twice from solution in a bromide. Marignac's purification consisted solely in crystallization of barium bromate and Scott's in distillation of hydrobromic acid.

Of the methods employed in these early determinations, that involving the analysis of metallic halides is least suited for the purpose, on account of the danger of occlusion of metallic salts by the precipitated silver bromide. That such an error actually exists to a slight extent is shown by the fact that the average of the "indirect" determinations is slightly larger than the average of the "direct" determinations. Obviously, if silver bromide is precipitated by means of either ammonium bromide or hydrobromic acid, occluded ammonium salts or free acids could be easily expelled by fusion of the bromide. This precaution was observed in most of the determinations recorded on page 202, and is absolutely essential for the complete elimination of water from the salt. Stas and Marignac both fused the silver bromide in their syntheses, but this operation was omitted by Scott, who dried the bromide at 180°. Scott's statement that the loss on fusion of silver bromide which had been dried at 180° was due to the presence of asbestos is contradicted by the experiments recorded later in this paper, in which the loss on fusion amounted to about one one hundredth of a per cent in the case of silver bromide which had been dried in a similar fashion and which was almost entirely free from asbestos.

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[12] Loc. cit., page 62.

[13] Attention has already been called to these points by Richards and Wells, These Proceedings, **41**, 440 (1906).

From this brief discussion of the more important errors which may have influenced previous determinations of the atomic weight of bromine, it is evident that some uncertainty still exists as to the true value of this constant. In the hope of throwing new light upon the subject, experiments were carried out by two of the methods outlined above, with especial precautions to insure purity of materials and to eliminate known possible errors in the experimental methods.

Both the methods chosen — synthesis of silver bromide from a weighed amount of silver, and conversion of silver bromide into silver chloride — have already been recently tested in this laboratory,[14] and have been found to be at least as satisfactory as any.

PURIFICATION OF MATERIALS.

Bromine. — In purifying bromine for this research, the principles set forth on page 204 of this paper were applied; but in some cases the purifying processes were repeated after the product was apparently pure, in order to make certain that further treatment had no effect.

Sample I was first completely dissolved in calcic bromide which had been made from about one third of the original material by means of lime and ammonia, and was then distilled from the solution. The product was covered with several times its volume of water, and was converted into hydrobromic acid by means of pure hydrogen sulphide which had been generated from ferrous sulphide with dilute sulphuric acid, and which had been thoroughly washed with water. After filtration from the precipitated sulphur and bromide of sulphur, the acid was boiled for some time, with occasional addition of small quantities of recrystallized potassium permanganate to eliminate the iodine. Finally the residual hydrobromic acid was heated with an equivalent amount of recrystallized permanganate, and the bromine was condensed in a flask cooled with ice.

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[14] Baxter, These Proceedings, **40**, 419; **41**, 73. Richards and Wells, Publications of the Carnegie Institution, No. 28.

Sample II was first converted into hydrobromic acid by means of red phosphorus and water, and the hydrobromic acid was then distilled, after having been boiled with an excess of bromine. An equivalent amount of permanganate was added, and the bromine liberated was separated from the solution by distillation. About one fourth of the product was next transformed into calcic bromide by means of ammonia and lime which was free from chloride, and the remaining three fourths of the bromine were dissolved in the calcic bromide and distilled. Still a third distillation from a bromide was carried out by reducing the product of the second distillation with hydrogen sulphide and subsequently oxidizing the hydrobromic acid with the purest recrystallized potassium permanganate, after boiling the acid with several small portions of permanganate to eliminate last traces of iodine.

Sample III was obtained by preparing calcic bromide from a portion of Sample II and distilling the remainder of Sample II from solution in this bromide.

In the case of Sample IV the processes of reduction to hydrobromic acid with hydrogen sulphide and oxidation of the hydrobromic acid with pure permanganate were four times repeated. After each reduction the hydrobromic acid was boiled with free bromine to remove iodine.

Sample V was three times reduced with hydrogen sulphide and oxidized with permanganate. One fourth the product was converted into calcic bromide and the remainder was dissolved in this calcic bromide and distilled.

Thus Sample I was twice distilled from a bromide; Sample II was treated three times in the same way; and Samples III, IV, and V four times.

Shortly before use each sample was distilled and converted into ammonium bromide by slow addition to an excess of redistilled ammonium hydroxide. The solution was then boiled to expel the excess of ammonia.

Silver. — Several different samples of silver were employed, many of which have already been used in atomic weight researches in this laboratory, and have shown evidence of great purity. For details concerning the purification the papers referred to should be consulted.

Sample A was employed in a determination of the atomic weight of iodine.[15] This specimen had been twice precipitated as chloride and once electrolyzed.

Sample B was used in experiments upon the atomic weight of iodine[16] and of manganese.[17] It was precipitated once as chloride, electrolyzed once, and finally precipitated as metal with ammonium formiate.

Sample C also was employed in a determination of the atomic weight of manganese, and was purified by recrystallizing silver nitrate, seven times from nitric acid and five times from aqueous solution. Finally the silver nitrate was reduced by means of ammonium formiate.

Sample D was prepared for the determination of the atomic weights of cadmium[18] and manganese, by one precipitation as chloride, one precipitation with ammonium formiate, and one electrolysis.

Sample E was first purified in part by precipitation as chloride, in part by precipitation with ammonium formiate. The combined material was then subjected to two electrolyses.

In all cases the electrolytic crystals were fused in a boat of the purest lime, contained in a porcelain tube, in a current of electrolytic hydrogen. After the buttons had been cleansed with dilute nitric acid and dried at 200°, they were cut into fragments of from four to eight grams either by means of a clean chisel and anvil or with a fine jeweller's saw. The latter method was employed in the case of Samples D and E, because it proved easier completely to free the silver from surface contamination with iron by etching the fragments with nitric acid, than when a chisel was used. The cleansing process with nitric acid was repeated until the solution thus obtained, after precipitation with hydrochloric acid and evaporation, proved free from iron. That every trace of iron could be removed by this treatment was proved by testing for iron the evaporated filtrates from several of the analyses subsequently recorded in this paper. Negative results were obtained in all cases.

After thorough washing with water and drying at 100°, the pieces of metal were heated to about 400° in a vacuum, and were preserved over solid potassium hydroxide in a desiccator.

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[15] Baxter, These Proceedings, **40**, 420 (1904).

[16] Baxter, Ibid., **41**, 79 (1905).

[17] Baxter and Hines. This paper will soon be published.

[18] Baxter and Hines, Jour. Amer. Chem. Soc., **28**, 772 (1906).

The Ratio of Silver to Silver Bromide.

The ratio of silver to silver bromide was determined as follows: Weighed quantities of silver were dissolved in the purest redistilled nitric acid diluted with an equal volume of water, in a flask provided with a column of bulbs to catch possible spatterings. However, during the solution of the silver the temperature was kept so low that almost no gas was evolved, and hence there was very little danger from this source. Next the acid solution of the silver was diluted with an equal volume of water, and was heated until free from nitrous acid and oxides of nitrogen. After still further dilution, the solution was added slowly with constant agitation to a dilute solution of an excess of ammonium bromide in a glass-stoppered precipitating flask, and the whole was violently shaken for some time to promote coagulation. By adding the silver solution to the bromide, occlusion of silver nitrate was almost wholly precluded. In some experiments the solutions were as dilute as twentieth normal, in others as concentrated as fourth normal. The final results seemed to be independent of the concentration of the solutions. At the end of about twenty-four hours the flask with its contents was again shaken, and then it was allowed to stand until the supernatant liquid was perfectly clear. The precipitate of silver bromide was collected upon a weighed Gooch crucible, after thorough washing by decantation with water, and was dried in an electric oven, first for several hours at 130°, finally for about fourteen hours at 180°. Then it was cooled and weighed.

The operations of precipitation and filtration were performed in a large cupboard lighted with red light, and if the flask was taken out of this cupboard it was enveloped in several thicknesses of black cloth.

Even after the prolonged drying, traces of moisture were retained by the salt, and could be expelled only by fusion. This was done by transferring the bulk of the silver bromide, freed as completely as possible from asbestos, to a small porcelain crucible which was weighed with its cover. The silver bromide was then fused by heating the small crucible, contained in a large crucible to prevent direct contact with the flame of the burner. A temperature much above the fusing point of silver bromide was avoided so that volatilization of the salt could not take place. This treatment must have eliminated occluded ammonium salts as well as water. Finally, in order to convert any occluded silver nitrate, metallic silver, or silver sub-bromide into silver bromide, the salt was again fused in a current of dry air containing bromine vapor. This treatment seldom produced any measurable effect either upon the weight or the appearance of the salt, which was perfectly transparent and of a light yellow color even after the first fusion in air.

A few shreds of asbestos displaced from the crucible, together with an occasional trace of silver bromide which escaped the crucible, were collected upon a tiny filter paper which was then ignited in a porcelain crucible. Before being weighed the ash was either treated with a drop of nitric and hydrobromic acids and again heated, or else was heated for some minutes in a current of air and bromine.

The filtrate and washings were evaporated to small bulk. The precipitating flask and all other glass vessels used in the analysis were rinsed with ammonia and the rinsings were added to the evaporated filtrate and wash waters. The whole was then tested in a nephelometer for silver and the quantity found was estimated by comparison with standard silver solutions. In most cases the correction thus obtained was less than one tenth of a milligram.

The asbestos which formed the felt in the Gooch crucible, after having been shredded, was digested for some hours with aqua regia and was then thoroughly washed with water. Before the empty crucible was weighed, the felt was ignited with a Bunsen burner. Crucibles thus treated and then heated to 180° after being moistened with water did not change in weight.

In the following table are cited all the analyses which were completed without accident. Vacuum corrections of -0.000031 for every apparent gram of silver and of $+0.000041$ for every apparent gram of silver bromide are applied.[19] The atomic weight of silver is assumed to be 107.930.

The platinum plated brass weights were standardized from time to time and were found to retain their original values within a very few hundredths of a milligram in all cases.

THE RATIO OF SILVER BROMIDE TO SILVER CHLORIDE.

The ratio of silver bromide to silver chloride was determined much as described in previous papers upon the atomic weight of iodine.[20] Pure silver bromide was prepared by precipitation of silver nitrate with an excess of ammonium bromide. The silver employed was

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[19] The specific gravity of the weights was determined to be 8.3.

The specific gravity of silver has been found to be 10.49. Richards and Wells, Publications of the Carnegie Institution, No. 28, 11.

The specific gravity of fused silver bromide has been found to be 6.473. Baxter and Hines, Amer. Chem. J., **31**, 224.

[20] Baxter, These Proceedings, **40**, 432 (1904); **41**, 75 (1905).

TABLE I.—THE ATOMIC WEIGHT OF BROMINE. — SERIES I.
Ag : AgBr Ag = 107.930

Number of Analysis.	Sample of Silver.	Sample of Bromine.	Weight of Silver in Vacuum.	Weight of Silver Bromide in Vacuum.	Loss on Fusion in Bromine.	Weight of Asbestos.	Dissolved Silver Bromide.	Corrected Wt. of Silver Bromide.	Ratio $\frac{Ag}{AgBr}$.	Atomic Weight of Bromine.
			grams.	grams.	grams.	grams.	grams.	grams.		
1	A	II	4.71853	8.21362	0.00027	0.00007	0.00021	8.21363	57.4476	79.946
2	B	II	5.01725	8.73387	0.00020	0.00019	0.00007	8.73393	57.4455	79.952
3	A	II	5.96818	10.38799	0.00023	0.00034	0.00122	10.38932	57.4453	79.953
4	C	II	5.62992	9.80000	0.00017	0.00039	0.00017	9.80039	57.4459	79.951
								Average,	57.4461	79.950
5	A	IV	8.13612	14.16265	0.00002	0.00047	0.00024	14.16334	57.4449	79.954
6	C	IV	5.07238	8.82932	0.00013	0.00043	0.00035	8.82997	57.4451	79.954
7	B	IV	4.80711	8.36838	0.00015[1]	0.00000	0.00004	8.36827	57.4445	79.956
								Average,	57.4448	79.955
8	C	V	4.27279	7.43771	0.00010[1]	0.00009	0.00007	7.43776	57.4473	79.947
9	A	V	5.86115	10.20299	0.00007[1]	0.00004	0.00003	10.20299	57.4454	79.953
10	B	V	7.91425	13.77735	0.00010	0.00010	0.00001	13.77736	57.4439	79.958
11	D	V	6.40765	11.15461	0.00000	0.00006	0.00001	11.15468	57.4436	79.959
12	D	V	6.38180	11.10942	0.00025	0.00012	0.00001	11.10930	57.4456	79.952
13	D	V	6.23696	10.85722	0.00005	0.00004	0.00001	10.85722	57.4453	79.953
								Average,	57.4452	79.953
14	E	I	9.18778	15.99383	0.00014	0.00019	0.00004	15.99392	57.4455	79.953
15	E	I	8.01261	13.94828	0.00018	0.00009	0.00007	13.94826	57.4452	79.953
16	E	I	10.48638	18.25467	0.00025[1]	0.00007	0.00001	18.25452	57.4454	79.953
								Average,	57.4454	79.953
17	E	III	8.59260	14.95790	0.00014	0.00016	0.00005	14.95797	57.4450	79.954
18	E	III	8.97307	15.62013	0.00004	0.00007	0.00006	15.62022	57.4452	79.953
								Average,	57.4451	79.954
		Total.	121.67653	. . .	. . .	. . .	. . .	211.81305	57.4452	79.953
						Average of all 18 determinations,			57.4453	79.953
						Average of last 7 determinations,			57.4453	79.953

[1] Fusion in bromine lost, after the salt had been fused in air.

purified either by precipitation as chloride and reduction with invert sugar, or by electrolysis, or by precipitation with ammonium formiate. The metal was then fused before a blowpipe upon a crucible of the purest lime, and the buttons were thoroughly cleansed with nitric acid. No further purification was considered necessary since the weight of the metal was of no consequence.

After the silver bromide had been washed by decantation with water, in some cases it was collected in a Gooch crucible in which a disk of filter paper was employed instead of asbestos, and after drying at 100° it was carefully separated from the filter paper. In other cases the precipitate was transferred to a platinum dish, and was drained with a platinum reverse filter[21] with a disk of filter paper. In still others a platinum Gooch crucible with small holes was found to be sufficiently effective as a filtering medium without the use of either asbestos or filter paper.

Before being weighed the silver bromide was fused in a current of air saturated with bromine in a weighed quartz crucible. The air was purified by passing successively over beads moistened with silver nitrate solution, over sodium carbonate, and finally over concentrated sulphuric acid which had been heated to its boiling-point with a small quantity of recrystallized potassium dichromate to eliminate volatile and oxidizable impurities. The air was then passed through dry bromine in a small bulb. This apparatus was constructed entirely of glass with ground joints. The tube which conducted the gases into the crucible passed through a Rose crucible cover of glazed porcelain in all experiments except Analyses 28 to 31, in which a quartz cover was employed. The quartz crucibles were always contained in large porcelain crucibles while being heated. They remained almost absolutely constant in weight during the experiments. The bromine was in each case a portion of the sample from which the silver bromide had been made.

Next the bromide was heated barely to fusion in a slow current of chlorine, generated by the action of hydrochloric acid upon manganese dioxide, and dried by means of concentrated sulphuric acid. The apparatus for this purpose also was constructed wholly of glass. When the bromine was apparently completely displaced, the silver chloride was heated in the air for a few minutes to expel dissolved chlorine, and then was cooled and weighed. A repetition of the heating in chlorine seldom affected the weight of the salt more than a few hundredths of a milligram, although occasionally a third heating was necessary to effect this result.

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[21] Cooke, These Proceedings, **12**, 121.

That no loss of silver chloride by volatilization took place is certain for two reasons. In the first place the cover of the crucible and the delivery tube for the bromine when rinsed with ammonia and the solution treated with a slight excess of hydrochloric acid gave no visible opalescence in the nephelometer. In the second place the weight of the chloride became constant without difficulty. It has already been shown that silver chloride which has been fused in chlorine, if subsequently heated in air, retains no excess of chlorine.[22]

The following vacuum corrections were applied: silver bromide, + 0.000041; silver chloride + 0.000071.[23] The atomic weight of chlorine referred to silver 107.930 is assumed to be 35.473.

Aside from the close agreement of all the results of Series I, the fact is to be emphasized that of the last seven analyses, which were consecutive, only two differ from the average of the series, 79.953, by as much as one one thousandth of a unit. Furthermore, there is no evidence of any dissimilarity in the different preparations of bromine. Material which has received only two distillations from a bromide gives values no lower than bromine which has been thus treated four times. The various specimens of silver also show no difference in purity.

In the case of Series II, the extreme variation of the results is only four thousandths of a unit, and only one of the thirteen experiments yielded a value which differs from the average by more than one one thousandth of a unit.

Finally, the difference between the averages of Series I and II is only seven ten thousandths of a unit. It is extremely unlikely that constant errors could have affected both series equally, so that this striking agreement is strong proof that both series are free from such errors.

It has already been pointed out that the average of Stas's syntheses, 79.954, probably represents with considerable accuracy the atomic weight of bromine, and that certainly his determinations are more accurate than those of later experimenters. His syntheses are few in number, however, and differ among themselves by several thousandths of a unit, so that they do not define within this amount the constant in question. Their average, however, confirms the value

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[22] Baxter, These Proceedings, **40**, 432 (1904); Richards and Wells, Publications of the Carnegie Institution, No. 28, page 59.

[23] Richards and Stull have found the density of fused silver chloride to be 5.56.

TABLE II. — THE ATOMIC WEIGHT OF BROMINE.

SERIES II.

AgBr : AgCl.

Ag = 107.930 Cl = 35.473

Number of Analysis.	Sample of Bromine.	Weight of Silver Bromide in Vacuum.	Weight of Silver Chloride in Vacuum.	Ratio $\frac{\text{AgBr}}{\text{AgCl}}$.	Atomic Weight of Bromine.
		grams.	grams.		
19	II	8.03979	6.13642	131.0176	79.953
20	II	8.57738	6.54677	131.0170	79.952
21	II	13.15698	10.04221	131.0168	79.952
			Average,	131.0171	79.952
22	IV	12.71403	9.70413	131.0167	79.952
23	IV	13.96784	10.66116	131.0162	79.951
			Average,	131.0164	79.952
24	V	13.08168	9.98469	131.0174	79.953
25	V	12.52604	9.56059	131.0175	79.953
26	V	11.11984	8.48733	131.0170	79.952
27	V	8.82272	6.73402	131.0172	79.953
			Average,	131.0173	79.953
28	I	11.93192	9.10721	131.0162	79.951
29	I	12.53547	9.56767	131.0190	79.955
			Average,	131.0176	79.953
30	III	17.15021	13.09009	131.0167	79.952
31	III	10.31852	7.87572	131.0168	79.952
			Average,	131.0168	79.952
	Total,	153.94242	117.49801	131.0170	79.952
Average of all 13 experiments				131.0171	79.952
Average of Series I and II				. . . . .	79.953

obtained in this paper. From all the experiments here described the number 79.953 seems to be the most probable value for the atomic weight of bromine.

In conclusion, attention may be called to the fact that a diminution in the atomic weight of bromine raises slightly all atomic weights resulting from the analysis of metallic bromides by precipitation with silver.

I am deeply indebted to the Carnegie Institution of Washington and to the Cyrus M. Warren Fund for Research in Harvard University for assistance in pursuing this investigation.

Chemical Laboratory of Harvard College,
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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE, E. L. MARK, DIRECTOR. — No. 184.

THE OPTIC CHIASMA OF TELEOSTS: A STUDY OF INHERITANCE.

By A. P. Larrabee.

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THE OPTIC CHIASMA OF TELEOSTS: A STUDY OF INHERITANCE.

BY A. P. LARRABEE.

Presented by W. E. Castle. Received June 28, 1906.

INTRODUCTION.

IN the majority of teleosts, as has been pointed out by several investigators, the condition of the optic chiasma is markedly different from that found in other vertebrates. In these teleosts the nerve fibres of the two optic nerves do not interlace at the chiasma, but each nerve remains distinct, so that the two can easily be dissected apart at that point. Conflicting statements regarding the condition of the chiasma are found, some writers stating that the nerve running to the right eye is dorsal, others making the opposite statement. In recent years Parker (**:03**) has shown that one condition is about as common as the other. He made a dissection of one hundred specimens, each of ten different species of symmetrical teleosts. Of these, 514 had the right nerve dorsal and 486 the left nerve dorsal. In each case the right or left nerve signifies the nerve running to the right or left eye respectively. He showed, too, that this dimorphism is not correlated with sex. To quote from his paper giving his conclusions on this point: "In Fundulus, of the 51 specimens with the left nerves dorsal, 29 were females and 22 males; and of the 49 with the right nerves dorsal, 29 were females and 20 males. Of the 43 specimens of Tautogolabrus with the left nerves dorsal, 26 were females and 17 males; and of the 57 with the right nerves dorsal, 26 were females and 31 were males. These figures show clearly that there is no close correspondence between the crossing of the optic nerves and sex."

The question whether this dimorphism is due to racial differences was left undecided. The writer will presently answer this question in the negative, and at the same time discuss the relation of the dimorphism to heredity, and in the brook trout the effects of altered action of gravity during development, together with the results obtained from the study of monstrous specimens and from the development of the optic nerves. The work on the inheritance of the optic chiasma was begun by Dr. W. E. Castle and Mr. F. E. Pomeroy, and was carried on and completed by the writer under the direction of Dr. Castle, to whom the writer wishes to acknowledge his great indebtedness for the material placed at his disposal and for aid and criticism during the progress of his studies. He is also grateful to Mr. Pomeroy for the use of results obtained by him at the beginning of this investigation. Thanks are due also to the Massachusetts Commissioners of Fisheries and Game, and in particular to Mr. Arthur Merrill, superintendent of the fish-hatchery at Wilkinsonville, Massachusetts, for supplying and rearing the brook trout material; also to the officers of the Bureau of Fisheries of the United States Department of Commerce and Labor who supplied and reared at Wood's Hole, Massachusetts, the young codfish used.

OBJECT OF THE INVESTIGATION AND THE METHOD EMPLOYED.

The writer has already hinted at the object for which this investigation was undertaken. The main object was to test the relation of heredity to the dimorphism of the chiasma, and especially to determine the applicability of Mendel's Law, should heredity prove to be a factor in this dimorphic condition. In order to determine whether or not heredity is a factor, matings of selected individuals were made, and the effects determined by dissecting the young. For this purpose two species of fish were used, the common brook trout (*Salvelinus fontinalis* Mitchill) and the codfish (*Gadus morrhua* L.). Since the spawning season of these two is mainly during the months of November and December, they were especially convenient forms with which to work. In making these matings, the eggs of a female were commonly divided into two or three lots, and each lot fertilized artificially with the milt of a different male. The parents were then dissected to determine the position of the nerves in the chiasmata; the different lots of eggs were carefully marked, and the necessary data recorded for future use. This part of the work was done by Dr. Castle; the study of the first generation of brook trout and of the first lot of young codfish was begun by Mr. Pomeroy and completed by the writer, who also carried out the remaining studies described in this paper and prepared this report for publication.

In preserving the two species of fish for study, different methods of fixation were found necessary. The young trout were of sufficient size to allow dissection with needles under the dissecting microscope. They were killed and fixed in Kleinenberg's picro-sulphuric solution, washed out in 70 per cent alcohol, and finally preserved in 85 per cent alcohol. The young cod were too minute to be dissected in the same way as the trout. It was necessary, therefore, to imbed these in paraffine, and section, using the compound microscope to follow the course of the optic nerves. Vom Rath's platinic-osmic-acetic fluid was found to be the best fixative for use with the cod.

Throughout this paper the writer will designate the optic nerves according to the method employed by Dr. Parker, that nerve being named *right* which runs to the right eye; the one to the left eye, *left*. For convenience, too, the specimens with the right nerve dorsal will be termed *rights*, and similarly those with the left nerve dorsal, *lefts*.

RESULTS OF SELECTED MATINGS IN THE BROOK TROUT (*Salvelinus fontinalis* Mitchill).

First Generation.

The first generation of the trout includes the young regarding which the condition of the chiasmata of one or both parents, but not of the grandparents, was known. The results obtained from this generation are shown in Tables I, II, and III. In all, there are eleven lots, four of which were obtained in 1901, the remaining ones in 1902. In each year where a male or female was used for more than one lot, the fact can be recognized by the correspondence of the numbers following the sex signs. The numbers of the two years, however, relate to different individuals. The control lot includes a number of young taken at random from the fry at the hatchery without any knowledge of the condition of the crossing of the nerves in the parents. This lot, comprising young of various parents, is assumed to represent the average condition of the trout in their natural state.

The proportion of rights and lefts found in the eleven lots of the first generation corresponds closely with that obtained from the control lot. The percentage of rights in both cases is practically the same, — 56 per cent. With the exception of two lots, there is a slight excess of rights in each case. In the nine lots where this excess is found, the rights outnumber the lefts by from 0.4 per cent to 20 per cent. The difference between the percentages of rights and of lefts in the series as a whole is 11.8 per cent, which closely approximates the difference of 12.2 per cent found between the two kinds in the control lot. Nothing in the net results indicates that in one case we are dealing with the young of selected parentage and in the other with young taken at random.

Tables II and III show from another point of view the results obtained from the first generation. In Table I the results are given as a whole, but in these two tables the results are grouped according to the condition of the chiasmata in the parents. Lots C and D, 1901, have to be omitted from Tables II and III, as the crossing in the female is not known. In comparing the results in these two tables, it is interesting to note

TABLE I.

Results of Selected Matings in the Brook Trout (*Salvelinus fontinalis* Mitchill).

First Generation.

Lot.	Nature of Mating.	Total Number of Young dissected.	Right Nerve[1] Dorsal.	Left Nerve[2] Dorsal.	Per cent R.	Per cent L.
A, 1901	$♀_1$ R × $♂_1$ L	792	471	321	59.4	40.6
B, “	$♀_1$ R × $♂_2$ R	346	188	158	54.3	45.7
C, “	$♀_2$? × $♂_3$ L	677	380	297	56.1	43.9
D, “	$♀_2$? × $♂_4$ R	798	479	319	60.0	40.0
1, 1902	$♀_1$ R × $♂_1$ L	122	62	60	50.8	49.2
2, “	$♀_1$ R × $♂_2$ R	105	59	46	56.0	44.0
3, “	$♀_1$ R × $♂_3$ L	212	103	109	48.6	51.4
4 “	$♀_2$ R × $♂_1$ L	168	91	77	54.1	45.9
5, “	$♀_2$ R × $♂_2$ R	420	212	208	50.2	49.8
6, “	$♀_3$ R × $♂_4$ R	100	48	52	48.0	52.0
7, “	$♀_3$ R × $♂_5$ L	225	126	99	56.0	44.0
Total		3965	2219	1746	55.9	44.1
Control		650	365	285	56.1	43.9
Grand total		4615	2584	2031	56.0	44.0

[1] Nerve to right eye.

[2] Nerve to left eye.

that, although the difference between the two groups is small, yet, in the case of the young whose parents were both rights, the tendency towards equality is slightly greater. In other words, among the young whose parents were both rights, the proportion of rights is smaller than among those which had but one parent with the right nerve dorsal.

TABLE II.

Lots from Table I in which the Condition of the Chiasma was the same (R × R) in Both Parents.

Lot.	Total No. dissected.	Right Nerve Dorsal.	Left Nerve Dorsal.	Per cent R.	Per cent L.
B, 1901	346	188	158	54.3	45.7
2, 1902	105	59	46	56.0	44.0
5, "	420	212	208	50.2	49.8
6, "	100	48	52	48.0	52.0
Total	971	507	464	52.2	47.8

TABLE III.

Lots from Table I in which the Condition of the Chiasma was Unlike in the two Parents.

Lot.	Total No. dissected.	Right Nerve Dorsal.	Left Nerve Dorsal.	Per cent R.	Per cent L.
A, 1901	792	471	321	59 4	40.6
1, 1902	122	62	60	50.8	49.2
3, "	212	103	109	48.6	51 4
4, "	168	91	77	54.1	45.9
7, "	225	126	99	56.0	44.0
Total	1519	853	666	56.2	43.8

It is evident from the close correspondence of the percentages shown in Tables II and III with those of the control lot (Table I), that if the Mendelian principles of heredity can be applied at all to the case of the optic chiasma, it is an example of alternative dominance, for neither condition is uniformly dominant over the other. The results shown in Table III, where the crossing of the nerves was different in the two parents, correspond closely with those to be expected if dominance is alternative. If such is the case, it is necessary to assume either that the parents were a "pure" right and a "pure" left, respectively, or that both were heterozygotes. On the former assumption, the results to be expected from the Mendelian formula would be equal numbers of rights and lefts, all heterozygotes, thus:

$$\begin{array}{l} R + R = \text{gametes of first parent.} \\ L + L = \text{'' \quad '' second '' .} \\ \hline 2\,R\,(L) + 2\,L\,(R) = \text{offspring.} \end{array}$$

If both parents were heterozygotes, the apparent result would be the same, but half each of the right and of the left offspring would be "pure," the other half heterozygous, thus:

$$\begin{array}{l} R + L = \text{gametes of first parent.} \\ R + L = \text{'' \quad '' second '' .} \\ \hline R + R\,(L) + L\,(R) + L = \text{offspring.} \end{array}$$

If one parent were a pure right and the other a heterozygote, the proportion of rights to be expected would be 75 per cent, thus:

$$\begin{array}{l} R + R = \text{gametes of first parent.} \\ R + L = \text{'' \quad '' second '' .} \\ \hline R + R + R\,(L) + L\,(R) = \text{offspring.} \end{array}$$

The last assumption evidently is untenable, for in none of the lots studied is this proportion approximated.

This principle of alternative dominance could be applied to the results in Table II only by assuming that both parents were heterozygous. On this assumption alone could the observed almost equal proportions of right and of left offspring be explained.

The first generation furnishes no direct evidence which would allow a positive assumption that the facts are to be explained by Mendel's Law.

Further, the results of the first generation show conclusively that prepotency is not found in any individual. In Lots 1, 2, 4, and 5, 1902, the eggs of two trout were divided each into two lots, making four in all. These were fertilized by two males so as to get as many combinations as possible. The following table (IV) gives the crosses together with the percentage of rights resulting from each:

TABLE IV.

Lot.	Nature of Mating.	Per cent R.
1, 1902	♀$_1$ R × ♂$_1$ L	50.8
2, "	♀$_1$ R × ♂$_2$ R	56.0
4, "	♀$_2$ R × ♂$_1$ L	54.1
5, "	♀$_2$ R × ♂$_2$ R	50.2

An examination of this table makes it evident that no prepotency is found in any individual or in either sex.

TABLE V.

Results of Selected Matings in the Brook Trout
(*Salvelinus fontinalis* Mitchill).

Second Generation.

Lot.	Nature of Mating.	Total Number of Young dissected.	Right Nerve Dorsal.	Left Nerve Dorsal.	Per cent R.	Per cent L.
1, 1903	♀$_1$ R × ♂$_1$ R	39	21	18	53.8	46.2
1 a, "	♀$_1$ R × ♂$_2$ R	8	4	4	50.0	50.0
2, "	♀$_2$ L × ♂$_3$ L	15	8	7	53.3	46.7
2 a, "	♀$_2$ L × ♂$_4$ L	28	7	21	25.0	75.0
3, "	♀$_3$ L × ♂$_5$ L	41	21	20	51.2	48.8
4, "	♀$_4$ R × ♂$_6$ R	19	10	9	52.6	47.4
4 a, "	♀$_4$ R × ♂$_7$ R	20	10	10	50.0	50.0
5, "	♀$_5$ R × ♂$_8$ R	69	33	36	47.8	52.2
6, "	♀$_6$ R × ♂$_9$ R	96	51	45	53.1	46.9
Total		335	165	170	49.2	50.8

Results of the Second Generation.

The second generation of trout (Table V) were descended from Lot B, 1901. This lot included offspring from parents both of which had the right nerve dorsal in the chiasma. The condition of the crossing in the more remote ancestors is unknown, but from the results of the first generation and of the control lot, it may safely be assumed that they were about equally divided between rights and lefts.

The trout used for the second generation were small and immature. Consequently the number of eggs produced was small, and the number of young hatched proportionately smaller than in ordinary lots. Six females and nine males were used. The eggs of three of the females were divided into two lots each, making nine lots in all. The young trout were taken soon after hatching and fixed, as before, in Kleinenberg's picro-sulphuric solution.

TABLE VI.

Group.	Total.	Number of "Rights."	Number of "Lefts."	Per cent R.	Per cent L.
A. (L × L)	84	36	48	43	57
B. (R × R)	251	129	122	51	49

In these nine lots there were 335 specimens, of which 165 were rights and 170 were lefts. These 335 trout can be divided into two groups according to their parentage. Group A includes those whose parents were both lefts; Group B, those whose parents were both rights. The results are summarized in Table VI.

The excess of lefts in Group A at first thought seems significant, but is doubtless due to the smallness of the lot (compare the result given by Lot 2 a, 1903). This lot gave the same proportion (75 per cent lefts to 25 per cent rights) which might be expected were this a case of dominance under Mendel's Law, but the number of young is so small as to make the chances of error very great. Not only the small number included in Group A throws doubt on the significance of the result, but also the results of the larger group (B). In this latter group there are 251 individuals, of which 129 or 51 per cent are rights. We are dealing here with individuals whose parents and grandparents were rights, and yet the result is practical equality of rights and lefts.

When discussing the results of the first generation, the writer stated that if Mendelism applied to this dimorphism, the case must be one of alternative dominance, and the suppositional grounds were stated on which that claim might be based. In discussing the group which included the young of trout both of which were rights, it was stated that both parents must be heterozygotes. Assuming this to be the case, it would follow that of the rights produced in the so-called first generation, one half would be pure and the other half heterozygous, i. e., rights having the left condition recessive. According to the laws of chance, out of a considerable number of these rights, there would be one chance out of three of obtaining a "pure" right. Among the parents of Group B there were four females and six males. Applying the law of chance stated above, it is improbable that out of ten specimens not one should prove to be a "pure" right, — an animal which would give at least 75 per cent of right offspring. These results lead inevitably to the conclusion that the Mendelian principles of heredity do not apply to the dimorphism of the chiasmata of the trout, thus confirming the doubts caused by the results of the first generation.

The trout in Group B, whose parents and grandparents were rights, afford a basis for comparison with the results to be expected from another theory of heredity, namely, the one advanced by Galton ('89). According to this theory, the parents contribute one half the hereditary influences, the grandparents one quarter, the great-grandparents one eighth, and so on.

In Group B there were 251 individuals, whose parents and grandparents were rights. According to the law of Galton, the influence of these more immediate ancestors would be 75 per cent. The remoter ancestors would probably be divided about equally between rights and lefts. This would increase the influence of the rights to 87½ per cent. Of the 251 trout, then, about 87½ per cent, or 219 individuals, should have the right nerve dorsal. Actually there were 51 per cent, or 129 rights. Such a discrepancy as this between the calculated and observed results clearly indicates the inapplicability of Galton's Law.

The results of selective breeding in the trout clearly indicate that neither dimorphic condition is a racial character. All the trout came from the same locality, and although bred to the second generation, no effect of selection on the dimorphism was evident, unless possibly a tendency to make the two conditions more nearly equal.

THE RESULTS OF SELECTED MATINGS IN THE CODFISH (*Gadus morrhua* L.).

The results obtained from the codfish have little significance in themselves, but they serve to confirm the conclusions drawn from the study of the brook trout.

In the examination of the codfish, made by Dr. Parker, 60 out of 100 individuals had the right nerve dorsal. Table VII shows a similar result for several selected matings of different sorts. Comparing this table with Table I, where the results of the first generation of trout are tabulated, a similarity is found with the conditions observed in that species. The range of variation in the percentage of rights is greater in the case of the cod than in the trout. Lot D, 1901, is specially noteworthy. With both parents lefts, there is among the young an excess of rights instead of lefts, as might be expected. From the similarity of results in the two species, the conclusions which were drawn in the case of the trout are made more certain. Selective breeding again fails to bring out any evidence that either condition is hereditary.

TABLE VII.

RESULTS OF SELECTED MATINGS IN THE CODFISH (*Gadus morrhua* L.).

Lot.	Nature of Mating.	Total Number of Young dissected.	Right Nerve Dorsal.	Left Nerve Dorsal.	Per cent R.	Per cent L.
A, 1901	♀$_1$ R × ♂$_1$ R	241	159	82	66.0	34.0
B, "	♀$_1$ R × ♂$_2$ L	168	113	55	67.3	32.7
C, "	♀$_2$ L × ♂$_1$ R	150	82	68	54.7	45.3
D, "	♀$_2$ L × ♂$_2$ L	249	145	104	58.3	41.7
9, 1902	♀$_4$ R × ♂$_6$ R	142	70	72	49.3	50.7
10, "	♀$_4$ R × ♂$_7$ L	102	52	50	50.9	49.1
Total		1052	580	552	55.2	44.8

RESULTS OF INVESTIGATION OF THE CHIASMATA IN MONSTROUS SPECIMENS OF *Salvelinus fontinalis*.

Double monsters among trout fry are of common occurrence. These vary greatly in the degree and kind of malformation. The specimens used for this investigation varied from those having two heads and one body to those with heads and bodies entirely distinct but with a yolk sac in common. For convenience the individuals have been placed in four groups.

Group I includes the double-headed specimens which were divided only anterior to the dorsal fin.

Group II includes double-headed specimens which were divided posterior to the dorsal fin, but had a portion of the vertebral column in common.

Group III includes specimens in which the two bodies were united throughout a portion or the whole of the tail region, but were elsewhere distinct.

Group IV includes specimens with two young entirely distinct and on opposite sides of the yolk sac.

The bearing which these dicephalous specimens have upon the subject under discussion can be shown by briefly considering some of the theories advanced and some of the results obtained by those who have made a study of this condition. Among other causes of dicephaly, polyspermy has been advanced by several writers as a probable cause. If this occurred, it is evident that the heredity of these monstrosities would be more complicated than that of normal individuals.

Schmitt (:02), in a paper on the gastrulation of double trout embryos, stated that the germinal area of the egg which produced a double embryo was no larger, and contained no more material, than the one which gave rise to a normal trout. In the formation of the gastrula two invaginations occurred simultaneously in different parts of the blastoderm. This observation indicates that polyspermy had not occurred; for if it had occurred, there would have been an increase in the material contained in the germinal area. This change would also occur if the egg were fertilized by a double-headed sperm, one of the tentative causes for duplicity advanced by some writers.

Hans Spemann (:01, :01^a), in experimenting with the eggs of the salamander Triton, was able to produce dicephalous individuals, or even two distinct individuals, by means of a ligature of hair tied across the blastoderm. The extent of the duplicity depended on the tightness of the ligature. A similar result has been obtained by Loeb and others, although by different methods.

From these various observations it is evident that an increase in the amount of sperm, either by polyspermy or otherwise, is not, at least commonly, the cause of dicephaly. Accordingly in two-headed trout, the hereditary characters of the divided parts should be the same. It would follow, then, that if the dimorphic condition is hereditary, the condition of the crossing should be the same in each head. By referring to Table VIII, it will be seen that in more than one half of the double-headed specimens dissected the crossings in the two heads were opposite. This is very significant, for it indicates that the dimorphic condition of the chiasma is not hereditary. If heredity is not a factor, what does determine the crossing of the nerves in the chiasmata? In attempting to answer this question some other possible factors were examined, but with negative results.

TABLE VIII.

Results obtained from Dissection of Monstrous Specimens of *Salvelinus fontinalis*.

Group.	Right Nerve Dorsal in Both Heads.	Left Nerve Dorsal in Both Heads.	Right Dorsal in one Head, Left in the other.	Per cent R + R.	Per cent L + L.	Per cent R + L.
I.	24	8	21	45.0	16.0	39.0
II.	10	6	25	24.4	14.7	60.9
III.	10	4	23	27.0	11.0	62.0
IV.	11	7	22	27.5	17.5	55.0
Total	55	25	91	32.2	14.6	53.2

Results from Study of the Development of the Optic Nerve.

Histological investigation was made of the optic nerves in different stages of development. For this purpose eggs were brought from the hatchery at Wilkinsonville, Massachusetts, and kept in a trough of running water. Eggs were taken each day and fixed in Vom Rath's fluid, which proved to be the best fixative for this purpose. As is well known, the great majority of nerve fibres in the optic nerve develop from the retinal cells and grow backward to the brain. The object of this investigation was not, however, to study the mode of development, but rather to find out whether both nerves developed at the same time. In nearly all cases the nerve fibres had reached the brain by the tenth day. Whenever the nerve fibres could be traced, they developed at the same time, for the fibres from both eyes were equally developed. The dimorphism is not due to an earlier development of one of the optic nerves.

THE EFFECT OF GRAVITY UPON THE CROSSING OF THE OPTIC NERVES.

O. Schultze ('95) in some experiments with frogs' eggs, found by compressing the eggs between glass plates so as to force the blastoderm to remain at some point other than its normal one, that he produced abnormal specimens. The degree of abnormality varied according to the angle made by the axis of the normal egg with the attraction of gravity. I have carried on similar experiments with eggs of the brook trout, but with no results so far as abnormalities were concerned. The compressed eggs died within a week or ten days. But the experiments showed conclusively that gravity could have no effect on the development of the nerves, and therefore could not cause the dimorphism; for in all cases the blastoderm finally came to the top of the egg, in spite of the pressure. If the blastoderm remained at the top under such conditions, it certainly would do the same under normal conditions. In this position the effect of gravity on the two sets of developing nerve fibres would be the same, and consequently gravity could not be a factor in the question under consideration.

THE LAWS OF CHANCE APPLIED TO THE RESULTS.

The writer has now shown that the dimorphism under consideration is not hereditary, is not due to an earlier development of one of the optic nerves, and is not caused by the action of gravity. It seems to be begging the question to say that it is a matter of chance whether one nerve is uppermost or not; for this means that the real cause of the dimorphic condition is unknown and, whatever it may be, it was not affected by the conditions of the experiments performed. Admitting that a "chance result" has this significance, the writer wishes to show that the results obtained from the dissections are really chance results. If so, the chances would be equal, in the normal trout, that either the right or the left nerve should be uppermost. In other words, the so-called rights and lefts should occur in equal numbers. Referring to the results obtained from the first and second generations of the trout, and to the control lot, also to the results obtained from the cod, it will be seen that the observed results closely approximate such a relation. Of Salvelinus, 4950 specimens in all were dissected. Of these, 2749 had the right nerve dorsal, and 2201 the left nerve dorsal. Of Gadus, 1132 specimens were dissected; 580 of these had the right nerve dorsal, and 552 the left nerve. In both cases, but especially in the first, there is an excess of rights. In view of the results obtained from the selected matings, this excess cannot be regarded as very significant. In the case of the trout, where it was greater than in the cod, it amounts to six per cent only.[1]

In the case of the two-headed trout the conditions are more complicated. In these, three combinations are possible. Either the right or the left nerve may be dorsal in each head, or the right nerve dorsal in one head, the left nerve dorsal in the other. According to the laws of chance, in one half of the specimens the crossings in the two heads should be unlike, while in the other half the crossing should be similar in the two heads. Further, of those having the crossings similar, one half should have the right nerve dorsal in each head, the other half the left nerve dorsal. Of the 171 two-headed trout dissected, 85 approximately should have crossings opposite in the two heads, and 42 each should have the right or the left nerve dorsal in both heads. Actually, 91 had the crossings opposite, 55 had the right nerve dorsal in both heads, and 25 the left nerve dorsal in both heads. Adding the last two groups together, the numbers of those having the crossings similar and of those with the crossings opposite are almost exactly equal, yet the right nerve, as in other categories of cases, is oftener uppermost than the left one. This fairly close correspondence with the expected chance result leads the writer to the conclusion that the position of the optic nerves in the chiasmata of the symmetrical teleosts is a matter of chance and not due to any of the possible causes which have been discussed in this paper.

SUMMARY.

1. The Mendelian principles of heredity do not apply to the dimorphic condition of the optic chiasma in symmetrical teleosts.
2. Galton's law likewise is inapplicable.
3. The dimorphic condition is not inherited.
4. The dimorphism is not due to an earlier development of one of the optic nerves.
5. Gravity has no effect on the character of the crossing of the optic nerves.
6. The condition of the crossing is a matter of chance.

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[1] *Note by W. E. Castle.* It is, however, somewhat remarkable that in both species studied the rights should with such uniformity predominate, even though the excess is not great. Selected matings evidently do not affect the relation one way or the other, but it persists nevertheless in nearly all matings and in the control. The case may perhaps belong in the same category as the more frequent occurrence of polydactylism in guinea-pigs upon the left side of the body (Castle, :06).

BIBLIOGRAPHY.

Castle, W. E.

:06. The Origin of a Polydactylous Race of Guinea-pigs. Carnegie Institution of Washington, Publication No. 49, pp. 17–29.

Galton, F.

'89. The Average Contribution of each Several Ancestor to the Total Heritage of the Offspring. Proc. Roy. Soc., London, Vol. 61, pp. 401–413.

Parker, G. H.

:03. The Optic Chiasma in Teleosts and its Bearing on the Asymmetry of the Heterosomata. Bull. Mus. Comp. Zoöl. Harvard Coll., Vol. 40, No. 5, pp. 219–242, 1 pl.

Schmitt, F.

:02. Über die Gastrulation der Forelle mit Berücksichtigung der Concrescenztheorie. Verhandl. d. deutsch. Zool. Gesellsch., 12te Jahresversammlung, Leipzig, pp. 64–83, 7 Fig.

Schultze, O.

'95. Die künstliche Erzeugung von Doppelbildungen bei Frosch Larven mit Hilfe abnormer Gravitations-wirkung. Arch. f. Entwickelungsmech., Bd. 1, pp. 269–305, Taf. 11 u. 12.

Spemann, H.

:01. Experimentelle Erzeugung zweiköpfige Embryonen. Sitzungsb. d. phys.-med. Gesellsch. Würzburg, Jahrg. 1900, pp. 2–9.

Spemann, H.

:01ª. Entwickelungsphysiologische Studien am Triton-Ei. Arch. f. Entwickelungsmech., Bd. 12, pp. 224–264, Taf. 5, 24 Textfig.

ERRATUM

Riddle (:06) in Proc. Am. Acad. Arts and Sciences, Vol. 42, No. 10

P. 193 line 16 from bottom } in place of "uni-
P. 194 line 4 from top } nucleate" read
"multi-nucleate"

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FLUORESCENCE AND MAGNETIC ROTATION SPECTRA OF SODIUM VAPOR, AND THEIR ANALYSIS.

By R. W. Wood.

With Five Plates.

Investigations on Light and Heat made and published, wholly or in part, with Appropriation from the Rumford Fund.

FLUORESCENCE AND MAGNETIC ROTATION SPECTRA OF SODIUM VAPOR, AND THEIR ANALYSIS.

By R. W. Wood.

Presented by C. R. Cross. Received August 7, 1906.

Previous work, which has been recorded in the Philosophical Magazine,[1] convinced me that a careful study of the remarkable optical properties of the vapor of metallic sodium would, in time, furnish the key to the problem of molecular vibration and radiation. This opinion has been strengthened by the work of the past year, and though much remains to be done, it seems best to place the results already obtained on record. In no other case that I know of is the molecular mechanism so completely under the control of the operator. Its periodicities can be studied in a variety of ways: by absorption, by cathode-ray stimulation, by excitation with light, either white or monochromatic, and lastly by its remarkable selective magnetic rotation of the plane of polarization.

The vapor is, in every case, that obtained by heating metallic sodium in steel or porcelain tubes, usually highly exhausted. From a study of the dispersion of the vapor, it seems probable that we may be dealing with clusters of molecules with which a certain amount of hydrogen may be associated.

As I have shown in a previous paper,[2] if a pool of sodium is heated in a highly exhausted horizontal tube, the top of which is cooler than the bottom, the vapor has an enormous optical density close to the surface of the pool, and a very small density along the roof, the non-homogeneous layer acting as a prism. The only way in which I can reconcile this state of things with the kinetic theory is to assume that the vapor leaves the metal in the state of molecular clusters, which gradually break up into smaller clusters and eventually into molecules. This is of course only an hypothesis, and I mention it in the present paper merely to indicate that our vibrating mechanism may be an aggregate and not a single molecule. It is also possible that hydrogen atoms are associated with the sodium, for the work on the dispersion indicated that there was present always a small trace of some gas other than sodium, which no amount of pumping would remove; that is, it appeared to be tangled up in the sodium vapor, condensing with it in the cooler parts of the tube. All of this is, however, irrelevant, for we are for the present merely engaged with the study of a certain remarkable vibratory mechanism, and for the present need not concern ourselves whether it is a molecule, a cluster of molecules, or a compound molecule.

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[1] Magneto-optics of Sodium Vapor, Phil. Mag., Oct., 1905. The Fluorescence of Sodium Vapor, Phil. Mag., Nov., 1905.

[2] A Quantitative Determination of the Dispersion of Sodium Vapor, These Proceedings, **40**, 335.

We will begin by a description of the various spectra which we shall study and compare in the present paper. The spectrum region with which we are concerned lies between wave-lengths 4600 and 5700, i. e. the region of the green-blue channelled absorption spectrum.

THE ABSORPTION SPECTRUM.

Photographed with the twelve-foot concave grating, the absorption spectrum is found to consist of a multitude of fine lines, to the number of about 1500 in the region specified. Its appearance has been found to be profoundly affected by the presence of hydrogen or any inert gas. It is shown on Plate 1, Figure 1, *g*, photographed in the second order with a twelve-foot concave grating. In hydrogen at atmospheric pressure its appearance is shown by spectra *f* and *h*, the fluted appearance being very marked. In a high vacuum its appearance is shown by spectrum *b*, the flutings having entirely disappeared. The chief change appears to lie in the increased absorbing power of certain lines or groups of lines. Spectrum *d* is taken under nearly the same conditions, the sodium vapor being less dense, however. A careful study of the effect of the inert gas has not yet been made, and it is mentioned here only on account of its relation with the subject of the paper. Comparatively few of the absorption lines have any relation with the fluorescent and magnetic rotation spectra, the *ones concerned, however, being those which are affected by the presence of the inert gas.*

Moreover, both the fluorescence and magnetic rotation disappear, i. e. cannot be excited, when the sodium vapor is formed in an atmosphere of hydrogen or other inert gas. Of this matter more will be said later on.

THE MAGNETIC ROTATION SPECTRUM.

It was found last year that a number of vapors, showing fine and sharp absorption lines, when placed in a powerful magnetic field, rotate the plane of polarization for wave-lengths agreeing with that of the absorption lines, not all of the absorption lines showing this rotatory power, however. The arrangement of the apparatus for showing the bright-line magnetic spectrum of sodium is shown in Figure A.

A piece of thin seamless steel tubing of such a diameter as to slip easily through the hollow cores of the electromagnet, from which the conical pole-pieces have been removed, is procured. A short piece of small brass tubing is brazed into one end, through which the tube is exhausted.

A lump of sodium the size of a walnut is melted in an iron crucible, and poured out into a V-shaped trough made of thin sheet iron. As soon as the bar is solid it is placed in the iron tube, one end of which has been previously closed with a small piece of plate glass cemented on with sealing-wax. The tube is introduced into the magnet, the sodium bar pushed to a position midway between the helices, and the other end closed with a piece of glass in a similar manner. The ends of the tube should be coated while hot with sealing-wax before the introduction of the sodium. One has then only to wave a Bunsen flame over them and press on the piece of glass, previously heated; the sealing-wax should come into optical contact with the glass to insure an air-tight joint. The tube is now connected with an air-pump which will produce a vacuum of a millimeter or two. If the air-pump leaks, it is a good plan to place a glass stopcock between the pump and tube to prevent the entrance of traces of air after exhaustion. For purposes of demonstration it is sufficient to heat the tube gradually with a Bunsen burner turned down low. In the present work, however, where constancy of temperature was essential, electrical heating was invariably used.

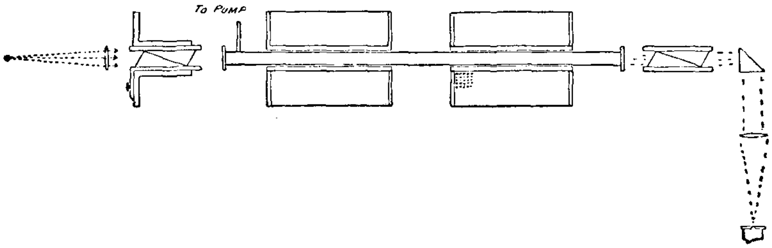

FIGURE A.

The light from an arc-lamp, made parallel by a lens, is passed through a Nicol prism, the steel tube, and a second nicol, after which it is brought to a focus by means of a second lens upon the slit of a spectroscope. With the steel tube cooled below the point at which sodium vapor forms, the nicols are set for complete extinction, and the field of the spectroscope becomes dark. The tube is now heated and the magnet turned on, the air-pump being worked occasionally to remove the hydrogen which is given off from the sodium. A bright yellow spot will appear on the slit of the spectroscope, which is seen to be made up of radiations chiefly in the immediate vicinity of the D lines. The phenomena at the D lines have been fully described in the paper already alluded to (Magneto-Optics).

When the vapor acquires a considerable density, a most magnificent bright-line spectrum appears in the red and green-blue region. Each bright line corresponds to a dark line in the absorption spectrum, but only a small percentage of the dark lines appears to exercise a rotatory power. Some of the strongest absorption lines are absolutely unrepresented in the magnetic rotation spectrum, which indicates that there is some radical difference in the absorbing mechanism.

It is with the bright-line spectrum in the green-blue region that we are now concerned. This spectrum has been photographed with the large, three-prism long focus spectrograph, and also with the twelve-foot concave grating. Reproductions of the prism spectrograms are given on Plate **2**, *f* and *m*. The magnetic spectrum made with the large grating and the absorption spectrum recorded on the same plate are reproduced on Plate **1**, *c* and *d*.

Only about sixty lines appear in this spectrum in contrast to the 1500 in the absorption spectrum. The intensities are very variable, and apparently bear no relation to the intensities of the corresponding absorption lines. The rotatory lines in many cases coincide with the heads of the groups of absorption lines, though the centre of the line appears to be slightly displaced beyond the head of the group of absorption lines. The displacement is, however, very slight, not more than half the width of the line. A list of the wave-lengths of all the lines visible on the negative is given on page 239. The approximate intensities are represented by numerals, 10 indicating the maximum intensity and 1 the minimum.

At first sight there appears to be no regularity whatever in the distribution of the lines, except perhaps above wave-length 502, where they appear to be about equally spaced in small groups of three or four lines each. Without the aid of the fluorescence spectra of the vapor excited by monochromatic light, it is doubtful whether any regular series of lines could be found in the magnetic spectrum, for, as has been subsequently found, more than half of the lines in the series are absent, and there are six or more series present. The fluorescence spectrum with white-light excitation is shown on Plate **1**, *e*, which is from a negative made with the twelve-foot grating. As will be seen, the bright lines coincide with the bright lines of the magnetic spectrum, though much broader. It will be easier to explain how the series were picked out after we have commenced the study of the fluorescence. I have added to Plate **1** spectrum *i*, the magnetic spectrum with the series

GREEN ROTATION SPECTRUM.

1	5225.34
1	5218.49
1	5212.02
1	5186.70
1	5179.71
1	5172.98
1	5171.98
1	5169.04
1	5165.85
1	5147.50
1	5140.71
3	5133.73
3	5126.54
2	5119.34
1	5095.70
2	5094.78
7	5087.31
7	5079.78
2	5071.58
1	5052.83
1	5049.56
5	5048.49
7	5040.65
2	5033.54
2	5025.66
3	5003.12 broad
10	5001.57
5	4979.34
2	4970.85
1	4967.10
1	4964.39
9	4962.85
1	4958.62
4	4933.93
5	4932.64
3	4924.32
4	4912.10
1	4904.67
1	4903.38
1	4896.65
1	4894.58
1	4892.77
2	4883.81
3	4865.59
2	4839.56
9	4837.49
2	4819.43
1	4814.60
1	4812.68
3	4810.16
3	4802.62
5	4792.67
3	4782.89
1	4777.00
1	4766.94
6	4756.69
1	4752.04
2	4738.51
4	4727.52
1	4716.90
1	4715.63
1	4703.78
2	4692.54
2	4670.30

indicated. There appear to be five distinct series and a number of lines which thus far have not been brought into any definite relation with one another. These series we will number 1, 2, 3, 4, and 5. All the lines belonging to the first series have one dot under them, those belonging to the second have two dots, etc. These series are shown separated on the chart (Plate **5**) at the top. Absent lines are indicated thus : $\overset{\wedge}{\vee}$.

The fifth series is at the top, the fourth next, and so on down, the extra lines being indicated in the lower row. This arrangement is considered provisional: it is the best that I can do at the present time, and I believe that it is correct in the main. We shall see presently, however, that photographs of the fluorescence stimulated by monochromatic radiation will have to be made with the large concave grating before we can be absolutely sure of all of the lines. We will drop the magnetic spectrum for the present and consider

THE FLUORESCENCE SPECTRUM.

In the previous paper I have described some of the remarkable changes which take place in the distribution of energy in the fluorescence spectrum of sodium vapor when the wave-length of the exciting light is changed. With white-light stimulation the general appearance of the spectrum is shown in Plate **4**, Figure 6, A. There is, in addition, a broad double band at the position of the D lines, and a red-orange spectrum which, when the vapor is dense, is distinctly banded. In the present paper we shall be concerned chiefly with the portion figured above, for it is in this region that most of the remarkable changes occur. As will be seen, it is comprised between wave-lengths 460 and 570, and is devoid of any apparent regularity in the distribution of its lines, except in the region above $\lambda = 505$ where we have lines spaced with considerable regularity, the spacing becoming less as the wave-length increases. The distribution of intensity in this portion of the spectrum is such as to give it a fluted appearance, the flutings being most conspicuous in the region between $\lambda = 505$ and $\lambda = 535$. With white-light stimulation the flutings cannot be made out above 540, as can be seen from Plate **4**, Figure 6, B, in which the upper limit of this part of the spectrum is shown. The fine lines are present in this region, becoming, however, less and less distinct as the upper limit of the spectrum is approached. If, now, instead of stimulating the vapor with white light, we employ blue light in the region 460–465 obtained from a spectroscope for the excitation, the fluorescent spectrum presents a totally different appearance (Figure 6, C). The blue region, corresponding in its range to that of the exciting light (indicated by a double arrow), appears as before, and the upper limit of the spectrum between wave-lengths 540 and 565, *the intermediate portion being entirely absent*, as shown in the lower spectrum of Figure 6, C. Furthermore, at the upper or yellow end, there now appear the flutings, which were absent when the fluorescence was stimulated with white light.

If, now, we gradually increase the mean wave-length of the exciting light, the region of maximum intensity in the fluorescence spectrum moves down from the yellow into the green, as is shown by the remaining photographs in Figure 6, C. Moreover, as I pointed out in the previous paper, the positions of the fluted bands change slightly, the positions of the individual lines which make up the bands remaining fixed however, the shift resulting from a change in the distribution of intensity among the lines. The reason of this curious phenomenon will appear when we come to the study of the fluorescence spectrum excited by strictly monochromatic radiations.

The spectrum stimulated by white light I have named the "complex fluorescent spectrum," for it has been found that it is a superposition of a number of simpler spectra, any one of which can be independently excited by suitably controlling the wave-length of the stimulating light. Indications of something of this sort were found last year, and were described in the preliminary paper. An insufficient number of photographs were obtained, however, at the close of the university year, to make anything like a complete analysis of the complex spectrum possible.

During the past winter and spring a careful study has been made of the relations existing between the complex fluorescent spectrum, the absorption spectrum, and the bright-line rotation spectrum described in the earlier paper. The fluorescent spectrum has at last been photographed with the twelve-foot concave grating, enabling a study to be made of its more minute structure.

Some very remarkable effects have been observed with monochromatic stimulations obtained by the isolation of certain lines from metallic arcs, which yield comparatively simple fluorescent spectra made up of widely separated sharp lines, placed in many instances at nearly equal intervals along a normal spectrum. A given series of lines can be brought out by stimulating with light of any wave-length corresponding to that of some line in the series, but when the stimulations occur at certain points, some of the lines may be absent, gaps appearing in the series. The most conspicuous example is the case of stimulation with the cadmium line 480, which will be considered in detail presently. It will be remembered that certain lines are absent in each series, in the magnetic spectrum.

The apparatus employed in the experiments was essentially the same as that used in the earlier work. It consisted of a seamless tube of thin steel three inches in diameter and thirty inches long, with a steel retort at its centre in which a large amount of sodium could be stored. The retort was made by fitting two circular disks of steel to a short piece of tubing, just large enough to slip snugly into the larger tube. The circular ends of the retort were provided with oval apertures, as shown in Plate **3**, Figure 1. The retort was half filled with sodium, the molten metal being poured in through one of the apertures. It was then introduced into the tube and pushed down to the centre, after which the plate-glass ends were cemented on, as shown in the figure. This arrangement prevented the rapid diffusion of the vapor, and enabled a large supply of metal to be kept at the centre of the tube. The tubes used in the earlier work required re-charging after two hours' continuous operation, while the retort tube could be operated for several hundred hours on a single charge.

The tube was exhausted with a Fleuss pump and heated at the centre with a large burner, the ends being kept cool by jackets of absorbent cotton which dipped into pails of water.

The illuminating beam of either white or monochromatic light was focussed just within one of the oval apertures of the retort, falling upon the opposite wall a little to one side of the other aperture. By covering the further end of the tube with a black cloth, the fluorescent spot showed against the dead black background of the second oval aperture, and its spectrum was therefore uncontaminated with the exciting radiations.

A large three-prism spectrograph was constructed for photographing the spectra. The prisms were of clear dense flint four inches in height, and the focal length of the lenses thirty-six inches.

Since only lenses such as are used for telescopes were available, the spectrum lines are not so sharp as one would wish, except near the axis of collimation. By adjusting things so that the centre of the fluorescent spectrum fell at this point, the definition was pretty fair throughout its extent, and wave-lengths could be determined with an error not greater than one or two Ångström units.

The photographs of the complex spectrum of the fluorescence excited by sunlight, obtained with this instrument (Plate **4**, Figure 6), showed peculiarities which made it appear of the utmost importance to study the spectrum under higher dispersion. The green fluorescent spot had, after repeated improvements in the apparatus, attained such brilliancy that I felt sure that records could be obtained with the twelve-foot concave grating. An all-day exposure was found to be sufficient, the resulting spectrogram, with the iron comparison lines and the wave-length scale, being reproduced on Plate **1**, Figure 2. The scale was printed separately, and slight errors occur, due to stretching and shrinking of the prints. They are not greater in any case than 1.5 A. E. This plate shows the minute structure of the complex spectrum, and enables us to measure the wave-lengths of the bright lines with far greater accuracy than could be done with the plates made with the prism spectrograph. As much of the detail is lost during the process of reproduction, I have prepared a very careful drawing of one of the groups of bands at wave-length 5200 (Plate 4, Figure 5). The drawing was made from a print with the aid of a hand magnifier, and the peculiarities shown are found throughout the entire spectrum.

The bright lines are sharp, and quite as narrow as the iron lines of the comparison spectrum. We must remember, however, that the slit was not very narrow, 0.2 mm. perhaps, and it is quite possible that a further contraction would not decrease the width of the fluorescent lines. Each bright line is in general accompanied by two lateral wings, which terminate quite sharply, the narrow spaces between every two adjacent wings appearing as narrow dark lines. These wings do not in general appear with strictly monochromatic stimulation. In the work of last year, when studying the remarkable changes which occur in the spectrum stimulated with the fairly homogeneous light furnished by the monochromatic illuminator with very narrow slits (see previous paper), I observed that as the wave-length of the light was very gradually altered, the fluorescent lines appeared with wings first on one side and then on the other, the change in the appearance of the line reminding one of a flag flying first on one side of the mast and then on the other. With the strictly monochromatic illumination obtained with the isolated metallic arc lines, the fluorescent lines are usually devoid of wings, though in some instances the wings are found, and sometimes the wings appear without the lines. These circumstances appear to indicate that the wings are due to the stimulation of the electron by frequencies slightly greater and slightly less than its own natural frequency.

I have not yet had time to repeat last year's experiments with the monochromatic illuminator, and plan to make a further study of the changes which accompany very gradual changes in the wave-length of the exciting light. The observations are very difficult and uncertain, as the light furnished by the monochromatic illuminator is not very bright when its slits are made as narrow as possible and the fluorescence spectrum can only be observed by carefully rested eyes. Probably by using a large prism spectroscope as an illuminator better results can be obtained. What is most desired is a light siren!

ANALYSIS OF THE COMPLEX SPECTRUM. STIMULATION WITH MONOCHROMATIC LIGHT.

It was found impracticable to use the monochromatic illuminator for the study of the simple spectra which made up the complex spectrum. Even with its slits very narrow, the wave-length range of the emitted light was wide enough to cover several of the absorption lines of the vapor. The earlier work had shown that the light of the cadmium spark was capable of exciting fluorescence, and experiments were accordingly started with metallic arcs. Just at this time came the very opportune invention of the fused quartz metallic arc lamps by Stark, working in the Heraeus laboratory at Hanau. Two of these lamps were immediately ordered, one filled with cadmium, the other with zinc. Their form is shown on Plate **3**, Figure 1. The lamp is kept in communication with a mercury pump during its operation and stands in a disk of water. The cadmium lamp worked well on a circuit of 110 volts direct, but the zinc lamp gave better results on the 220. They are started by a small induction coil, one terminal of which is connected to the negative pole of the lamp, the other twisted around the quartz U tube. A blast lamp is directed against the tube until the portion above the metal electrodes is red hot, the coil is then started, and the arc usually forms at once. As exposures of eight or ten hours were often necessary, and the lamps have a trick of going out every half-hour or so, an automatic starter was devised, which turned on the coil the moment the lamp went out. As soon as the arc struck again, the coil was stopped. This arrangement is figured on Plate **3**, Figure 2, and consisted of a small electromagnet in circuit with the lamp, which pulled a steel spring away from a brass screw as long as the lamp burnt. The spring and screw were inserted in the primary circuit of the coil.

The cadmium lamp burns with a greenish-blue light of dazzling brilliancy, the zinc lamp with a curious purple light, which causes all the woodwork in the room to appear blood-red, while most other objects appear bluish white or purple. Both lamps excite a brilliant fluorescence of the sodium vapor when their images are thrown upon the oval aperture of the retort. In this case the fluorescence is excited by several different radiations. Various devices were used for picking out one line at a time. The cadmium radiations which are capable of exciting fluorescence have wave-lengths 5086, 4800, and 4678. Color screens and the Fuess monochromatic illuminator, as well as the thin crystals of chlorate of potash (described in the Phil. Mag. for June), were tried; also a block of quartz, cut perpendicular to the axis, placed between two nicols. The arrangement which gave the best results, and appeared to be accompanied with the least loss of light, is the one figured on Plate **3**, Figure 2. One vertical tube of the lamp is used as the source, the light from which, after collimation, passes through a large bisulphide of carbon prism, and is focussed upon the retort by an achromatic telescope objective with an aperture of 12 cms. and a focal length of 2 metres. The dispersion of the prism was sufficient to separate completely the monochromatic images of the lamp, any one of which could be thrown into the aperture, the light passing by the edge of the 90° prism by means of which the fluorescent light was reflected through a lens and thence upon the slit of the spectroscope.

The sources of light which have thus far been successfully employed in stimulating the fluorescence of the vapor are the following: quartz arc lamps containing cadmium, zinc, and thallium; ordinary arcs between lead, silver, bismuth, and copper electrodes; lithium, sodium, and barium arcs, and vacuum tubes containing helium and hydrogen.

Unfortunately the quartz lamps are very expensive, and become almost useless after a run of about thirty hours, the surface becoming granular and an opaque black deposit forming on the inner walls. As exposures of eight hours are sometimes necessary, it will be seen that lamps at $30 apiece, with an average life of thirty hours, make the investigation an expensive one.

The photographs of the fluorescent spectra obtained with monochromatic stimulation are reproduced on Plate **2**.

After each exposure the D lines were recorded on the plate, so that the different spectra could be brought into coincidence for purposes of comparison: the D lines will be found at the extreme right in each spectrum. The photographs have been reproduced as negatives, and the point or points coinciding with the wave-lengths of the stimulating light are indicated by arrows. A large scale drawing or chart of the most interesting of these spectra, together with others too faint for reproduction, has been made on cross-section paper, the points of excitation being indicated by arrows, as in the photographs (Plate **5**).

Drawings of the complex spectrum and the magnetic rotation spectrum made from the photographs obtained with the large concave grating will be found at the bottom of the chart: the former is a positive. At the top I have given a composite drawing which represents a superposition of all the simple fluorescent spectra thus far obtained. Immediately below it will be found the spectra excited by the complete radiation of the cadmium and zinc tubes. In each case there are three different exciting radiations simultaneously applied, yet it is almost impossible to find two fluorescent lines which coincide. The other spectra are excited for the most part by a single monochromatic radiation, the wave-length of which is indicated by the arrow. I have not yet obtained photographs of the fluorescent spectra excited by the separated radiations of the copper arc or by the separated zinc lines 468 and 472; consequently these have been drawn together. It is possible, however, by comparing the spectrum excited by copper with the one excited by zinc 4811, to make a guess as to which lines belong together.

We will begin by a study of the spectra excited by the cadmium radiations.

Cadmium Stimulation.

Photographs of the fluorescent spectrum obtained with the cadmium-arc excitation are shown on Plate **2**, *h*, *i*, *j*, *k*.

Of these, *i* and *k* are excited by all three cadmium lines; the former taken with a much narrower slit than the latter. Spectrum *h* was taken when the sodium vapor was excited by the line 480; observed visually, it consists of twelve bright lines, in groups of two and four, as shown on the chart immediately above the complex spectrum at the bottom. Midway between the groups very faint lines can be perceived if the eye is carefully rested. The strong lines are arranged thus: two, absent line, four, absent line, two, absent line, four. The absent lines, or, more correctly speaking, the faint lines, evidently belong to the same series, and taken collectively the lines will be found to be very nearly equidistant, measured along a *normal spectrum.*

The wave-lengths of the lines in this series, as determined from measurements of the plates obtained with the prism spectrograph, are as follows:

λ.	λ differences.
4760	
4800	34
4838	38
. . .	$\frac{70}{2}$ 35
4908	
4946	38
4983	37
5019	39
. . .	$\frac{76}{2}$ 38
5095	
5133	38
. . .	$\frac{74}{2}$ 37
5207	
5245	38
5283	38
5321	38

These wave-lengths I consider to be accurate to within about 2 A. E. or $\frac{1}{3}$ of the distance between the D lines.

It is, of course, of the utmost importance to determine the law which governs the spacing of the lines in the simple spectra. A criterion may perhaps be obtained by referring to the magnetic rotation spectrum of the vapor, the lines of which correspond in general to the lines of the fluorescence spectrum. This spectrum has been photographed with a large concave grating and the wave-lengths determined certainly to within a tenth of an Ångström unit. The strong lines at the following points of the spectrum form a series analogous to the series obtained with the cadmium 480 series.

λ.	λ differences.
5119.34	
5079.78	39.56 A. E.
5040.65	39.13
5001.57	39.08
4962.85	38.67
4924.32	38.52

This we have called the first series.

The wave-length differences are, in this case, much more nearly constant, and decrease progressively.

The lines of this series are especially conspicuous in the magnetic rotation spectrum (Plate **1**, Figure 1, *c*), hence I have mentioned it first; they appear in the fluorescence excited by the lead line 5001, as will be seen by reference to the chart, Plate **5**.

If now we try to fit one of the magnetic series to the cadmium 480 fluorescence series, we find that the third magnetic series coincides with it between 5019 and 5134, while in the violet region it coincides with the fourth magnetic set. I do not feel sure whether this peculiarity is due to slight errors in the determination of wave-lengths or not. I think not, however, for I have very carefully superposed the two negatives (cadmium fluorescence and magnetic rotation spectra), both taken with the same instrument, and find the same disagreement. We cannot be sure of anything, however, until the cadmium series has been photographed with the grating. During the coming year I expect to photograph the fluorescence spectrum excited by cadmium and zinc radiations with the large concave grating. It will then be possible to determine the wave-lengths of the lines to within a tenth of a unit.

One of the most remarkable facts connected with the appearance of the lines of a series is that the distribution of energy among the individual lines depends upon the point of excitation. Unfortunately there are very few arc lights bright enough to excite fluorescence. It was found, however, that the silver line 5207, which coincides with one of the fluorescent lines of the cadmium 480 series, was bright enough for the purpose. The silver was carefully freed from copper, as the neighboring copper lines are powerful exciters; in their absence, no prismatic separation was necessary, as the rest of the silver lines were inoperative. A photograph of the fluorescence spectrum obtained with silver stimulation is shown on Plate **2**, *g*. The series in this case *has no gaps* in it, the line at 5170, which is absent with cadmium excitation, coming out strong (see chart as well). The monochromatic illuminator, with its slits reduced to the width of a hair, was arranged to furnish light of wave-lengths corresponding to other lines in the series, and photographs obtained which are recorded on the chart. It will be seen that faint or missing lines occur in each case, but that their position varies with the point of excitation. If we consider each line caused by a single electron or vibrator, the phenomena suggest that the vibrators are united in some way, perhaps in a closed ring, and that when the system is set in vibration there are nodal points, the position of which depends upon the point in the chain where the periodic force is applied. Moreover, as has already been pointed out, if the force is applied at the "high frequency" portion of the chain, the regions excited are those of highest and lowest frequency, the intermediate portion appearing to be at rest. This is especially noticeable in the case of the bismuth excitation. (Plate **2**, *e*, and chart.)

In addition to the lines enumerated above, there are a number of others at the upper end. These do not appear to be distributed with the same regularity, though some of them may form an extension of the series, or more probably may be the beginnings of other series. In general it has been found that in the simple spectra the lines are regularly spaced between the extreme violet end and a point at about $\lambda = 5350$. Above this point the spacing is generally very irregular, and it is difficult to unravel the spectrum. Of this more will be said later.

We will next take the fluorescent spectrum excited by the green cadmium line 5086.

This spectrum is remarkable in that it is made up of eleven pairs of lines regularly spaced (Plate **2**, *j* and chart). The other two cadmium lines appear on the plate, as the spectrograph was not shielded from the diffused light from the lamp.

A series in the magnetic spectrum coincides with the series formed by the shorter λ member of each pair. The wave-lengths and differences are given in the following table:

λ.	λ differences.	λ.	λ differences.
5165.85		. . .	$\frac{77.64}{2}$ 38.82
5126.54	39.31	4970.85	
5087.31	39.23	4932.64	38.21
5048.49	38.82	4894.58	38.06

I am quite at a loss as to how the series formed by the other members of the doublets is originated. It appears to coincide with the series excited by helium 5014, as will be seen by reference to the chart. It appeared at first as if the exciting line might lie between two adjacent fluorescent lines, and in that way excite a double series, but cadmium 5086 is slightly on the short λ side of the magnetic line 5087.3, while the wave-length of the other line of this pair is 5092, i. e. on the long λ side.

This is the only case recorded where a spectrum of doublets is excited by monochromatic stimulation, though I am of the opinion that the copper line 5152 behaves in the same way.

What is still more remarkable is the fact that if the excitation is at a different point we no longer get doublets. The lithium line at 4971 takes hold of one of the more refrangible components of one of the doublets, but only a single series of lines appears in the fluorescence spectrum (see chart). The other series, i. e. the less refrangible components, can be separately excited by stimulation with the helium line 5014 (see chart). If we are dealing with anything in the nature of electron doublets, we should expect both the lithium and helium radiations to excite a fluorescence showing double lines.

If we try to explain the phenomena by assuming two chains of electrons fastened together at the point 5086, we must account for the fact that the 5086 vibrator excites the other chain when it is acted upon by light of its own frequency, but not when it is vibrated by the lithium radiation acting at a different point on the chain. I have adopted this hypothesis of electron chains merely to aid in describing the physical phenomena, and not with much hope that it will explain anything.

It seems much more likely that the different lines represent vibrations of different frequencies of the same system. We must not try to make the molecule too much like a piano. The vibrations may be ripples running over its surface or they may be unlike anything with which we are familiar. If we had never seen a bell, it would be difficult to work out the theory of its very complicated vibrations from a study of a set of simple pendulums. Possibly stimulation at some other points might give rise to the double lines.

I attempted to do this with the monochromatic illuminator, but without success. The band of exciting light cannot well be made much narrower than the distance between the components of the doublets. Even with the instrument set at 5086, I could detect no evidence of the doublets. I am planning to investigate this matter further with a larger monochromatic illuminator designed to furnish more nearly monochromatic light.

In addition to the eleven pairs of lines in the fluorescence spectrum excited by cadmium 5086 there are two strong lines at wave-lengths 5305 and 5341. These seem to belong to the same series, and the former has a faint companion, the two forming a doublet. The line 5341 is also accompanied by a companion, which, however, is so faint as to be barely distinguishable.

The spectrum excited by the more refrangible of the two blue cadmium lines 4678 is reproduced only on the chart. It consists of a regular series of five lines in the blue region, and a large number of irregularly spaced lines of widely different intensities in the yellow-green region. None of these lines appear to be represented in the magnetic spectrum.

This spectrum illustrates well the characteristic peculiarity of the sodium fluorescence spectrum, that stimulation at the more refrangible end excites powerful fluorescence at the opposite end. The lines which form the regular series we may call directly excited, the others in the yellow region indirectly excited. The latter in all cases seem to be irregularly spaced. The great problem to solve is to determine the nature of the mechanism and find out how the low-frequency vibrators are set agoing by the stimulation of the high-frequency ones, while they remain quiescent when the stimulation is at the middle of the spectrum. Speculations on these points must be deferred for the present.

Zinc-arc Excitation.

The complete fluorescent spectrum excited by all three of the zinc lines (i. e. the total radiation of the lamp) is shown near the top of the folding chart, just below the cadmium fluorescence. It is scarcely possible to find a coincidence of two lines. The two spectra placed side by side make a striking picture of the variation produced by different excitations of the same fluorescing medium. A photograph of the spectrum is reproduced on Plate **2**, *c*.

Exciting with the zinc line 4811 alone gives us the spectrum shown on Plate **2**, *b*. The other two zinc lines are of course present on account of diffused light. Referring now to the chart, we find that the violet end of the spectrum agrees pretty well with the fifth magnetic series, though other lines are present. The two strong lines at 5188 and 5225 also appear to belong to the same series. The lines on the whole are much less regularly distributed than in the case of the cadmium 480 excitation. The three wide pairs between 523 and 535 are peculiar to this excitation.

The fluorescence excited by the other two zinc lines, 4680 and 4722, is also recorded on the chart. These lines are so close together that it was found impossible to illuminate the vapor with light from but one of them, and have at the same time sufficient illumination to excite much fluorescence. I have not yet found much evidence of regularity in the distribution of the lines in this case, though there is undoubted evidence of two series in the immediate vicinity of the exciting lines. I have indicated with the letters A and B the lines which appear to belong together (compare with helium excitation). We have an enormous number of lines in the yellow-green region, since we have a double stimulation at the opposite end of the spectrum. There seems to be some regularity here, but it is difficult to say which lines belong together.

Bismuth Excitation.

The light of the bismuth arc makes a beautiful stimulus for the fluorescence, since it contains but a single operative line, the strong one at 4724. It gives rise to a very regular series in the blue-violet region, the lines appearing to fall midway between the lines of the third and fourth magnetic series (Plate **2**, *e*, and chart). Though the wave-length of the exciting line is only two Ångström units longer than that of the zinc line 4722, the spacing of the series in the two cases is quite different. The same thing has been noticed in the case of the shortest cadmium and zinc lines, which makes it seem possible that interesting results may be obtained by altering the wave-length of the exciting line, either by pressure or a powerful magnetic field. Experiments in this direction will be made next winter.

In both of these cases, in each of which we have excitation by lines of nearly the same wave-length, the wider-spaced series is produced when the stimulation is by the longer wave-length. It remains to be determined whether we take hold of different absorption bands and excite entirely different series, or whether we stimulate the same vibrator in each case, the spacing of the resulting lines depending upon how nearly we approach its natural period in our exciting vibrations.

In addition to the regularly spaced lines in the violet, we have a complex assortment of lines in the yellow-green region, the intervening portion being totally devoid of lines. One of these lines, $\lambda = 5300$, has a broad diffused wing, and it is perhaps worthy of remark that in the spectrum excited by the two zinc radiations we have a hazy doublet at this point, in the spectrum excited by zinc 4811, a single line, and in the spectrum excited by cadmium 480, two faint lines. Some of the other lines have wings, as will be seen from the chart, and at wave-length 546 we find a broad hazy band. All of these peculiarities complicate things, and I have drawn attention to them merely to show that we must not expect to explain matters by too simple a mechanism.

A word or two about the bismuth arc may not be out of place. Various plans were tried, such as immersion of the electrodes in water, burning in the carbon arc, etc. The best arrangement was found to be a shallow iron dish about 4 cms. in diameter (pounded from a piece of thin sheet iron), filled nearly full of molten bismuth, and kept hot over a small burner. The dish of metal formed the positive electrode, the negative being a bar of iron which could be raised or lowered by a rack and pinion. The arc required constant attention, fresh metal being put into the dish every ten or fifteen minutes, and as exposures of eight hours were necessary, it will be seen that an enormous amount of very fatiguing work was necessary in all cases where open-air arcs were used.

Copper Excitation.

I have been unable thus far to obtain photographs of the fluorescence excited by the separated copper radiations. The lines are close together, and the arc climbs about over the electrodes. I hope next year to improve matters in this respect. On Plate **2**, *a*, is seen the fluorescence excited by the total copper radiation. Only the three green lines are operative in stimulating the vapor. The lines in the fluorescence spectrum appear to bear no very definite relation to the lines of the magnetic spectrum, as will be seen by the chart. There are many coincidences, however, with lines in the spectrum excited by zinc 4811, and by zinc 468 and 472. By comparing the three spectra I have made a provisional determination of the lines which belong together in the spectrum excited by the copper radiation. These lines are indicated by crosses and vertical dashes placed above them; other lines, which do not appear in the zinc spectra, have not been marked. I suspect that excitation with copper 5152 will produce a doublet at this point, and probably at other points, just as does cadmium 5086. An attempt will be made to verify this surmise. I have on one or two occasions, when trying to stimulate the vapor with this isolated line, been of the impression that I saw doublets distinctly, but at the time I attributed it to incomplete separation of the exciting lines.

Lead Excitation.

A lead arc, operated in a manner similar to the one described for bismuth, was used for exciting the vapor. The only line operative was the one at 5001, and it gave rise to a well-marked series of fluorescence lines, which coincided exactly with the first series of the magnetic rotation spectrum. It is worthy of remark that one of the extra lines of the magnetic spectrum lies very close to the exciting line, yet none of these lines appeared in the fluorescence spectrum. The fluorescence excited by lead was very feeble, and even with an eight-hour exposure the lines were very faint.

HELIUM EXCITATION.

A large "end on" helium tube with a 3 mm. bore was constructed for the investigation. This tube could be run continuously with an induction coil yielding a heavy ten-inch spark. An exposure of about twelve hours was given. Two of the helium lines are operative: line 5015 gives a well-marked series, the lines of which fall exactly midway between the lines of the second and third magnetic series; line 4713 gives a good series in the blue and at least six distinct lines in the yellow-green, the wave-lengths of which can be seen from the chart. The exciting line in this case coincides with one of the lines in the spectrum excited by zinc 468, and there is perfect agreement in position between the fluorescent lines in both cases, not only in the blue, but also in the yellow-green region. The lines in the spectrum excited by line 468, or at least as many as could be identified, have been marked. The identification was of course made by comparison with the spectrum excited by helium.

LITHIUM EXCITATION.

An arc was caused to play between a carbon rod and a large carbon block on which the lithium salt was placed. The image of the red flame was projected upon the window of the retort and excited a bright fluorescence. Two of the lithium lines were operative, — one at 4601, the shortest monochromatic stimulation thus far found, which gives the series in the violet (see chart), and a large number of lines in the yellow-green; and another at 4971, which gives a beautiful series in the green, coinciding exactly with the second magnetic series. The 4971 stimulation should be especially interesting, since there are several lines in the magnetic spectrum very close to it. The line is unfortunately not very bright, and the fluorescence lines were so feeble that they could only be measured with difficulty.

The lines in the yellow-green region are also of considerable interest, since they result from a single monochromatic stimulation applied practically at the extreme lower end of the fluorescence spectrum. In fact, this line is considerably below the limit of the fluorescence spectrum as usually seen with white-light stimulation: this limit is not far from wave-length 4670, which is the shortest thus far detected in the magnetic spectrum photographed with the grating, though faint lines are visible even below 4600 on negatives made with the prism spectrograph. There seems to be evidence of a number of series in the yellow-green region, the spacing, however, decreasing with increasing wave-length, just the opposite of the state of things which holds in the blue-green region. It is more likely, however, that the apparent decreasing of the spacing as the yellow end of the spectrum is approached is due to other series similar to those which are found in the green and blue, the nearness of the lines resulting from the large number of superposed series. With white-light stimulation no trace of the lines can be seen above wave-length 555, and they are so faint as to be almost indistinguishable for a considerable distance below this point. The broad flutings seen in the spectrum stimulated with white light are doubtless to be referred in some way to the circumstance that the lines of the different series get into and out of step periodically: they may thus be considered analogous to the bands seen when two diffraction gratings of slightly different spacing are superposed.

Barium Excitation.

The fluorescence excited by the barium arc appears to be due chiefly to the line 4934, which coincides with one of the extra lines in the magnetic spectrum. Line 4932 of the second magnetic spectrum is also very near it, and we find that the fluorescence spectrum contains lines which coincide with the magnetic lines of the second series, as well as lines which coincide with some of the extra magnetic lines. The barium arc contains a good many other fainter lines which may give rise to some of the fluorescent lines. It will be necessary to repeat the experiment with the 4934 line isolated.

Sodium Excitation.

As I showed in the earlier paper referred to, if we stimulate the vapor with intense sodium light, we obtain a yellow fluorescence which the spectroscope shows to be made up of two lines in the position of the D lines. We have here a re-emission of light of the same wave-length as the exciting light, and nothing else. This I have called resonance radiation, as we may find that it is different from fluorescence, though the two are doubtless intimately related. As there are several pairs of lines in the ultra-violet which belong to the same series as the D lines, it seemed of great importance to determine whether these appeared in the spectrum of the fluorescence excited by the sodium flame. The sodium tube was provided with a quartz window, and the light of the oxyhydrogen flame, heavily charged with sodium, focussed upon the aperture of the retort with a glass lens. White light from the arc was also used, as this excites the D line vibrations in the fluorescence. The spectrum was photographed with a small quartz spectrograph, and though the D lines were greatly overexposed, no traces of any of the ultra-violet doublets were found on the plate. Conversely, illumination with ultra-violet light in the region of the first ultra-violet pair of lines failed to produce any visible fluorescence. It was hoped that a faint yellow fluorescence might be produced in this way, due to emission in the region of the D lines. I have not yet tried stimulation with D_1 and D_2, alone, to see whether both D lines appear in the fluorescence. This will be a very difficult experiment, and I am saving it for the last. It will settle the question as to whether the principal series of sodium is a series of doublets or two series of single lines.

CATHODE-RAY EXCITATION.

The cathode rays, I find, excite a fluorescence similar to white light. The lines of the principal and subordinate series appear as well, some of them of overpowering intensity. The apparatus for the electrical excitation is shown on Plate **3**, Figure 3.

It consisted of a steel tube 3 cms. in diameter and 35 cms. in length, one end closed with a glass plate, the other cemented with sealing-wax to a glass tube carrying the cathode. The mercury pump was kept in continuous operation to remove the hydrogen liberated from the sodium. On looking into the tube through the glass window a blazing spot of yellow light 2 cms. in diameter was seen at the point where the cathode rays entered the vapor. Its spectrum was photographed with the prism spectrograph, and is reproduced on Plate **2**, *n*. In addition to the fluorescent spectrum, and the sodium lines of all three series, the hydrogen lines come out strong. I have never been able to eliminate them entirely. Very few experiments have been made on the electrical excitation, but some very curious phenomena have been observed. In some cases, by looking into the tube in an oblique direction, it was seen that at the point where the cathode rays entered the mass of vapor there was a bright green spot of fluorescent light, while at the point of exit there was an orange-yellow spot, *the intervening space being non-luminous.* Seen in a direction oblique to the direction of the rays, the two spots were seen completely separated. This I consider a very remarkable circumstance, and a spectroscopic study of the two spots of light will undoubtedly yield very fruitful results.

Unfortunately the condition is a difficult one to keep fixed, for the phenomenon only appears when the density of the sodium vapor is just right and the surrounding vacuum high. As I have shown in the paper on the dispersion of sodium vapor, we can have a dense mass of the metal vapor, bounded on each side by a very high vacuum, a very anomalous condition from the point of view of the kinetic theory of gases. My impression is that the green spot will show the fluorescent spectrum, and the yellow spot the lines of the principal and subordinate series, as found in the sodium arc, but as yet I have not found time to make even a visual examination. Several attempts have been made, but by the time the image of the spot was thrown upon the slit in the proper direction to pass the light through the prisms, and the eye brought to the instrument, the conditions in the tube changed.

It is difficult to account for the absence of luminosity of the centre of the mass and the two bright spots. Perhaps the condition under which the rays excite fluorescence exists only where the vapor mass is in contact with the vacuum, i. e. in the region where the hypothetical clusters of molecules are breaking up and flying to the cooler walls of the tube. Even assuming this to be the fact, the difference in the color of the two spots is still to be accounted for. Possibly the cathode rays excite the green spectrum, while the canal rays travelling towards the cathode excite the orange-yellow luminescence. I have made one experiment with a similar tube arranged so as to deliver a stream of canal rays against the vapor. The luminescence was bright yellow, but the tube cracked before a spectroscopic examination was made.

On the other hand, it may be that whatever causes the green luminescence is removed from the ray bundle by absorption, the residue exciting the yellow luminescence at the point of exit. If this is the case, we should expect the same amount of yellow light in each spot, and I am of the opinion that the green light is much too pure for this to be the case. Further experimenting will be necessary before it is possible to draw any very definite conclusions.

In the spectrum excited by the cathode rays the D lines are of immense brilliancy, running together into a single band of light. On each side of this are seen three or four symmetrically placed bands, decreasing in brilliancy as they recede from the D lines in each direction. No trace of these bands appears in the magnetic spectrum, which in this region shows only fine lines arranged in narrow groups, which do not coincide with the bright bands of the cathode luminescence.

A photograph of these bands is reproduced in Plate **4**, Figure 4.

They have some connection with the D lines, I feel sure, for they are symmetrically arranged on each side of them. If the photograph had been made with a grating, we should of course call them ghosts. It may be that they are analogous to satellite lines, but if they are, we are certainly dealing with the phenomenon on a grand scale, for the fourth one is not far from the sodium doublet at 5688! All of these points will be more fully investigated during the coming year.

Other Possible Excitations.

It has occurred to me during the preparation of this paper that very interesting results would be obtained by exciting the fluorescence with the light selectively rotated by the vapor in a magnetic field, i. e. by the magnetic bright-line spectrum. This light is fairly intense, and it would be interesting to see whether the intensity distribution among the excited fluorescence lines was the same as in the magnetic spectrum.

What I most need, however, is a set of screens which will enable me to separate lines such as those of copper without resorting to the systems of prisms and lenses. A good collection of solutions of the rare earths would probably be very useful in the work. Erbium, praseodymium, and neodymium I have, but I should feel very grateful for the loan of any others which might prove serviceable, or for any suggestions regarding other possible sources of monochromatic light. As I have said before, the instrument most needed is a light siren!

Composite Excitation.

At the top of the chart just below the magnetic series will be found a spectrum containing about two hundred lines. This is a composite drawing made by superposing all of the drawings made of the simple spectra excited by monochromatic stimulation. It contains many lines not found in the complex spectrum excited by white light. In the latter, between wave-lengths 5000 and 5100, we find but ten or a dozen lines, while in the composite spectrum there are at least twenty. This circumstance is of interest in connection with the periodic dark regions of the complex spectrum, which give it a fluted appearance. The formation of these flutings requires further study, as their position shifts as we alter the wave-length of the exciting light, which in this case is a rather broad, isolated band from the continuous spectrum obtained with the monochromatic illuminator. The phenomenon was more fully described in the earlier paper, but requires further study.

THE SERIES IN THE MAGNETIC SPECTRUM.

As we have seen, the complex fluorescent spectrum is made up of six or more series of lines, the individual lines of each series being about 38 Ångström units apart, the spacing becoming less as we pass from yellow towards violet. The fact that the lines in the magnetic spectrum coincide with the lines of the complex spectrum makes it seem certain that the same series will be found there. By comparing the various fluorescent spectra with the magnetic spectrum, and by measuring carefully the distances between the lines of the latter, I have made a provisional assignment of the magnetic lines thus far observed into five series.

The wave-lengths and wave-length differences are given in the tables on page 259.

As will be seen by reference to the chart, the first series has the largest *average* spacing, and the fifth the smallest, the "scale," if the term is allowed, decreasing gradually from the first to the fifth, the two coming into coincidence at about wave-length 4860.

Doubtless these series could be extended to wave-length 5500 or thereabouts by making use of the grating photograph of the complex fluorescence obtained by white-light stimulation. The lines are, however, so diffuse in their nature, with overlapping wings and other peculiarities, that I have not yet attempted any further extension. I think that by employing a much denser vapor the magnetic spectrum can be considerably extended, and as the lines are much sharper in this case, an extension of the series will be an easy matter. The other series necessary to give the close spacing found in the yellow may be discovered in this manner.

A theoretical discussion of the results will have to be deferred to a subsequent paper. In fact, I prefer to have this side of the subject attacked by those who have given especial attention to the theory of molecular radiation. The absence of many lines in each series in the magnetic spectrum, and the absence of certain lines in the fluorescent spectrum, are especially suggestive. We have similar conditions in other series of lines, as is well known, but the present case is, so far as I know, the only one in which we can, by varying the exciting conditions, bring about a change in the position of the absent lines. It appears to me that the data furnished us by sodium vapor ought in the end to enable us to choose between the various theories proposed to account for spectrum series.

The investigations recorded in this paper have been made possible

FIRST SERIES.

λ.	λ differences.
5119.34	
5079.78	. . . . . 39.56
5040.65	. . . . . 39.13
5001.57	. . . . . 39.08
4962.85	. . . . . 38.67
4924.32	. . . . . 38.52

SECOND SERIES.

λ.	λ differences.
5165.85	
5126.54	. . . . . 39.31
5087.31	. . . . . 39.23
5048.49	. . . . . 38.82
. . . .	$\frac{77.64}{2} = 38.83$
4970.85	
4932.64	. . . . . 38.21
4894.58	. . . . . 38.06
. . . .	$\frac{75.15}{2} = 37.57$
4819.43	
?4782.89	. . . . . 36.54
. . . .	$\frac{112.89}{3} = 37.63$
. . . .	
4670	

THIRD SERIES.

λ.	λ differences.
5211.71	
5172.82	. . . . . 38.89
5133.73	. . . . . 39.09
5094.78	. . . . . 38.95
. . . .	$\frac{115.34}{3} = 38.45$
. . . .	
4979.34	
. . . .	$\frac{75.96}{2} = 37.98$
4903.38	
4865.59	. . . . . 37.79
. . . .	$\frac{113.55}{3} = 37.85$
4752.04	
4715.63	. . . . . 36.41

FOURTH SERIES.

λ.	λ differences.
5219.00	
5179.71	. . . . . 39.29
5140.71	. . . . . 39.00
. . . .	$\frac{115.05}{3} = 38.35$
. . . .	
5025.66	
. . . .	$\frac{113.56}{3} = 37.85$
. . . .	
4912.10	
. . . .	$\frac{74.61}{2} = 37.30$
4837.49	
. . . .	$\frac{109.97}{3} = 36.68$
. . . .	
4727.52	
?4692.54	. . . . . 34.98

FIFTH SERIES.

λ.	λ differences.
5225.34	
5186.70	. . . . . 38.64
5147.50	. . . . . 39.20
. . . .	$\frac{76}{2} = 38.00$
5071.50	
5033.54	. . . . . 38.00
. . . .	$\frac{74.92}{2} = 37.46$
4958.62	
. . . .	$\frac{74.81}{2} = 37.40$
4883.81	
. . . .	$\frac{73.65}{2} = 36.82$
4810.16	
. . . .	$\frac{71.66}{2} = 35.83$
4738.5	

EXTRA LINES.

5096.00
5052.83
5049.56
5003.12
4964.39
4967.10

through very generous aid given from the Rumford Fund by the American Academy of Arts and Sciences.

I am also under great obligation to my assistant, Mr. F. L. Cooper, for the many hours of very fatiguing work which he devoted to the research.

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RESULTS OF THE FRANCO-AMERICAN EXPEDITION TO EXPLORE THE ATMOSPHERE IN THE TROPICS.

By A. Lawrence Rotch.

RESULTS OF THE FRANCO-AMERICAN EXPEDITION TO EXPLORE THE ATMOSPHERE IN THE TROPICS.

By A. Lawrence Rotch.

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It has long been believed that the ascending currents above the thermal equator proceed immediately over the northeast and southeast trade-winds as southwest and northwest anti-trades. It is possible that part of the anti-trade sinks down over the high barometric pressure in the North and South Atlantic oceans and returns with the trade-winds, but the greater portion descends north and south of the origin of the trades and continues to the poles as the prevailing southwest or northwest winds of the North and South Temperate zones, respectively. The facts upon which this theory is based, as regards the Atlantic Ocean, are mainly observations upon the Peak of Teneriffe, where the southwest wind can be observed the whole year; and although it is lower in winter than in summer, there are no observations which prove that this anti-trade ever reaches the surface of the ocean.

The author, after he had made the first meteorological observations over the ocean, with kites flown from a transatlantic steamer in 1901, concluded that a vessel, which could be navigated at will, would enable meteorological data to be obtained with kites independently of the natural wind.[1] He suggested the application of this method to the exploration of the atmosphere in the trade-wind region at the Glasgow meeting of the British Association in 1901,[2] and at the Berlin Congress for Scientific Aëronautics in 1902.[3] In order to organize such an expedition, applications for aid were addressed in 1902 to the Prince of Monaco, and in 1903 to the Carnegie Institution, but without receiving the desired assistance. However, Professor Hergesell, President of the International Commission for Scientific Aëronautics, succeeded in interesting the Prince of Monaco in the scheme, and upon his yacht, the "Princesse-Alice," during the summer of 1904, kite-flights were made in the region bounded by Spain, the Azores, and the Canaries. Although a height exceeding that of the Peak of Teneriffe was attained several times, the southwest current, which had been reported on this mountain, was not found. Thereupon Professor Hergesell stated that he believed this current was due to the disturbing influence of the island, and that in the region explored the interchange of air takes place through the northwest current, which he had observed.[4]

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[1] Science, **14**, 412 and 896.

[2] Report Brit. Ass. Adv. Sci., Glasgow, 1901, Transactions Section E.

[3] Protokoll der 3. Versammlung der Internationalen Kommission für wissenschaftliche Luftschiffahrt, Beilage 11.

These conclusions, which involved a fundamental principle of meteorology, namely, the existence of the upper return-trades, seemed to demand further investigation, and accordingly M. Teisserenc de Bort and the writer undertook to execute this through their assistants, Messrs. Maurice of Trappes Observatory and Clayton of Blue Hill Observatory. Mr. Clayton made the voyage from Boston to Gibraltar, via the Azores, in June, 1905, on the White Star steamer "Romanic," and flew kites six times to an average height of about 1000 meters. The steam-yacht "Otaria," of 350 tons, purchased and equipped by M. Teisserenc de Bort for exploring the atmosphere, and having on board Messrs. Maurice and Clayton as the scientific staff, went during the months of July and August, 1905, at the mutual expense of her owner and the writer, from the Mediterranean, via Madeira, the Canary and Cape Verde islands, to latitude 10° north, longitude 30° west, returning via the Azores and Corunna (Spain) to Havre. Seventeen kite-flights were made over the ocean besides two in the harbor of Santa Cruz (Teneriffe) to study the sea-breeze, and another off Corunna (Spain) during the total solar eclipse of August 30, 1905. In this way continuous records of barometric pressure, air-temperature, relative humidity, and wind-velocity were obtained from sea-level up to the extreme height of 2200 meters, although wind-velocity was recorded to an altitude of 3100 meters. The direction of the wind was obtained by measuring the azimuth of the kites. Direct observations were made by Mr. Clayton upon the Peak of Teyde (Teneriffe) to a height of 3700 meters, and upon the Peak of Fogo (Cape Verdes) to 2200 meters, and observations of clouds enabled the wind to be ascertained about 500 meters higher. To attain a greater height in the free air than was possible with kites, the "Otaria" was provided with large paper balloons, eleven of which, filled with hydrogen gas, were liberated from the islands of St. Michaels (Azores), Madeira,

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[4] Conférence de la Commission Internationale pour l'Aérostation Scientifique, St. Pétersbourg, 1904, Supplément 7.

Teneriffe (Canaries), and St. Vincent (Cape Verdes). Since these balloons were only intended to show the atmospheric drift, they did not carry self-recording instruments, and their direction and velocity at increasing heights were determined from angular measurements at the ends of a base-line laid off on the lee shore of the islands mentioned. One balloon, carrying a self-recording barometer and thermometer, was launched from the yacht off the island of Palma (Canaries), but, though its drift was observed, the balloon could not be recovered. It was found possible sometimes to follow the balloons in the telescope until they reached a height of 11,000 or 12,000 meters.

The observations obtained with the kites at 500 and 1000 meters, and the simultaneous observations at sea-level, are given in Table I, which is divided into two parts, the first containing the observations made in an east-southeast direction across the Atlantic, and the second part those made in a southerly direction within the northeast trade and on its borders. West of the Azores, on the westerly slope of the permanent area of high barometric pressure, the observations between longitudes 69° and 39° show a slow decrease of temperature with increase of height, amounting to 4.5° per 1000 meters. In the lower 500 meters the decrease is only 0.24° per 100 meters, owing to inversions of temperature within the first few hundred meters in half the flights. In the next 500 meters there is the more rapid decrease of 0.66° per 100 meters. Upon the easterly and southeasterly slopes of the high pressure, between longitudes 25° and 19°, latitudes 38° and 33° north, the temperature falls at the adiabatic rate of one degree per 100 meters in the lower 500 meters, and then declines more slowly, namely, 0.20° per 100 meters, up to 1000 meters. The adiabatic rate appears to prevail over the ocean at night as well as in the daytime, and the bases of the cumulus clouds generally are not higher than 500 meters. The relative humidity decreases with height on the west of the high pressure, and increases to above 500 meters on the southeast side. In the former region southwest winds prevail, and in the latter locality northeast winds, the southwest winds turning to the left when facing them up to 500 meters, with increasing velocity up to 1000 meters, and the northeast winds turning to the right and increasing slightly in velocity up to 500 meters, but diminishing above that level. The mean directions and velocities in this table are resultants derived geometrically. In the column for wind-direction, a plus sign before the figures representing the differences per hundred meters indicates a turning towards the right hand, and a minus sign a turning towards the left.

The observations on the northern edge of the northeast trade, that is,

TABLE I.

METEOROLOGICAL CONDITIONS ABOVE THE ATLANTIC.

1. *West and East of the Barometric Maximum, between Long. 69° and 16° W of Greenwich, Lat. 42° and 33° N.*

Date, 1905.	Place.		Temperature (C°).			Relative Humidity.			Wind Direction.			Wind Velocity (Meters per Second).		
	Lat. N.	Long. W.	0 Meters.	500 Meters.	1000 Meters.	0 Meters.	500 Meters.	1000 Meters	0 Meters.	500 Meters.	1000 Meters.	0 Meters.	500 Meters.	1000 Meters.
									°	°	°			
June 3	42	69	11.0	12.2	7.5	86	73	87	. .	. .	. .	. .	. .	. .
“ 5	41	54	17.1	15.3	. .	85	73	. .	N 35 W	S 52 W	. .	6.0	2.0	. .
“ 6	41	47	15.8	15.3	13.4	96	88	52	S 14 W	S 28 W	S 32 W	8.5	12.3	12.0
“ 7	40	39	19.8	16.0	13.4	90	84	73	S 9 W	S 0	S 20 W	5.5	8.5	5.7
Mean	41	52	15.9	14.7	11.4	89	79	71	S 60 W	S 20 W	S 28 W	6.6	7.3	9.0
Δ 100 m.	. .	. .	. .	−.24	−.66	. .	−2.0	−1.6		−8	+2	. .	+.14	+.34
June 9	38	25	16.4	12.0	. .	57	83	. .	N 13 W	N 24 W	. .	10.0	13.3	. .
“ 10	38	19	17.5	12.5	9.8	68	78	60	. .	. .	. .	. .	. .	. .
July 3	33	16	23.0	16.7	13.3	71	88	100	N 28 E	N 45 E	N 50 E	8.0	10.0	10.7
“ 4	33	16	20.8	16.0	16.3	. .	. .	. .	N 28 E	N 2 E	N 8 W	7.0	8.3	6.5
Mean	35	19	19.4	14.3	13.1	65	83	80	N 12 E	N 23 E	N 28 E	7.8	8.3	7.5
Δ 100 m.	. .	. .	. .	−1.02	−.22	. .	+3.6	−.6	. .	+2	+1	. .	+.10	−.16

2. *Within the Northeast Trade and North and South of it, between Lat. 36° and 10° N, Long. 17° and 30° W.*

Aug. 20	36	24	23.2	18.9	. .	84	96	. .	N 56 E	N 56 E	. .	7.0	7.0	. .
" 19 P.	35	22	23.8	19.1	15.9	80	93	90	N 56 E	N 56 E	N 56 E	8.0	7.1	13.5
" " A.	35	19	23.6	19.0	14.7	79	92	96	N 64 E	N 54 E	N 64 E	7.0	6.8	10.0
" 18 P.	34	17	23.7	18.8	16.5	84	94	92	N 33 E	N 26 E	N 38 E	6.0	6.0	5.5
" " A.	34	19	23.2	18.3	15.8	83	94	90	N 38 E	N 45 E	N 50 E	8.0	10.0	10.7
Mean	35	20	23.5	18.8	15.7	82	94	92	N 50 E	N 48 E	N 54 E	7.0	7.0	9.5
Δ 100 m.	. .	. .	. .	−.92	−.62	. .	+2.4	−.4	. .	0	+1	. .	.00	+.50
Aug. 14	31	17	21.7	16.8	23.7	85	100	18	N 6 E	N 19 E	N 27 W	13.0	14.0	5.0
" 8	28	19	23.5	25.6	. .	67	29	. .	N 67 E	N 84 E	. .	1.6	2.2	. .
" 9	28[1]	17	23.2	19.8	23.9	72	58	41	SE	Calm	Calm	1.0	0	0
" 10	28[1]	17	23.6	22.7	21.1	82	86	85	NE	NE	Calm	3.0	1.3	0
July 12	27	17	20.3	20.0	28.9	84	67	7	. .	. .	. .	. .	. .	. .
Aug. 4	24	20	23.0	18.3	19.0	76	93	52	N 28 E	N 30 E	N 60 E	8.0	8.7	9.0
July 15	19	23	22.7	19.4	22.6	89	73	52	. .	. .	. .	. .	. .	. .
" 20 P.	16	26	24.3	19.5	21.2	83	87	48	N 50 E	N 45 E	N 57 E	6.0	6.0	10.5
" " A.	16	26	23.4	18.1	. .	90	99	. .	. .	. .	. .	. .	. .	. .
" 27	15[2]	24	25.7	21.7	28.2	68	83	13	E	var.	NE	8.0	1.7	9.5
" 28	15[2]	24	25.2	22.8	20.5	60	66	66	N	var.	S	7.0	3.0	1.5
Mean	22	22	23.3	20.4	23.2	78	77	42	N 30 E	N 32 E	N 44 E	4.2	4.3	5.6
Δ 100 m.	. .	. .	. .	−.58	+.56	. .	−.2	−.7	. .	0	+3	. .	.00	+.26
July 24	11	26	26.0	21.7	18.7	83	96	93	N	. .	N	3.0	. .	5.0
" 22	10	30	24.0	20.5	18.0	96	100	100	S 35 W	S 35 W	S 35 W	. .	. .	. .
Mean	10	28	25.0	21.1	18.3	89	98	96	. .		. .	. .	. .	. .
Δ 100 m.	. .	. .	. .	−.78	−.56	. .	+2.2	−.4	. .		. .	. .	. .	. .

[1] Peak of Teneriffe.

[2] Peak of Fogo.

between latitudes 36° and 34° north, show a rapid fall of temperature with height (7.8° per 1000 meters) which is fastest (0.92° per 100 meters) within the first 500 meters. The relative humidity rises nearly to saturation within the first 500 metres, but does not attain this point, the flat cumulus clouds found in the region being probably formed by the condensation of water-vapor rising from the ocean through a slow motion of diffusion. The wind, which is generally northeast, changes its direction but little with altitude, although it increases in velocity above 500 meters. The same features are shown by the only two observations south of the northeast trade, in about latitude 10° north. Within the trade-wind region, between latitudes 31° and 15°, the vertical distribution of temperature and moisture is quite different. Near its origin the trade has the character of a descending current, that is, small vapor contents and little cloud, which is the flat cumulus typical in northern regions of high pressure and descending air. In approaching the equator, that is, south of latitude 24° north, the trade presents characteristics of an ascending current, the relative humidity increases much with height, the sky is cloudy, and there are frequent rains, often accompanied by thunder-storms. The decrease of temperature within the trade region is less than 1°C. in 1000 meters, there being a fall of 0.58° in the first 500 meters and a rise of 0.56° in the next 500, on account of the inverted temperature-gradients which occur near 1000 meters at the upper limit of the trade-wind. Its depth varies from day to day between 300 and 1500 meters, and appears to be greatest in the afternoon and least at night. The upper portion is damp, with cumulus and strato-cumulus cloud, above which the wind falls light and the relative humidity sinks nearly to zero, coinciding with the rise in temperature, which frequently carries it much above that at sea-level. With increasing altitude there is a gradual shifting of the wind, when facing it, to the right, accompanied by an accelerated velocity, up to at least 1000 meters. The conditions at greater heights were deduced by Mr. Clayton as follows: Above the surface-trade is a current some 2000 meters in depth, varying in direction between northeast and northwest, but coming always from a direction to the left of the lower wind when facing it. This current is extremely dry and potentially warm, and its velocity usually much exceeds that of the lower wind. The third stratum, which begins at a height of about 3000 meters, moves from east, south or southwest, being generally from the east in equatorial regions, and from the south between latitudes 15° and 30° north (see Table II). As observed on the Peak of Teneriffe this stratum was dry in its lower portion, but with a larger vapor contents than the air immediately below. Alto-

TABLE II.

Winds above the Trade-wind Region of the Atlantic, between Lat. 37° and 10° N, Long. 15° and 26° W of Greenwich.

1905. Aug. 22	St. Michaels (Azores)	NE to 800 meters; NW to at least 4200 meters.
" "	Madeira	WNW to 1800 m.; NW and SW to 11,500 m.; SW at 11,600 m.
" "	"	NE to 2900 m.; NW and NE to 4200 m.; WSW to at least 12,500 m.
July 17	Teneriffe (Canaries)	NE trade to 400 m.; NW to 3500 m.; WSW to at least 7500 m.
" 9	"	NE trade (variable) to 2000 m.; NNW to 4000 m.; SSE and SW to at least 5700 m.
" 10	"	NE trade to 3300 m.; SW and NW to 5200 m.; S and SE to at least 11,000 m.
Aug. 10	"	NE trade to 3200 meters; SE and S to 5300 m.; SSW to at least 5900 m.
" 11	"	NE trade to 2300 m.; S and SSW to at least 3900 m.
" 9–10	Peak of Teyde (Teneriffe)	E, 1 m. per sec., to 1000 m.; N, 4 m. per sec., at 2000 m.; N, 7 m. per sec., at 2500 m.; E, 3 m. per sec., at 3000 m.; S, 4 m. per sec., at 3500 m.; S at 4000 m. (?)
Aug. 13	At sea off Palma (Canaries)	NE trade to 2600 m.; NW to 3400 m.; WSW at 3400–4200 m.; SW to at least 6500 m.
July 17	St. Vincent (Cape Verdes)	NE trade (variable) to 4100 m.; variable to 5100 m.; SE and SSE to at least 11,000 m.
" 18	"	NE trade to 1300 m.; ESE to at least 2360 m.
" 29	"	NE trade to 650 m.; NW (variable) to 1900 m.; SW and SSW to 7500 m.; ESE and NE to 11,700 m.; S to at least 13,600 m.
" 27–28	Peak of Fogo (Cape Verdes)	E, 7–3 m. per sec., to 500 m.; NE, 6 m. per sec., at 1000 m.; N, 6 m. per sec., at 1500 m.; N, 5 m. per sec., at 2000 m.; ENE at 2500 m.; E at 3500 m. (?)
" 25	Lat. 13° N, Long. 24° W	NE, 4–3 m. per sec., to 1000 m.; E, 16 m. per sec., to 2500 m.
" 24	Lat. 11° N, Long. 26° W	N, 3–5 m. per sec., to 1600 m.; E at 3000 m. (?) by ACu clouds; ESE at 11,000 m. (?) by CiS clouds.

cumulus and alto-stratus clouds were seen floating in it at a height of perhaps 4000 or 5000 meters, and from them light sprinkles of rain fell occasionally. On the Peak of Teneriffe, in passing into this upper current, a rise of temperature was noted, which was less than that encountered above the surface-trade.

The winds at great heights in and near the trade-wind region are given in Table II. They were obtained by pilot-balloons launched from the islands, excepting the one from the yacht, and the last figures for each ascension show the maximum height at which the balloon was sighted. The means of direct observations of wind-direction and velocity at definite heights, obtained during the ascents and descents of the peaks on the tropical islands of Teneriffe and Fogo, and the drift of clouds passing at estimated heights above these mountains, are given. Observations of the direction and velocity of the wind, obtained in two kite-flights south of the trade-wind region, complete the table. In that portion of the Atlantic investigated by the Franco-American Expedition, the atmospheric circulation was found to be as follows: (1) North of Madeira, and near the Azores, the upper winds, as was already known by observations of clouds, are chiefly from west and northwest, this region being generally to the north of the barometric maximum over the ocean and beyond the zone of the trades. (2) The winds blowing towards the equator are from northeast to east in the lower region, and generally from northwest to northeast above 1000 meters. (3) The return currents from the equator, or anti-trades, are formed by winds having a southerly component, being generally southwest in the latitude of the Canaries, and southeast near the Cape Verdes, thus showing the influence of the earth's rotation. The law of the vertical succession of winds, as formulated by Abercromby,[5] namely, a shifting in the northern hemisphere of the upper winds to the left-hand, when one's back is towards the wind, is found not to hold true always, the right or left-handed rotation depending upon the origin of the wind, and, presumably, upon the distribution of the pressure at high levels.

The vertical distribution of temperature and relative humidity revealed by these observations up to a height of 4000 meters is nearly the same as that found by Professor Hergesell during the cruises of the "Princesse-Alice," in 1904 and 1905.[6] Most of his observations of direction of the upper currents differ radically, however, in showing no southerly component, although one balloon, launched west of the Canaries, gave the same direction as that obtained near these islands, meeting interlaced currents from the southeast and southwest, above the northeast trade. From the distribution of pressure on the earth's surface it would be supposed that the upper anti-trade ought to be especially regular in the region between Cape Verde and the Canaries; but this idea is contrary to the belief of Professor Hergesell that the upper southeast and southwest winds observed near these islands, and long considered to furnish a demonstration of the return-trade, are due to local disturbing causes. To settle this question, Messrs. Teisserenc de Bort and the author again sent the "Otaria," during the winter of 1906, to the south and west of the region which had been explored by them the preceding summer. Since this paper was presented to the Academy, Messrs. Maurice and Nilsson, constituting the scientific staff of the "Otaria," have communicated the results of their atmospheric soundings, made from the vessel to the westward of the Canaries, and these results appear in Table III. The longitudes are from Greenwich.

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[5] Nature, **36**, 85.

[6] Comptes Rendus de l'Académie des Sciences, 30 janvier, 1905; Meteorologische Zeitschrift, November, 1905; Bulletin du Musée Océanographique de Monaco, 30 novembre, 1905.

TABLE III.

Winds observed in February, 1906, above the Atlantic, Southwest of the Canaries.

Feb. 13	Lat. 28° N, Long. 18° W. ENE to 2850 m., NW to 3680 m., SW to the culminating point of the balloon, 5300 m.
" "	Lat. 27° N, Long. 18° W. ENE to 1800 m., SSE to 2100 m., N to 2250 m., SW to 2500 m., NW stratum 500 m. thick, then SW to 5100 m.
" 14	Lat. 26° N, Long. 19° W. NE changing to N up to 1350 m., NW to 2600 m., WSW and SW to 5100 m.
" "	Lat. 26° N, Long. 19° W. NE to 1300 m., NW and W to 3150 m., strong SW to 3300 m. Cirrus clouds (4 observations) from S 50° W.
" 15	Lat. 25° N, Long. 20° W. NE to 2300 m., NW to 3000 m., SW to 3250 m., WNW changing to W up to 3950 m., SW to 4150 m. Alto-cumulus clouds from NE, cirrus (2 observations) from S 30° W.
" 16	Cirrus clouds from S 45° W.

It is seen that the upper anti-trade is shown both by the balloons and by the drift of the clouds, the stratified conditions giving place to the southerly wind between 3000 and 4000 meters. Therefore the classic observations of the return-trade, which were long ago made on the Peak of Teneriffe, indicate a general phenomenon, and agree with those obtained over the open ocean by the present expedition.

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AN APPROXIMATE LAW OF FATIGUE IN THE SPEEDS OF RACING ANIMALS.

BY A. E. KENNELLY.

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RACES between swift men, or between swift horses, have been of the greatest interest in all times. Olympia and Epsom Downs are known to fame by the races they have witnessed. Olympian races, recently revived, are of international interest.

It is strange that, judging from encyclopedias and text-books on athletics, there is very little published information concerning the speeds at which races are run. Apparently, all that is known by our books on these matters is that short races are run at higher speeds than long races. Every one knows that a contestant in a mile or kilometer race runs at a lower speed than a sprinter in a 100-yard or 100-meter dash.

There has, however, been accumulated during the last century, and particularly during the last fifty years, a considerable fund of publicly recorded information concerning the record times in which races of stated length have been run. Athletes are, for example, generally familiar with the records of the 100-yard and the mile runs; namely, 9.6 and 252.75 seconds, respectively. A reduction of either of these record times by even one per cent would be a matter of world-wide importance and the hero of the new record would be famous among the inhabitants of the temperate zones.

This paper presents the data which the writer has been able to collect upon record speeds in various kinds of racing, as well as the conclusions that seem to be warranted thereby. It will be seen that the records align themselves closely to a simple mathematical relation. It is not pretended that the records conform rigorously to this mathematical relation. Such a condition could hardly be expected from the performances of different animals at different times and in different parts of the world. It is claimed, however, that any one who will analyze the records presented will be able to satisfy himself that they approximate to the said mathematical relation for practical purposes within satisfactorily small limits of deviation.

We may commence with horse-racing records.

Horses Trotting.

Table I, the data of which are taken from page 259 of "The World Almanac and Encyclopedia" for 1906, gives in column I the date of the record, in column II the distance run, or length of the course, and in column III the best record time. For these data "The World Almanac" is made responsible. These data have been checked, however, by those given in the same publication for preceding years. No event has been rejected. The best records for 1, 2, 3, 4, 5, 10, 20, 30, 50, and 100 miles of trotting are taken. They are stated to be World's records, and at least one, — the 4-mile event — is stated to have been made in England.

Commencing with the above data, column IV shows the distances expressed in meters. The meter and kilometer are so much simpler to deal with numerically than the foot, yard, furlong, and mile, that it is worth while to reduce all distances to meters. Column V gives the average speed at which the record was made, expressed in meters per second. Thus, taking the first event, the mile (1609.3 meters) was trotted in 118.5 seconds. This represents an average speed of $1609.3 \div 118.5 = 13.58$ meters per second (30.4 miles per hour; or 44.5 feet per second).

Turning now to Figure 1, the abscissas are laid off both in miles and in kilometers, as far as 20 miles (32.2 kilometers). The ordinates represent speeds both in meters per second and in miles per hour. Another scale of ordinates gives the record time of each run in seconds. It is seen that the speeds, taken from column V, drop from 13.58 meters per second (30.4 miles per hour) at 1 mile (1609 meters) to 9.18 meters per second (20.6 miles per hour) at 20 miles. The average speed of the trotting horse that made the 20-mile record was therefore 67.6 per cent, or about two-thirds of that of the trotting horse which made the 1-mile record.

Taking next the time ordinates, the rising line in Figure 1 closely follows the first six successively increasing times. It is evident that both the speed-distance line and the time-distance line are curves, when thus plotted. The curvature of these curves is greatest near the start, or over the short courses, and diminishes as the course increases.

If, however, the speed and the time with respect to distance be plotted on logarithm paper, as in Figure 2, instead of on ordinary cross-section paper, as in Figure 1, the points fall approximately upon straight lines.

The above fact is the gist of this paper. That is to say, if we consider the three quantities L, T, and V, or length of course, record

TABLE I.

ANALYSIS OF WORLD'S TROTTING RECORDS (HARNESS RACING).

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
Date.	Distance, miles.	Time, seconds, T	Distance L, meters.	Speed V, $\frac{\text{meters}}{\text{seconds}}$	log. L.	log. T.	log. V.	log T' computed.	T' computed.	deviation, $T' - T$.	Per cent, $100\left(\frac{T'-T}{T}\right)$.
1905	1	118.5	1,609.3	13.58	3.2066	2.0737	1.1329	2.0774	119.5	1.0	0.8
1902	2	257.0	3,218.7	12.52	3.5077	2.4099	1.0978	2.4162	260.7	3.7	1.4
1893	3	415.5	4,828.0	11.62	3.6838	2.6186	1.0652	2.6143	411.4	−4.1	1.0
1899	4	598.0	6,437.4	10.77	3.8087	2.7767	1.0320	2.7548	568.6	−29.4	4.9 } 1.8
1893	5	750.75	8,046.7	10.72	3.9056	2.8755	1.0301	2.8638	730.8	−20.0	2.8
1893	10	1,575.0	16,094.0	10.22	4.2066	3.1973	1.0093	3.2024	1,594.0	19.0	1.2
1865	20	3,505.0	32,187.0	9.18	4.5077	3.5447	0.9630	3.5414	3,477.0	−28.0	0.8
1857	30	6,479.0	48,280.0	7.45	4.6838	3.8115	0.8723	3.7393	5,484.0	−995.0	15.0
1846	50	14,140.5	80,467.0	5.69	4.9056	4.1504	0.7552	3.9888	9,746.0	−4394.5	31.0
1853	100	32,153.0	160,930.0	5.01	5.2066	4.5072	0.6994	4.3274	21,250.0	−10,900.0	34.0

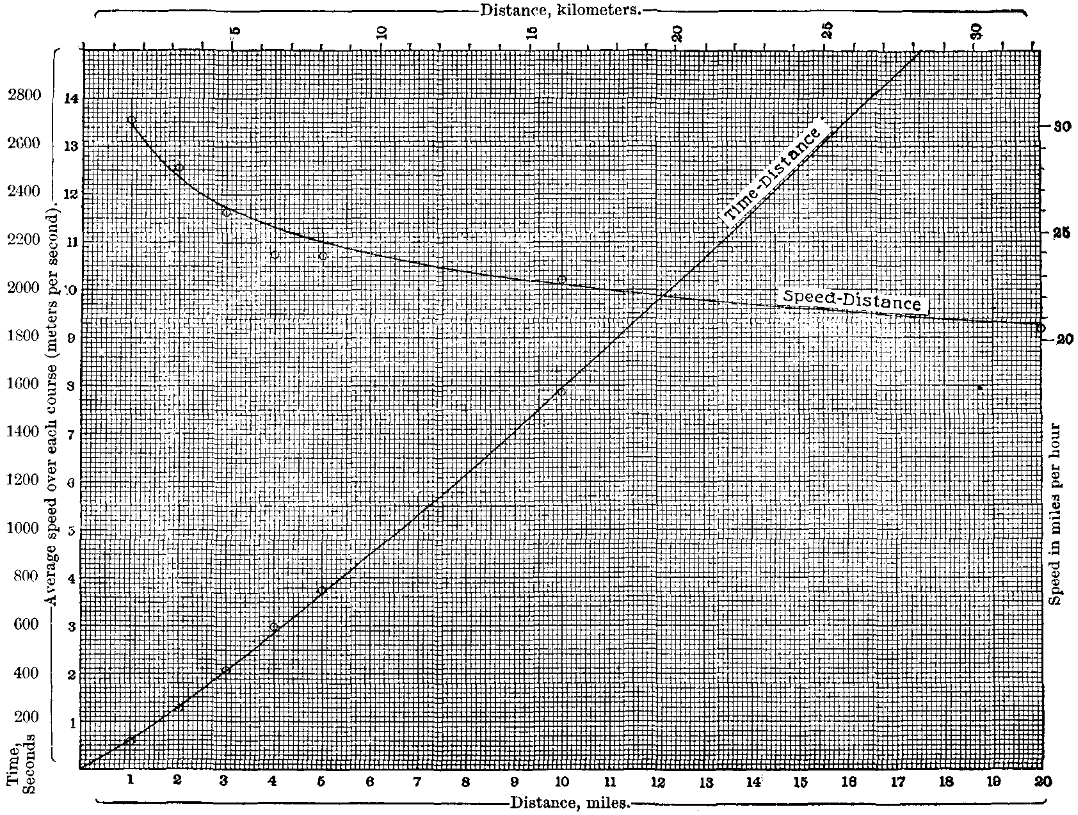

FIGURE 1. *World's Trotting Records.*

time, and average speed of the run, in a series of events, any one, say T, plotted against either of the others, say L, the observations are found to fall approximately upon a straight line, with logarithm paper. In other words, the curves plotted on plain rectangular paper from the same records are approximately simple exponential curves, of the type $y = x^n$.

Instead of plotting the quantities in the ordinary way upon logarithm paper to produce straight lines, we may perform the equivalent operation of plotting the logarithms of the quantities upon ordinary scaled paper, and produce similar straight lines. That is, we may plot any one of the quantities log L, log T, and log V against either of the other two. For some purposes the latter method is to be preferred, although it takes more time. Its application is presented in Figure 3, where log T and log V are both plotted as ordinates against log L as abscissas for all of the data of Table I. Columns VI, VII, and VIII in the table contain the common logarithms of the entries in columns IV, III, and V respectively. It is seen in Figure 3 that the speeds fall closely upon the descending straight line, as far as the 20-mile distance, as already seen in Figure 2. Beyond the 20-mile distance, the speeds fall off markedly and are much too low to meet the line. Table I indicates, however, that these long-distance records of 30, 50, and 100 miles respectively, were made about 50 years ago, whereas the short-distance records are of recent date. At the dates indicated (1846, 1853, 1857) the short-distance trotting records were by no means so good as they are to-day. It is reasonable to assume that if these deviating long distances were attempted to-day, their records would be materially improved.[1]

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[1] Since this paper was written, the writer has been indebted to Prof. E. L. Mark for a photographic curve-sheet pertaining to a paper presented by Prof. Francis E. Nipher to the St. Louis meeting in 1903 of the American Association for the Advancement of Science. The curve-sheet shows the steady reduction in the record times of the trotting-horse mile and also of the running-horse mile at different dates between 1840 and 1903. The curves indicate a final limit to the trotting mile at 98 seconds and a final limit to the running mile at 91.5 seconds. The equations to the curves do not appear on the sheet, but have been computed by the writer, from the curves, as follows: —

At any epoch y years after 1840, the trotting-horse mile record approximates to

$$T_y = 98\ (1 + 0.56 \times 10^{-0.00526\,y}) \qquad \text{seconds} \qquad \text{(a)}$$

and for the running-horse mile record:

$$T_z = 91.5\ (1 + 0.154 \times 10^{-0.00993\,z}) \qquad \text{seconds} \qquad \text{(b)}$$

where z is the epoch in years after A.D. 1863.

The ascending line in Figure 3, connecting log T and log L, represents the sequence of record times with satisfactory precision, as far as 20 miles, with the exception, perhaps, of the 4-mile event, in which the time is long and the speed low. As shown in Figure 1 or Table I, the speed in the 4-mile trot is only half of one per cent greater than the speed in the 5-mile trot. It should be relatively faster by more than this amount, to judge by the speeds in the other events, and this means that it should lie nearer the straight line of time-distance in Figure 3.

Beyond 20 miles, the points deviate markedly from the rising straight line of Figure 3 in the direction of excessive time, or low speed, as already considered.

The rising straight line of Figure 3 represents the equation

$$\log T = \tfrac{9}{8} \log L - 1.53 \qquad (1)$$

while the falling straight line of speed-distance corresponds to

$$\log V = 1.53 - \tfrac{1}{8} \log L. \qquad (2)$$

That is, the rising line makes with the axis of abscissas an angle of 48° 22′, whose tangent is $\frac{9}{8}$; while the falling line makes with the same axis an angle of $-7° \, 7' \, 30''$, whose tangent is $-\frac{1}{8}$.

Equation (2) implies that at $L = 1$, or upon a course one meter long, the speed of trotting would be 33.9 meters per second, the logarithm of this number being 1.53. It would be impossible for a horse to reach any such speed on such a very short course, even with flying start, if only owing to inertia and the large effort required for initial acceleration. The initial velocity of equation (2) is therefore a fictitious quantity of merely theoretical interest. The speed curve of Figure 1 and the straight speed lines of Figures 2 and 3 mark a satisfactory application of equation (2) between the limits of 1 mile and 20 miles. Columns IX and X of Table I give the computed time for each event, as determined by equation (1) or the rising lines in Figures 1, 2, and 3. Column XI gives the deviation or discrepancy between the

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From formula (a) or from the curve-sheet, the ratios of reduction in the trotting-horse mile record time to 1905 are,

at 1846	0.82
" 1853	0.844
" 1857	0.858

Using these correcting ratios, the 30-mile trotting record is brought on to the logarithmic straight line in Figure 3; while the 50-mile and 100-mile records are only brought about half-way towards that line, as indicated on the figure.

computed record time T' seconds and the published record time T seconds, while column XII expresses this discrepancy in percentage of the record time T. Thus, the 4-mile event should have been trotted in 568.6 seconds by the formula, as against the published record of 598 seconds, a discrepancy of 29.4 seconds or 4.9 per cent of the published record. It is seen that between the limits of 1 mile and 20 miles (1.61 and 32.2 kilometers) the average discrepancy between the recorded time and the time taken from the equation (1) or the ascending lines in Figures 2 and 3 is 1.8 per cent. The discrepancy is much greater in the three longest events and reaches 34 per cent in the 100-mile trot. Owing to the age of these three records, however, it is submitted that they may properly be set aside. At all events, between the limits of 1 mile and 20 miles the straight logarithmic line of times agrees with the published records to an average of 1.8 per cent. If the suspected 4-mile record were set aside, the average discrepancy without regard to sign would come down to 1.1 per cent.

In Figure 4, drawn to uniform scale, the average speeds are continued to 100 miles of course-length, or beyond the limits of Figure 1. The curve of speeds corresponds to equation (2) or to

$$V = \frac{33.9}{L^{\frac{1}{8}}} \quad \text{meters per second.} \qquad (3)$$

The figure shows the discontinuity which exists between the speeds over the three longest courses and those over courses up to 20 miles, as taken from column V, Table I.

Horses Running.

The records for running-horse races appear in columns I and II of Table II. They are taken from page 258 of "The World Almanac" for 1905, which gives the records for 33 courses between $\frac{1}{4}$ mile (402.3 meters) and 4 miles (6437 meters) on American turf, revised to December 1, 1904. Column III gives the distances in meters and column IV the average speed of each run. The times and the speeds are plotted against distance to uniform scale in Figure 5 and to logarithmic scale in Figure 6. In the latter case the entries in columns V, VI, and VII are used. It is to be noticed that in Figure 5, with uniform ruling, the observations follow curves, whereas in Figure 6, with logarithmic ruling (or logarithms of the quantities on uniform ruling), the observations fall substantially on straight lines. The two curves drawn in Figure 5 respectively correspond mathematically to the two straight lines drawn to meet the observations in Figure 6.

TABLE II.

HORSES RUNNING.

Analysis of Record Performances on American Turf.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
Distance, miles.	Time T, seconds.	Distance L, meters.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.	log T.	log L.	log V.	log T' computed.	T' computed, seconds.	$T'-T$ discrepancy, seconds.	$100\left(\frac{T'-T}{T'}\right)$ per cent discrepancy.	Mean per cent.
0.25	21.25	402.3	18.93	1.3274	2.6046	1.2772	1.3028	20.08	−1.17	5.5	
0.375	34.0	603.5	17.75	.5315	.7807	.2492	.5009	31.69	−2.31	6.8	
0.4375	40.5	704.1	17.38	.6075	.8476	.2401	.5761	37.68	−2.82	7.0	
0.5	46.0	804.7	17.49	.6628	.9056	.2428	.6414	43.79	−2.21	4.8	
0.5625	51.6	905.2	17.54	.7126	.9567	.2441	.6990	50.01	−1.59	3.1	
0.625	56.75	1005.8	17.72	.7540	3.0025	.2485	.7504	56.29	−0.46	0.8	
0.6875	62.5	1106.4	17.71	.7959	.0439	.2480	.7970	62.66	0.16	0.3	
0.75	68.0	1207.0	17.75	.8325	.0817	.2492	.8395	69.10	1.10	1.6	
0.8125	78.4	1307.6	16.68	.8943	.1165	.2222	.8787	75.63	−2.77	3.5	
0.875	83.5	1408.1	16.86	.9217	.1486	.2269	.9148	82.19	−1.31	1.6	
0.9375	92.2	1508.9	16.37	.9647	.1787	.2140	.9486	88.84	−3.36	3.6	
1.0000	95.5	1609.3	16.85	.9800	.2066	.2266	.9800	95.50	0	0	

1.0114	100.0	1627.6	16.28	2.0000	.2115	.2115	.9855	96.72	–3.28	3.3
1.028	101.2	1655.0	16.35	.0052	.2188	.2136	.9937	98.56	–2.64	2.6
1.040	102.6	1673.3	16.31	.0111	.2236	.2125	.9990	99.98	–2.62	2.6
1.0625	104.6	1709.9	16.35	.0195	.2330	.2135	2.0097	102.2	–2.4	2.3
1.125	111.0	1810.5	16.31	.0453	.2578	.2125	.0376	109.0	–2.0	1.8
1.1875	117.4	1911.1	16.28	.0697	.2813	.2116	.0741	118.6	1.2	1.0
1.25	122.8	2011.7	16.38	.0892	.3036	.2144	.0891	122.8	0	0
1.3125	133.0	2112.3	15.88	.1239	.3248	.2009	.1131	129.7	–3.3	2.5
1.375	137.6	2212.9	16.08	.1386	.3450	.2064	.1357	136.7	–0.9	0.7
1.5	150.25	2414.0	16.07	.1768	.3827	.2059	.1781	150.7	0.45	0.3
1.625	165.2	2615.2	15.83	.2180	.4175	.1995	.2273	168.8	3.6	2.2
1.75	177.0	2816.3	15.91	.2480	.4497	.2017	.2535	179.3	2.3	1.3
1.875	199.0	3017.5	15.16	.2989	.4796	.1807	.2871	193.7	–5.3	2.6
2.000	206.5	3218.6	15.59	.3149	.5077	.1928	.3188	208.4	1.9	0.9
2.125	222.0	3419.8	15.40	.3464	.5340	.1876	.3483	223.0	1.0	0.5
2.25	229.2	3621.0	15.80	.3602	.5588	.1986	.3762	237.8	8.6	3.8
2.5	264.5	4023.3	15.21	.4224	.6046	.1822	.4278	267.8	–3.3	1.2
2.625	298.5	4224.5	14.15	.4749	.6258	.1509	.4516	282.9	–15.6	5.2
2.75	298.75	4425.7	14.82	.4753	.6460	.1707	.4743	298.1	–0.65	0.2
3.00	324.0	4828.0	14.90	.5105	.6838	.1733	.5169	328.8	4.8	1.5
4.00	431.0	6437.0	14.93	.6345	.8087	.1742	.6574	454.4	23.4	5.4

} 1.9

Figure 2. *World's Trotting Records Plotted on Logarithm Paper.*

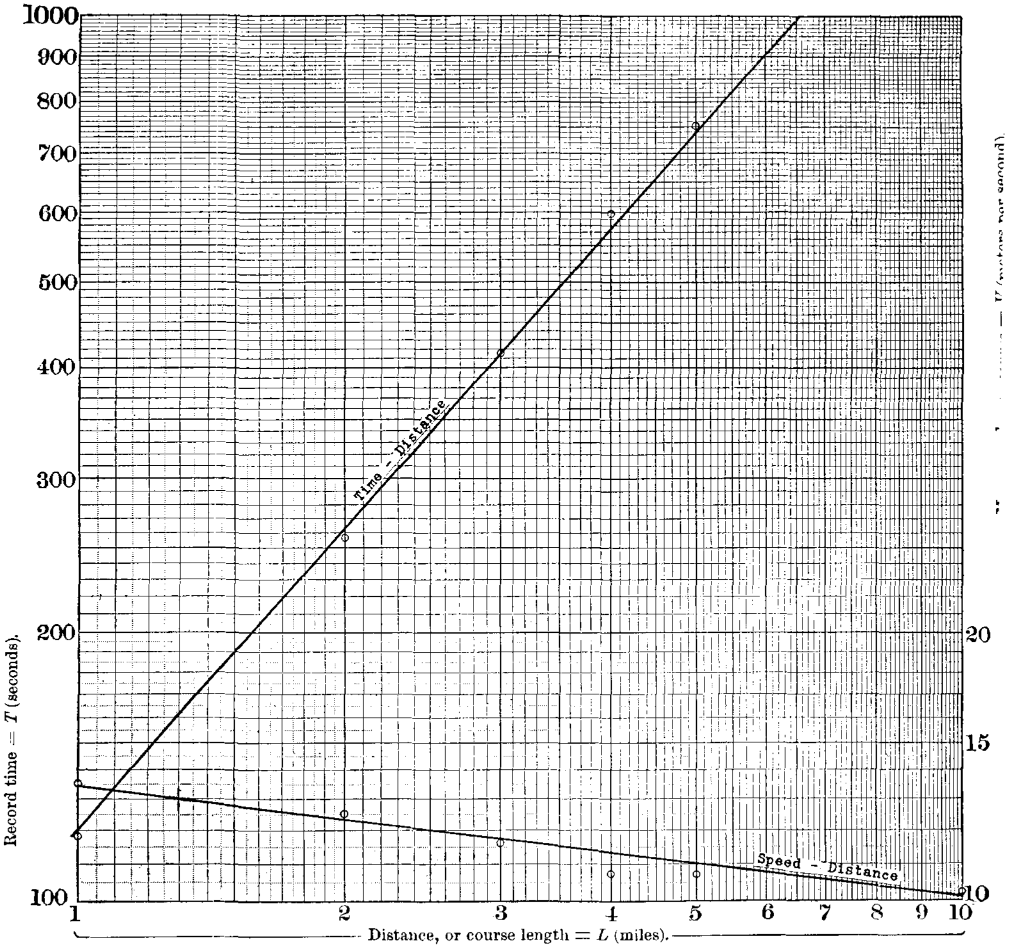


FIGURE 3. *World's Trotting Records to 100 Miles.*

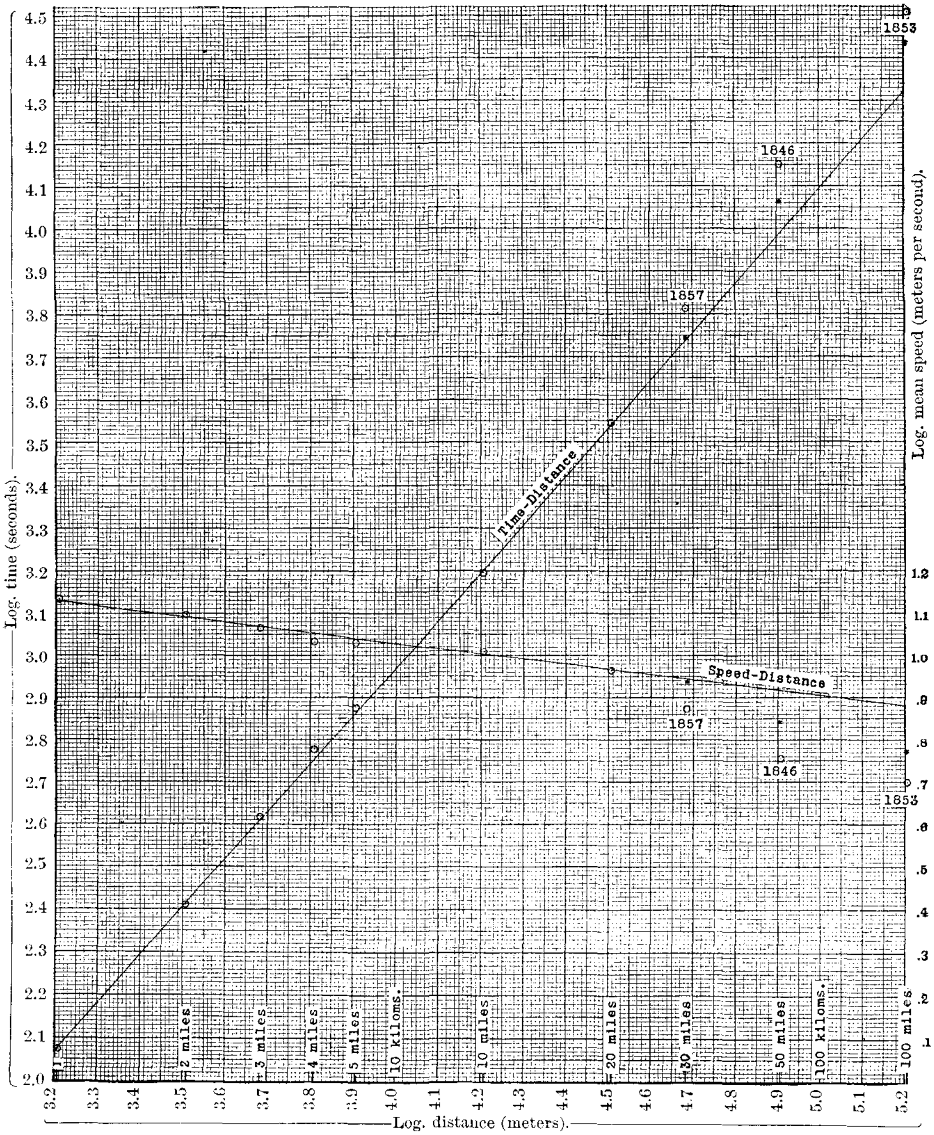


Figure 4. *World's Trotting Records to 100 Miles.*

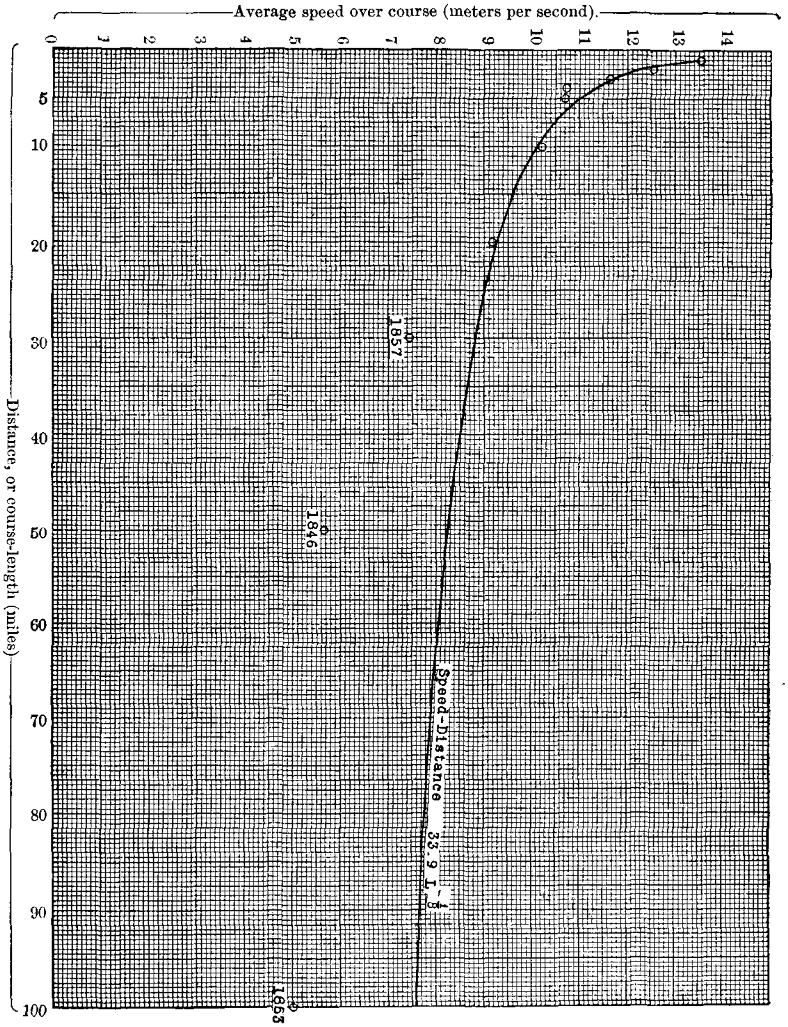


FIGURE 5. *Running Horses, Speeds and Record Times.*

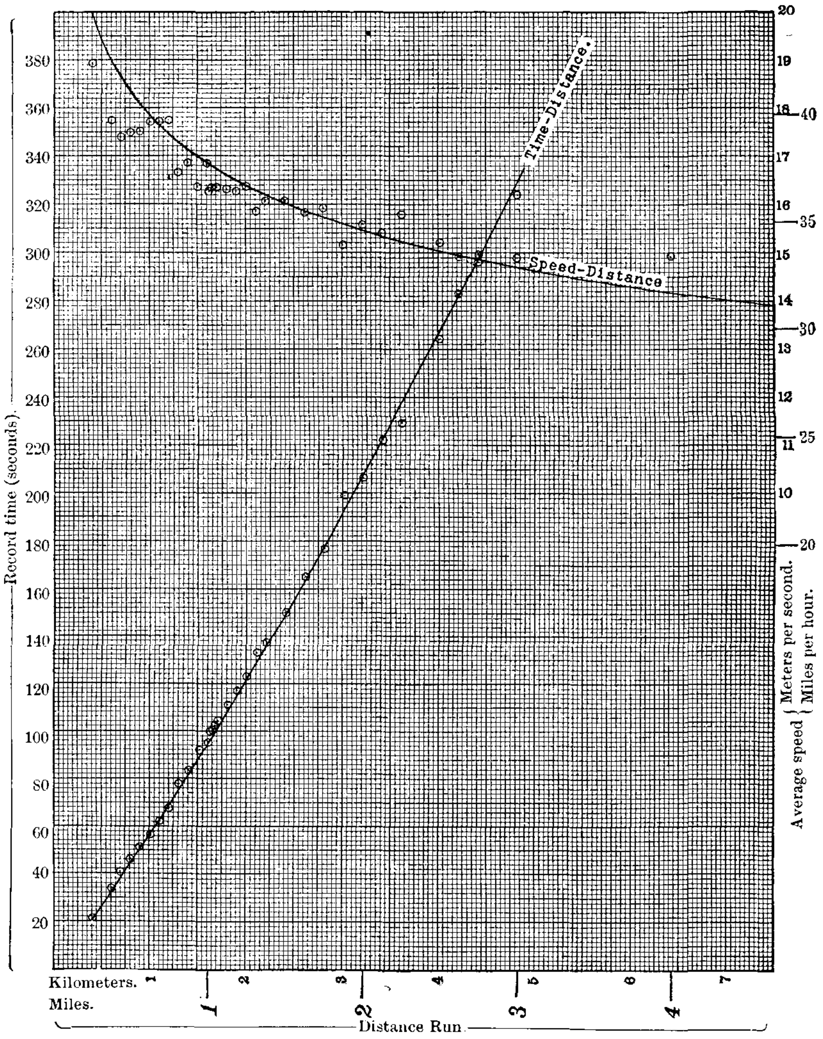
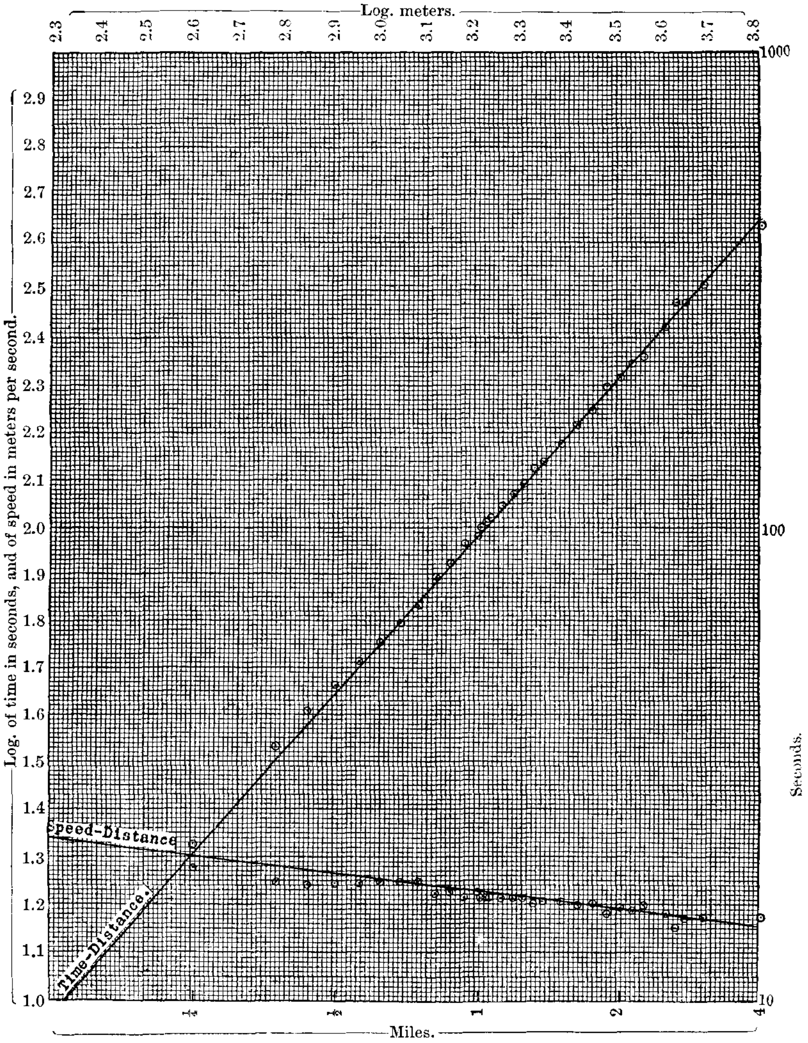


FIGURE 6. *Speeds and Record Times of Running Horses (Logarithmic Co-ordinates).*



Referring to Figure 6, it is to be observed that the record speeds are low, or the times high, with respect to the straight lines up to 1000 meters (0.62 mile). It is possible that this discrepancy on the short courses may be due to the inertia of the horses; that is to say, the horses may be supposed to lose time, or to miss attainment of speed, over the short runs below a kilometer, owing to the effort required to start their bodies into motion from rest. This time lost in acceleration is ignored in the straight-line logarithmic law, which assumes that the animal starts with full speed. For courses of over 1 kilometer, the discrepancy disappears.

Moreover, horse-races are stated to be rarely run over distances less than 5 furlongs (1006 meters); so that the question of the discrepancy over short courses is of but little practical importance.

The ascending straight line in Figure 6, drawn to meet the points as fairly as may be, is carried through the 1-mile record, and makes an angle with the axis of distance of 48° 22′, whose tangent is $\frac{9}{8}$. The falling line is also carried through the 1-mile record and makes an angle with the same axis of — 7° 7′ 30″, whose tangent is — ^{1}.

The straight lines of Figure 5 correspond to, or determine, the following equations:

$$\log T = \tfrac{9}{8} \log L - 1.6274 \qquad (4)$$

and

$$\log V = 1.6274 - \tfrac{1}{8} \log L. \qquad (5)$$

These are also respectively equivalent to:

$$T = \frac{L^{\frac{9}{8}}}{42.4} \quad \text{seconds} \qquad (6)$$

and

$$V = \frac{42.4}{L^{\frac{1}{8}}} \quad \text{meters per second.} \qquad (7)$$

By comparing equations (7) and (3) it appears from them that the record speeds of trotting horses is less than the record speeds of running horses over a given distance in the ratio of 33.9 to 42.4, or by 20 per cent of the latter.

On courses longer than 1 kilometer, the straight lines of Figure 6 fit the observations of Table II with satisfactory precision, some observations lying on one side and others on the other. The greatest discrepancies are at 2¼ miles and at 4 miles. Figure 5 shows in its upper line that both of these performances were remarkable. The speed over the 2¼ mile course appears from the data to have been greater than the speed in the 2-mile event. Again, the speed in the 4-mile race was actually higher than the speed in the 3-mile race.

Column IX of Table II gives the time that should correspond to each event, according to formulas (4) or (6). Column X gives the deviation from the record time. Column XI expresses the deviations in percentage of the record time. The mean percentage deviation between observed and computed record times between the limits of 1 kilometer (5 furlongs) and 6.44 kilometers (4 miles) is shown in column XII to be 1.9 per cent. Consequently, we may expect by following the ascending straight line of Figure 6 to predict the record time of any race between these limits to within 2 per cent on the average.

The mean percentage deviation between observed and computed record times for the entire series of events, i. e. between $\frac{1}{4}$ mile and 4 miles (0.4 to 6.44 kilometers), is 2.4 per cent.

Horses Pacing.

The record data for pacing horse-races in harness on American turf appear in columns I, II, and III of Table III. They are taken from page 260 of "The World Almanac" for 1906. The entries in the succeeding columns, IV to VIII, are then found in the manner previously described. In Figure 7 the logarithm of the time, or log T in column VII, is plotted against the logarithm of the distance, or log L in column VI. The logarithm of the mean speed in each event, or log V in column VIII, is also plotted. Taking the ascending time-distance line, it is a straight line ruled through the 2-mile record point, so as fairly to conform with the other record points. It makes an angle with the distance axis of 48° 22′ or $\tan^{-1} \frac{9}{8}$. Referring to the descending speed-distance line, it is a straight line drawn through the 2-mile record point, so as fairly to conform with the other record points. It makes an angle with the distance axis of $-7° \; 7' \; 30''$ or $\tan^{-1} - \frac{1}{8}$. The two straight lines include between them an angle of approximately 55° 30′ 30″.

The straight lines of Figure 7 correspond respectively to the following equations:

$$\log T = \tfrac{9}{8} \log L - 1.5363 \qquad (8)$$

$$\log V = 1.5363 - \tfrac{1}{8} \log L. \qquad (9)$$

These in turn correspond respectively to the following:

$$T = \frac{L^{\frac{9}{8}}}{34.38} \quad \text{seconds.} \qquad (10)$$

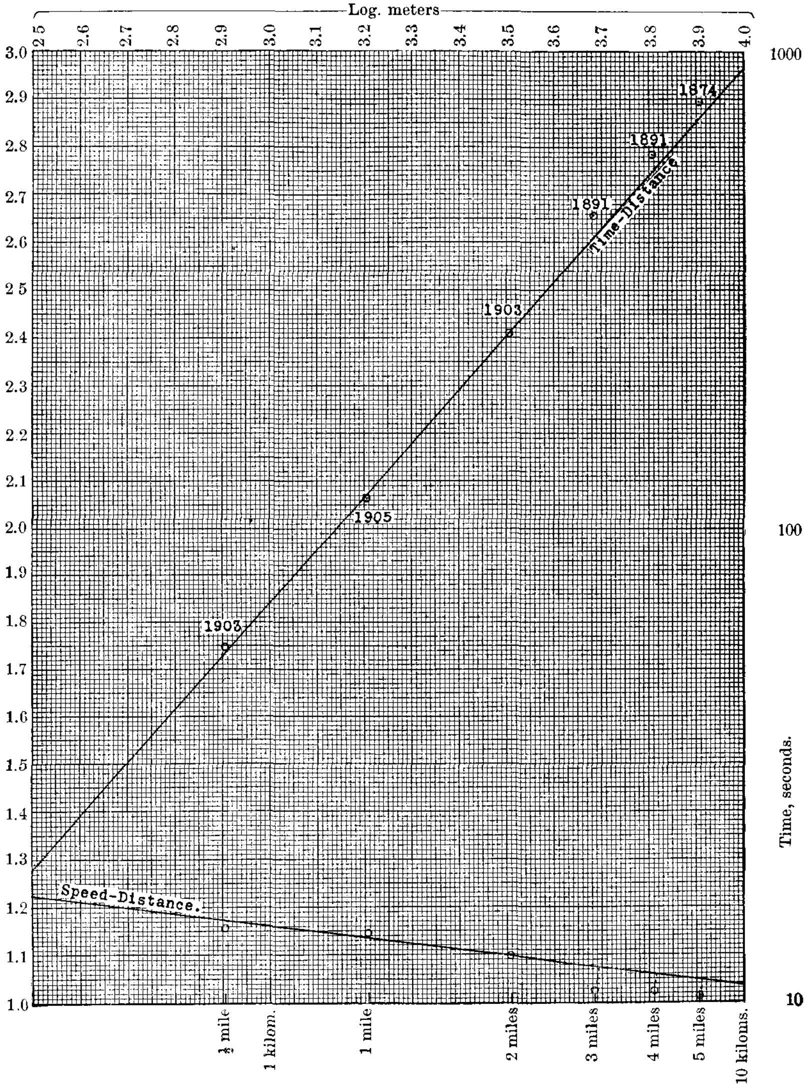
$$V = \frac{34.38}{L^{\frac{1}{8}}} \quad \text{meters per second.} \qquad (11)$$

TABLE III.

Analysis of Pacing Horse Records in Harness on American Turf.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	
Date.	Distance, miles.	Time T, seconds	Distance L, meters.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.	log L.	log T.	log V.	log T' computed.	T' computed, seconds	$T'-T$, deviation.	$100\left(\frac{T'-T}{T}\right)$ per cent deviation.	Means.
1903	$\frac{1}{2}$	56.0	804.7	14.37	2.9056	1.7482	1.1574	1.7325	54.0	−2.0	3.5	
1905	1	115.25	1609.3	13.97	3.2066	2.0616	1.1450	2.0711	117.8	2.55	2.2	1.9
1903	2	257.0	3218.7	12.53	3.5077	2.4099	1.0978	2.4099	257.0	0.0	0.0	
1891	3	453.25	4828.0	10.65	3.6838	2.6563	1.0275	2.6080	405.5	−47.75	10.5	5.4
1891	4	610.0	6437.4	10.55	3.8087	2.7853	1.0234	2.7485	560.4	−49.6	8.1	
1874	5	783.5	8046.7	10.27	3.9056	2.8940	1.0116	2.8575	720.3	−63.2	8.1	

FIGURE 7. *Pacing Records Plotted to Logarithmic Co-ordinates.*



An inspection of column X, Table III, containing the computed record times according to formulas (8) and (10), shows that the ½-mile record was 2 seconds longer than the computed time, the mile 2½ seconds shorter, and the 3-mile, 4-mile, and 5-mile events considerably longer. These deviations appear in column XI, and are given in percentages of the respective actual records in column XII. The ½-mile deviation of 3½ per cent may be explained by inertia on short courses. The deviations of 10½, 8, and 8 per cent on the long distances are not explainable in such a manner. The 3-mile and 4-mile records date from 1891, and the 5-mile record from 1874. Reference to the records of 1892, as given on page 273 of "The World Almanac" for 1894, shows that at that time the pacing records were low by comparison with those at the present date. Thus the mile record (1.6 kilometers) in pacing was 124 seconds, and the 2-mile record (3.2 kilometers) was 287.75 seconds, which are in deviation from the modern records of those events, according to Table III, by 7.6 and 11.9 per cent respectively. Consequently, it seems fair to say that these records of 3, 4, and 5 miles, which are 8 to 10.5 per cent off the straight lines in Figure 7, would have been close to similar lines drawn for records in 1894. In other words, the 1-mile and 2-mile records have steadily improved since 1891, whereas the 3-mile, 4-mile, and 5-mile records remain as they were at that date.

If we eliminate the 3-mile, 4-mile, and 5-mile records from consideration, the average discrepancy between the computed and observed record times is seen in column XII to be 1.9 per cent. If, however, we have to consider all of the events, the average discrepancy is 5.4 per cent.

Summing up the analysis of horse-racing as presented in Tables I, II, and III with their accompanying curve-sheets, it is submitted that the logarithmic straight lines meet the observations, within reasonable limits of range, to 2 per cent of average discrepancy. Moreover, these corresponding straight logarithmic lines are parallel to each other for running, pacing, or trotting. This means that there is a similar law of fatigue in each of these styles of progression.

MEN RUNNING.

The first two columns of Table IV give the world's records of running races. The first four events, up to 45 yards inclusive, are taken from the Amateur Athletic Union Records, as given on page 262 of "The World Almanac" for 1906. The remaining events, up to 623 miles inclusive, are taken from page 242 of "The World Almanac" for 1904. For each event the best record has been selected, whether the holder

TABLE IV. — MEN RUNNING.

Analysis of Best Professional and Amateur World's Records.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
Distance, yards.	Time T, seconds.	Distance L, meters.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.	log L.	log T.	log V.	T' computed.	$T'-T$ deviation, seconds.	$100\left(\frac{T'-T}{T}\right)$ deviation per cent.
20	A 2.8	19.5	5.188	1.2622	0.4472	0.7150	1.55	−1.25	−44.5
35	A 4.0	32.0	8.000	.5052	.6021	.9031	2.90	−1.1	−27.5
40	A 4.4	36.6	8.312	.5632	.6435	.9197	3.37	−1.03	−23.5
45	A 5.2	41.2	7.914	.6144	.7160	.8984	3.85	−1.35	−26.0
50	5.25	45.7	8.706	.6600	.7202	.9398	4.33	−0.92	−17.5
60	A 6.4	54.9	8.572	.7393	.8062	.9331	5.32	−1.08	−16.9
70	A 7.2	64.0	8.890	.8062	.8573	.9489	6.33	−0.87	−12.1
75	7.25	68.6	9.458	.8362	.8604	.9758	6.84	−0.41	−5.7
80	A 8.0	73.2	9.145	.8643	.9031	.9612	7.36	−0.64	−8.0
100	9.6	91.4	9.524	.9611	.9823	.9788	9.5	−0.1	−1.0
110	11.0	100.6	9.144	2.0025	1.0414	.9611	10.5	−0.5	−4.5
120	A 11.4	109.7	9.625	.0403	.0569	.9834	11.6	0.2	1.8
125	A 12.4	114.3	9.207	.0580	.0934	.9646	12.2	−0.2	−1.6
130	12.125	118.9	9.804	.0751	.0837	.9914	12.7	0.6	4.8
131.5	12.4	120.2	9.696	.0800	.0934	.9866	12.9	0.5	4.0
135	13.2	123.4	9.352	.0915	.1206	.9709	13.3	0.1	0.8
140	13.5	128.0	9.482	.1072	.1303	.9769	13.8	0.3	2.2
150	14.5	137.2	9.458	.1372	.1614	.9758	14.9	0.4	2.8
180	A 18.0	164.6	9.144	.2164	.2553	.9611	18.3	0.3	1.7
200	19.5	182.9	9.380	.2622	.2900	.9722	20.6	1.1	5.7
220	A 21.2	201.2	9.490	.3036	.3263	.9773	23.0	1.8	8.5
250	A 24.6	228.6	9.294	.3591	.3909	.9682	26.5	1.9	7.7
300	30.0	274.3	9.144	.4382	.4771	.9611	32.5	2.5	8.3
350	A 36.4	320.0	8.792	.5052	.5611	.9441	38.7	2.3	6.3
400	A 42.2	365.8	8.668	.5632	.6253	.9379	45.0	2.8	6.6
440	A 47.0	402.3	8.561	.6046	.6721	.9325	50.1	3.1	6.6
500	A 57.8	457.2	7.910	.6601	.7619	.8982	57.8	0	0
600	A 71.0	548.6	7.727	.7393	.8513	.8880	71.0	0	0
660	A 82.0	603.5	7.361	.7807	.9138	.8669	79.0	−3.0	−3.7
700	89.0	640.1	7.203	.8069	.9494	.8575	86.5	−2.5	−2.8
800	A 104.4	731.4	7.007	.8642	2.0187	.8455	98.0	−6.4	−6.1
880	113.5	804.7	7.089	.9056	.0550	.8506	109.2	−4.3	−3.8
1000	A 133.0	914.4	6.874	.9611	.1239	.8372	126.0	−7.0	−5.3
1320	A 182.8	1207.0	6.602	3.0817	.2620	.8197	172.2	−10.6	−5.8
miles									
1	252.75	1609.3	6.367	.2066	.4027	.8039	238.1	−14.6	−5.8
1.25	330.0	2011.6	6.095	.3035	.5185	.7850	299.0	−31.0	−9.4
1.5	403.5	2414.0	5.997	.3837	.6058	.7779	376.7	−26.8	−6.6
1.75	A 488.2	2816.0	5.768	.4496	.6886	.7610	446.8	−41.4	−8.5
2	551.5	3218.7	5.837	.5077	.7415	.7662	519.4	−32.1	−5.8
2.5	A 726.0	4023.0	5.542	.6046	.8609	.7437	667.6	−58.4	−8.0
3	859.5	4828.0	5.618	.6838	.9342	.7496	819.6	−39.9	−4.6
3.5	1004.2	5633.0	5.607	.7507	3.0020	.7487	974.6	−29.6	−3.0
4	1165.6	6437.0	5.525	.8088	.0665	.7423	1133.0	−32.6	−2.8
4.5	A 1345.0	7242.0	5.385	.8599	.1287	.7312	1293.0	−52.0	−3.9
5	1480.0	8047.0	5.436	.9056	.1703	.7353	1456.0	−24.0	−1.6
5.5	A 1662.6	8851.0	5.150	.9470	.2208	.7262	1621.0	−41.6	−2.5
6	1790.0	9656.0	5.394	.9848	.2529	.7319	1787.0	−3.0	−0.2
6.5	A 1976.4	10,462.0	5.293	4.0196	.2959	.7237	1956.0	−20.4	−1.0
7	2085.0	11,265.0	5.404	.0518	.3191	.7327	2126.0	41.0	2.0

TABLE IV. — *Continued.*

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
Distance, miles.	Time T, seconds.	Distance L, meters.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.	log L.	log T.	log V.	T' computed.	$T'-T$ deviation, seconds.	$100\left(\frac{T'-T}{T}\right)$ deviation per cent.
7.5	A 2298.0	12,070	5.252	4.0817	3.3614	0.7203	2297	–1	0
8	2420.0	12,875	5.320	.1097	.3838	.7259	2470	50	2.1
8.5	A 2613.0	13,680	5.236	.1361	.4171	.7190	2645	32	1.2
9	2721.0	14,484	5.322	.1608	.4347	.7261	2820	99	3.7
9.5	A 2931.0	15,289	5.217	.1844	.4670	.7174	2998	67	2.3
10	3066.6	16,094	5.248	.2066	.4866	.7200	3175	108.4	3.5
10.5	3229.0	16,900	5.214	.2279	.5097	.7182	3355	126	3.9
11	3388.0	17,703	5.225	.2480	.5299	.7181	3534	146	4.3
11.5	3543.0	18,508	5.224	.2674	.5494	.7180	3716	173	4.9
12	3722.5	19,312	5.187	.2858	.5709	.7149	3898	175.5	4.7
13	4231.0	20,922	4.946	.3206	.6264	.6942	4266	35	0.8
14	4572.0	22,531	4.928	.3528	.6601	.6927	4637	65	1.4
15	A 4804.6	24,140	5.023	.3827	.6817	.7010	5010	205.4	4.3
16	5286.0	25,750	4.872	.4108	.7231	.6877	5389	103	1.9
17	5655.0	27,360	4.838	.4371	.7524	.6847	5768	113	2.0
18	A 6010.0	28,970	4.819	.4619	.7789	.6830	6147	137	2.3
19	A 6360.0	30,580	4.807	.4854	.8035	.6819	6537	177	2.8
20	A 6714.0	32,190	4.794	.5077	.8270	.6807	6926	212	3.1
21	A 7570.0	33,790	4.464	.5288	.8791	.6497	7315	–255	–3.4
22	A 7968.0	35,410	4.444	.5491	.9013	.6478	7709	–259	–3.2
23	A 8390 0	37,010	4.411	.5683	.9238	.6445	7918	–472	–5.6
24	A 8825.0	38,620	4.377	.5869	.9457	.6412	8504	–321	–3.6
25	A 9224.0	40,240	4.363	.6047	.9649	.6398	8904	–320	–3.5
30	11,709.0	48,280	4.124	.6838	4.0685	.6153	10,930	–779	–6.6
40	16,647.0	64,370	3.868	.8088	.2214	.5874	15,110	–1537	–8.6
50	21,304.5	80,470	3.777	.9056	.3285	.5771	19,410	–1895	–8.9
60	27,033.0	96,560	3.572	.9848	.4319	.5529	23,830	–3203	–11.9
70	32,595.0	112,650	3.456	5.0518	.5132	.5386	28,350	–4245	–13.0
80	38,030.0	128,750	3.386	.1098	.5801	.5297	32,950	–5080	–13.4
90	43,215.0	144,850	3.352	.1609	.6356	.5253	37,610	–5605	–13.0
100	48,390.0	160,933	3.325	.2066	.6848	.5218	43,320	–5070	–10 5
110	55,245.0	177,030	3.204	.2480	.7423	.5057	47,130	–8115	–14.7
120	60,490.0	193,120	3.192	.2858	.7817	.5041	51,980	–8510	–14.1
130	68,685.0	209,220	3.046	.3206	.8369	.4837	56,890	–11,795	–17.2
140	75,030.0	225,310	3.003	.3528	.8752	.4776	61,830	–13,200	–17.6
150	80,905.0	241,400	2.984	.3828	.9080	.4748	68,380	–12,525	–16.7
200	126,568.0	321,900	2.543	.5077	5.1024	.4053	92,370	–34,200	–27.0
300	209,826.0	482,800	2.302	.6838	.3218	.3620	145,700	–64,126	–25.8
383	288,625.0	616,400	2.135	.7899	.4606	.3293	192,300	–96,325	–33.4
450	343,578.0	724,210	2.108	.8599	.5361	.3238	230,000	–113,578	–33.0
500	393,509.0	804,700	2.005	.9056	.5949	.3107	258,900	–134,600	–34.1
560	451,485.0	901,200	1.996	.9548	.6547	.3001	294,100	–157,385	–34.8
623	510,030.0	1,003,000	1.966	6.0013	.7078	.2935	331,700	–179,300	–34.5
. .	7.0	60	8.57	1.7782	0.8451	.9331			
. .	10.8	100	9.25	2.0000	1.0334	.9666			
. .	21.6	200	9.25	2.3010	1.3345	.9665			
. .	49.2	400	8.13	2.6021	1.6920	.9101			
. .	116.0	800	6.90	2.9031	2.0645	.8386			
. .	245.8	1500	6.10	3.1761	2.3906	.7855			
5	1571.6	8047	5.12	3.9056	3.1962	.7094			
. .	10,288.0	40,000	3.90	4.6021	4.0123	.5898			

was amateur or professional. Amateur records are indicated in column II by the letter A. The last eight events are taken from records of the Olympian Games, as published in the Boston "Transcript" for May 12, 1906. These international contests have been held in 1896, 1900, 1904, and 1906. The best of these four records has been taken for each event.

In Figure 8, log T (see column VI, Table IV) is plotted as ordinates against log L (column V) as abscissas. The circles mark the world's records and the crosses the Olympian records. In order to keep the chart within reasonably small dimensions, retaining a fairly extended scale, the observations are made to cross the chart twice, by employing dual scales of ordinates and abscissas. The straight line is drawn through the 500-yard record to meet the remaining observations. It runs off the sheet the first time at log $T = 2.71$ and log $L = 3.5$. It then recommences at the left hand of the upper line and finally leaves the sheet at log $T = 4.9$ and log $L = 5.45$. These parts of what would be a single straight line on a larger sheet make an angle of 48°22′ or $\tan^{-1}\frac{9}{8}$ with the axis of abscissas. The record points conform closely to this line, swinging slightly from one side of it to the other. They lie above it as far as log $L = 2.0$. They lie beneath from log $L = 2.0$ to log $L = 2.66$, and again from log $L = 4.0$ to 4.5. In the remaining parts they lie above. In other words, the straight line hits the record path at log $L = 2.1$, 2.7, 3.75, and 4.5.

Table IV gives in column VIII the values of the record times corresponding to the straight line in Figure 8. Column IX gives the deviation from the world's record for each event and column X the percentage deviation. The percentage deviation commences at −44.5 for the shortest run (19.5 meters). It dwindles to 0 at about 110 meters. It reaches a maximum of 8.5 near 200 meters, returns to 0 at 500 meters, swings over to −8.5 at 2800 meters, returns to 0 near 10 kilometers, swings to 4.9 at 18 kilometers, crosses the zero point near 30 kilometers, and then steadily increases numerically and in the negative direction until it is −34.8 at 900 kilometers.

The large discrepancies below 100 yards may be attributed to inertia. This is indicated in Figure 9, where the speeds of the events are plotted up to 201 meters. It is evident from the dotted line following the records that runners attain their maximum apparent speed in the neighborhood of 110 meters (120 yards). At shorter distances the retarding effect of inertia prevents a higher average speed in the run from being developed. At longer distances, the starting retardation is reduced in effect, but fatigue acts in its place. The simple logarithmic law, represented by the heavy line, takes no account of inertia and assumes a maximum speed on an indefinitely short course. It is probably not worth while going to the complication of correcting the formula by a term or terms introducing a retardation at the start, since races below 100 yards are rare, and the deviation between the actual and computed speeds disappears beyond 100 yards.

At the other end of the line in Figure 8 the deviations become large after log $L = 5$ or beyond 100 kilometers. Beyond 150 miles the deviations exceed 20 per cent. It is possible that the discrepancy may here be accounted for by reason of the fact that somewhere in this neighborhood the runner stops at intervals to take food, or rest, and there is no longer a continuous performance enacted. According to the logarithmic straight line, the speeds on courses between 450 and 623 miles (724 and 1003 kilometers) are only two-thirds of what should be expected.

Summing up the entries in column X, without regard to sign, it is found that if all the events are included from 19.5 meters to 1003 kilometers, the average deviation is 8.9 per cent. If, however, the summation be limited to the range from 100 meters to 100 kilometers, the average deviation is 4.3 per cent. This appears to be a satisfactory result, considering that the range of distance is 1000 to 1, and the range of record times 2500 to 1.

From a practical standpoint it may be inferred from an inspection of either Table IV or Figure 8, that it should be easier for trained athletes to beat the world's records between 600 meters and 9 kilometers, or between 30 and 1000 kilometers, than to beat the records between 100 meters and 600 meters, or between 10 kilometers and 30 kilometers. Expressed in another way, we should expect the degree of physical exhaustion in record runs over the short courses up to 600 meters to be more severe than on the courses from that distance up to 9 kilometers. The existing one-mile or three-mile record does not seem to be so severe as the records from 100 yards to 500 yards. Whatever mathematical conclusions may be drawn from the data, the belief seems unavoidable that a study of Figure 8 will be useful to athletes training for running with a view to breaking records.

The crosses in Figure 8, representing Olympian records, while useful as a check, do not serve the results in establishing the straight lines, because these Olympian records are inferior to the corresponding world's records, excepting that for the 100-meter race.

The line of Figure 8 corresponds to the following formulas :

$$\log T = \tfrac{9}{8} \log L - 1.2307 \qquad (12)$$

FIGURE 8. *World's Running Records.*

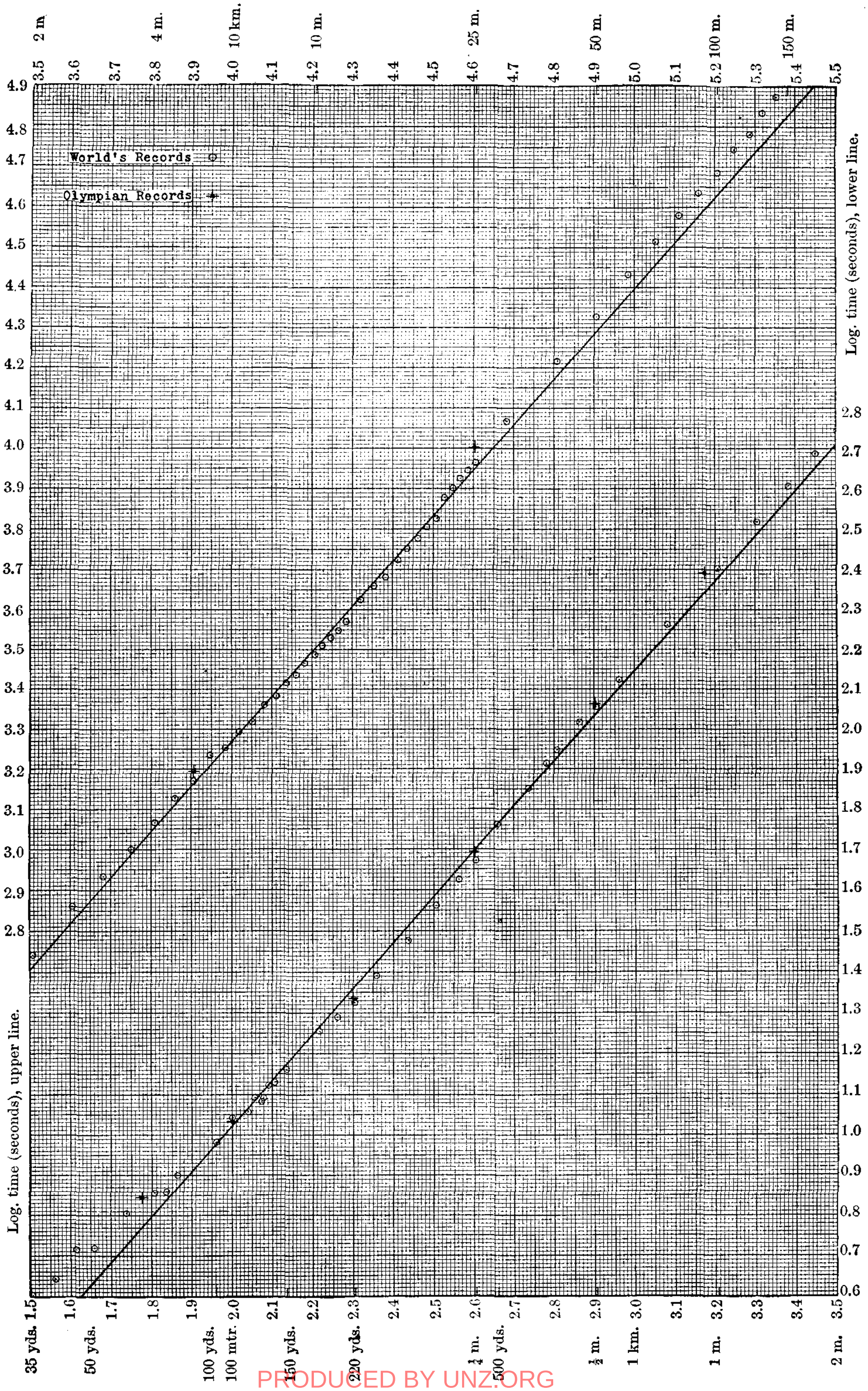
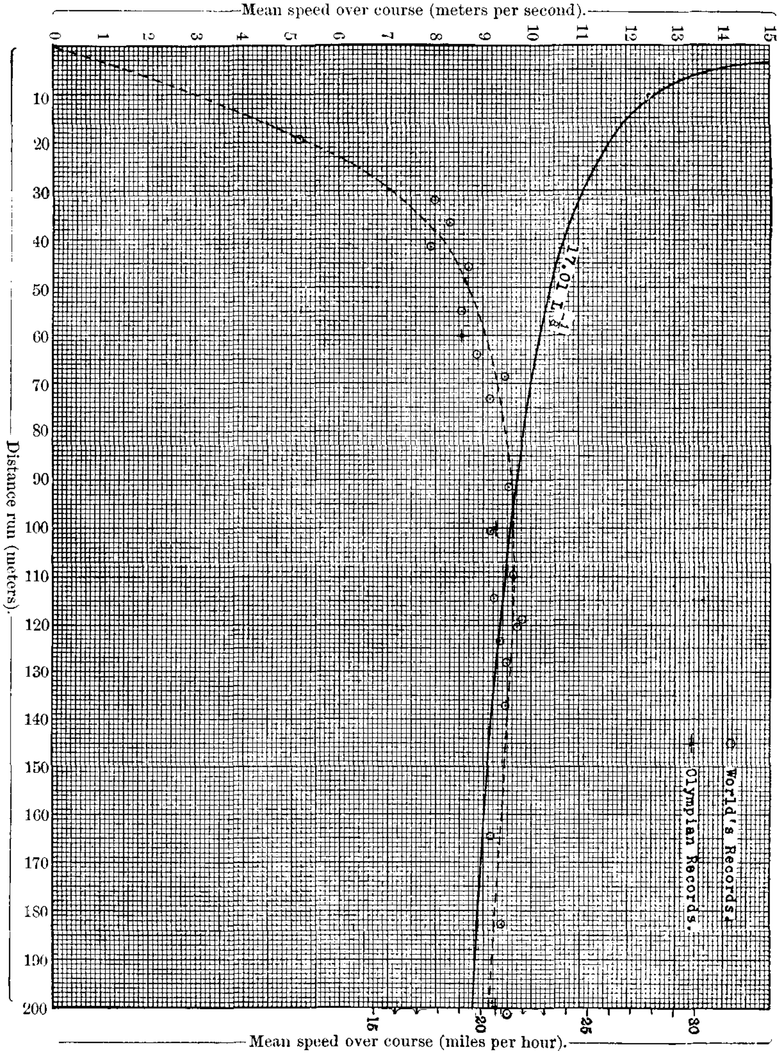


FIGURE 9. *Mean Running Speeds over Short Courses.*



NOTE. — Figure 8, on the opposite page, expresses the relation between time and distance to logarithmic co-ordinates. The scale of abscissas at the top of the figure gives the logarithm of distance in meters for the upper line; that at the bottom, for the lower line.

or
$$T = \frac{L^{\frac{9}{8}}}{17.01} \quad \text{seconds.} \tag{13}$$

$$\log V = 1.2307 - \tfrac{1}{8} \log L \tag{14}$$

or
$$V = \frac{17.01}{L^{\frac{1}{8}}} \quad \text{meters per second.} \tag{15}$$

Men Walking.

The data for walking races are given in columns I and III of Table V. They are taken from page 244 of "The World Almanac" for 1904, and are world's records, amateur and professional. The best records in walking, unlike other athletic sports, seem to have been made entirely by professional walkers.

Figure 10 indicates the points found by plotting log T from column V against log L from column II. In order to economize space, the series of points is carried twice across the sheet to two sets of scales. The straight line, seen in two segments, is drawn through the 4-mile record point, and is drawn to meet the other points fairly. It makes an angle of 48° 22′ or $\tan^{-1} \frac{9}{8}$, with the axis of distances. The entries in column VIII correspond to points on the straight line. The computed times T' of each event are thus obtained and set down in column IX. The last column gives the percentages of deviation between the actual and computed record. The observations commence above the line, or the record times for the 1-mile and 2-mile events are long, or the speeds low, by comparison with the 4-mile and the 50-mile records. This can hardly be accounted for by inertia, as in horse-racing; because at the low speed of walking, the retardation, due to starting from rest, must disappear in less than 100 meters.

The points fall below the line between 4 miles and 30 miles, representing faster speeds than the computed to the extent of nearly 6 per cent. Between 40 and 70 miles the agreement between the observed and computed times is close. Beyond 70 miles the speed falls below that computed by the logarithmic line. The deviations do not exceed 5 per cent until beyond 120 miles.

The mean percentage of deviation, without regard to sign, is 5.6 per cent over the entire series of events, and 3.4 per cent between the limits of 1 mile and 120 miles.

There seems to have been scarcely any improvement in walking records during the ten years preceding the date at which these have been selected (1904).

FIGURE 10. *World's Walking Records.*

Time-distance to logarithmic co-ordinates.

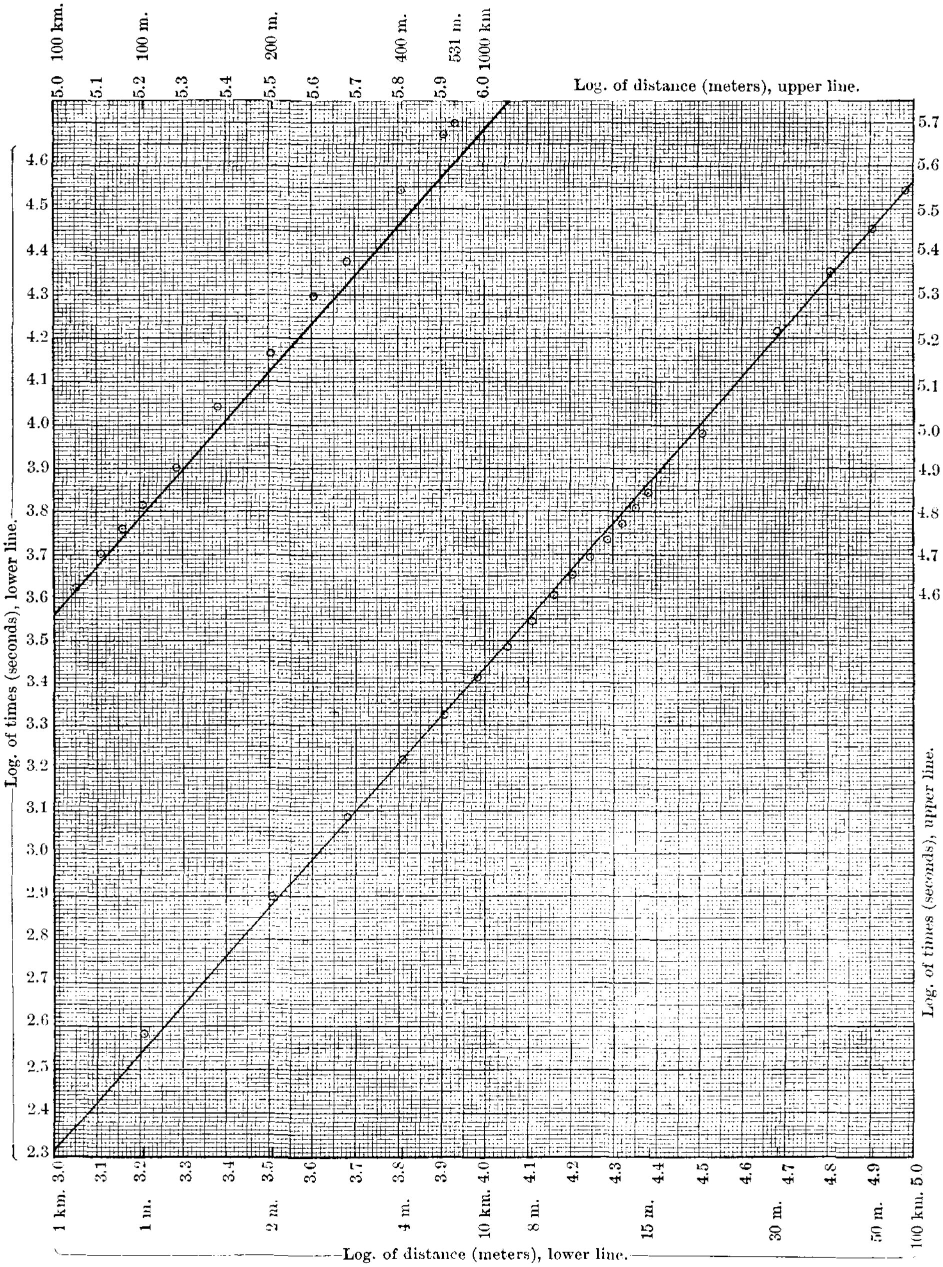


TABLE V.

MEN WALKING.

Analysis of World's Best Walking Records, Amateur or Professional.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
Distance, miles.	Distance, L, meters.	Time, T, seconds.	log L.	log T.	log V.	V, $\frac{\text{meters}}{\text{seconds}}$.	log T', computed.	T', computed.	$T'-T$ deviation.	$100\left(\frac{T'-T}{T}\right)$ percentage deviation.
1	1,609.4	383.0	3.2066	2.5832	0.6234	4.201	2.5428	349.0	–34.0	–8.9
2	3,218.7	794.0	.5077	2.8998	.6079	.054	.8816	761.4	–32.6	–4.1
3	4,828.0	1,221.5	.6838	3.0869	.5969	3.953	3.0797	1,201.0	–20.5	–1.7
4	6,437.4	1,658.0	.8087	.2196	.5891	.882	.2202	1,660.0	2.0	0.0
5	8,046.7	2,110.0	.9056	.3243	.5813	.813	.3292	2,134.0	24.0	1.1
6	9,656.1	2,581.0	.9848	.4118	.5730	.741	.4183	2,620.0	39.0	1.2
7	11,265.0	3,064.0	4.0517	.4863	.5654	.676	.4936	3,116.0	52.0	1.7
8	12,875.0	3,517.0	.1097	.5462	.5635	.660	.5588	3,621.0	104.0	3.0
9	14,484.0	4,034.0	.1608	.6057	.5551	.590	.6163	4,133.0	99.0	2.5
10	16,094.0	4,485.0	.2066	.6518	.5548	.588	.6678	4,654.0	169.0	3.8
11	17,703.0	4,958.0	.2480	.6953	.5527	.570	.7144	5,181.0	223.0	4.5
12	19,312.0	5,434.0	.2858	.7351	.5507	.554	.7569	5,713.0	279.0	5.1

13	20,922.0	5,926.5	.3206	.7727	.5479	.531	.7961	6,253.0	326.5	5.5
14	22,531.0	6,431.5	.3528	.8083	.5445	.504	.8323	6,797.0	365.5	5.7
15	24,140.0	6,956.0	.3827	.8424	.5403	.470	.8659	7,343.0	387.0	5.6
20	32,187.0	9,597.0	.5077	.9821	.5256	.354	4.0066	10,150.0	553.0	5.8
30	48,280.0	16,494.0	.6838	4.2173	.4665	2.928	.2047	16,110.0	–384.0	–2.3
40	64,374.0	22,610.0	.8087	.3543	.4544	.847	.3452	22,140.0	–470.0	–2.1
50	80,467.0	28,456.0	.9056	.4542	.4514	.828	.4542	28,460.0	4.0	0.0
60	96,561.0	34,847.0	.9848	.5422	.4426	.771	.5433	34,940.0	93.0	0.3
70	112,650.0	41,915.0	5.0517	.6224	.4293	.687	.6186	41,550.0	–365.0	–0.9
80	128,750.0	50,513.0	.1097	.7034	.4063	.549	.6838	48,280.0	–2,233.0	–4.4
90	144,840.0	57,550.0	.1608	.7600	.4008	.517	.7413	55,120.0	–2,430.0	–4.2
100	160,940.0	65,330.0	.2066	.8151	.3915	.463	.7928	62,060.0	–3,270.0	–5.0
120	193,120.0	79,585.0	.2858	.9009	.3849	.426	.8819	76,190.0	–3,395.0	–4.3
150	241,400.0	110,188.0	.3827	5.0421	.3406	.191	.9909	97,930.0	–12,258.0	–11.1
200	321,870.0	146,790.0	.5077	.1667	.3410	.193	5.1316	135,400.0	–11,390.0	–8.3
250	402,340.0	198,744.0	.6046	.2983	.3063	.024	.2406	174,000.0	–24,744.0	–12.5
300	482,800.0	239,400.0	.6838	.3791	.3047	.017	.3297	213,500.0	–25,900.0	–10.8
400	643,740.0	348,663.0	.8087	.5424	.2663	1.846	.4702	295,300.0	–53,363.0	–15.3
500	804,670.0	470,025.0	.9056	.6721	.2335	.712	.5792	379,500.0	–90,525.0	–19.3
531	854,559.0	499,710.0	.9317	.6987	.2328	.709	.6086	406,100.0	–93,610.0	–17.8

The straight line of Figure 10 corresponds to the following equations:

$$\log T = \tfrac{9}{8} \log L - 1.0646 \qquad (16)$$

and

$$\log V = 1.0646 - \tfrac{1}{8} \log L \qquad (17)$$

or

$$T = \frac{L^{\frac{9}{8}}}{11.6} \text{ seconds} \qquad (18)$$

and

$$V = \frac{11.6}{L^{\frac{1}{8}}} \text{ meters per second.} \qquad (19)$$

Men Rowing.

Table VI is compiled from the data contained in the article on "Rowing" appearing on page 208, Vol. X, of the "Universal Encyclopedia" published in 1900. The entries in columns I, II, III, and V of table VI are taken directly from that article. The remaining columns give the deductions therefrom, as in preceding cases. The logarithms of the times are plotted against the logarithms of the distances in Figure 11. The 8-oar line is drawn through the 4-mile record point. The 1-mile record is the only point seriously off the line, in the direction of low speed. The 1-mile speed is seen in column VI to be lower than the 4-mile speed. The 4-oar line is also drawn through the 4-mile record point. The 3-mile event is the only one seriously off this line, in the direction of high speed. The speed over this course was 5.15 meters per second, which is only about 7 per cent short of the record speed for the 1-mile event. The single-pair-of-sculls line is drawn through the 5-mile record point. The ¼-mile and the 1-mile events are the only ones seriously off this line. It is possible that the quarter-mile is affected by starting inertia. The mean deviation for the entire series is 6 per cent. If we discard the 1-mile 8-oar event and the ¼-mile singles event, the mean deviation of the remaining series is 4.3 per cent. This seems to be a good showing for the logarithmic straight line considering the extent to which both wind and tide are capable of influencing rowing speeds.

All three straight lines in Figure 11 are drawn to make an angle of $48^\circ\ 22'$ with the axis of distances.

The formulas deducible from the three straight lines of Figure 11 are presented in Table VII.

TABLE VI

Men Rowing.

Analysis of Records of American Oarsmen.

I. Type.	II. Date.	III. Distance, miles.	IV. Distance L, meters.	V. Time T, seconds.	VI. Speed V, $\frac{\text{meters}}{\text{seconds}}$.	VII. log L.	VIII. log T.	IX. log V.	X. log T' computed.	XI. T' computed, seconds.	XII. $T'-T$ deviation, seconds.	XIII. $100\left(\frac{T'-T}{T}\right)$ per cent deviation.
8-oars	1883	1	1609.3	304.75	5.281	3.2066	2.4839	0.7227	2.4054	254.3	−50.5	−16.5
	1889	1.5	2414	400	6.035	.3827	.6021	.7806	.6035	401.3	1.3	0.3
	1891	2	3218.7	583.5	5.516	.5077	.7660	.7417	.7442	554.9	−28.6	−4.9
	1891	3	4828	867.5	5.564	.6838	.9383	.7455	.9423	875.6	8.1	0.9
	1888	4	6437.4	1210	5.320	.8087	3.0828	.7259	3.0828	1210	0	0
4-oars	1883	1	1609.3	291	5.530	3.2066	2.4639	0.7427	2.4929	311.1	20.1	6.9
	1887	2	3218.7	747	4.309	.5077	.8733	.6344	.8317	678.8	−68.2	−9.1
	1875	3	4828	937.25	5.151	.6838	.9719	.7119	3.0298	1071	133.8	14.6
	1871	4	6437.4	1480	4.349	.8087	3.1703	.6384	.1703	1480	0	0
	1860	5	8046.7	1844.75	4.362	.9056	.2659	.6397	.2793	1902	57.3	3.1
	1871	6	9656.1	2360.6	4.090	.9848	.3730	.6118	.3684	2390	29.4	1.2
single-sculls	1891	¼	402.3	57	7.058	2.6046	1.7559	0.8487	1.8352	68.4	11.4	20.0
	1872	1	1609.3	301	5.347	3.2066	2.4786	.7280	2.5224	333	32.0	9.6
	1868	2	3218.7	680	4.734	.5077	.8325	.6752	.8612	726.4	46.4	6.8
	1890	3	4828	1171	4.123	.6838	3.0686	.6152	3.0593	1146	−25.0	−2.1
	1883	4	6437.4	1677.5	3.837	.8087	.2247	.5840	.200	1585	−92.5	−5.5
	1879	5	8046.7	2036.25	3.951	.9056	.3088	.5968	.3088	2036	0	0

FIGURE 11. *American Rowing Records.*

Time-distance to logarithmic co-ordinates.

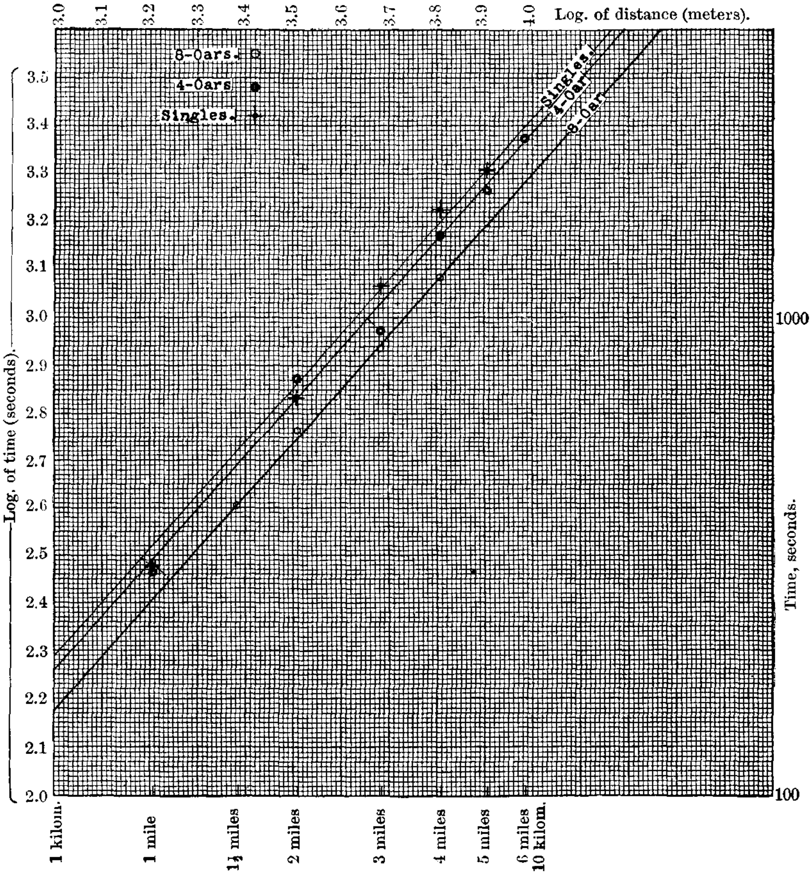


TABLE VII.

QUANTITATIVE RESULTS OF ANALYSIS OF ROWING RECORDS.

8-oars.	4-oars.	Singles.
$\log T = \frac{9}{8} \log L - 1.202$	$\log T = \frac{9}{8} \log L - 1.1145$	$\log T = \frac{9}{8} \log L - 1.085$
$\log V = 1.202 - \frac{1}{8} \log L$	$\log V = 1.1145 - \frac{1}{8} \log L$	$\log V = 1.085 - \frac{1}{8} \log L$
$T = \frac{L^{\frac{9}{8}}}{15.92}$ seconds	$T = \frac{L^{\frac{9}{8}}}{13.02}$ seconds	$T = \frac{L^{\frac{9}{8}}}{12.16}$ seconds
$V = \frac{15.92}{L^{\frac{1}{8}}} \frac{\text{meters}}{\text{second}}$	$V = \frac{13.02}{L^{\frac{1}{8}}} \frac{\text{meters}}{\text{second}}$	$V = \frac{12.16}{L^{\frac{1}{8}}} \frac{\text{meters}}{\text{second}}$

It is seen that for any given distance over 1 mile, the speeds of 8*s*, 4*s*, and singles are in the ratio 15.92 : 13.02 : 12.16, or 100 : 81.8 : 76.4, to the degree of approximation supported by the analysis.

MEN SWIMMING.

The data for swimming have been taken from the world's records given on page 252 of "The World Almanac" for 1905, setting forth 38 events from 25 yards to 4000 yards by amateurs and by professional swimmers of all nations. These data appear in columns I and II of Table VIII. The remaining columns work from these data as in preceding tables.

In Figure 12, log T' is plotted against log L, and also log V against log L. The ascending time-distance straight line is drawn through the 220-yard (201.2-meter) record point, and thence in a direction to conform fairly well with the other record points. It is drawn to make an angle of 48°.22′ or $\tan^{-1} \frac{9}{8}$ with the axis of distances. The descending straight line of speed-distance is likewise drawn through the 220-yard record point, and to make an angle of $-7° \, 7' \, 30''$ or $\tan^{-1} - \frac{1}{8}$ with the same axis.

Referring to the latter line, it will be observed that there is no visible retardation of speed over the short courses even down to 25 yards (22.9 meters). On the contrary, the speeds for the first four events, up to 60 yards (55 meters), inclusive, are higher than correspond to the logarithmic straight line. This may be accounted for by the fact that swimming speeds are only 15.5 per cent of men's running speeds at any given distance beyond 100 meters; so that the retardation at starting must be much less than in running. Moreover, it is possible that the

TABLE VIII.

MEN SWIMMING.

Analysis of Best Professional and Amateur Records.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
Distance, yards.	Time T, seconds.	Distance L, meters.	V, $\frac{\text{meters}}{\text{seconds}}$.	log T.	log L.	log V.	log T' computed.	T' computed seconds.	$T'-T$ deviation, seconds.	$100\left(\frac{T'-T}{T}\right)$, per cent deviation.
25	A 12.2	22.86	1.874	1.0864	1.3591	0.2727	1.1094	12.87	0.67	5.5
45	A 23.0	41.15	.789	.3617	.6144	.2527	.3966	24.93	1.93	8.4
50	A 24.6	45.72	.859	.3909	.6601	.2692	.4480	28.05	3.45	14.0
60	A 34.0	54.86	.614	.5315	.7393	.2078	.5371	34.44	0.44	1.3
80	P 51.0	73.15	.434	.7076	.8642	.1566	.6776	47.6	–3.4	–6.7
90	A 51.6	82.3	.595	.7126	.9154	.2028	.7352	54.35	2.75	5.3
100	A 58.0	91.44	.577	.7634	.9611	.1977	.7866	61.18	3.18	5.5
110	A 61.0	100.6	.649	.7853	2.0025	.2172	.8332	68.11	7.11	11.7
120	A 72.2	109.7	.520	.8585	.0403	.1818	.8751	75.01	2.81	3.9
150	A 96.6	137.2	.420	.9850	.1372	.1522	.9847	96.54	–0.06	–0.1
200	A 140.0	182.9	.306	2.1461	.2622	.1161	2.1254	133.5	–6.5	–4.6
220	A 148.6	201.2	.354	.1720	.3036	.1316	.1720	148.6	0	0
250	P 177.8	228.6	.285	.2499	.3591	.1092	.2344	171.5	–6.3	–3.5
300	A 214.6	274.3	.278	.3316	.4382	.1066	.3121	205.2	–9.4	–4.4
330	O 252.3	301.8	.196	.4019	.4797	.0778	.3700	234.5	–17.8	–7.1

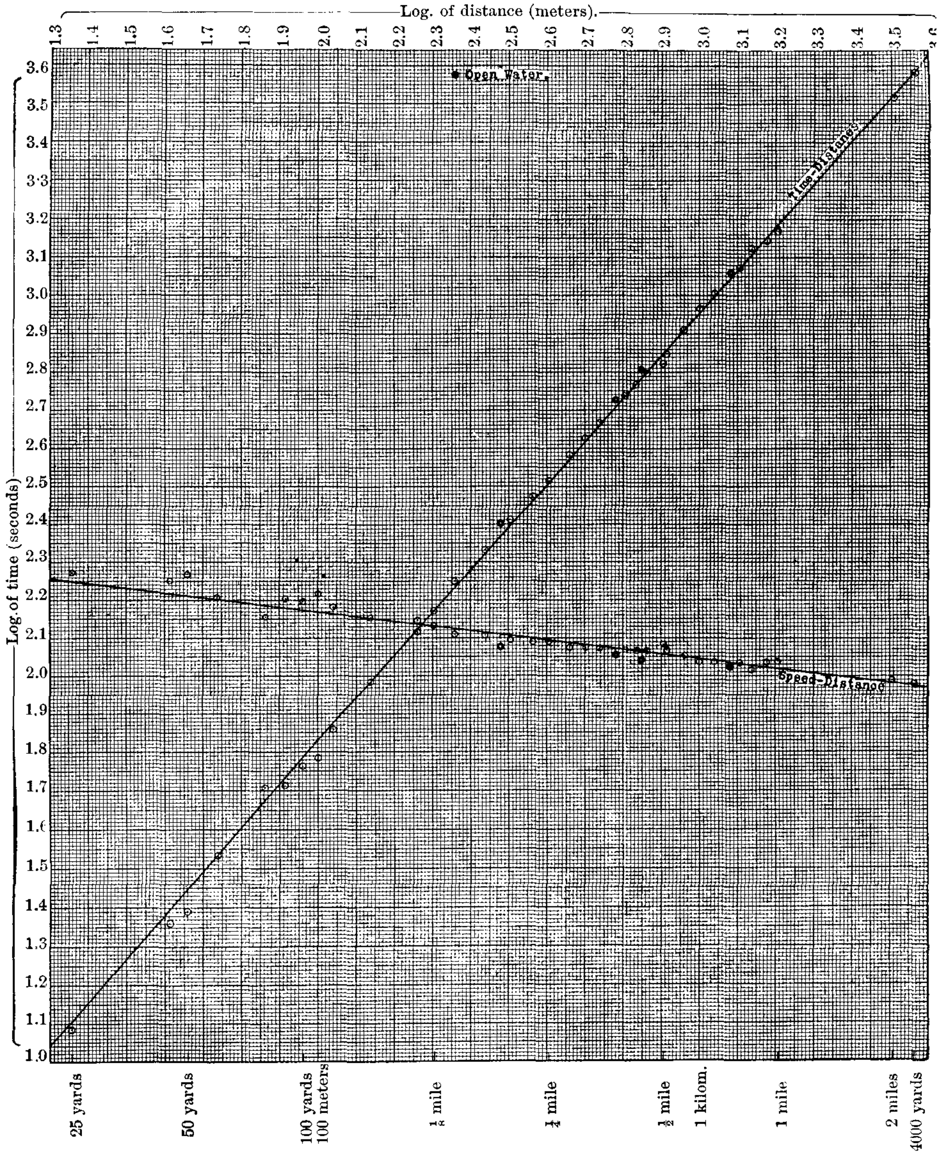
350	P 258.0	320.0	.240	.4116	.5052	.0936	.3988	250.5	–7.5	–2.9
400	A 297.0	365.7	.231	.4728	.5631	.0903	.4639	291.0	–6.0	–2.0
440	A 326.2	402.3	.233	.5135	.6046	.0911	.5106	324.0	–2.2	–0.7
500	A 383.6	457.2	.192	.5838	.6601	.0763	.5730	374.1	–9.5	–2.5
550	A 423.4	502.9	.188	.6268	.7015	.0747	.6196	416.5	–6.9	–1.6
600	A 465.0	548.6	.180	.6675	.7393	.0718	.6621	459.3	–5.7	–1.2
660	O 533.0	603.5	.132	.7267	.7807	.0540	.7087	511.3	–21.7	–4.1
700	A 547.4	640.1	.169	.7383	.8062	.0679	.7374	546.3	–1.1	–0.2
750	A 587.4	685.8	.167	.7689	.8362	.0673	.7711	590.4	3.0	0.5
770	O 642.0	704.1	.097	.8075	.8476	.0401	.7840	608.1	–33.9	–5.3
800	A 628.0	731.5	.165	.7980	.8642	.0662	.8026	634.8	6.8	1.1
880	P 664.5	804.6	.211	.8225	.9056	.0831	.8492	706.6	42.1	6.3
900	P 706.0	823.0	.166	.8488	.9154	.0666	.8602	724.8	18.8	2.7
1000	A 812.8	914.4	.125	.9100	.9611	.0511	.9116	815.8	3.0	0.4
1100	A 925.4	1005.8	.087	.9663	3.0025	.0362	.9582	908.2	–17.2	–1.9
1200	A 1011.0	1097.2	.085	3.0048	.0403	.0355	3.0007	1002.0	–9.0	–0.9
1320	O 1148.5	1207.0	.051	.0601	.0817	.0216	.0473	1115.0	–33.5	–2.9
1400	A 1183.0	1280.2	.082	.0729	.1073	.0344	.0761	1192.0	9.0	0.5
1500	A 1324.6	1371.6	.035	.1221	.1372	.0151	.1098	1288.0	–36.6	–2.8
1650	A 1390.4	1508.7	.085	.1431	.1786	.0355	.1563	1433.0	42.6	3.1
1760	A 1476.2	1609.3	.090	.1691	.2066	.0375	.1887	1544.0	67.8	4.6
3520	A 3294.0	3218.7	0.9772	.5177	.5076	1.9899	.5265	3362.0	68.0	2.1
4000	A 3824.0	3657.0	0.9564	.5825	.5631	.9806	.5889	3881.0	57.0	1.5

A denotes an amateur record. P, a professional record. O, a professional record in open water.

Except as above indicated, the records are stated to have been made in baths or bathing houses, as distinguished from open water.

FIGURE 12. *Swimming Records.*

Time-distance and speed-distance to logarithmic co-ordinates.



starting plunge made by the swimmer may actually advance him, relatively speaking, on the shortest courses.

With the exception of the four open-water events, on all of which the speed is distinctly low, and the 80-yard event, which has an unduly low speed (about the same as in the 150-yard event), the observations cling closely to the straight line. The speeds at both the half-mile and the 1-mile events appear to be distinctly higher than the rest.

Table VIII indicates that the mean deviation, without regard to sign, of all the events is 3.5 per cent.

The formulas pertaining to the straight lines in Figure 12 are

$$\log T = \tfrac{9}{8} \log L - 0.4196 \tag{20}$$

$$\log V = 0.4196 - \tfrac{1}{8} \log L \tag{21}$$

$$T = \frac{L^{\frac{9}{8}}}{2.628} \qquad \text{seconds} \tag{22}$$

$$V = \frac{2.628}{L^{\frac{1}{8}}} \qquad \text{meters per second.} \tag{23}$$

Men Skating.

Table IX gives the analysis of 24 yard and mile events and also 5 metric events, between the limits of 50 yards and 100 miles. The data appearing in columns I and II of this Table are taken from the records of the Amateur Athletic Union revised in 1905 and published on page 265 of "The World Almanac" for 1906. They represent, therefore, amateur records.

In Figure 13, log T has been plotted against log L, crossing the sheet twice for economy in space. The straight line drawn in two sections is carried through the 1-mile record point. It also runs very near to the 2-mile, 4-mile, 500-meter, and 5000-meter points. There is a wide deviation of the points from the line between 50 yards (45.72 meters) and 440 yards (402.3 meters). This may be attributed to the influence of inertia in acceleration at the start. Column IV of Table IX shows in fact that the maximum speed over a course is not reached until the 440-yard event (402.3 meters), when it attains 11.43 meters per second (25.5 miles per hour). The entries in column IV also reveal considerable relative variation, and do not descend with the same degree of uniformity as is manifested by the corresponding speed entries in Table VIII, or other tables. Perhaps this variability in the

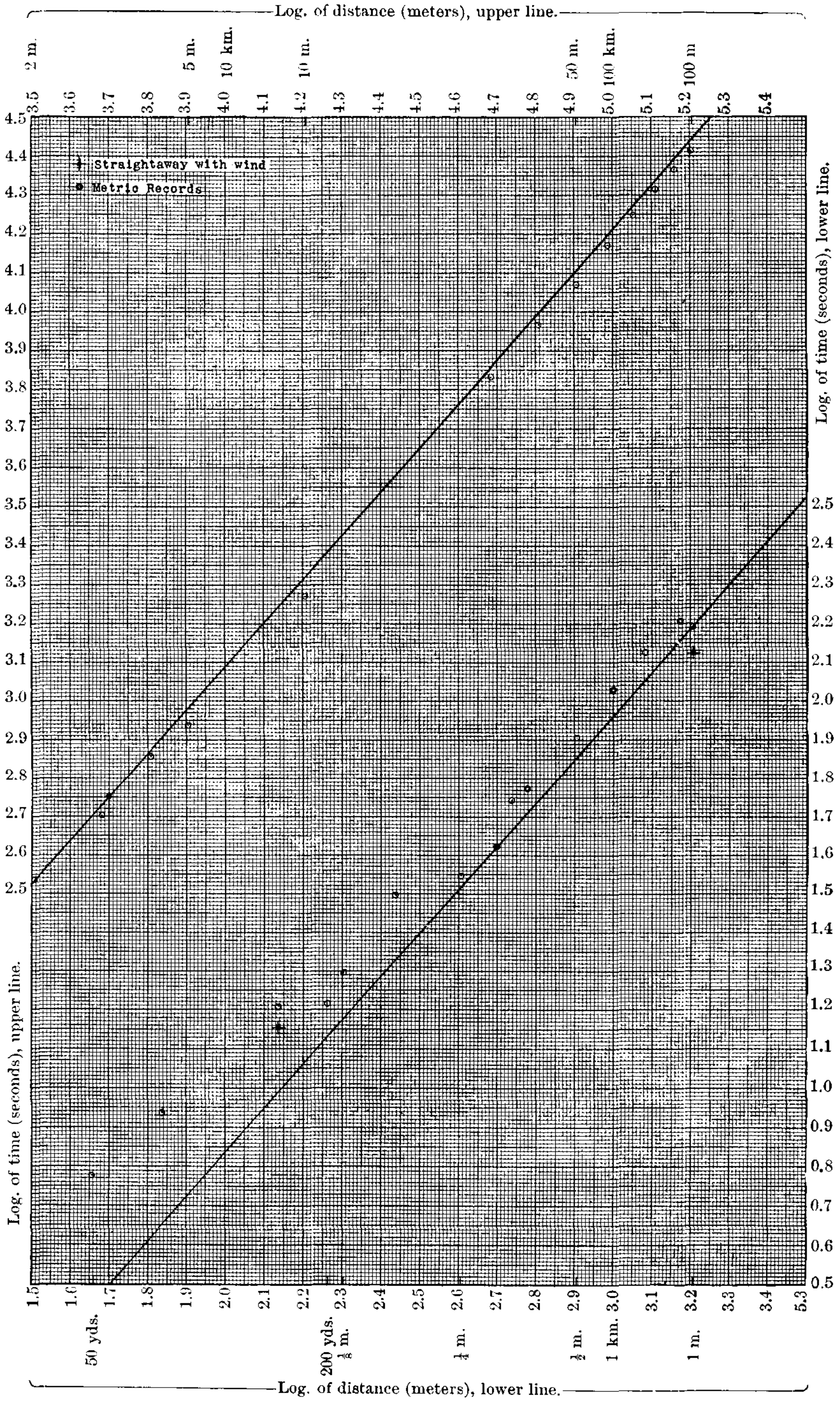
TABLE IX. — MEN SKATING.

Analysis of Amateur Skating Records.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
Distance, yards.	Time T, seconds.	Distance L, meters.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.	log L.	log T' computed.	log T.	log V.	T' seconds, computed.	$T'-T$ deviation, seconds.	$100\left(\frac{T'-T}{T}\right)$ deviation per cent.	Mean percentage deviation.
50	6.0	45.72	7.62	1.6601	0.4533	0.7782	0.8819	2.84	−3.16	−52.0	7.6
75	8.6	68.58	7.974	.8362	.6514	.9345	.9017	4.48	−4.12	−48.0	
150	15.88	137.16	8.635	2.1372	.9901	1.2009	.9363	9.78	−6.1	−38.3	
200	16.4	182.88	11.150	2622	1.1307	.2148	1.0474	13.51	−2.9	−17.8	
220	19.8	201.2	10.160	.3036	.1773	.2967	.0069	15.04	−4.8	−24.0	
300	31.4	274.3	8.736	.4382	.3287	.4969	0.9413	21.32	−10.1	−32.1	
440	35.2	402.3	11.43	.6046	.5159	.5465	1.0581	32.8	−2.4	−6.8	
600	55.25	548.6	9.929	.7393	.6674	.7423	0.9970	46.5	−8.8	−15.8	
880	80.4	804.6	10.01	.9056	.8545	.9053	1.0003	71.5	−8.9	−11.1	
1,320	133.0	1,207.0	9.075	3.0817	2.0526	2.1239	0.9578	112.9	−20.1	−15.1	
miles											
1	156.0	1,609.3	10.32	.2066	.1931	.1931	1.0135	156.0	0	0	
2	342.6	3,218.7	9.395	.5077	.5319	.5348	0.9729	340.3	−2.3	−0.7	
3	503.0	4,828.0	9.598	.6838	.7300	.7016	.9822	537.1	34.1	6.8	
4	720.5	6,437.4	8.934	.8087	.8705	.8576	.9511	742.2	21.7	3.0	
5	864.0	8,046.7	9.314	.9056	.9795	.9365	.9691	954.0	90.0	10.4	
10	1,871.2	16,093.0	8.600	4.2066	3.3181	3.2721	.9345	2,080.0	208.8	11.1	
30	6,800.0	48,280.0	7.100	.6838	.8550	.8325	.8513	7,161.0	361.0	5.3	
40	9,286.0	64,374.0	6.932	.8087	.9955	.9678	.8409	9,897.0	611.0	6.6	
50	11,759.4	80,467.0	6.842	.9056	4.1045	4.0704	.8352	12,720.0	960.0	8.2	
60	14,820.6	96,561.0	6.515	.9848	.1936	.1709	.8139	15,620.0	800.0	5.4	
70	17,715 6	112,654.0	6.358	5.0517	.2689	.2483	.8034	18,570 0	854.0	4.8	
80	20,515.0	128,748.0	6.275	.1097	.3341	.3121	.7976	21,580.0	1,065.0	5.2	
90	23,157.6	144,841.0	6.254	.1608	.3916	.3647	.7961	24,640.0	1,482.0	6.4	
100	25,898.0	160,935.0	6.213	.2066	.4531	.4133	.7933	28,390.0	2,492.0	9.6	
meters											
500	41.8	500.0	11.95	2.6990	1.6221	1.6218	1.0772	41.9	0.1	0.2	
600	59.6	600.0	10.07	.7782	.7112	.7752	1.0030	51.4	−8.2	−13.8	
1,000	107.0	1,000.0	9.346	3.0000	.9607	2.0294	0.9706	91.4	−15.6	−14.6	
1,500	160.8	1,500.0	9.328	.1761	2.1588	.2063	0.9698	144.1	−16.7	−10.4	
5,000	565.4	5,000.0	8.843	.6990	.7471	.7524	0.9466	558.6	−6.8	−1.2	

FIGURE 13. *Amateur Skating Records.*

Time-distance to logarithmic co-ordinates.



records is due in some measure to the influence of wind, which is particularly active on skating speeds.

Column XII of Table IX indicates that the mean percentage deviation of the series of computed times against record times is 7.6 per cent between 500 meters and 100 miles (160.9 kilometers). If the same test is applied to the entire series between 50 yards and 100 miles, the mean percentage of deviation, without regard to sign, is 13.3 per cent.

The following formulas pertain to the straight line of Figure 13 :

$$\log T = \tfrac{9}{8} \log L - 1.4143 \qquad (24)$$

$$\log V = 1.4143 - \tfrac{1}{8} \log L \qquad (25)$$

or

$$T = \frac{L^{\frac{9}{8}}}{25.96} \quad \text{seconds} \qquad (26)$$

$$V = \frac{25.96}{L^{\frac{1}{8}}} \quad \text{meters per second.} \qquad (27)$$

Men Bicycling.

The data for bicycling have been taken from pages 267 and 268 of "The World Almanac" for 1906, and are embodied in columns I and II of Table X. The first five events are professional paced records against time. The next series of events, from 2 miles to 100 miles, inclusive, are professional motor-paced records, in competition. The remainder are stated to be American competition, professional, paced, hour records. The average speeds over the distances are set down in column V. It will be seen that the speed between 2 miles (3.22 kilometers) and 30 miles (48.28 kilometers) was almost precisely uniform at 23.5 meters per second. In fact, it appears from the table in "The World Almanac" that all of these records were made by one and the same individual, on one and the same day (May 31, 1904), at Charles River Park, Mass.. Again, from 31 miles (49.89 kilometers) to 50 miles (59.59 kilometers), the speeds are nearly uniform at 22.8 meters per second (51 miles per hour), and these appear to have been likewise made at Charles River Park by the same rider on the same day (September 1, 1903). There is very little fall of speed between the 50-mile event and the 2-mile event, or apparently but little fatigue as far as 50 miles. Beyond 50 miles, however, the speeds fall off and fatigue is indicated.

TABLE X.

MEN BICYCLING.

Analysis of Professional Bicycling Paced Records.

I.	II.	III.	IV.	V.
Distance, miles.	Time, h. m. s.	Distance L, meters.	Time T, seconds.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.
0.25	20.0	402.3	20.0	20.12
0.33	27.8	536.4	27.8	19.3
0.5	41.0	804.7	41.0	16.92
0.66	58.6	1,072.8	58.6	18.30
1	1 06.2	1,609.3	66.2	24.30
2	2 19.0	3,218.7	139.0	23.16
3	3 31.6	4,828.0	211.6	22.82
4	4 43.0	6,437.4	283.0	22.75
5	5 51.0	8,046.7	351.0	22.93
6	7 00.2	9,656.1	420.2	22.98
7	8 07.6	11,265.4	487.6	23.10
8	9 14.2	12,874.8	554.2	23.23
9	10 22.0	14,484.1	622.0	23.28
10	11 29.2	16,093.0	689.2	23.35
11	12 36.2	17,703.0	756.2	22.88
12	13 43.0	19,312.0	823.0	23.46
13	14 50.4	20,921.0	890.4	23.50
14	15 57.2	22,531.0	957.2	23.54
15	17 03.4	24,140.0	1,005.4	24.00
16	18 10.6	25,749.0	1,090.6	23.61
17	19 17.4	27,359.0	1,157.4	23.64
18	20 24.2	28,968.0	1,224.2	23.66
19	21 30.8	30,577.0	1,290.8	23.68
20	22 37 6	32,187.0	1,357.6	23.71
21	23 44.6	33,796.0	1,424.6	23.72
22	24 51.8	35,406.0	1,491.8	23.73
23	25 59.0	37,015.0	1,559.0	23.75
24	27 07.6	38,624.0	1,627.6	23.73
25	28 14.2	40,234.0	1,694.2	23.75
26	29 22.6	41,843.0	1,762.6	23.73
27	30 30.2	43,452.0	1,830.2	23.74
28	31 37.4	45,062.0	1,897.4	23.75
29	32 48.0	46,671.0	1,968.0	23.72
30	33 52.6	48,280.0	2,032.6	23.77
31	36 26.0	49,890.0	2,186.0	22.82
32	37 37.2	51,499.0	2,257.2	22.81
33	38 48.8	53,109.0	2,328.8	22.81
34	39 57.6	54,718.0	2,397.6	22.82
35	41 07.6	56,327.0	2,467.6	22.82
36	42 18.2	57,937.0	2,538.2	22.83
37	43 28.2	59,546.0	2,608.2	22.83
38	44 39.2	61,155.0	2,679.2	22.82
39	45 49.4	62,765.0	2,749.4	22.83
40	47 00.0	64,374.0	2,820.0	22.83

TABLE X. — *Continued.*

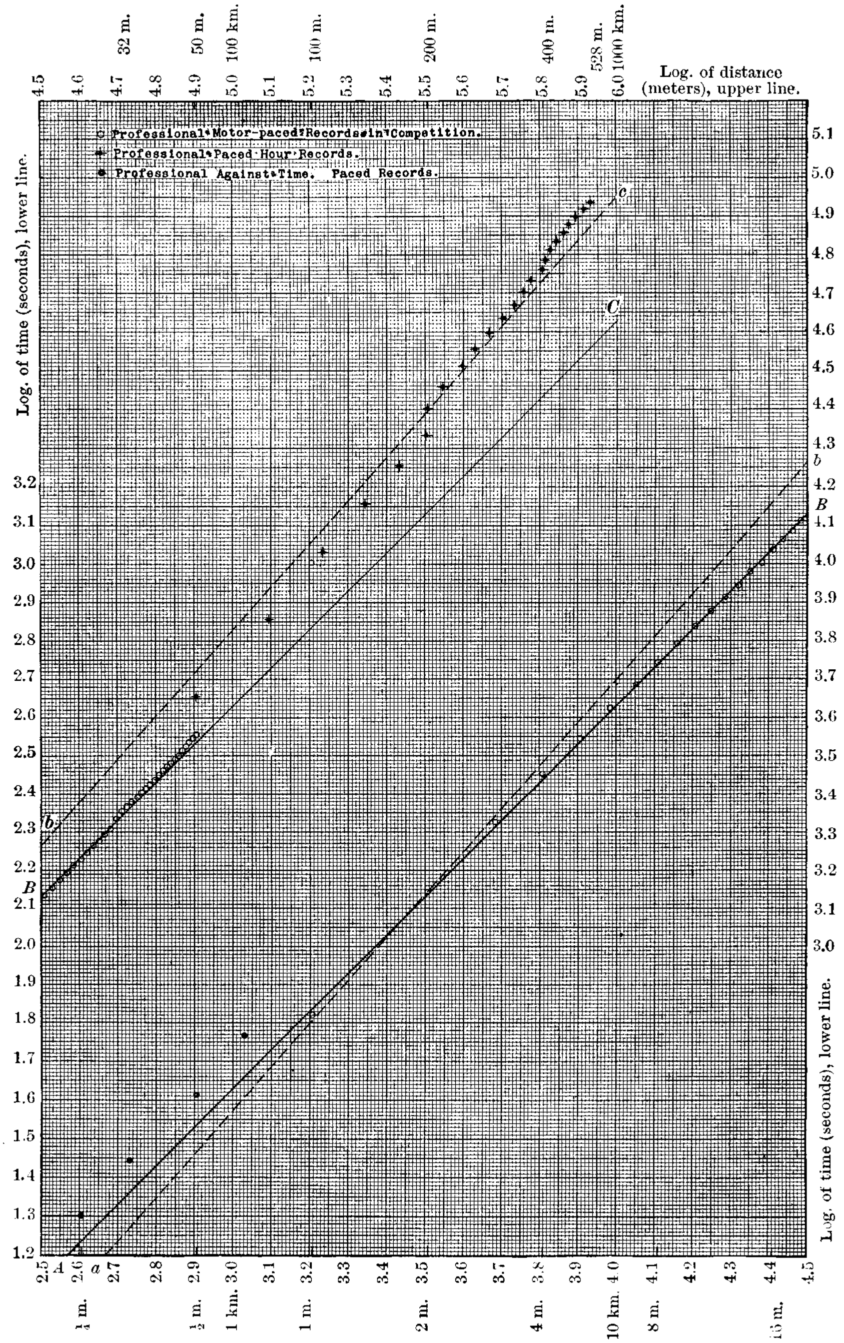
I.		II.			III.	IV.	V.
Distance, miles.		Time, m.	h.	s.	Distance L, meters.	Time T, seconds.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.
41			48	10.8	65,983.0	2,890.8	22.82
42			49	21.2	67,593.0	2,961.2	22.82
43			50	31.2	69,202.0	3,031.2	22.83
44			51	41.2	70,811.0	3,101.2	22.83
45			52	20.8	72,421.0	3,170.8	22.84
46			54	23.8	74,030.0	3,263.8	22.68
47			55	49.6	75,639.0	3,349.6	22.58
48			57	21.2	77,249.0	3,441.2	22.45
49			58	43.2	78,858.0	3,523.2	22.38
50			59	59	80,467.0	3,599.0	22.36
100		2	48	11.8	160,930.0	10,091.8	15.58
miles	yds.						
50	3	1			80,470.0	3,600.0	22.35
77	440	2			124,322.0	7,200.0	17.27
106	900	3			171,409.0	10,800.0	15.87
137	275	4			220,728.0	14,400.0	15.33
168	910	5			271,198.0	18,000.0	15.07
197	220	6			317,241.0	21,600.0	14.69
199	220	7			320,456.0	25,200.0	12.71
218	440	8			351,230.0	28,800.0	12.20
246	440	9			396,300.0	32,400.0	12.23
265		10			426,480.0	36,000.0	11.84
289		11			465,100.0	39,600.0	11.75
312	880	12			502,920.0	43,200.0	11.64
335	1540	13			540,535.0	46,800.0	11.55
355		14			571,314.0	50,400.0	11.33
372		15			598,670.0	54,000.0	11.09
397	220	16			639,100.0	57,600.0	11.09
403	440	17			649,000.0	61,200.0	10.60
416		18			669,500.0	64,800.0	10.33
432		19			695,200.0	68,400.0	10.16
450	1540	20			725,600.0	72,000.0	10.08
466	660	21			750,600.0	75,600.0	9.928
485	220	22			780,700.0	79,200.0	9.858
507	1320	23			817,100.0	82,800.0	9.870
528	925	24			850,600.0	86,400.0	9.846

Figure 14 shows log T plotted against log L, as in preceding cases. The observations fall near to a straight line, $A\ B$, $B\ C$, as far as 50 miles. This line makes an angle of 45° with the axis of distances. For the purposes of comparison a broken line ab, bc is drawn through the 2-mile record point at an angle of 48° 22′ to correspond with the lines in all of the other figures. The contrast is very noticeable. Beyond 100 miles (160.9 kilometers) the points tend to follow the direction of the broken line, but not with any degree of precision.

The inference to be drawn from Table X and Figure 14 is that

FIGURE 14. *Bicycling Records.*

Time-distance to logarithmic co-ordinates.



we have no means of discovering from these records what the highest speed of a bicycle rider may be. It is inconceivable that there should be no fatigue for 50 or 30 miles. If the record speed over 30 miles be 23.77 meters per second, the speed over 2 miles should be much greater. If the same law of fatigue held for bicycle riders as for runners, walkers, swimmers, and skaters, the speed at 2 miles should be approximately $\sqrt[8]{15}$ or 1.40 times greater; viz., 33.3 meters per second, or 74.4 miles per hour. There is no proof, however, that the same law of fatigue applies, and the air resistance at such high speeds might influence the results. It is, however, evident that the speed at 2 miles, or similar distances, is kept down abnormally to that at 30 miles. The explanation suggests itself that the records are all made on a circular track of considerable lateral inclination. The cyclist, on short runs, perhaps attains the highest speed that he dares and not the highest speed that his muscles could develop. When travelling at 23.75 meters per second (53 miles per hour), careful steering must be needed to keep on the track, and perhaps the records indicate the limit of steering nerve rather than the limits of speed and endurance below 30 or 50 miles. The case is somewhat similar to that of automobiles in this respect. The track records of heavy-weight gasolene automobiles, as given in "The World Almanac" for 1906, indicate speeds of 30 meters per second (67 miles per hour) at 1 mile, and hardly any reduction up to 10 miles, or no sensible fatigue within those limits. At 1000 miles (1609 kilometers) the speed is 20.35 meters per second (45.5 miles per hour). But on the straightaway courses, as distinguished from track courses, the speed averaged 46.77 meters per second (104.5 miles per hour) at 1 mile, and fell off distinctly with distance at a rate very similar to the fatigue rate of racing animals. Until, therefore, we have a wide straightaway course, say 5 kilometers long, provided for cyclists, of as good quality throughout as is presented in circular tracks, the cyclist's maximum speed will remain a matter of doubt.

Summary of Results.

A summary of the results of the various analyses in regard to accuracy is presented in Table XI. Column I refers to the table considered. Column II, the character of the race. The total number of records in each table appears in column III. The range of distances covered is given in column IV, both in miles and in kilometers. The sum total of the percentage deviations, without regard to their sign, as found for each table, including every record, is given in column V. The quotient of the sum in column V, by the number of records in

TABLE XI.

SUMMARY OF DEVIATIONS.

I.	II.	III.	IV.		V.	VI.	VII.	VIII.		IX.	X.
Table.	Racers.	Total number of Records.	Range of Distances.		Sum total of percentage Deviations without regard to Sign.	Average percentage Deviation over entire series of Records.	Number of properly acceptable Records.	Range of Distances.		Total percentage Deviations.	Average percentage Deviations.
			Miles.	Kilometers.				Miles.	Kilometers.		
I	Horses trotting	10	1–100	1.61–161	92.9	9.29	7	1.00–20	1.61–32.2	12.9	1.84
II	Horses running	33	$\frac{1}{4}$–4	0.40–6.4	80.3	2.43	28	0.63–4	1.00–6.4	53.1	1.89
III	Horses pacing	6	$\frac{1}{2}$–5	0.80–8.0	32.4	5.40	3	$\frac{1}{2}$–2	0.80–3.2	5.7	1.90
IV	Men running	92	$\frac{1}{80}$–623	0.02–1000	807.1	8.77	69	$\frac{1}{22}$–60	0.07–96.5	286.3	4.15
V	Men walking	32	1–531	1.60–855	178.8	5.59	25	1.00–120	1.60–193	83.7	3.35
VI	Men rowing	17	$\frac{1}{4}$–6	0.40–9.7	101.5	5.97	15	1.00–6	1.60–9.7	65.5	4.36
VIII	Men swimming	38	$\frac{1}{70}$–2.3	0.02–3.7	133.8	3.52	38	$\frac{1}{70}$–2.3	0.02–3.7	133.8	3.52
IX	Men skating	29	$\frac{1}{35}$–100	0.05–161	384.7	13.26	22	$\frac{1}{3}$–100	0.55–161	165.7	7.53
		257			1811.5	7.05	207			806.7	3.9

column III, gives the mean percentage deviation for each table in column VI, or the mean difference between the computed record time and the actual record time in percentage of the latter. Reasons have been given in connection with each table why certain records should be left out of consideration. The remainder are discussed in columns VII, VIII, IX, and X. These contain what may be called the net results, while columns III, IV, V, and VI contain the gross results.

In the net results, horses come out the best, and nearly equally well for trotting, running, or pacing, viz., 1.9 per cent mean deviation. The horses come out much better than the men in this comparison. It should be observed, however, that the ranges of distances covered by the horse-races are relatively small, — 20, 6.4, and 4, — whereas with men the ranges are successively 1320, 120, 6, 185, 300, — much greater. Perhaps if the ranges covered in the horses' performances had been similar to those covered in the men's performances, the disparity in precision would disappear. In the men's performances the net average deviation is about 4 per cent, except in skating, where it is 7.5 per cent. The net average deviation of all the 207 records is 3.9 per cent.

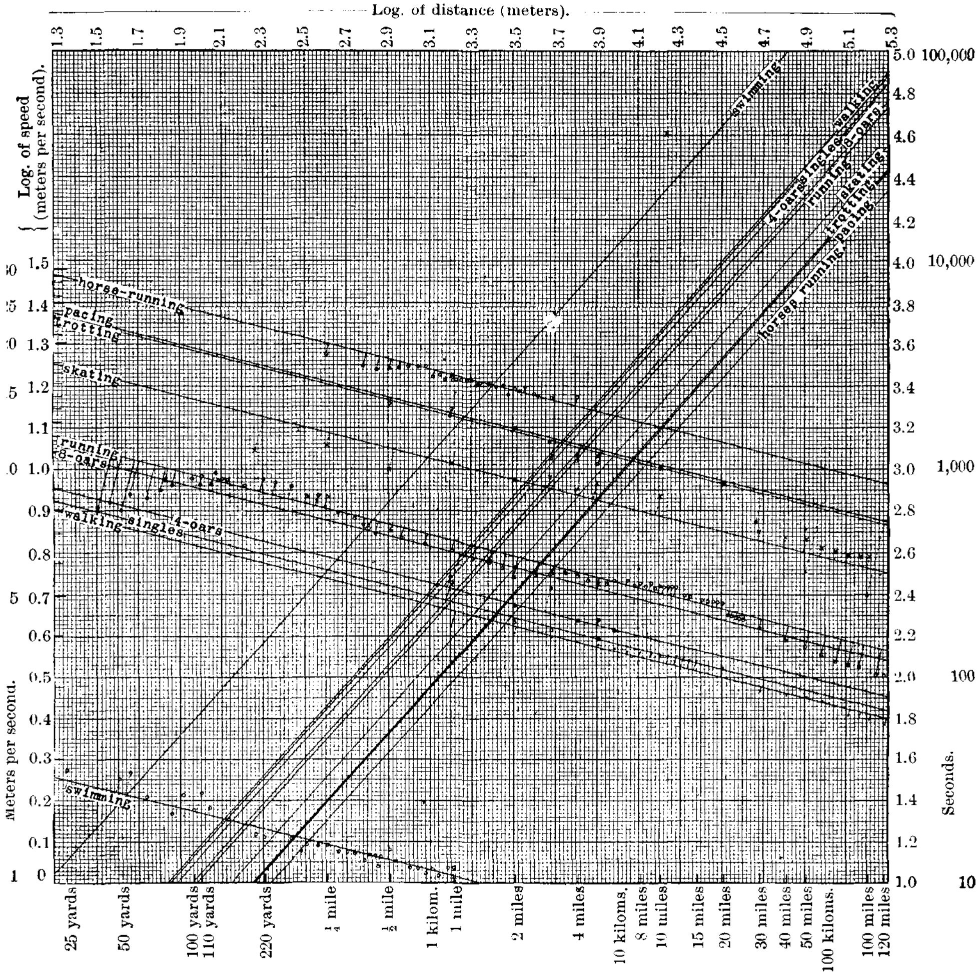
Considering the gross results of columns III, IV, V, and VI, the lowest deviation is found in trotting horses (2.43 per cent) followed by men swimming (3.52 per cent). The greatest deviation is in skating (13.26 per cent). The mean deviation of the whole series of 257 records, rejecting none, is 7.05 per cent.

It is submitted that the summary in Table XI demonstrates the proposition that the records in races of men and of horses approximately follow straight lines when plotted on logarithm paper; because the average percentage deviation of all the records is only 7 per cent from the line, and excluding 50 of the records as unreliable for reasons assigned, this average deviation falls to 4 per cent. The record time that should belong to any given distance within the usual limits, and for any of the events considered, except bicycling, can thus be assigned with these probable degrees of accuracy.

It is not so remarkable that the records of any one event, such as men running, should approximately conform to a logarithmic straight line; but it is remarkable that the straight lines should be parallel, or substantially parallel, in all of these eight classes of events, including three gaits in quadrupeds and three gaits in bipeds, besides motion in the water and over ice. Figure 15 collects all of the logarithmic straight lines on one sheet to a reduced scale. The ascending parallel straight lines are time-distance lines. The descending parallel straight

FIGURE 15. *Speed-distance and Time-distance Lines.*

Ascending lines, time-distance ; descending lines, speed-distance.



EXPLANATION. — At the right of the Figure the numerals 1.0 to 5.0 are logarithms of time (seconds).

lines are speed-distance lines. The mean speeds of the various events in Table I to IX inclusive are plotted with relation to the descending lines in such a manner as to reveal their deviations as clearly as possible.

Deductions from the Results.

Starting with the approximate equation

$$T \cong \frac{L^{\frac{9}{8}}}{c} \text{ seconds} \qquad (28)$$

as borne out in the preceding discussions of records in eight different kinds of races, where c is a constant for each type of race, it follows that as the length of the course, L, increases, the time occupied in the race will increase according to the following table :

TABLE XII.

Effect of Increase of Distance upon Increase in Time.

Increase in Distance.	Increase in Time.	Increase in Distance.	Increase in Time.
2	2.181	70	119.1
3	3.442	80	138.4
4	4.757	90	158.0
5	6.114	100	177.8
6	7.506	200	387.8
7	8.928	300	770.5
8	10.38	400	845.9
9	11.85	500	1087.0
10	13.33	600	1335.0
20	29.08	700	1588.0
30	45.90	800	1845.0
40	63.43	900	2106.0
50	81.54	1000	2371.0
60	100.1		

Thus, if the equation (28) is correct, doubling the distance means increasing the time by 118 per cent, and increasing the distance 60 times increases the time 100 times. Since the equation (28) is not set up as a rigid law, but as a statistical approximation, or approximative law, we cannot expect to find the relations of Table XII accurately presented by all the events. For instance, if we limit ourselves to the first statement that twice the distance should be covered in 2.18 times the time, we can examine all the cases of pairs of such events in the records already considered and find what the ratios of time are when the distances are as 2 : 1. The answer to this question is contained in the following table:

TABLE XIII.

ANALYSIS OF TIME RATIOS OVER PAIRS OF COURSES IN RATIO OF 2 : 1.

Table.	Type of Racers.	No. of Pairs.	Range of Distances covered.		Sum of Ratios.	Mean Ratio.	Highest Ratio.	Lowest Ratio.
			Miles.	Kilometers.				
I	Horses trotting	5	1–100	1.61–161	11.092	2.218	2.327	2.098
II	Horses running	19	0.25–4.0	0.4–6.4	40.523	2.133	2.244	2.000
III	Horses pacing	3	0.5–4	0.8–8.0	6.662	2.221	2.374	2.058
IV	Men running	57	0.03–300	0.02–483	123.754	2.171	2.593	1.818
V	Men walking	18	1–500	1.61–805	39.678	2.204	2.375	2.073
VI	Men rowing	8	0.25–6	0.4–9.6	17.950	2.244	2.518	1.981
VIII	Men swimming	23	0.014–2	0.023–3.2	50.337	2.189	2.436	2.016
IX	Men skating	13	0.03–100	0.05–161	27.198	2.092	2.556	1.760
	General average	146	. . .	. . .	317.194	2.173	2.593	1.760

Taking, for example, the case of running men with reference to Tables IV and XIII, there are 57 pairs of records in which one distance is just double the other, commencing at 20–40 yards and ending with 150–300 miles. The sums of all the time ratios for these 57 pairs of events is 123.754, representing an average value of 2.171, as against 2.181 required by Table XII from equation (28). The highest ratio in any of these pairs is 2.593, and the lowest 1.818.

It will be seen that men running and men swimming come nearest in their averages to the ratio 2.181; while men skating deviate the most (4 per cent). Taking all the 146 pairs of double distances presented in the entire set of records, and without rejecting any, the average ratio of all is 2.173, which is within 0.4 per cent of the value 2.181 required in equation (28). This result constitutes an independent demonstration of the approximate accuracy of that equation. If pairs at short distances affected by starting retardation, or at very long distances, had been rejected, the agreement would have been still closer.

Equation (28) leads to the following :

$$T \cong \frac{c^8}{V^9} \text{ seconds} \qquad (29)$$

or the time varies approximately inversely as the ninth power of the speed in the race. If we assume that the racer reaches the winning post virtually exhausted, so far as affects racing effort, then the time of exhaustion varies inversely as the ninth power of the speed. The computed effect of increasing speed is given in the following table :

TABLE XIV.

COMPUTED INFLUENCE OF SPEED UPON THE TIME OF EXHAUSTION.

Speed.	Time.	Reciprocal.	Speed.	Time.	Reciprocal.
1.0	1.000	1.0	1.5	0.0260	38.4
1.01	0.915	1.09	1.6	0.0146	68.7
1.1	0.424	2.36	1.7	0.00843	119
1.2	0.194	5.16	1.8	0.00504	198
1.3	0.0943	10.6	1.9	0.00310	322
1.4	0.0484	20.7	2.0	0.00195	512

Table XIV shows that if a racer increases his speed 10 per cent, he brings down his running time from 100 to 42.4, or 2.36 times ; and if he doubles his speed, he becomes exhausted in 0.195 per cent of the original time, or 512 times more quickly.

The records do not show that any one racer would be exhausted 512 times as soon if he doubled his speed, because there is no evidence at hand as to the behavior of any single individual at doubled speeds. If, however, any one racer could be trained to take the record speed at each event in the entire series, — that is to say, if a runner could be trained to take every race from the 20-yard dash to the 50-mile run at world's record speeds in each, — then this ideal athlete would be exhausted in a time approximately as the inverse ninth power of the speed. It is reasonable to assume that what would be true of this ideal athlete also tends to be true of any normal athlete, even though his range of speeds and distances be limited.

As an example of this rapid rate of exhaustion, take the 20-mile (32.19 kilometers) running event in Table IV. The speed at which this race was run averaged 4.794 meters per second as shown in column IV. At twice this speed or 9.588 meters per second, there is no exact distance in the table; but the nearest is 131.5 yards (120.2 meters) at 9.696 meters per second. The time of the 20-mile event was 6714 seconds in column II. The time of the 131.5 yard event was 12.4 seconds, or 541 times less, as against 512 times in Table XIV.

Again, consider the last event in Table VIII of swimming records. The 4000-yard (3657-meter) event was finished in 3824 seconds at a speed over the course of 0.9564 meters per second. If we take a speed 1.5 times greater than this, or 1.4346 meters per second, we find one near to it in the table; namely, 1.420 meters per second (column XI) in the 150-yard (137-meter) event. According to Table XIV, the time of exhaustion at 1.5 times greater speed is 38.4 times less than the original. The time should therefore be 3824 ÷ 38.4 = 99.6 seconds. The actual time of the event is given as 96.6 seconds.

DISTANCE

Equation (28) leads to the following expression for L:

$$L \cong c^{\frac{8}{9}}\ T^{\frac{8}{9}} \quad \text{meters.} \tag{30}$$

That is, as more and more time is allowed for racers to occupy in an event, the distances they will traverse will not be directly proportional to the time, but will vary as the eighth power of the ninth root of the time, approximately. A few numerical values are given in the accompanying table:

TABLE XV.

DISTANCES TRAVERSED WITH INCREASING RACING TIME.

Time.	Distance.	Time.	Distance.
1	1	20	14.3
2	1.85	50	32.4
3	2.66	100	60.0
4	3.43	200	111.0
5	4.18	500	251.0
10	7.74	1000	464.0

It is thus indicated that with 500 times more time, the distance covered will be only 251 times greater. As an example, we may take the 4-mile (6.44 kilometers) walking event of table V. It occupied 1658 seconds. If we increase the time twenty times, or to 33,160 seconds, we should expect from Table XV that the distance covered would be 57.2 miles (92 kilometers). The nearest event to this in Table V is the 60-miles (96.6 kilometers), occupying 34,847 seconds or 21 times the original time, which is a satisfactory agreement.

Another consequence of equation (28) is expressed :

$$L \simeq \frac{c^8}{V^8} \text{ meters} \qquad (31)$$

or the distance covered in a race varies approximately as the inverse eighth power of the speed adopted. That is, if an athlete could be trained to take any distance from the shortest to the longest at the record speed for the event ; then the distance which this athlete would be able to run before being exhausted would be as the inverse eighth power of his speed over the course. A few numerical values are given in the accompanying table :

TABLE XVI.

Distances capable of being traversed as the Speed is increased.

Speed, V.	Distance, V^{-8}.	Reciprocal, V^8.	Speed, V.	Distance, V^{-8}.	Reciprocal, V^8.
1.0	1.	1.	1.5	0.0390	25.6
1.01	0.923	1.08	1.6	0.0233	43.0
1.1	0.467	2.14	1.7	0.0143	69.8
1.2	0.233	4.30	1.8	0.00907	110.0
1.3	0.123	8.16	1.9	0.00589	170.0
1.4	0.0678	14.8	2.0	0.00391	256.0

The table shows that if the speed is doubled, the distance that can be run, before exhaustion, is reduced 256 times according to (31).

As an instance, the 20-mile (32.19 kilometers) running event of Table IV, already referred to, may be selected. The speed over the course was 4.794 meters per second. At 9.588 meters per second the distance should be $\frac{20}{256} = 0.0781$ mile $= 125.7$ meters. Table IV shows that at 9.696 meters per second, the nearest to the required speed, the distance run was 131.5 yards, or 120.2 meters.

SPEED.

Equation (28) leads to the expression for speed:

$$V \cong c\,L^{-\frac{1}{8}} \simeq \frac{c}{L^{\frac{1}{8}}} \quad \text{meters per second} \qquad (32)$$

already illustrated in formulas (3), (7), (11), (15), and (19). It means that the speed of racing over courses of different lengths varies inversely as the eighth root of the length, approximately. Table XVII gives a few numerical applications of this rule.

TABLE XVII.

EFFECT OF INCREASING DISTANCES UPON THE SPEED OVER THE COURSE.

Distance, L.	Speed, $L^{-\frac{1}{8}}$.	Reciprocal, $L^{\frac{1}{8}}$.	Distance, L.	Speed, $L^{-\frac{1}{8}}$.	Reciprocal, $L^{\frac{1}{8}}$.
1	1.	1.	100	0.562	1.78
2	0.917	1.09	200	0.516	1.94
3	0.872	1.15	300	0.490	2.04
4	0.840	1.19	400	0.473	2.12
5	0.815	1.22	500	0.460	2.17
6	0.799	1.25	600	0.449	2.23
7	0.784	1.28	700	0.441	2.27
8	0.771	1.30	800	0.433	2.31
9	0.760	1.32	900	0.427	2.34
10	0.750	1.33	1000	0.422	2.37
20	0.688	1.45	2000	0.387	2.59
30	0.654	1.53	3000	0.368	2.72
40	0.631	1.59	4000	0.355	2.82
50	0.613	1.63	5000	0.345	2.90
60	0.599	1.67	6000	0.337	2.97
70	0.588	1.70	7000	0.331	3.02
80	0.578	1.73	8000	0.325	3.08
90	0.570	1.75	9000	0.320	3.12

According to these results the speed has to be reduced 1.33 times to race 10 times the distance, and 1.78 times to race 100 times the distance. Thus, taking the 100.6-meter event (110 yards) in Table IV, the speed over the course is 9.144 meters per second. At 96.56 kilometers (60 miles), nearly 1000 times greater distance, the speed has fallen to 3.572 meters per second, or to 39.1 per cent of the former. According to Table XVII, the speed should fall to 42.2 per cent on increasing the distance 1000-fold.

Another consequence of formula (28) is expressed thus :

$$V \cong c^{\frac{8}{9}} T^{-\frac{1}{9}} \cong \frac{c^{\frac{8}{9}}}{T^{\frac{1}{9}}} \quad \text{meters per second} \qquad (33)$$

or the speed over the course approximately varies inversely as the ninth root of the racing time.

In Table XVIII the speeds of the various racers are set down as computed for courses of 1 kilometer and of 1 mile. Thus over 1-kilometer courses the speed of the running horse is 17.88 meters per second (column VI), and the time for the race 55.9 seconds. At the end of the series come swimmers, with a speed of 1.108 meters per second and an inferred kilometer-time of 902.4 seconds. Table VIII shows that the time for 1.006 kilometers was 925.4 seconds. Turning to the mile range, the speed of the running horse is 37.7 miles per hour over the 1-mile range. His mile-time is 95.5 seconds, both computed and recorded. The speed of the swimmer is 2.336 miles per hour, and the mile-time 1544 seconds as computed, and 1476.2 seconds as observed. The speed of a running man is almost precisely half that of the trotting horse, for distances above 1 kilometer where starting retardation ceases to affect the horse. The speed of a professional walker is very nearly the speed of a professional rower (singles).

It is to be noted that all these speeds are average speeds over the courses. There is no evidence among the records to show what the speed was at different points in the course. So far as concerns anything appearing in the data, the speed of a runner, for example, which averages 7.17 meters per second over a 1-kilometer course, might be 10 meters per second in the first part and 5 in the last part, or *vice versa*. Evidence is lacking to show what the facts are, and they are of great importance to the science of athletics. The speed of a world's-record type of trained runner might be determined at any or all points of a course, either by securing a light recording chronograph on the back of his belt, with a thread payed out as he ran, or by pacing the runner with a light motor-car carrying an automatic speed

TABLE XVIII.

SPEEDS OVER COURSES AT 1-KILOMETER AND 1-MILE DISTANCES.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
Table.	Racers.	log *c*.	*c*.	Speed at 1 Kilometer.		Computed Time for 1 Kilometer, seconds.	Speed at 1 Mile.			Computed Time for 1 Mile, seconds.	Record Time for 1 Mile, seconds.
				log.	$\frac{\text{meters}}{\text{seconds}}$.		log.	$\frac{\text{meters}}{\text{seconds}}$.	$\frac{\text{miles}}{\text{hours}}$.		
II	Horses running	1.6274	42.4	1.2524	17.88	55.9	1.2266	16.85	37.70	95.5	95.5
III	Horses pacing	1.5363	34.38	1.1613	14.50	68.98	1.1355	13.66	30.56	117.8	115.25
I	Horses trotting	1.53	33.9	1.155	14.29	70.0	1.129	13.46	30.13	119.5	118.5
IX	Men skating	1.4143	25.96	1.0393	10.95	91.35	1.0135	10.31	23.08	156.0	156.0
IV	Men running	1.2307	17.01	0.8557	7.173	139.4	0.8299	6.759	15.12	238.1	252.75
VI	Men 8-oars } rowing	1.202	15.92	0.827	6.714	148.9	0.8012	6.327	14.15	254.3	304.75
VI	Men 4-oars	1.1145	13.02	0.7395	5.489	182.2	0.7137	5.173	11.57	311.1	291.0
VI	Men singles	1.085	12.16	0.710	5.129	195.0	0.6842	4.833	10.81	333.0	301.0
V	Men walking	1.0646	11.6	0.6896	4.893	204.4	0.6638	4.611	10.32	349.0	383.0
VIII	Men swimming	0.4196	2.628	0.0446	1.108	902.4	0.0188	1.044	2.336	1544.0	1476.2

recorder, or by noting on a chronograph the times of the runner's passage past a suitable number of fixed points along the track.

Although nothing can be stated directly from the data in this paper as to the degree of uniformity or of variation in the speed of a record-making trained racer, yet if it is proper to apply the inference drawn from a long series of complete races to the speed conditions during the operations of any one taken singly, then it should follow that the speed of a record-maker is very nearly uniform throughout the whole course. If, as appears from the whole series, the time of exhaustion varies inversely as the ninth power of the velocity, and this condition applies within the limits of any single race, then it is easily seen that the quickest way to reach the winning post is to take at the outset that speed which will just produce exhaustion at the goal, and keep to that speed throughout the course. The penalty for raising the speed at any part would be a degree of untimely exhaustion far outweighing the benefit gained. Trainers commonly direct practising athletes to spurt, or accelerate, near the end of the run. This advice must be sound on any theory, because to slacken speed at the end, if there is any balance of running energy left, would be absurd. The runner naturally expends all the available energy balance on the last lap; but if he is able to accelerate to any appreciable extent, it must mean that he has kept too much energy in reserve, and he would have done better to adopt a higher general speed. If, on the contrary, his pace falls to any appreciable extent at the end, he would have economized time by maintaining a lower general speed. Experimental evidence to test this theory would be of great interest. If the theory is correct, athletes, in training for a given event, ought to be motor-paced, the speed of the pacing motor being set uniform. In the earlier practice, this motor-speed should be, say, 15 per cent less than the desired record-speed, and the athlete should train to keep close to the motor. As the training progressed, the uniform speed of the motor over the course should be raised, say, 1 per cent at a time. Of course these suggestions advance beyond the warrant of evidence at this time.

Conclusions.

An analysis of the various national and international appended racing records, as above detailed, leads to the following conclusions, for trotting, pacing, and running horses, as well as for running, walking, rowing, skating, and swimming men:

(a) The time varies approximately as the ninth power of the eighth root of the distance. Doubling the distance means increasing the time 118 per cent (Table XIII).

(b) The time occupied in a record-making race varies approximately inversely as the ninth power of the speed over the course. Doubling the speed cuts down the racing time 512 times (Table XIV).

(c) The distance covered increases approximately as the eighth power of the ninth root of the time. Doubling the time of the race allows of increasing the course length by 85 per cent (Table XV).

(d) The distance covered increases approximately as the inverse eighth power of the speed over the course. Doubling the speed cuts down the distance that can be covered 256 times (Table XVI).

(e) The speed over the course varies approximately as the inverse eighth root of the distance. Doubling the distance brings down the speed about 9.3 per cent (Table XVII).

(f) The speed over the course varies approximately as the inverse ninth root of the racing time.

It may be noted that all of the statements (a) to (f) are different aspects of one and the same fact.

(g) If any of the three quantities L, T, and $V = \frac{L}{T}$ be plotted on logarithm paper as ordinates to either of the other quantities as abscissas, the record points will fall on, or near to, a straight line (Figures 2, 3, 6, 7, 8, 10, 11, 12, 13, and 15).

(h) Athletes aspiring to break racing records might succeed better in attacking those whose points fall below the straight lines of speed against distance, or above the straight lines of time against distance, rather than those whose points fall on the opposite sides of those lines.

(i) The records presented on bicycling do not determine the proper highest speed of cycling below 30 miles (48 kilometers), since there is apparently no reduction in speed by fatigue up to that distance.

(j) With the exception of bicycling, as above noted, the law of fatigue in racing is the same, or very nearly the same, with horses as with men, in air or in water, as indicated by the records analyzed in this paper.

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AN EXPERIMENTAL STUDY OF THE IMAGE-FORMING POWERS OF VARIOUS TYPES OF EYES.

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"Die wirkliche Naturwissenschaft begann damit, dass man, anstatt über das Wesen der Schwerkraft zu fabuliren, die näheren Umstände der Bewegung des fallenden Steines, des Pendels u. s. f. genau bestimmte und möglichst genau und einfach beschrieb. In der Biologie, speziell in Bezug auf die uns hier interessirenden *mechanischen* Lichtwirkungen kann die Aufgabe des Forschers auch nur darin bestehen, die durch das Licht ausgelösten thierischen Bewegungen ihrer Abhängigkeit nach näher zu bestimmen und zu beschreiben." (Loeb, **'90**, p. 20.)

I. INTRODUCTION.

THE structure of eyes has been long and carefully studied, both as to gross anatomy and finest histological detail. This is especially true of the human eye, but the eyes of other vertebrates, and of the invertebrates as well, have received a large share of attention. The ability of various eyes to form more or less accurate images of external objects has been for the most part inferred from the application of physical laws to the knowledge of their structure. In the case of man, however, there is less need for inference, since his common experience in seeing is a constant demonstration of the ability of his eyes to form images. Nevertheless, even here certain factors which add to the efficiency of the eye as a visual organ, but are in no way concerned with the actual physical formation of the image on the retina, must be taken into account. For example, judgments based upon experience involving other elements than mere image-formation, such as the mental superposition of the images of the two eyes, giving a stereoscopic effect, the action of the ciliary muscles and other mechanisms for accommodation, and the influence of other senses, especially that of touch. The filling out of the blind spot of the retina by the mind is a good example of the influence of experience upon the interpretation of the actual sense impression. It is not unlikely that the eye of a baby is capable of forming practically as good an image as that of an adult; but the baby lacks the experience and training of the adult which would enable it to form proper judgments, and consequently it appears to have little if any conception of distances and space relations in general.

The images formed by other eyes than the human can also in a measure be studied from direct observation. Thus the image of distant objects may be seen upon the retina of the freshly removed eye of an albino rabbit, since the absence of pigment leaves the posterior portion of the eyeball semi-transparent, and the relaxed eye is accommodated to distant vision. Or, the posterior portion of the eyeball of a pigmented eye, such as that of a frog, may be cut away, and by aid of a microscope the image may be projected so that it can be observed directly, or thrown upon a screen. This has likewise been accomplished with the eyes of some invertebrates; thus Exner (**'91**) succeeded in taking a remarkable photograph through the eye of a fire-fly, while Parker (**'95**) demonstrated empirically that the compound eyes of Astacus form a single image rather than a number of separate images.[1] Again, in the eyes of many vertebrates the decolorizing effect of light upon the visual purple, after protracted exposure, may be seen upon the retina, and the monochromatic image thus formed may even be permanently fixed by a proper treatment with alum solution (Kühne, **'79**, p. 299) or platinum chloride (Stern, : **05**).

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[1] The multiple photographs obtained through the corneal facets of insect eyes do not represent the retinal image formed by these eyes, since in obtaining such photographs only one portion of the dioptric apparatus is used.

By means of the ophthalmometer and other optical appliances, Beer has made an extensive study of accommodation in the eyes of the lower vertebrates and cephalopods. This faculty of accommodation may in a general way be related to image formation, since it is probable that an eye that is capable of accommodating differently to near and to distant objects must be able to form fairly distinct images of those objects. The converse, however, is not necessarily true, since there are eyes which appear to have considerable image-forming power, but show no mechanism for accommodation. The evidence that the compound eyes of insects can accommodate seems insufficient, and there is no evidence of this power in the eyes of any of the other invertebrates except the cephalopods, unless perhaps the movements of the eyes of certain copepods may be considered as such.

All the methods which have been enumerated for determining the exact nature of the images formed by eyes have certain obvious defects. Especially is this true in the more lowly organized types of eyes, where comparisons with the human eye cannot be so closely drawn. A study of structure alone cannot give an exact basis for this determination, since it is not always easy to interpret the optical properties merely from the structure. This is well illustrated in the variety of opinions that have existed as to "mosaic" vision in insects and other animals with so-called compound eyes. Empirical tests with fresh eyes lose much of their value on account of the changes that take place in the tissues immediately their blood supply is cut off and they are removed from the animal. The most serious of these changes are loss of tonus and coagulation of the fluids. The image on the retina, due to the change in the visual purple, is not definite enough to be of much use for settling this question ; and the inferences to be drawn from the action of the eye in accommodation can be of only the most general nature.

It has not been the purpose of the work described in the following pages to furnish a more exact method of determining the precise image-forming powers of eyes. The aim has been, rather, to treat the formation of images from the point of view of their relation to the animal as a living organism, — to determine in what way the ability to form a more or less perfect image affects the responses of the animal to light, and what relation, if any, this result has to the normal habits of the creature, and to its behavior under experimental conditions. This investigation was suggested to me by Professor G. H. Parker, the suggestion being an outcome of his study of the phototropism of the mourning-cloak butterfly, in the course of which he found that this insect is able to discriminate between light derived from a large luminous area and that from a small one, even when the light from the two sources is of equal intensity as it falls on the animal, and that it usually flies toward the larger areas of light. He took the biological significance of this reaction to be that "this species remains in flight near the ground because it reacts positively to large patches of bright sunlight rather than to small ones, even though the latter, as in the case of the sun, may be much more intense." (Parker, :**03**, p. 467.) It is evident that such a reaction as that described can be taken as a rough measure of the image-forming capacity of the eyes of the butterfly studied, but a comparison of different animals on this basis can be made only in a very broad and general way.

Except for the work of Parker just mentioned, there appear to be few if any direct references in the literature to experiments or observations calculated to determine the difference in the reactions of animals to luminous (or illuminated) fields of different sizes; at least, few if any in which other factors, such as intensity and color of light, have been eliminated. An experiment performed by Loeb ('**90**, p. 47, Versuch 2) upon a species of crepuscular moth (*Sphinx euphorbiae*) perhaps comes closest to the conditions of the present investigation. Specimens of the moth were brought into a room illuminated at one side by a window, while upon the opposite wall was placed a kerosene lamp. Here, then, were conditions with a large area of light at one side and a small source of light at the other. No comparison was made of the relative intensities of the lights, but as the experiment was performed at the approach of twilight it is to be presumed that the light from the window was much less intense than it would have been in the middle of the day, so that the light from the lamp was relatively more intense. Animals liberated at a point midway between the window and the lamp flew to the window; and it was not until they were brought within about a meter of the lamp that they flew in its direction. Loeb regards this result as due entirely to the relative intensities of the light received from the two sources; but from the experiments to be described later, it will be seen that the larger area of the window was undoubtedly an important factor irrespective of, or at least in addition to, the intensity of the light.

A number of observations have been made on the tendency of animals to go toward, and to collect in, shaded areas. Mitsukuri observed that certain Japanese marine snails (species of Littorina) gathered in largest numbers in the shadow of certain objects which he used in his experiments (Mitsukuri, :**01**, p. 1, Experiment 2), and he furthermore explained the movement of these snails landward at the time of flood tide, when they are negatively phototropic, as due to the fact that, on account of the bank, with bushes, grass, rocks, etc., less light came from that side. Somewhat similar observations have recently been made upon another species of Littorina (*L. rudis*) by Bohn (:05), who found that snails moving in an illuminated field took a course determined by the relative amount of light coming from different directions, and that if black or white vertical screens were placed in the field, the course of the animals was correspondingly deflected; they were "attracted" or "repulsed" by the screens, as the case might be. Bohn emphasizes the importance of the size of the screen, but makes no attempt to determine whether the greater influence of a large screen is due to the formation of a correspondingly larger image upon the animal's retina or merely to the fact that it reflects a greater amount of light if it is white, and absorbs more if it is black. He apparently made no attempt to determine exactly his light intensities, either absolute or relative, and consequently his results are rather crude and only qualitative in this respect.

Torelle (:03, pp. 470, 471) made a number of tests of frogs with reference to shadows and to dark objects, and found that, although ordinarily positively phototropic, they move out of the sunlight into the shadow, even when by so doing the movement is away from, or at right angles to, the direction of the ray (p. 487). When once in the shadow, however, the animals turned and faced the sun-illumined area. This occurred when the shadow was that of a building or merely that beneath a box raised a short distance from the ground. These results in their bearings upon the present investigation will be discussed more fully in considering the results of my own experiments.

The greater part of my experimental work consisted in testing the reactions of various suitable animals to two sources of light, differing in area, but of equal intensity at a point midway between them, where the animals were exposed. It was planned so far as possible to select animals with representative types of eyes, such as direction eyes (planarians, etc.), compound or mosaic eyes (insects, crustaceans, etc.), and camera eyes (vertebrates), and, in addition, to conduct parallel experiments upon eyeless forms (such as the earthworm) which are known to be sensitive to photic stimulation. For the purpose of the experiments it was obviously necessary to use only animals which were decidedly positive or negative in their reactions to light, since those which were normally irresponsive or indifferent to light coming from one direction only could not be expected to show evidence of discriminating between luminous areas of different size. Furthermore, in so far as possible, animals which normally react positively to light, and also those which are negative, were used in order to determine whether, as would be expected *a priori*, the responses (evidences of discrimination between the two lights) were reciprocal in the two cases. Forms which would require to be experimented with under water were avoided on account of the practical difficulties involved, — not only optical complications due to the reflection and refraction of light by the water and the containing vessel, but also on account of the difficulty of directly testing the lights at the middle point in order to determine whether that received from one source was exactly equal to that received from the other.

Before passing on to a description of the apparatus most used, I wish to express my indebtedness to Professor G. H. Parker, to whom, as has already been mentioned, the study owed its inception, and under whose direction, in connection with one of his courses, it was carried on for a year. The later prosecution of the work was under the direction of Professor E. L. Mark; to him I owe my gratitude for much valuable criticism and advice, since to his attention to accuracy of method both in experimentation and in the deduction of conclusions therefrom must depend in large part any merit which the present contribution may possess.

II. Description of Apparatus.

In all the experiments on the reactions of animals to two lights of different areas, as well as in certain of the other lines of investigation, the same general apparatus was used. Special devices were found necessary in working with each species of animal; these can best be explained in connection with the accounts of experiments with the respective species. Although the apparatus was changed in some of its details and added to from time to time, the changes and additions were small and comparatively unimportant, the general plan and arrangement of the apparatus remaining essentially the same throughout. It will be described in its final form, reference being made to such additions or alterations as seem to be worthy of mention.

The apparatus was installed in a long, narrow room (7 meters by about 2 meters) commonly used for photographic purposes. The single window of this room could conveniently be made light tight, while an antechamber at the entrance provided against the admission of stray light at that end of the room. The side walls were of brick, the roughness of which tended to give less definite reflections than smoother surfaces would have done, and the whole interior of the room was painted dead black. Furthermore, practically everything in the room which did not from its nature need to be otherwise was painted black to prevent the reflection of light.

The definite arrangement and relation of the parts of the apparatus as viewed in vertical elevation may be seen in Figure 1, while Figure 2 shows the horizontal plan. Against the north wall of the room was placed a large wood-top table (*T*); at the right of this a lamp (*Sm*), giving as nearly as practicable a point of light; and at the left, at an equal distance from its centre, the apparatus (*Lg*, *Lg'*) designed to furnish the larger illuminated area (*g*). A third lamp (*V*) was placed directly above the middle point of the line joining the other two lights,[2] a line which may be designated as the *directive axis* (*a*, Figure 2).

The lamp giving the small light (*Sm*) consisted of a wooden box (22 × 22 × 35 cm.) painted black, and having a Nernst filament arranged before a small opening in the side facing the table. By this arrangement a definite and clear-cut luminous area was secured. Since there was no reflecting surface immediately back of the filament, the light given off in that direction traversed the blackened box and was absorbed by the distant walls, two of which were set at an angle to each other, as shown in Figure 1, to prevent, as far as possible, reflection of light.

The filament used on this lamp was the regular 110-volt "single glower" Nernst filament, which is about 15 mm. long and has a diameter of approximately 1 mm.

The illuminating apparatus (*Lg* and *Lg'*) furnishing the large light was more complicated. The lamp proper (*Lg'*) was similar to *Sm*, except that two 220-volt "six glower" filaments were used on a circuit of the strength indicated.[3] The light of these two filaments was many times stronger than that of the small lamp. In front of the lamp *Lg'* was a long box (*Lg*,) lined throughout with white glazed paper, except the end nearer the table, which was closed with a plate of ground glass (*g*). The end farthest from the table made an angle of 45° with the sides of the box (see Figure 2). An opening (*o*) in the side of the box near the end farthest from the table admitted the light from the lamp *Lg'*, which fell at an angle of 45° upon the white paper covering

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[2] The word "lights" is here used to designate the luminous areas which furnished the light used in the experiments. In the case of the lamps *Sm* and *V*, these were Nernst filaments used directly; in the case of *Lg* it was a large square of ground glass illuminated by light from behind. The lamp *V* was not used in the experiments described in the present paper.

[3] The 220-volt current was secured by using a transformer on the same (110-volt) circuit that supplied the smaller lamp.

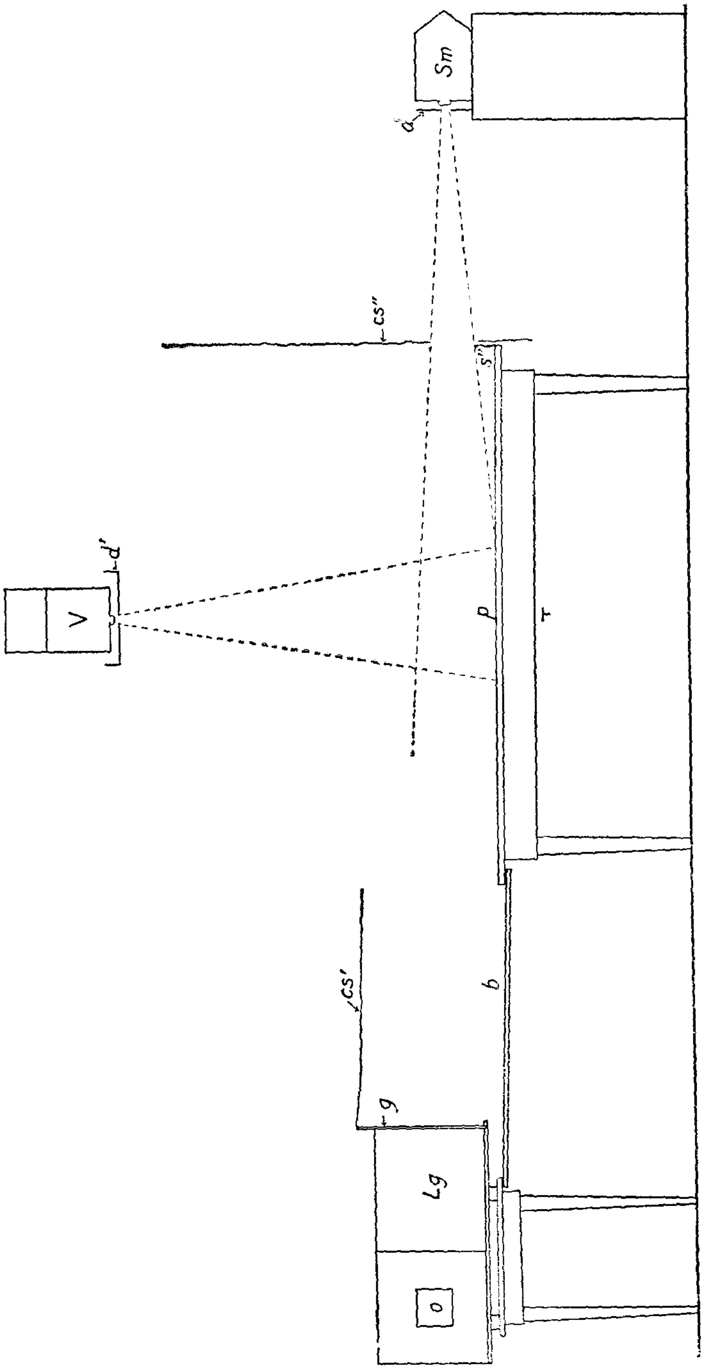
Sm
a
cs''
s''
V
d'
p
T
b
cs'
g
Lg
o

FIGURE 1.

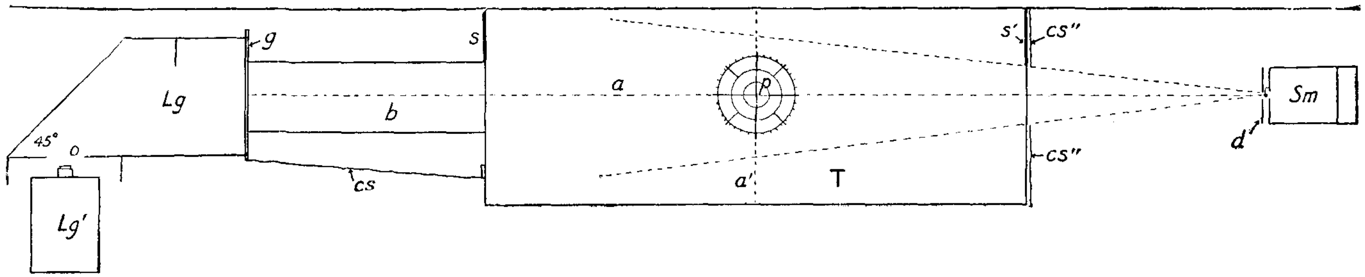

FIGURE 2.

FIGURES 1 and 2. General scheme of apparatus. Figure 1, elevation; Figure 2, plan: *a*, directive axis; *a′*, normal axis; *b*, board connecting table tops; *cs*, *cs′*, *cs″*, cloth screens; *d*, *d′*, diaphragms; *g*, ground glass; *Lg*, box of apparatus furnishing large luminous area; *Lg′*, lamp shining into interior of *Lg* through *o*, an opening in the side of *Lg*; *p*, working position; *Sm*, lamp giving small area of illumination; *T*, table; *V*, lamp designed to throw light down vertically.

the oblique end of the box, and was thus reflected directly toward the ground glass. The white-paper lining of the sides aided in securing the desired diffusion of light. In this way a remarkably even illumination of the large sheet of ground glass was obtained. By means of a square mat of opaque paper a clear field 41 cm. square was produced, and this constituted the larger of the two areas of illumination used in the experiments. The ground glass was first placed with the unground surface toward the table, but later the ground side was turned in that direction, in order to avoid the reflection of light from the small lamp opposite.

The lamp (*V*), sending down vertical rays, was essentially like the small lamp (*Sm*), being similarly provided with a single 110-volt filament. There was arranged in front of it, however, a sliding diaphragm (not shown in the figure), by means of which the size of the orifice, and therefore the amount of light coming from it, could be regulated at will.

The filament of the small lamp (*Sm*) and the ground glass (*g*) of the large lamp (*Lg*) were each exactly 2 meters from the central point (*p*) of the table. For convenience this middle point may be spoken of as the *working point* or *working position*. All light, except that coming directly from the sources described, was excluded, as far as possible, from the working position by means of diaphragms (*d*), screens of heavy pasteboard or wood (*s*, *s'*, *s''*), and black cloth (*cs*, *cs'*, *cs''*). The intensity of illumination of the ground-glass surface could now be easily regulated by varying the distance of the lamp *Lg'* from the oblique reflecting surface in the box *Lg*. With the aid of a Lummer-Brodhun photometer placed at the working position (*p*) it was an easy matter, by varying the distance from *o* of the light *Lg'*, to make the intensities of the two lights exactly the same.

It is now necessary to compare the respective areas of the two sources of illumination. The small filament was 1 mm. in diameter and 15 mm. long, but at each end was a small knob where the platinum wires were attached to the filament, and when the latter was at white heat these glowed as well. Thus the total length of glowing surface was about 17 mm. Since the illuminated surface of the ground glass was 41 cm. square, as has been stated, its area was 168,100 sq. mm. If in the small lamp we regard the filament only, we shall have an area of 15 sq. mm., and therefore the ratio of the two areas will be that of 11,207 to 1. If, however, the length of the smaller light is considered as 17 mm., the ratio becomes 9,888 to 1. Since the ratio cannot be obtained with great accuracy, and the exact proportion is of only minor importance, we may, for all practical purposes, regard the ratio as 10,000 to 1.

The intensity of the light from either source was found at the end of the investigations to be only about 1.25 candle meters. There were two reasons why this intensity was not constant throughout the experiments. In the first place, the voltage appeared at times to be very inconstant, causing considerable fluctuation in the intensity of the lights. These fluctuations were usually of sufficient amount to be noticeable; but since both lights were on the same circuit, the *relative* intensities were but little affected; however, at such times the lights were tested with the photometer, in order to make sure that they had not changed relatively, and that they were still balanced at the working point. It was found by experience that with small fluctuations their change in relative intensity was inappreciable, it being only in cases of considerable fluctuations that it was necessary to make any readjustment. The other change in intensity was due to a gradual deterioration of the filaments, and therefore was more constant. The light intensity employed at the beginning of the investigation was very close to 5 candle meters, from which, as stated above, it gradually ran down to as little as 1.25 candle meters in the later experiments. The whole change from this cause was probably not so great, however, as sometimes occurred in the temporary fluctuations due to varying voltage. We seem justified in leaving these comparatively small inconstancies out of consideration, since it is not probable that they were of sufficient amount to influence materially the results on any of the animals used. At most, these differences could probably have made only a slight difference in the percentage of reactions and none in their character, since Adams (:**03**) has shown that in the earthworm, for example, it is only at very low intensities, near the point where the animal changes from negative responses to positive, that small differences of intensity have a relatively great effect. At the higher intensities considerable range in the intensity produces relatively little change in the percentage of responses. Furthermore, the animals employed in these experiments were selected for their decided reactions to light of about the intensity used, and it is probable that in no case where positive results were obtained was this near the point where these forms change the character of their response.[4]

By means of a microspectral photometer the qualities of the two lights were also tested and compared. The differences in the spectral components were found to be so slight as to be negligible. A sheet of clear glass, equal in thickness to the ground glass of the large light, was at first placed in front of the small light, in order to make the conditions of the two as similar as possible; but it was found to be of little use, and was of such obvious disadvantage, because of the considerable dispersal and reflection of the light which it caused, that it was later dispensed with entirely, and consequently is not represented in the figures of the apparatus.

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[4] The common European garden snail (*Helix pomatia*) was found to be so inconstant in its responses that it was not suitable for the purpose of these investigations. This may have been because the light used was near the *critical intensity* for this species, though it is more probable that the changes in response depended entirely upon certain physiological states of the animal independent of the light (see p. 391).

The vertical light (*V*), which has been described, was used only in connection with the small light (*Sm*) in studying chiefly the reactions of animals to lights of equal intensity, but coming from different directions, — in this case one light striking the animal from directly above, the other coming to it horizontally. This combination of lights was also employed in studying the reactions to shadows and to sudden differences of intensity. These experiments will be described in another paper.

Other appliances and conveniences will be mentioned in their proper places. It should be stated here, however, that screens were provided by means of which the light from any of the lamps could be immediately shut off from the working position at will.

At the point on the table exactly midway between the lights was painted a white line, and this may be spoken of as the *normal axis* of the apparatus (*a′*, Figure 2), since it is at right angles to the directive axis already mentioned. At the beginning of each trial in an experiment the subject was usually placed, headed in one direction or the other, on this line, its long axis coinciding with the line. For the sake of brevity in description, this is called the *normal position*. Any line running parallel to the normal axis — nearer to one light or the other — may be said to run in a *normal direction*. The centre of the *working position* was at the point where the normal and directive axes crossed, and the working area included a small horizontal surface, the extent of which was determined by the distance the animals were allowed to move. Three concentric circles, the inner with a radius of 5 cm., the second with a radius of 10 cm., and the outer with a radius of 15 cm., were described with their centres at this point. Each of the circles was divided by short cross lines into arcs of 10°, and by means of this device the angles at which the animals deviated from the normal axis and crossed any of the circles could be read directly with ease and accuracy.

III. Experiments.

As stated in the Introduction, the experiments were undertaken with the idea of ascertaining, as far as possible, to what extent complexity in the organization of eyes is correlated with the reactions to luminous areas of different size but of equal total luminosity. The character and relative percentage of phototropic responses were used as measures of the reactions. The method employed can perhaps best be explained by an example. Let us suppose that an animal which is decidedly positive in its ordinary reactions to directive light is placed midway between two luminous areas of exactly equal shape, size, and intensity, and in such a position that one light is at its right, the other at its left. Let us assume, further, that each luminous area is 1 cm. square, has an intensity of 100 c. p., and is situated at a distance of 2 meters from the animal. The measure of the light then impinging upon either side of the subject would be 25 C.M. (candle meters). We should expect one of two results: (1) The animal being equally stimulated upon both sides would go straight ahead without turning; or (2) owing to chance or random movement, it would become turned slightly more towards one light, which would thus have a more direct effect than the other, and the animal would then continue crawling towards this light. But since the chance of random movements in one direction is as great as in the other, the number of times that the animal would go towards each of the lights should, in a large number of trials, be equal, and we should have essentially a balanced condition as before.

Now let us enlarge one of the areas to, say, 100 cm. square, but keep the total amount of light given by it the same as before. Its area is now 10,000 times as great as before, and consequently the intensity of the light radiated from a single square centimeter is now only 0.01 c. p. The whole amount of light received by the animal is, however, the same as before, namely, 25 C.M. upon each side. If the animal is without image-forming organs, — in other words, without eyes, — it has no obvious means of appreciating the increase in size of one of the areas, and we should expect the reactions to be the same as when the lights were of equal size, that is, the animal would be indifferent. In this case, the skin (or certain scattered cells in the skin) is the sensitive surface, and since there is no apparatus for concentrating the light from the large area, the amount of light received by any point on the skin on either side of the animal is equal to that received by any other. This is evident from the fact that light from every one of the 10,000 areas (each a centimeter square) which make up the large area falls upon each point of the surface of the animal; the intensity of the light from any single square centimeter of the area is only 0.0025 C.M., but since there are 10,000 such radiating squares the total intensity is 25 C.M.

In an animal possessing eyes capable of forming good images of external objects the conditions are very different. In this case light from all parts of the large area cannot fall upon every point in the sensitive surface (here the retina), but the light from each part of the field retains its position relative to that from other parts, and falling upon the retina in this order covers there an area similar in shape and relative intensity of illumination to the external one. Such is the image. The small light, which we are considering as only 1 cm. square, would likewise form an image on the retina, and this, with the light at the given distance, would cover a certain small area, which we may denote by x. x, then, is the size of the retinal image of a luminous object 1 cm. square at a distance of 2 meters. The light received on this area would have a certain intensity, which may be designated by y. Now, since the large light has 10,000 times the area of the smaller one, its image on the retina (making no allowance for aberration or other optical defects) would be 10,000 times as large as the image of the small light, or 10,000 x; and the intensity on any single area x would be only $\frac{1}{10000}$ y. It is obvious, therefore, that under these circumstances we have entirely different conditions of stimulation on the two sides — that is, in the two eyes — of the animal. On the retina of one eye only a very small area (x) is stimulated, but the light has a considerable intensity, which we have called y. The retina of the other eye is stimulated over a much larger area (10,000 x), but each area (x) receives in this case a light intensity of only $\frac{1}{10000}$ y.

This is of course only an approximation to the relative influence of the two sources of illumination. It is probable that even in the most highly organized eyes, owing to aberration and other defects, the actual conditions are far from those here assumed. It must be remembered, furthermore, that from a physiological standpoint the retina cannot be accurately divided into areas, such as we have assumed x to be, corresponding to external areas, but that its physiological action, as well as its finer structure, must be considered in terms of visual elements — the ommatidia in the compound eye, the rods and cones in the vertebrate eye. The matter is further complicated by the fact that these visual elements may not have a uniform distribution over the whole retina, nor do we know that those of different parts of the organ are equally sensitive to light stimulation.

In spite of these defects and deviations from the suppositional case, we are safe in assuming (1) that the image of the larger light is spread over a larger area on the retina, or better, that a larger number of visual elements receive light, and (2) that the intensity of the stimulus acting on any one visual element is very much less than that which acts on the one element, or very few elements, which are stimulated by the small light. Analogy with the reception of stimuli by sense organs in general makes the assumption reasonable that the difference between even a very weak light and no light falling on a visual element may have a much more stimulating effect upon the animal than the same, or even a greater, difference in the amount of the light at higher intensities. Such being the case, we should expect an animal to react more strongly to that stimulus which fell upon the larger number of visual elements — that an animal normally positive, for example, would be more strongly positive to the large light than to the small, and similarly that a negative animal would tend more often to move away from the larger than from the smaller luminous area. Parker's (:03) results give evidence that in Vanessa such is really the case. In the following pages are recorded the results of experiments made under more uniform and more precisely determined conditions upon a variety of animals, some negative and others positive, some with eyes and others without.

The kinds of animals experimented upon were not taken entirely at random, but species whose reactions to a single light were definite and well known were selected as far as possible. The number of animals from which selection could be made was greatly limited owing to the optical difficulties with animals living in water. Only those were used, consequently, which could be studied in the air. In addition to the animals with which a considerable series of experiments was finally made, a large variety of animals were tested in a more or less complete manner, to determine their suitability for the work. These included a number of insects, such as the water strider (Hygrotrechus), an elater, a small species of cockroach, and certain kinds of bees and flies, several myriapods and spiders, the European slug (*Limax maximus*), the horned toad (Phrynosoma), and two species of salamanders (*Plethodon glutinosus* and *P. erythronotus*). Some of these did not appear from their movements to be responsive to light, others were inconstant in their responses, and still others were too inactive for the purposes of this work. Those from which more or less satisfactory data were obtained are as follows: 1. The common dungworm or earthworm (*Allolobophora foetida* [Sav.]). 2. A large land planarian (*Bipalium kewense* Moseley). 3. The mealworm (larva of *Tenebrio molitor* Linn.). 4. The sow bug (*Oniscus asellus* Linn.). 5. The cockroach (*Periplaneta americana* Linn.). 6. The mourning-cloak butterfly (*Vanessa anti-*

opa Linn.). 7. The water scorpion (*Ranatra fusca* Pal. B.). 8. The pomace fly (*Drosophila ampelophila* Loew). 9. The common garden snail of Europe (*Helix pomatia* Linn.). 10. The cricket frog (*Acris gryllus* Le Conte), and the green frog (*Rana clamata* Daudin).

Below are set forth the results of the experiments on these animals in the order given above. The statement of results is followed by a discussion of their significance.

1. *Earthworm* (*Allolobophora foetida* [Sav.]).

In connection with observations upon the reactions of a series of animals having eyes of different kinds and of varying degrees of complexity, it seemed desirable, for the sake of comparison, to make similar observations upon a form without eyes. Of all the forms available, the earthworm appeared to be the best suited to this purpose. A number of observers have worked with earthworms of various genera, so that their behavior in relation to light is better known than that of many other animals. Although there appear to be differences in the degree of sensitiveness of different species, the behavior of the several kinds that have been studied seems to be essentially the same.

That earthworms ordinarily respond negatively to light of moderate intensities was early remarked by Hoffmeister ('45), and subsequently by Darwin ('81), and has been abundantly confirmed by all later workers. For the purpose of the present investigation it was sufficient that the worms react in this manner in a large percentage of cases, and that they lack eyes, — that is, any organs for focussing the light upon special sense cells. The fact that they possess such special cells as the so-called light cells, — demonstrated by Hesse ('96) and shown for Perichaeta by Harper (:05), — has no bearing in this connection, since there is no contrivance for concentrating the light upon these structures. Nor does it matter that the animal is sensitive upon all parts of its body, though to a different degree in the different regions, as shown by Parker and Arkin (:01). Indeed, it is unimportant to know in what manner the reaction is brought about; whether it is a direct response, in the sense of the tropism theory, or a result of trial and error, as maintained by Holmes (:05) and confirmed for moderate intensities of light by Harper (:05), since the problem is merely to determine, as indicated by the direction in which the animal moves, whether it is more strongly stimulated upon one side than upon the other.

Method of Experimentation. The luminous areas, as previously explained, were fixed, one at the left and one at the right of the experimentation table. Earthworms in good condition, which had previously been kept in the dark, were tested, each in its turn, first, with the lights separately, and then with both acting at the same time. In each experiment the worm was placed in the normal position, — that is, with its long axis at right angles to the rays of light (see p. 346), — and allowed to crawl a distance of 15 cm., i. e., until its anterior end crossed the outer circle marked on the table (see p. 346, and Figure 2, p. 343). The course and the angle at which the worm crossed the circle were immediately recorded on blank diagrams differing from the table diagrams only in the absence of one of the three circles. These circles were divided, as on the table diagram, into arcs of 10° each. When the worm had been tested five times with the head started in one direction, it was then tested an equal number of times with the head started in the opposite direction. Experiments of this kind and number were made in succession with five different worms, such experiments — 50 in number — constituting a *set*. Three such sets of experiments were made, and they may be designated as *A*, *B*, and *C* respectively. Sets *B* and *C* were performed under precisely similar conditions, and may be considered together; in set *A* the conditions differed from those of sets *B* and *C* only in the method of starting the worm. This method proved to be much less satisfactory than the one employed in sets *B* and *C*.

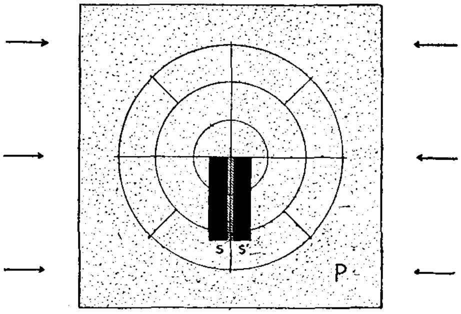

FIGURE 3. Apparatus used in earlier experiments with *earthworm*: *P*, ground-glass plate; *s*, *s′*, small glass strips forming runway.

In the experiments of set *A*, a ground-glass plate (Figure 3, *P*) was placed on the diagram on the table, and the worm was allowed to crawl on the roughened glass surface. The plate was kept moist by the frequent use of a damp sponge, and when thus moistened was sufficiently transparent to show the circles and radii on the table below. Two strips of thick glass (*s*, *s′*) — made opaque by being painted, except where the worm was to come in contact with them — were placed parallel to the normal axis, with one of their ends at the directive axis, and with a space between them only a little wider than the diameter of the worm's body. This formed a trough, or runway, in which the worm was placed and allowed to start crawling, whereupon a third piece of glass was used to cover the runway. Thus, when the worm emerged from the end of the runway, its anterior end projecting out, it was subjected to the lights in the desired manner. To start the worm headed in the opposite direction, the glass strips were moved on the plate to a similar position on the other side of the directive axis.

The results of 50 trials made in this way under each condition of illumination were as follows:

a. *Response to small light alone.* In 50 trials the animal turned 43 times away from the light (— reactions) and 7 times toward it (+ reactions). This gives an excess of 36 responses away from the light, or 72 per cent of the whole number. This may be taken as a measure of the negative phototropism of the worm under the given conditions (*cf.* Parker and Arkin, :**01**, p. 153).

b. *Response to large light alone.* Out of 50 responses 3 were toward the light (+), 44 away from the light (—), and 3 straight ahead or indifferent. The excess away from the light was 41, or 82 per cent.

c. *Response to simultaneous influence of both lights.* Out of 50 trials 31 resulted in a turning towards the large light, 15 towards the small light, and 4 were indifferent. Here there is an excess of 16 reactions, or 32 per cent, away from the small light and towards the large.

This result is far from what would be expected in the case of an eyeless animal. One would suppose, since, when both lights are used, the amount of light striking the worm on each side is the same, that the responses in each direction would be much more nearly equal in number. Another noticeable fact is the high percentage of negative reactions when only a single light was used. The highest average of negative responses obtained by Adams (:**03**) was 59 per cent. This was observed with a light of 48 C.M. intensity at the point where the worms were placed.[5]

In this set of experiments no mention has hitherto been made of the angle at which the worms turned. It was found upon an inspection of the records that in fully a third of all the trials made the animals upon leaving the runway had turned at an angle of 90°, confirming the suspicion awakened by watching them, that their natural thigmotactic response caused them in a large number of cases to turn sharply and follow the ends of the strips of glass. For the same reason they also appeared very reluctant to leave the runway, often drawing back completely after the anterior end had been extended a centimeter or two. It was only by stimulating such individuals at the posterior end that they could be induced to continue crawling.

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[5] It should be borne in mind that the present results are not strictly comparable with those of Parker and Arkin or of Adams, since those investigators recorded all definite head movements (i. e., crawling movements), while in these experiments the direction of the turning in an excursion of 15 cm. is used.

As it seemed possible that the factor of thigmotaxis might have vitiated in large measure the results, the second and third sets of experiments (*B*, *C*) were made under somewhat different conditions. The ground-glass plate, instead of being laid upon the table, was suspended about 2 cm. above it by means of strings leading upward to a common point, and thence to the ceiling in much the manner devised by Parker and Arkin (:01, p. 151). In this way the horizontal position

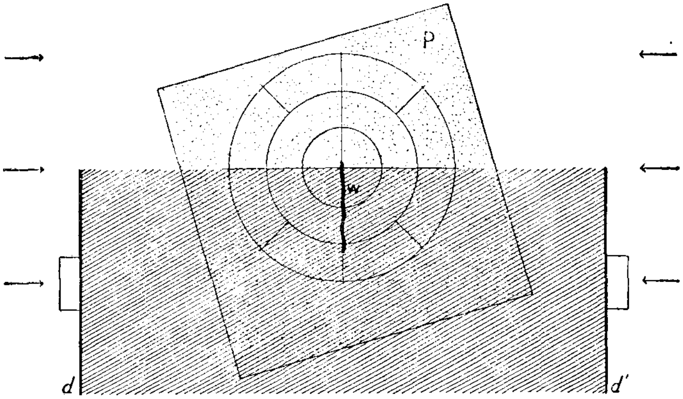
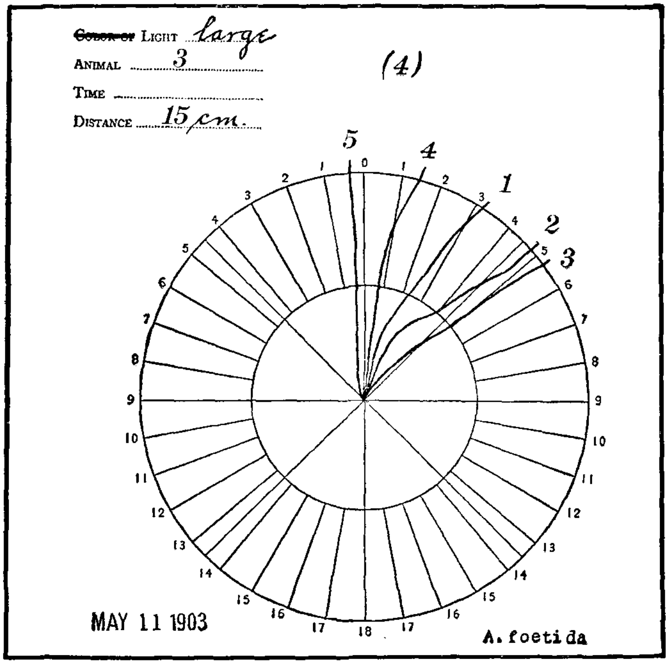


Figure 4. Apparatus used in later experiments with *earthworm*, and with the *land planarian*, *mealworm*, *snail*, and *slug*. *d*, *d'*, diaphragms or screens of heavy pasteboard ; *P*, ground-glass plate ; *w*, worm.

of the plate was maintained, but it could be moved in any direction in a practically horizontal plane, and could be rotated freely about its vertical axis. The worm could now be allowed to crawl freely on the glass plate. By means of the opaque screens *d* and *d'* (Figure 4), it was kept out of the influence of the light until it had straightened out and was moving well, whereupon, by revolving the plate if necessary, and moving it horizontally in the right direction, the worm (*w*) could be brought into the normal position, with its anterior end projecting about a centimeter beyond the directive axis, i. e., into the lighted area. In this way it crawled out into the lighted area at right angles to the direction of the rays of light, and without the disturbing thigmotactic influence of the sides of the glass runway existing in the previous experiments. When once in position, the glass plate was held stationary until the worm had crawled to the outer circle marked on the table, a distance of 15 cm. When it was desired to test the worm headed in the opposite direction, it was only necessary to shift the screens *d* and *d'* to the other side of the directive axis. This arrangement proved to be much more satisfactory than the one first employed, the results being free from the disturbing factor of thigmotaxis.



~~Color of~~ Light large
Animal 3
Time
Distance 15 cm.

(4)

MAY 11 1903

A. foetida

FIGURE 5. Record of five successive trials with an *earthworm* exposed to the influence of the large area of illumination and headed away from the observer. For fuller explanation see text.

The difference in the results in these two cases emphasizes the importance, in work of this nature, of bearing in mind and eliminating, as far as possible, all extraneous influences.

Before describing these experiments and their results, it may be well to describe in some detail the methods employed in keeping the records, since the same description will apply in the case of most of the animals studied. Figure 5 shows a record of five successive trials with a single worm, — in this case exposed to the influence of only one light (the one having the larger area), which was situated at the left. The worm is in all five cases headed away from the observer. It will be noticed that the circles on this printed blank are divided into divisions of 10° each, corresponding to the divisions of the circles on the table, so that it was an easy matter to plot the path of the crawling worm with considerable accuracy. The successive trials were numbered in the order in which they were made, from one to five. When the animals were headed in the opposite direction (toward the observer), similar blanks were used, but the zero point of the transverse, or normal, axis of the circles was the near end, not the far end. By this device the relation of the lights to the diagram is kept always the same; the large light being at the left, the small light at the right.

In giving numerical value to the angles at which the outer circle was crossed, the readings were, for convenience, grouped into as many classes as there were divisions of the circle. Each class was designated by one of the numbers, 0 to 18, and embraced all the records falling within 5° of the radius bearing the corresponding number. Its nature was further indicated by *Lg* or *Sm*, according as the radius in question lay on the side of the normal axis toward the large light or toward the small one. The dividing lines (radii) shown on the diagram therefore fall in the middle of their respective classes; thus, for example, the radius numbered 2, which marks a point 20° from the normal axis, is in the middle of class *Lg* 2, or *Sm* 2, as the case may be, and all crossings between 15° and 25° fall in this class. Crossings which happen to lie exactly midway between the lines numbered on the diagram are always put into the class which lies next to the right of the point of crossing; in other words, in the direction toward which the hands of a clock move. Thus a reading falling midway between 5 (50°) and 4 (40°) on the *Lg* side of the vertical would be put into class *Lg* 4; if it fell in a corresponding position on the *Sm* side of the vertical, it would be included in class *Sm* 5. When to the readings for an animal headed away from the observer are added those for the same animal headed in the opposite direction, — the latter would lie in the lower half of the circle, — the possible error resulting from this method of recording is counterbalanced and the effect thus eliminated.

From what has been said it can be seen at a glance that the five readings shown in the illustrative figure (Figure 5) fell into the following classes: one in class 0; one in class *Sm* 1; one in class *Sm* 3; and two in class *Sm* 5.

Something must be said with regard to the disposition to be made of those cases in which the worm turned more than 90°, and so crossed the circle at some point in the dark half of the field. There are three possible ways in which these cases might be treated. Either (1) ignore the response, (2) recognize as many classes in the dark field as in the light one, or (3) regard the response as equivalent to that of an animal crossing the circle in the light field at an equal distance from the directive axis. To ignore the response would obviously give unsatisfactory results, for such animals certainly exhibit strong responses either toward or from a given light. To create of them separate classes would also be undesirable, for the records should express the degree to which the worm turned from the normal, or, putting it the other way about, how closely it came into orientation in line with the axis of the lights. Since 90° (class 9) represents the extreme possibility of turning from the normal axis, the creation of further classes for crossings in the dark field would be misleading. The third possibility is obviously the one to be adopted. Thus, if an animal turned so far as to pass over the line at the point 13 on the side toward the small light, the trial was recorded in class *Sm* 5.

After the foregoing explanation the accompanying table (Table I) will be easily understood. This gives the results of 100 trials made with 10 individuals exposed to unilateral illumination, in this case to the large light. The 19 vertical columns, numbered each way beginning with 0 and going to 9, correspond to the classes described above. The individual numbered 3 (marked with an asterisk) is the same one that is represented in Figure 5. In the columns next beyond columns 9 are recorded the total number of crossings made on the *Lg* side of the 0 class and on the *Sm* side of that class, while in the columns next beyond these are indicated the number by which one of these exceeds the other. From this it can be seen at a glance that when the worms were headed away from the observer (left side to light), they crawled only 4 times toward the light (+ responses), 38 times away from the light (— responses), and 8 times straight ahead (indifferent responses). Thus there is an excess of 34 trials on the side of *Sm*. The records for the worms headed in the opposite direction (right side to light) are not so striking. These are: toward *Lg*, 17; toward *Sm*, 26; indifferent, 7, giving an excess of only 9 toward *Sm*. Adding these two sets together, we have: *Lg* (+), 21; *Sm* (—), 64; 0 (±), 15; excess in favor of *Sm*, 43. Since 100 trials were made altogether, 43 per cent may be taken as an index of the negative response of *Allolobophora foetida* to the large light alone under the conditions of the experiments.

The results of 100 trials under each of the three conditions, large light only, small light only, and both lights simultaneously, are summarized in Table II. The general arrangement of the data is similar to that in Table I, except that the details of individual reactions

TABLE I.

Reactions of *Earthworm* to Large Light. (*Experiments B and C combined.*)

	No. of the Individual.	Excess to *Lg.*	Total to *Lg.*	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	Total to *Sm.*	Excess to *Sm.*
Headed away from observer (left side to light).	1	..	..	..	..	..	..	..	..	..	..	..	..	..	4	..	..	1	..	..	..	..	5	
	2	..	..	..	..	..	..	..	..	..	..	..	1	1	1	1	1	..	..	..	..	..	4	
	3*	..	..	..	..	..	..	..	..	..	..	..	1	1	..	1	..	2	..	..	..	..	4	
	4	..	2	..	1	..	..	..	1	..	..	..	..	..	1	1	..	..	..	1	..	..	3	
	5	..	1	..	..	..	..	..	..	1	..	..	..	..	2	1	1	..	..	..	..	..	4	
	6	..	..	..	..	..	..	..	..	..	..	..	2	..	1	1	1	..	..	..	..	..	3	
	7	..	..	..	..	..	..	..	..	..	..	..	..	1	1	..	..	1	1	1	..	..	5	
	8	..	1	..	..	..	..	..	..	..	1	..	4	..	..	..	..	..	..	..	..	..	0	
	9	..	..	..	..	..	..	..	..	..	..	..	..	1	..	1	1	1	1	..	..	..	5	
	10	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	2	1	1	..	..	5	
Total	..	..	4	..	1	..	..	..	1	1	1	..	8	4	10	6	5	7	3	3			38	34
Headed toward observer (right side to light).	1	..	1	..	..	..	..	..	..	..	1	..	3	..	1	..	..	..	..	..	..	..	1	
	2	..	2	..	..	..	..	..	..	2	..	..	3	..	..	..	..	..	..	..	..	..	..	
	3	..	5	..	..	..	..	2	..	..	2	1	..	..	..	..	..	..	..	..	..	..	..	
	4	..	2	..	..	..	..	..	..	..	2	..	..	1	..	1	1	..	..	..	..	..	3	
	5	..	2	..	..	..	..	..	1	..	1	..	..	..	1	..	1	1	..	..	..	..	3	
	6	..	..	..	..	..	..	..	..	..	..	..	..	3	2	..	..	..	..	..	..	..	5	
	7	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3	1	..	1	..	..	..	5	
	8	..	..	..	..	..	..	..	..	..	1	..	..	1	1	1	1	..	1	..	..	..	5	
	9	..	3	..	1	..	..	..	..	..	..	1	1	1	..	..	..	..	..	..	..	..	1	
	10	..	2	..	..	..	..	1	..	1	..	..	..	..	1	1	1	..	..	..	..	..	3	
Total	..	..	17	..	1	..	..	3	1	3	7	2	7	6	6	6	5	1	2	..	..	..	26	9
Total, both positions	..	..	21	..	2	..	..	3	2	4	8	2	15	10	16	12	10	8	5	3	..	..	64	43

have been omitted. For the sake of brevity the worms are spoken of as "headed north" or "headed south," in place of "headed away from observer," and "headed toward observer," respectively.

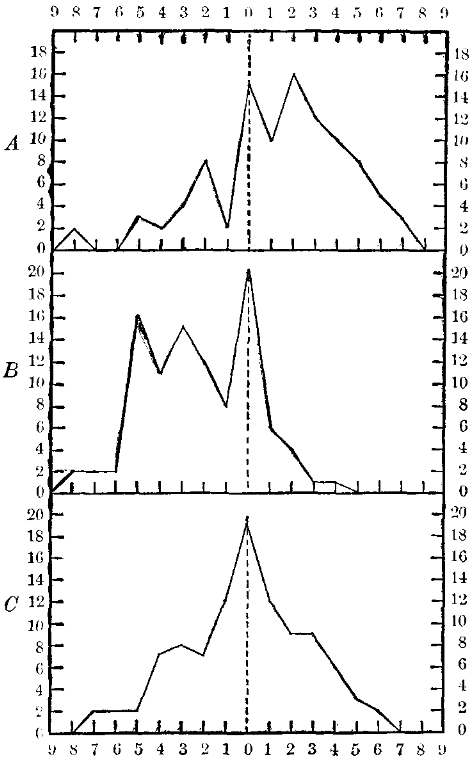


FIGURE 6. Frequency polygon showing the distribution of reactions of the *earthworm* to (*A*) the large light alone, (*B*) the small light alone, and (*C*) both lights used simultaneously (100 trials under each condition).

The first of these sets of records — that resulting from the use of the large light alone — has just been discussed in commenting on Table I. The per cent of negative reactions to the *small* light alone is even larger than that to the large light alone. It will be noted that the worms turned 68 times to *Lg* (—) and only 12 times to *Sm* (+), making an excess of 56 times, or 56 per cent, away from the light. In the case of *both* lights, however, the records in the two directions nearly balance. They read: to *Lg*, 40; to *Sm* 41; indifferent 19. This leaves an excess of only one record, or 1 per cent of the whole, in favor of the side toward the small light.

By regarding merely the total number of crossings at one side or the other of the normal, no account is taken of the degree of divergence from that line. This, however, is shown within limits of 10° by the segregation of the records into classes, as shown in the tables; but it may be more quickly perceived by the use of a graphic method — by plotting the records in the form of frequency polygons, as seen in Figure 6. Along the abscissae are laid off 19 divisions, corresponding to the 19 classes, as in the tables. Here, too, the relative positions of the lights are always the same — the large light at the

TABLE II.

REACTIONS OF *Earthworm* TO LARGE LIGHT ALONE, TO SMALL LIGHT ALONE, AND TO BOTH LIGHTS SIMULTANEOUSLY.
(*Experiments B and C combined.*)

Lights used.	Direction headed.	Excess to *Lg.*	Total to *Lg.*	CLASSES.																			Total to *Sm.*	Excess to *Sm.*
				9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9		
Large	No.	. .	4	. .	1	. .	. .	. .	1	1	1	. .	8	4	10	6	5	7	3	3	. .	. .	38	34
	So.	. .	17	. .	1	. .	. .	3	1	3	7	2	7	6	6	6	5	1	2	. .	. .	. .	26	9
Total	.	. .	21	. .	2	. .	. .	3	2	4	8	2	15	10	16	12	10	8	5	3	. .	. .	64	43
Small	No.	28	34	. .	2	1	1	7	5	9	6	3	10	3	1	1	1	. .	. .	. .	. .	. .	6	. .
	So.	28	34	. .	. .	1	1	9	6	6	6	5	10	3	3	. .	. .	. .	. .	. .	. .	. .	6	. .
Total	. .	56	68	. .	2	2	2	16	11	15	12	8	20	6	4	1	1	. .	. .	. .	. .	. .	12	. .
Both	No.	6	23	. .	. .	2	1	1	4	4	4	7	10	7	2	3	2	2	1	. .	. .	. .	17	. .
	So.	. .	17	. .	. .	. .	1	1	3	4	3	5	9	5	7	6	4	1	1	. .	. .	. .	24	7
Total	. .	. .	40	. .	. .	2	2	2	7	8	7	12	19	12	9	9	6	3	2	. .	. .	. .	41	1

left, the small at the right. The number of responses falling in each class are recorded on the corresponding ordinates. In the first polygon (Figure 6, *A*) are represented the experiments with the large light alone (as recorded in Table II), where, it will be noticed, the polygon lies mainly on the side away from the light; similarly the second polygon (Figure 6, *B*) lies chiefly on the side away from the small light, which was alone used in this case. The third figure (Figure 6, *C*) represents the data when both lights were used, and is remarkably symmetrical, with its mode at 0. This indicates a condition as nearly balanced as could well be expected.

These results on the earthworm would appear to admit of only one interpretation, namely, that the *intensity* of the light is the controlling factor in its reactions and not the size of the field from which the light is derived. The latter, in fact, appears to have no influence whatever. This is in exact accordance with what would be expected in the case of an eyeless animal. It possesses no organs so differentiated as to be able to discriminate space relations in objects beyond its touch; there is no mechanism for preserving these relations in space, as the light from the different parts of the object falls upon the sensitive portion of the animal, and consequently there is no suggestion of image formation. The light stimulates according to its intensity only, and since the intensity is the same on each side, the animal responds by approximately an equal number of deviations in each direction.[6] It may be asked why, if the amount of stimulation on the two sides is balanced, the worm does not always crawl in a straight line, deviating neither one way nor the other, and always cross the circle so as to fall in the 0 class. This is probably for the same reasons that with unilateral illumination the worms do not uniformly turn away from the light. Similarly Parker and Arkin (:**01**) pointed out that in their experiments only 30.2 per cent of the "head movements" were away from the light, 65.6 per cent being straight ahead and 4.2 per cent toward the light. These authors attributed the movements toward the light to the uneven surface of the paper on which the worms were allowed to crawl, and to other disturbing influences. In view of Harper's recent work (:**05**), it would seem that conditions of uneven or one-sided expansion or contraction of the animal might have some influence in producing these results, by exposing the deep-lying cells — which are supposed to be the organs stimulated — unequally to the light. Undoubtedly, too, these animals are not the simple mechanisms that we are often inclined to consider them, but are subject to certain internal conditions — physiological states, if we care to call them so — which are beyond our observation and control, and by their changes produce corresponding irregularities in the results. On this account it is important to use animals for experimentation that have previously been subjected for a considerable period to constant conditions.

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[6] Loeb (:**05**, p. 2, footnote) maintains that "if two sources of light of equal intensity and distance act simultaneously upon a heliotropic animal, the animal puts its median plane at right angles to the line connecting the two sources of light." This statement needs a number of qualifications.

2. *Land Planarian* (*Bipalium kewense* Moseley).

The so-called eyes of planarians are remarkably uniform throughout the group in general plan of construction. Each consists of one to several sense elements, or cells, backed by a pigmented cup, which is in turn composed of one or a few cells. These eyes have no apparatus for the formation of an image, but are constructed in such a way that generally light from only one direction can affect a single eye at a given time. For this reason they are often spoken of as *direction eyes*. In the genera Planaria, Dendrocoelum, and related forms, only two eyes are present; but in others, such as Polycelis and the land planarians, eyes may be present in great numbers.

Bipalium is a well-known genus, comprising a large number of species, mostly indigenous to Japan, China, India, Ceylon, and the Malay Archipelago. There is well established in certain greenhouses in Cambridge a species, the original home of which is unknown. This species was first described by Moseley (**'78**) from a specimen obtained in the Kew Gardens, England, and was named by him *Bipalium kewense.* Since that time it has been found in hothouses in widely separated localities in nearly all parts of the globe, and it is generally believed to have been unintentionally distributed with consignments of plants, either from its original home or from other places where it had become established. Woodworth (**'96**) reported it from Cambridge, Mass., in 1896. A form described from Landsdowne, Pa., by Sharp (**'91**) as *Bipalium manubriatum* is probably the same species; and Woodworth (**'98**) has since reported it from other localities in the United States (Baltimore, Md., Pittsburg and Allegheny, Pa., and Springfield, Ohio).

Bipalium kewense is the largest of the land planarians, the specimens ordinarily ranging from 12 cm. or 13 cm. to 25 cm. in length; and Woodworth (**'96**) records one 30 cm. long. These measurements are those of the worm in the extended condition, which is assumed in crawling. The body is capable of great contraction, so that a worm which is, when crawling, 25 cm. long, may contract to a length of only 5 or 6 cm. (see figures given by Bell, '**86**, pl. 18). The head when extended is semilunar, or shaped like a battle-axe (see Figure 9, p. 364), and is about twice as broad as the body immediately behind it. The eyes have been described in other species of Bipalium by Moseley ('**74**, p 144) and in *B. kewense* by Bergendal ('**87**) and Jänichen ('**96**, p. 275; Taf. 10, Figur 18; Taf. 11, Figur 21). Bergendal ('**87**, p. 49) describes the position and arrangement of the eyes as follows: "Eyes occur in this species in enormous numbers. They form a zone of three or four rows near the margin of the head, and are also placed on the sides (not on the back) of the whole body, even to the hindmost end." The largest eyes are on the head and immediately behind it, and they are closer together in those regions than on the body farther back. According to Jänichen ('**96**, p. 276), each pigment cup has the form of a hollow hemisphere, the convex side of which is turned inward, while the flat side, which is open, is directed outward. The pigment spot, which forms the background of the sensory cells, is composed of a single cell. From three to six flattened sense cells lie in the pigment cup, and, as in Polycelis, the eyes farther posteriorly are not only smaller, but usually have a smaller number of sense cells.

Hesse ('**97**) has discussed from a theoretical standpoint the manner in which the eyes of planarians probably function. He deals in this case with those forms which possess only two eyes. The diagrams which he gives ('**97**, p. 575) are reproduced in Figure 7, and require only a brief explanation. When there is only a single sense cell in the pigment cup, it may be stimulated by light coming from various angles, covering a considerable range, that is, by any light which is able to enter the cup. Thus the effect would be the same in the left of the two eyes represented in the diagram for light coming in any of the directions shown by the arrows in *A*, *B*, *C*, or *D*. If, however, there were a number of sense cells in the cup, only a part of these would be reached by the light coming from any one of the directions shown, and the light would strike different cells according to the direction from which it came. In the first case, the structure of the eye would not furnish any opportunity for determining the direction of the light within the range of the arrows in *A* to *D*; in the second case it would, since under the conditions shown in *A* the cells on one side of the cup (posterior) would receive the light, whereas in *D* those of the other side would receive it. This condition may be considered as the beginning of a crude image-forming apparatus, the eye approximating in a rough way the concave mosaic type. These are theoretical deductions from the anatomical conditions; no test has been made to show whether the physiological action supports the hypothesis. The eyes of Bipalium are so small, and their axes so variously directed, that it seems doubtful if the individual eyes can have more than a very general directive function in the reception of light. Although in general they open directly outward, — as mentioned by Jänichen ('96), and as may be seen in the photograph of a portion of the border of the head reproduced in Figure 8, — they are nevertheless not all accurately oriented in that way; furthermore, the position and direction of each eye naturally varies with the movements and contractions of the animal. In general, however, they do open outward, as is represented diagrammatically in Figure 9. It will be noticed that the eyes of any particular region are so arranged that they usually receive light exclusively from a direction normal to that part of the surface of the head beneath which they lie, as indicated by the arrows. The efficiency of

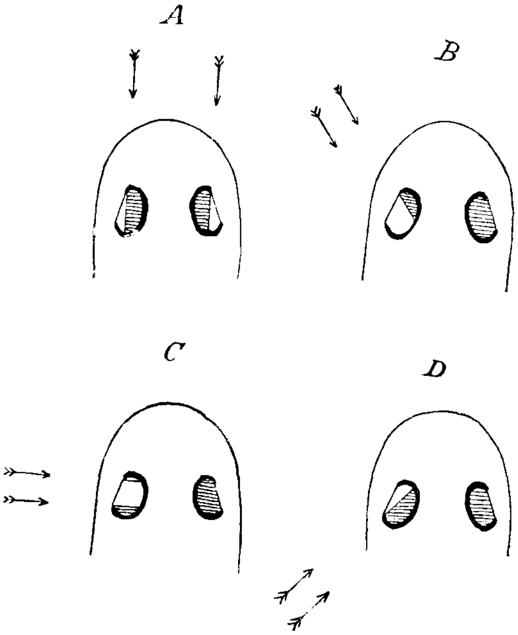

FIGURE 7. Various illuminations of the *planarian* eye with light coming from different directions. The optic cups are drawn too large in proportion to the size of the head. The arrows indicate the direction of the light; the portions of the interior of the cup not reached by the light are shaded. — From Hesse, '97.

this arrangement is greatly increased by the lateral extension of the head, which allows room for a greatly increased number of eyes in its anterior border. Taken as a whole, this arrangement might be roughly compared to a single convex mosaic eye, such as is found, for example, in the Entomostraca.

FIGURE 8. Photomicrograph of a portion of the margin of the head of *Bipalium*, showing the position and arrangement of the eyes. × 70.

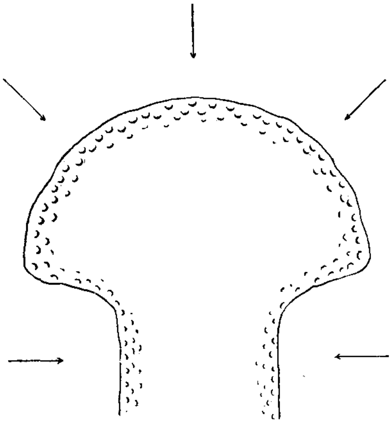

FIGURE 9. Diagram of the head of *Bipalium*, to show how light coming from different directions (indicated by the arrows) may affect only certain of the eyes.

Below are described the results of physiological tests with lights of different sizes, in an attempt to determine in how far the eyes may act in this way.

It is a well-known fact that most planarians avoid the light (the exceptions being certain chlorophyll-bearing forms), and they have long been used for work in phototropism. Bipalium appears never to have been used for experimental purposes, although the fact that the land planarians share with the water inhabiting forms aversion to light was remarked upon more than sixty years ago by Darwin ('**44**). That they are also nocturnal in their habits has been commented upon by various authors since that time. Bell ('**86**, p. 168) remarks that "There can be no doubt as to the sensitiveness of Bipalium to light," but he referred only to what he supposed was the effect of strong light in causing the worm to break up into a number of pieces, and not to the directive action of the light. As a matter of fact, *Bipalium kewense* is exceedingly sensitive to light, of even a very low intensity, falling upon it from the side, and responds immediately by turning away from the light. For this reason, and because it is easy to keep and to handle, it is an excellent animal for experimental purposes. Like most planarians, it creeps with an even, gliding motion, the head being slightly raised and waved to right and left, apparently in searching movements, as the worm crawls forward. In the daytime this Bipalium is usually to be found coiled up underneath flower pots which sit on the ground in warm, moist rooms of greenhouses; and it is probable that it comes out and moves around only at night. What it eats appears not to be known with certainty, but the common opinion seems to be that, like other turbellarians, it is carnivorous. Some of the related forms are known to eat earthworms, and certain authors believe the same to be true of *Bipalium kewense*.

Description of Experiments. The experiments on Bipalium were conducted in the same manner as those on the earthworm, except that the ground glass on which the animals were placed was supported about 1 cm. above the table by means of small wooden blocks, or feet, glued to the under surface of the glass at its four corners, instead of being swung free by strings from the ceiling. By lifting the plate very slightly, it could be turned easily in any direction. This had the advantage that when the worm was once in the proper position, the wooden feet could be brought in contact with the table again, the plate thus remaining perfectly steady and horizontal. When the worm was creeping well in the shaded area, the plate was moved so that about 5 mm. of the anterior end of the worm was brought out, in the normal position, into the influence of the light, and the record was taken when the animal's head reached the *first* circle, instead of the third, as in the case of the earthworm. It thus crawled a distance of only 4.5 cm. at each trial; but the reactions of Bipalium are usually so immediate and definite that this distance seemed sufficient for the purpose of the experiment.

The results of 10 trials each, with 10 worms (100 trials in all), when both lights were operative, are shown in Table III. It will be noticed that more than half of the 100 trials fall in the 0 class, and that only a single record falls more than two classes to right or left of that position. This means that in only one instance did a worm deviate more than 25° from the normal axis, thus showing a remakably well ballanced condition of all stimuli. The excess of responses towards *Sm* over those towards *Lg* is, however, comparatively large, being 17 per cent. The real nature of the results, with the conspicuous "mode" at 0, may be more readily appreciated when the data are plotted in a frequency polygon, as has been done in Figure 10. These results indicate that Bipalium has, to a slight extent, the ability to appreciate differences in area, since it responds by turning away from the larger luminous area more often than from the smaller. It is possible that a considerably larger area, even though the total amount of light were no greater, might have a more decided effect. If the arrangement of the eyes around the circumference of the head acts, as has been suggested, in a manner similar to a single so-called mosaic eye, forming roughly what Exner terms an apposition image, it can be seen that light must come from rather widely different directions in order to enter an appreciably larger number of the cups than would be affected by light from a single point. This is owing to the comparatively large semicircle of the head and the irregularity in arrangement of the pigmented cups, as well as to their shallowness and their great distance apart, as compared with the ommatidia of, e. g., an insect's eye. It is not suprising, then, that light from the different parts of a field only 41 cm. across and 2 meters away (the most extreme rays making with each other an angle of only about 12°) produces little noticeable effect in the reactions of such a worm.

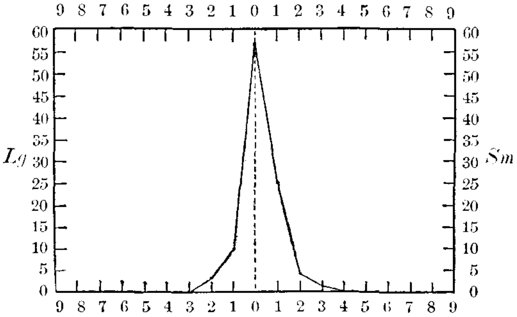

FIGURE 10. Frequency polygon constructed from the results with *Bipalium* shown in Table III (100 trials to both lights used simultaneously).

After the above set of experiments had been made it was found that a disturbing factor had influenced some of the work with Bipalium. This factor was a light draught of air sweeping the table

TABLE III.

REACTIONS OF *Bipalium kewense* TO BOTH LIGHTS.

Lights used.	Direction headed.	Excess to *Lg.*	Total to *Lg.*	CLASSES.																			Total to *Sm.*	Excess to *Sm.*
				9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9		
Both	No.	. . .	9	. .	. .	. .	. .	. .	. .	. .	3	6	28	11	1	1	. .	. .	. .	. .	. .	. .	13	4
	So.	. . .	4	. .	. .	. .	. .	. .	. .	. .	. .	4	29	14	3	. .	. .	. .	. .	. .	. .	. .	17	13
Total	. .	. . .	13	. .	. .	. .	. .	. .	. .	. .	3	10	57	25	4	1	. .	. .	. .	. .	. .	. .	30	17

from left to right; it came from a ventilator in the south wall of the room, which was sometimes open, sometimes closed. No record was made as to whether it was open or closed when the above experiments were made; but in the light of subsequent experiments made to test the effects of the draught the records themselves seem to show that it must have been closed. This appears from the fact that, in the experiments made to test the effect of this current of air, it was found that the worms were very sensitive to it, and that in all cases when the ventilator was open the "mode" for the reaction was pushed over to the right of the 0 class, the light being in these tests purposely non-directive, as it came from a lamp directly above the normal axis of the circle.

3. *Mealworm* (*Larva of Tenebrio molitor* Linn.).

The negative phototropism of the mealworm has been commented upon by Loeb ('**90**, pp. 84, 85), who found that these animals not only moved away from the side of a dish from which the light came but that they remained in the darkened half of the dish when the plane between the dark and the light portions coincided with the direction of the rays. A few responses to a single light as used in these experiments confirmed Loeb's conclusions that the animals are markedly negative in their reactions to light; consequently a large series of experiments was made to determine their reactions to the two lights of different area.

The larvae were placed, one at a time, upon the ground-glass plate used in the previous experiments, which, as in the case of Bipalium, was supported about 1 cm. above the table by means of small wooden feet. As with Bipalium, too, the records were taken when the larva reached the first, or inner, circle, so that it crawled a distance of only about 5 cm. In the earlier experiments the glass plate was kept moist in the belief that the moisture caused the larva to keep up its crawling motion; but later the plate was used dry without essential difference in the results. The animals usually started promptly and crawled rather rapidly and uniformly, as soon as placed on the plate, but in some instances they showed considerable hesitancy and made trials in various directions before going ahead.

They appear to have three characteristic modes of progression: (1) The most usual manner is by means of the legs alone, the posterior portion of the body being dragged along passively. This part may be held straight, but sometimes it is slightly curved to one side, in which case it is possible that its position influences to some extent the direction in which the animal turns. (2) The larva occasionally crawls by alternate elongation and contraction of the body (made possible by the telescoping movements of the segments), much as an earthworm crawls, and at such times the anterior end is often raised into the air and waved about more or less. (3) The larva may also move backward for a short distance at about the same rate that it normally crawls forward. It seldom goes backward more than two or three centimeters, and then starts forward again, apparently *always turning in one direction or the other*, and not going forward in the same line in which it was moving backward. This last reminds one somewhat of the motor reaction described for Protozoa and some other animals by Jennings; but whether it is used for the same purpose in response to strong stimulation or other conditions does not concern us here, since it is not the usual method of orientation to light of moderate intensity.

In the experiments with light falling upon one side only, the larvae were given five trials headed in one direction, and then five headed in the opposite direction; but in the trials with both lights at once they were headed alternately in the two directions, — one trial in one direction and the next in the opposite. Since in both cases there are the same number of trials in each direction, it is not probable that this difference of method affects the results. Only 50 trials each were made with the two lights separately, whereas 200 trials were made with both operating at the same time. The same five individuals were employed in the trials with the lights used separately and in 100 of the 200 trials with both lights together; in the remaining trials with both lights five other larvae were employed. The results of the experiments on the larvae of *Tenebrio molitor* are classified and summarized in Table IV.

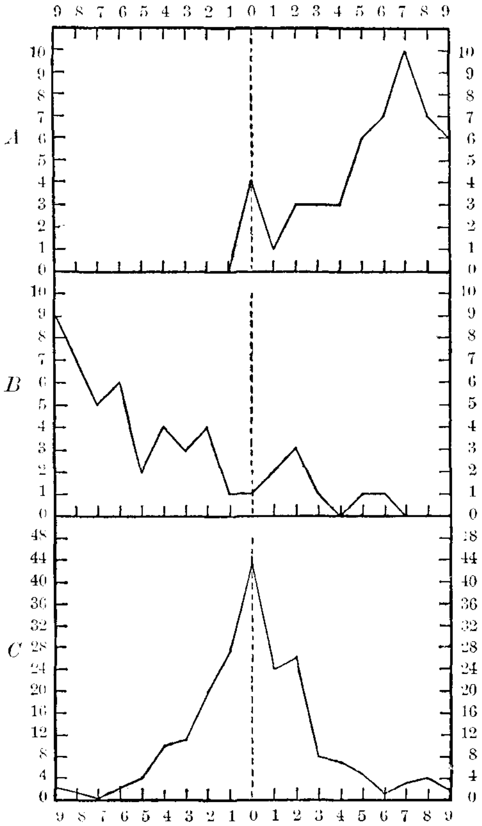

FIGURE 11. Frequency polygon constructed from results with the *mealworm* shown in Table IV. *A*, 50 trials to large light alone; *B*, 50 trials to small light alone; *C*, 200 trials to both lights used simultaneously.

The preponderance of reactions away from the light is strikingly shown in the first two sets of experiments, where unilateral illumination was tested. With the large light there was not a single positive reaction, and only 4 out of 50 fell in the 0 class, so that 46, or 92 per cent, of the reactions can be counted as distinctly negative responses. The records with the small light alone are somewhat more scattering, but are nevertheless preponderatingly negative. Only 8 records are towards the light, while in 41 out of the 50 trials the animals turned towards the dark, giving an excess of 33, or 66 per cent, in that direction. The responses when the two lights act simultaneously are almost equally balanced as between the two sides, by far the larger number of records on each side falling near the 0 class, which itself contains 43 of the 200 records, or nearly one fourth of the whole number. This result may be much more quickly and completely grasped from an inspection of Figure 11, *C*. On the side towards the

TABLE IV.

Reactions of the *Mealworm* to Large Light, to Small Light, and to Both Lights.

Lights used.	Direction headed.	Per cent of Excess to *Lg.*	Excess to *Lg.*	Total to *Lg.*	Classes.																			Total to *Sm.*	Excess to *Sm.*	Per cent of Excess to *Sm.*
					9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9			
Large	No.	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	1	. .	1	1	2	3	1	5	7	4	24	24	96
	So.	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	3	1	2	2	1	3	6	5	. .	2	22	22	88
Total	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	4	1	3	3	3	6	7	10	7	6	46	46	92
Small	No.	40	10	17	3	2	3	3	1	1	2	2	. .	1	2	2	1	. .	1	1	. .	. .	. .	7	. .	. .
	So.	92	23	24	6	5	2	3	1	3	1	2	1	. .	. .	1	. .	. .	. .	. .	. .	. .	. .	1	. .	. .
Total	. .	66	33	41	9	7	5	6	2	4	3	4	1	1	2	3	1	. .	1	1	. .	. .	. .	8	. .	. .
Both	No.	. .	. .	39	1	1	. .	. .	2	4	5	11	15	14	13	14	4	6	4	. .	. .	4	2	47	8	8
	So.	5	5	38	1	. .	. .	2	2	6	6	9	12	29	11	12	4	1	1	1	3	. .	. .	33	. .	. .
Total	. .	. .	. .	77	2	1	. .	2	4	10	11	20	27	43	24	26	8	7	5	1	3	4	2	80	3	1.5

large light are 77 records; on that towards the small light 80, giving an excess of only 3 records in the direction of the small light, which is only 1.5 per cent of the total number of trials.

Polygons *A* and *B* are plotted from the data of the experiments with one-sided illumination, when the large (*A*) and small (*B*) lights were used separately.

The conclusions to be drawn from the reactions of the larvae of *Tenebrio molitor* are, it would seem, as follows. These animals are almost uniformly negative to unilateral illumination in light of moderate intensity. The ability of the eyes to form distinctive images of objects differing considerably in size is wholly lacking, or at least practically so, as is to be inferred from the reactions of the animals when exposed to the simultaneous influence of the two light areas used in these experiments. It is perhaps worth noting that the excess of 1.5 per cent is away from the large light, and that in no case did the larvae turn towards the large light when that light was used alone, whereas a few did turn towards the small light when that was the only one used. Still, it would not be very surprising if in a repetition of this series of experiments the balance should lie on the opposite side of the zero class. The magnitude of this difference is probably too small to be of significance.

The above results are what would be expected in view of the rudimentary condition of the eyes in the mealworm. The eyes apparently consist of only two or three ocelli on each side of the head, arranged in a vertical row immediately behind the base of the antennae. Examination of the chitin immediately overlying them, even after it had been boiled in caustic potash, has shown no thickenings or differentiations that might serve as lenses. The eyes are so small that they can scarcely be seen without the aid of a lens.

Loeb ('90) has remarked on the relation existing between the reactions of these animals to light and moisture and their natural habits.

4. *Sow Bug* (*Oniscus asellus* Linn.).

This widely distributed, active little isopod may be found in abundance beneath stones, bark, pieces of wood, etc., in the woods in the vicinity of Cambridge. Its retiring habits, indicating an avoidance of light, and the ease with which it can be kept and handled in the laboratory, suggested that it might be a suitable form for use in these experiments. Handling often has at first an inhibitory effect as regards locomotion, but this is usually overcome in a short time, and the animals then normally start off as soon as released. The eyes are small, but still readily visible, each consisting of a group of about 20 ocelli situated on the side of the head at the base of the antero-lateral lobe.

The animals were oriented in the proper position by placing them within a small rectangular glass frame without top, the glass being covered with black paper to exclude the light until the pen thus made was lifted up. The frame was oblong, and just large enough to enclose the Oniscus easily without allowing it to turn around. With a little care the frame could be readily moved on the table (the glass plate was not used) to the position desired, and with the animal headed in either direction. When it was lifted off, the animal was left exposed to the lateral influence of one or the other or both of the lights, as the case might be, but free to move in any direction. The records indicate the place where it crossed the second of the three circles inscribed on the table, — the one with a radius of ten centimeters. As the animals usually took a straight, or only slightly curved course, this was practically the distance travelled by them in each of the trials.

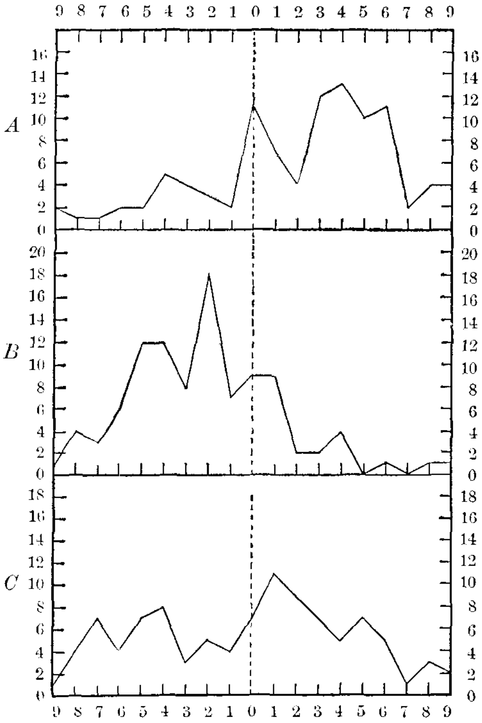

FIGURE 12. Frequency polygon constructed from results with *Oniscus* shown in Table V. *A*, reactions to large light alone; *B*, to small light alone; *C*, to both lights used simultaneously (100 trials under each condition).

The experiments with the lights used singly showed Oniscus to be decidedly negative in the character of its reaction, but by no means so strikingly so as the Tenebrio larva. In Oniscus, as may be seen from an inspection of Table V, only 45 per cent to 51 per cent of the reac-

TABLE V.

REACTIONS OF *Oniscus asellus* TO LARGE LIGHT, TO SMALL LIGHT, AND TO BOTH LIGHTS.

Lights used.	Direction headed.	Per cent of Excess to *Lg.*	Excess to *Lg.*	Total to *Lg.*	CLASSES.																			Total to *Sm.*	Excess to *Sm.*	Per cent of Excess to *Sm.*
					9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9			
Large	No.	. .	. .	9	1	. .	. .	. .	1	3	2	1	1	4	2	2	7	6	7	8	2	1	2	37	28	56
	So.	. .	. .	13	1	1	1	2	1	2	2	2	1	7	5	2	5	7	3	3	. .	3	2	30	17	34
Total . .	. .	. .	. .	22	2	1	1	2	2	5	4	3	2	11	7	4	12	13	10	11	2	4	4	67	45	45
Small	No.	50	25	35	. .	1	1	2	7	7	4	8	5	5	7	. .	2	1	. .	. .	. .	. .	. .	10	. .	. .
	So.	52	26	36	1	3	2	4	5	5	4	10	2	4	2	2	. .	3	. .	1	. .	1	1	10	. .	. .
Total . .	. .	51	51	71	1	4	3	6	12	12	8	18	7	9	9	2	2	4	. .	1	. .	1	1	20	. .	. .
Both	No.	. .	. .	15	1	2	3	1	2	2	1	2	1	4	7	7	1	5	3	5	. .	1	2	31	16	32
	So.	18	9	28	. .	2	4	3	5	6	2	3	3	3	4	2	6	. .	4	. .	1	2	. .	19	. .	. .

| Total . . | . . | . . | . . | 43 | 1 | 4 | 7 | 4 | 7 | 8 | 3 | 5 | 4 | 7 | 11 | 9 | 7 | 5 | 7 | 5 | 1 | 3 | 2 | 50 | 7 | 7 |

tions were away from the light in excess of those toward the light, whereas in Tenebrio the excess in this direction ranged from 66 per cent to 92 per cent of the total number of trials. The results of the experiments on Oniscus are represented graphically in Figure 12.

In the trials with both lights the records on the two sides are nearly balanced, there being an excess of only 7 per cent of the reactions away from the larger light. This excess again lies in the direction we should expect if the eyes enable the animal to discriminate between the two luminous areas, but, as in the case of Tenebrio, it is too small to be of great significance. It will be noticed by comparing the figures obtained with the large and small lights used separately that the result is the opposite of that obtained with the two lights acting simultaneously, and also opposite to the results in the case of Tenebrio, for in Oniscus a larger percentage turned away from the small light than from the large one. Since the experiments on Tenebrio and on Oniscus were performed at different times, it is possible that a difference in the (absolute, not relative) intensities of the lights at the two times may be responsible for the want of harmony in the results. When both lights were used at the same time, they were adjusted so that their intensities were equal, and therefore no errors could have arisen from this source.

It may be concluded from these experiments that, although Oniscus has apparently much better developed eyes than the larva of Tenebrio, its responses to light are of a considerably less definite character. This means that with unilateral illumination Tenebrio turns toward the light much oftener than Oniscus, and although in a frequency polygon representing the reactions to light the mode for each of the animals falls upon the side away from the light, in the case of Oniscus it falls much nearer to the median (indifferent) line than it does in Tenebrio. This may be seen by a comparison of Figures 11 and 12. The greater delicacy of the adjustment to light conditions in Tenebrio is also shown in the reactions to the two lights used simultaneously. There the responses of this animal are concentrated about the indifferent or zero class (Figure 11, *C*), whereas in Oniscus under similar conditions (Figure 12, *C*) they are scattered much more evenly through the nineteen classes. This should not be taken to mean that Tenebrio probably has better vision than Oniscus, but merely that in nature its phototropism is more strongly negative, a state which may be independent of the acuteness of vision, but is commonly related to the conditions of existence. Whatever evidence is afforded by these experiments as to the comparative ability of the eyes of Tenebrio larvae and of Oniscus to form images lies in the fact of the apparently greater efficiency of the eyes of the latter animal in distinguishing between a large and a small area — an efficiency which is indicated in one case by 7 per cent, in the other by 1.5 per cent. If Oniscus were naturally as sensitive in its responses to light as Tenebrio, this difference would probably be greater. In other words, the image-forming power of the eye of Oniscus probably exceeds that of Tenebrio more than appears in this method of experimentation, owing to the fact that even to unilateral illumination the responses of the former are so much less definite than those of the latter.

5. *Cockroach* (*Periplaneta americana* Linn.).

This cockroach, which abounds in slaughter-houses and other similar situations where there is much waste matter that can serve as food, possesses large compound eyes with a great number of ommatidia. Latter (:**04**, p. 88), in speaking of the related species, *P. orientalis*, says: "To what extent the compound eyes are capable of forming a distinct image of surrounding objects we cannot say, but it is evident that they are keenly sensitive to differences of light and shade from the speed with which a cockroach makes for dark corners and crevices when disturbed." This characteristic of cockroaches is very evident to any one who has ever observed them. If a number of them are in a glass jar, the cover may be removed with comparatively little danger that any of them will escape so long as the mouth of the jar is turned toward the light; but if its position is reversed, they will usually escape from it very quickly. These and similar observations led to the selection of this animal also for experimentation with the lights of different area.

Cockroaches are so active and comparatively difficult to handle that it took considerable experimenting to devise a scheme for getting them oriented in the proper position without the introduction of disturbing factors. If one attempts to liberate them from the hand, their struggles usually result in their being in an undesirable position when released, instead of exactly in the normal position. Finally, after employing several unsuccessful devices, the following was made use of. A glass cylinder some 15 centimeters in diameter and 20 centimeters high was placed endwise on the table and so arranged that from one side of the bottom a low, narrow glass runway led off for a distance of about 5 centimeters. This runway was large enough to allow a cockroach to pass through it easily, but not large enough to permit the insect to turn about. It was covered with black opaque paper, which cut off the light and made its interior dark. This apparatus could be placed with the axis of the runway directly over the normal axis of the diagram on the table and so oriented that its anterior end was directed either north or south at will and coincided with the directive axis. The animal to be tested was then dropped into the cylinder and made to move about until at length it entered the runway. When it came out at the other end of the runway, it was properly oriented, with one light at its right, the other at its left. Some of the most active individuals would run through this passage and emerge without stopping; but there was much individual variation in their behavior in this respect. By far the larger number of them stopped in the runway as soon as their antennae projected from the farther end, and there they would remain, waving the antennae about or cleaning them by drawing them through their jaws, but refusing to go farther without being stimulated from the rear. Such individuals were stimulated by pushing a small block of wood into the runway behind them until it came against the cerci or the hind legs. The block was made nearly as broad as the runway in order that it might strike the animal evenly and not stimulate one side more than the other. Usually the slightest touch was sufficient to start the insect from the runway, but some individuals seemed very reluctant to leave, and in a few instances, immediately they were outside, they turned about and regained the shelter they had just left. Most of them, however, as soon as outside, ran ahead steadily and rapidly, either going nearly straight or swerving to one side or the other; others would hesitate or even stop altogether, and then make sharp turns, often going off finally in a direction entirely different from that which they first took when they came out of the runway. The records indicate the points at which they crossed the outer circle — the one having a radius of 15 centimeters.

Table VI summarizes the results of 98 trials with the large light alone and 200 each with the small light alone and with the two lights used simultaneously. As in the preceding experiments, Figure 13 shows the records in graphic form.

These records show that Periplaneta is decidedly negative to light from one side, — 42.8 per cent of the reactions may be so regarded in the case of the large light and 55 per cent in that of the small light. Here the peculiarity will be noticed that, as in the case of Oniscus, there was a larger per cent of reactions away from the small light than away from the large one. Moreover, in the cockroach, as may be seen from the lower third of Table VI and from Figure 13, *C*, there were more turnings from the small light than from the large one, when both were used. This excess is very small, it is true, — only 9 reactions in

TABLE VI.

Reactions of the *Cockroach* to Large Light, to Small Light, and to Both Lights.

Lights used.	Direction headed.	Per cent of Excess to *Lg.*	Excess to *Lg.*	Total to *Lg.*	Classes.																			Total to *Sm.*	Excess to *Sm.*	Per cent of Excess to *Sm.*
					9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9			
Large	No.	. .	. .	12	. .	. .	2	. .	1	3	. .	1	5	3	2	9	8	1	4	4	2	1	3	34	22	44.8
	So.	. .	. .	12	. .	. .	2	3	2	2	. .	1	2	5	5	9	10	4	2	1	. .	1	. .	32	20	40.8
Total . .	. .	. .	. .	24	. .	. .	4	3	3	5	. .	2	7	8	7	18	18	5	6	5	2	2	3	66	42	42.8
Small	No.	47	47	66	. .	1	1	1	4	14	14	16	15	15	5	2	2	2	6	. .	. .	. .	2	19	. .	. .
	So.	63	63	77	4	3	2	5	7	19	16	16	5	9	3	4	2	3	. .	. .	2	. .	. .	14	. .	. .
Total . .	. .	55	110	143	4	4	3	6	11	33	30	32	20	24	8	6	4	5	6	. .	2	. .	2	33	. .	. .
Both	No.	. .	. .	33	1	2	1	1	1	4	2	6	15	27	23	8	6	1	. .	. .	2	. .	. .	40	7	7
	So.	16	16	40	. .	. .	2	. .	1	8	3	12	14	36	11	8	2	3	. .	. .	. .	. .	. .	24	. .	. .
Total . .	. .	4.5	9	73	1	2	3	1	2	12	5	18	29	63	34	16	8	4	. .	. .	2	. .	. .	64	. .	. .

200, or 4.5 per cent, — but it is difficult to explain why they are in the direction of the larger light, instead of the reverse. In the first set of 100 trials with both lights the records of particular individuals were not kept separate, and as it was thought that possibly some abnormal animal had influenced the total, the whole series was repeated on another set of five individuals, these matters being kept in mind; but the results were entirely comparable with those obtained in the first set of experiments. Such being the case, the two sets of records were combined, and are so represented in the table and the diagram.

The only plausible explanation of this unexpected result which offers itself at the present time is, that at least a portion of the animals were influenced by other factors as well as by the light from the two primary sources. Cockroaches, if undisturbed, are apt to make themselves at home wherever they are put, and usually soon settle down to cleaning their antennae or to making an inspection of their surroundings, apparently irrespective of such influences as steady directive light. It has already been mentioned how the animals came to rest at the end of the runway, waving the antennae about or cleaning them. Similarly, they

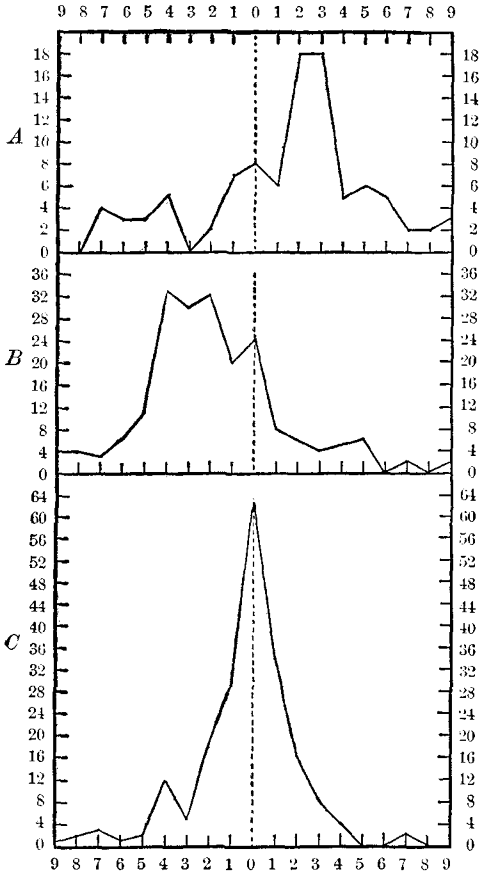

FIGURE 13. Frequency polygon constructed from results with *cockroach* shown in Table VI. *A*, 98 trials to large light alone; *B*, 200 trials to small light alone; *C*, 200 trials to both lights used simultaneously.

would sometimes stop outside the runway and go through the same operations.

As a rule the animals which react most constantly and uniformly to directive light are those that are under more or less unnatural conditions, as was the case with the earthworm, Bipalium, Tenebrio, and, to a less extent, Oniscus, when placed on the glass plate or on the table top. These animals usually come to rest under normal surroundings only when a considerable portion of the body, especially the dorsal side, is in contact with something. Cockroaches, on the other hand, may often be found at rest upon the walls of the dimly lighted cellars and basements where they live. They do not by any means spend all their time in cracks and crevices, though they usually retire to such places when more strongly illuminated. In many animals the ordinary reactions appear to have an inhibitory effect on the reaction to light, so long as the animal is not disturbed in any unusual way. Or the animal may simply come to rest, and yet respond immediately to such constant stimuli as directive light or gravity if it be disturbed. Carpenter (:**05**) has pointed out, for example, that pomace flies (Drosophila) will come to rest in the darker portions of the dish with their heads turned away from the light, but if they be disturbed by turning the dish slightly, they respond at once with their ordinary phototropic reaction. Mechanical agitation has a similar accelerating effect upon their reaction to gravity. As will be described later, a frog may sit for a considerable time with the axis of the body at right angles to the direction of a light, which apparently has no effect upon him; but upon being stimulated in any non-directive way he will usually turn at once and face the light, or may even hop toward it. Similarly, reactions to food, to contact stimuli, etc., may inhibit entirely the ordinary reactions to light. Certain pycnogonids cease entirely their efforts to go toward the light when the feet can grasp the stems of hydroids, among which these animals normally live; while if they are placed in a dish of water where they are unable to grasp any such familiar objects, they are strongly phototropic (Cole, :**01**). In fact, Loeb ('**90**, p. 21, *et seq.*) made special mention of the inhibitory effect of contact stimuli in his pioneer work on phototropism.

Instances need not be multiplied. The point is that the cockroaches, not being under especially unusual conditions, may have been influenced by other stimuli or by physiological conditions which in the case of animals less "at home" would have been overcome by the phototropic response. This may account for the fact that the slight excess of reactions was in a direction opposite to what one would expect. But the surprising thing is that this excess should have been so small, for in an animal with eyes comparatively so well developed we might expect more evidence of image-formation to be apparent in its reactions to the lights of different areas. An inspection of Figure 13, however, shows that the "mode" for the negative responses of the cockroach to unilateral light lies only 20° to 40° from the indifferent position, in which respect it agrees more closely with Oniscus (compare Figure 12) and differs markedly from the Tenebrio larva (Figure 11), although its eyes are undoubtedly vastly better adapted to ordinary vision than those of the mealworm. This means that the cockroach is less responsive to directive light than the mealworm, or at any rate that its responses are less definite and constant. Another factor which must be taken into consideration is the greater rapidity with which the former animal travels, so that in a given distance the light acts upon it during a much shorter interval than is the case with an animal which moves more slowly. Under these circumstances the records would tend to be less divergent; and undoubtedly this was an important factor in the reactions to both lights, where, it will be observed, the number of records in the 0 class was very large (Figure 13). The same factor would have a tendency to mask any differences there might be in the reactions to the two lights. This fault was partially corrected in the experiments by taking the records for the cockroach when it crossed the outer circle (15 cm. radius), whereas the records on Tenebrio were taken when it reached the innermost circle (5 cm. from the starting point).

The conclusions to be drawn from the experiments on the cockroach may, then, be stated briefly as follows: *Periplaneta americana* reacts negatively to directive light as used in these experiments in an excess of about 50 per cent (42 per cent to 55 per cent) of its responses. It possesses relatively large eyes, which, one would suppose from their structure, were capable of much better image-formation than those of any of the other forms so far employed; but the results of the experiments fail to confirm this point. The explanation is probably to be looked for in the fact that on account of the influence of other factors the reactions to light are masked, and probably to a certain extent inhibited.

6. *Mourning-cloak Butterfly* (*Vanessa antiopa* Linn.).

The work of Parker (:**03**) on Vanessa naturally suggested that animal as a favorable subject for these experiments. The following conclusions reached by Parker (:**03**, p. 467) are of importance in the present connection:

"3. V. antiopa creeps and flies toward a source of light, that is, it is positively phototropic in its locomotor responses.

"4. Its positive phototropism occurs with lights varying in intensity from 2 candle-power at 2 meters distance (0.5 candle-meter), to 250 candle-power at 2 meters distance (62.5 candle-meters). . . .

"10. V. antiopa does not discriminate between lights of greater or less intensity provided they are all of at least moderate intensity and of approximately equal size.

"11. V. antiopa does discriminate between light derived from a large luminous area and that from a small one, even when the light from these two sources is of equal intensity as it falls on the animal. These butterflies usually fly toward the larger areas of light."

It will be seen that Parker had already concluded what might be expected to result from experiments on these butterflies under the conditions of the present investigation. His conclusions were based in large part upon observations in the field, supplemented by a number of experiments conducted in the laboratory. These, while they seemed to establish the verity of his conclusions beyond a reasonable doubt, were largely qualitative in their character, and it seemed of interest to repeat them under conditions in which the intensities and comparative areas of the lights were accurately known. Specimens of *V. antiopa* were accordingly procured, and as it was desirable — considering the comparatively small space where the insects could be exposed equally to the two lights — to have them crawl instead of fly, the wings were clipped off a short distance from the body, leaving only short stumps, by means of which the animals could be easily handled. Two difficulties presented themselves. Many of the butterflies either made attempts to fly, which resulted in their flopping about helplessly on the table, or they feigned death when released and refused to move at all. It was found that by holding the animal by one wing-stump, it would usually struggle with its legs and the other wing-stump. If in such a state of activity it was placed on the table, it would seldom feign death, but would start at once to crawl off. Attempts to start the animals with exact orientation in the normal position were abandoned, since it was found that holding them by one wing was apt to impart a unilateral impetus to their first locomotor movements; furthermore, after starting to crawl they would often begin making attempts to fly, and so struggle about on their backs with their legs in the air, thus losing all orientation before they regained their feet and progressed again by creeping. For similar reasons no record was made of the angles at which the animals diverged from the normal; in fact, when they once began crawling well, they crawled, as a usual thing, directly toward the light. In some cases they would first crawl a short distance toward the small light and then turn and go toward the larger light. Since such results appeared to be due to chance orientation at the beginning of the trial, or after the insect had lost its orientation by fluttering, the records of the direction of its response were not made until it had crossed a line, on either side, at a distance of about 25 cm. from the normal axis. In the later trials no attempt was made to place the insects in normal orientations at the beginning of the trials, but they were dropped at random as nearly as possible at the central position on the table. About the same proportion between negative and positive responses was obtained as when the attempt had been made to orient the animals.

In all 164 trials were made in this way upon 7 different individuals of Vanessa, with the following results:

Toward large light.	Indifferent.	Toward small light.		Total.
143	1	20	=	164

Thus it will be seen that 87.2 per cent of the responses were toward the large light; 12.2 per cent were toward the small light; while only a single one, or 0.6 per cent could be called indifferent. The excess of responses toward the large light over those toward the small was 123, or just 75 per cent of the whole. This result therefore confirms Parker's conclusion that *Vanessa antiopa* discriminates between lights of different area falling with equal intensity upon the animal. That so many as 12 per cent of the responses were toward the smaller light is probably to be accounted for largely upon the chance orientation and the condition of making the record as soon as the insect had crossed a given line at a certain distance from the starting place. One or two instances were observed in which the butterfly actually turned and went back toward the large light after having passed this limit in the direction of the smaller one.

7. *Water Scorpion* (*Ranatra fusca* Pal. B.).

The interesting work by Holmes (:05[a]) on the reactions of the water bug Ranatra to light suggested this form as a favorable one for use in the study of image-formation. Except under certain conditions, Ranatra is very strongly positive to light, apparently of any intensity, and reacts to it with great uniformity and persistence. Holmes says (p. 315): "Light seems to dominate entirely this creature's behavior when the phototactic reactions are once started. It does not manifest any fear or awareness of any object in its environment save the light which it so strenuously seeks. Its excitement increases the longer it is operated with, and after a time it may be picked up without feigning death, or with only a momentary feint."

Diligent search in the streams and ponds in the vicinity of Cambridge failing to disclose any of these animals, several dozen specimens were procured from Ann Arbor, where Dr. Holmes obtained his, and where they are often very abundant. They were shipped to Cambridge in a small tin pail, with barely enough water to cover them, but arrived in good condition and were kept in aquaria in the laboratory for a number of weeks. They were fed occasionally on Asellus and whatever small water beetles or other aquatic insects could be obtained, and when the freezing of the ponds made such food difficult to obtain, it was found that mealworms (Tenebrio larvae) could be substituted with apparently as good results. These were offered to the Ranatras in a pair of tweezers, usually having been crushed slightly in order to make it easier for the bugs to insert their beaks through the hard outer covering of the larva; otherwise the Ranatras were often unable to penetrate the larvae anywhere unless they chanced to find the soft integument at the bases of the legs. Holmes gives, in addition to a description of their reactions to light, a good account of their general habits, a knowledge of which is always an invaluable preliminary to experimental work upon any animal.

Comparatively few preliminary experiments were necessary to confirm the majority of Holmes's results, including the death-feigning, the head movements and swaying movements in response to light, and the subsequent positive phototropism, as well as the negative response to light under certain conditions. This negative response was found, in part, to occur at times when there appeared to be no definite assignable cause; but in general it seemed to be due to an appreciably less active condition, brought about possibly in many cases by exhaustion or by lack of food. As Holmes says (:**05**^{a}, p. 317): "The negative reaction is associated with a condition of lowered photo-tonus. It is rarely shown except when the animal is in a condition of comparative sluggishness. When in great excitement, when its movements take place with quickness and vigor, Ranatra always shows a positive reaction." He found that the negative reaction usually followed prolonged exposure to darkness. At times, however, temporary periods of negative response appear in animals which are otherwise uniformly responding positively to light, under apparently similar conditions and with no obvious cause. It will readily be seen how this might be a disturbing influence in the experiments testing the responses to both lights; for, whereas a positive animal might be expected to turn toward the large light, if it discriminated between the two lights at all, conversely an animal in a condition of negative response would under similar circumstances turn toward the smaller light. For this reason, although it would not matter in testing the reactions of the animal to the two lights, whether it were positive or negative, it is exceedingly important that the insect should be in one or other of these states and remain so without change throughout the experiment. It is so much easier to keep Ranatra in the positive condition than in the negative one, and the insect reacts so much more definitely and decidedly when positive, that, so far as possible, only individuals in this condition were used. A number of specimens were taken from the water and placed on the table. Most of them immediately went into the death feint, as described by Holmes; but there were usually a few that remained active. Some of these were occasionally positive, though more often they were negative and began crawling away from the light. After a time the others began gradually to come out of their death feint, and these, in nearly all cases, were positive. They were allowed to crawl toward the light until they became fully active and could be picked up and handled without fear of changing their reactions. That individual was then selected for experimentation which appeared most strongly and persistently positive. If, as often happened, they reached such a state of excitement that they attempted to fly to the light, the method adopted by Holmes to prevent this was resorted to, namely, fastening down their wings with asphalt varnish.

No regular separate series of experiments were made to test the reactions of Ranatra to the lights singly, but trials of this kind were made from time to time in the course of the tests with both lights by screening off one or the other, and exposing the animal to the remaining light. Furthermore, the method used for bringing the animals into normal orientation at the beginning of each trial afforded a means of observing immediately any change in the character of the reactions. The preliminary orientation was accomplished in the following manner: Two runways were constructed by tacking strips of pasteboard, painted black, to the sides of wooden blocks on each of which was mounted a small incandescent lamp (Figure 14, *l*, *l'*) registered as 2 c. p. By means of a single switch, either of these lamps could be lighted at will simply by throwing the switch handle one way or the other. When the switch was midway, both lamps were thrown out of circuit. If it is now desired to test an animal to the action of two lights, *Lg* and *Sm*, having it first headed north, so that *Lg* is to its left and *Sm* to its right, the animal is picked up by its breathing tube and dropped into the runway *So* with its head pointing toward *No*. The switch has previously been thrown so that lamp *l* is lighted, and the animal, being positive to light and protected from lights *Lg* and *Sm* by the sides of the runway, starts crawling straight ahead toward *l*. As soon as its head emerges from the runway it comes under the influence of the lights at the sides, and the lamp *l*, which was used only to bring it out in proper orientation, is immediately switched off. If the animal now turns toward either of the side lights, the record is taken, *Lg* or *Sm*, according to the way it turns, the record not being taken, however, until it has crawled far enough to cross the outer (15 cm.) circle. Should it go straight ahead and enter the opposite runway, the record is indifferent, or 0. The next trial is made from the other runway, *No*, and so on alternately, the specimen being headed first in one direction and then in the other, in order to avoid the possible establishment of a habit of turning in the same direction. The likelihood of this occurring without proper precaution is indicated by Holmes's work (:**05**^{a}, p. 336). Still further variation in operation was obtained by occasionally running two successive trials in the same direction, thus shifting the order of alternation.

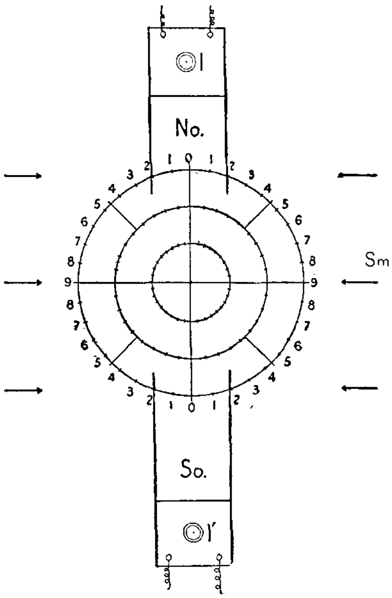

Figure 14. Apparatus employed for orienting *Ranatra*; *l*, *l'*, 2 candle power incandescent lamps; *No*, runway on north side of working position; *Sm*, side directed toward small light; *So*, runway on south side of working position.

A varying number of trials were made with 12 different individuals, the results of which are summarized in Table VII. The results of the trials with the single lights, which were made from time to time, are included for comparison with those to both lights, since they can with considerable accuracy be taken as a measure of the positive phototropism of the animal during the period it was being experimented with. A test in each of the two orientations was usually made with each of the lights separately at the beginning of the work with any individual. The insect was then tested with both lights for

TABLE VII.

SUMMARY OF REACTIONS OF *Ranatra* TO LARGE LIGHT ALONE, TO SMALL LIGHT ALONE, AND TO BOTH LIGHTS USED SIMULTANEOUSLY.

Lights used.	Large.			Small.			Both.			Total of Trials with each Individual.
Direction of Reaction.	*Lg*(+).	0	*Sm*(−).	*Lg*(−).	0	*Sm*(+).	*Lg.*	0	*Sm.*	
Individual a	4	. .	. .	. .	. .	4	6	. .	2	16
" b	10	. .	. .	. .	. .	8	23	2	11	54
" c	8	. .	. .	. .	1	7	23	. .	5	44
" d	25	. .	7	7	. .	21	23	. .	19	102
" e	18	1	1	3	1	20	77	14	17	152
" f	4	. .	. .	. .	. .	4	59	. .	13	80
" g	7	. .	3	4	. .	6	23	. .	5	48
" h	. .	. .	. .	. .	. .	. .	12	. .	. .	12
" i	4	. .	. .	. .	. .	4	35	2	13	58
" j	2	. .	. .	. .	. .	2	3	2	3	12
" k	. .	. .	. .	. .	. .	. .	6	. .	2	8
" l	4	. .	. .	. .	. .	4	46	2	28	84
Totals	86	1	11	14	2	80	336	22	118	670
Per cent of total trials	87.8	1	11.2	14.6	2.1	83.3	70.6	4.6	24.8	
Excess (per cent)	76.5	. .	. .	. .	. .	68.7	45.8			
Total of trials under each condition	98			96			476			670

a considerable number of trials; then tests were again made with the lights singly. This was not altogether necessary, since a change in the phototropic state of the animal could usually be told at once by its behavior in the runway. As it became negative, instead of going out toward the incandescent light in the other runway, it would attempt to turn about and go from it. Furthermore, whenever an animal was subjected to both lights and turned for a number of times toward the small one, it was immediately tested with the lights separately to find whether it reacted negatively under these conditions. This was found to be the case in enough instances to establish pretty thoroughly the conclusion that a change in the character of the animal's reactions was usually an explanation of a number of turnings to the small light. The negative condition, however, often appears to be very temporary and vacillating, and at such times the animals show much hesitancy in their reactions, sometimes heading first toward one light and then toward the other, as if undecided in which direction to go. When a specimen appeared to have become at all permanently negative in its reactions, the experiments with it were discontinued and another individual was taken in its place. It will be observed in Table VII that the per cent of plus reactions for the large light and for the small light were nearly equal, — 87.8 per cent for the one and 83.3 per cent for the other, — averaging about 85 per cent of the total number of reactions in the two cases. The excess of positive over negative reactions was 76.5 per cent and 68.7 per cent respectively. When both lights were used, the per cent of responses to the large light was somewhat smaller, but still comparatively large. This was 70.6 per cent of the total number of reactions, or 45.8 per cent in excess of the reactions to the small light. These figures are considerably smaller than the corresponding ones for Vanessa, in which, it will be recalled, 87.2 per cent of the reactions were toward the large light, while the excess in that direction was 75 per cent of the whole. This difference, however, is probably caused not so much by an inferiority of the eyes of Ranatra, in respect to image-formation, as by the variable character of its phototropic states. Ranatra has periods of negative phototropism, whereas Vanessa appears never to be negative in any of its locomotor responses to light.

From the experiments which have just been described, it is apparent that Ranatra is able to discriminate between a very small source of light and a luminous area 41 cm. square, even when the intensity of the light striking the two eyes is the same; and judging by the large per cent of reactions toward the large light by animals which are positive in response to unilateral illumination, it is reasonable to infer that the eyes are capable of forming images of considerable definiteness. Differences in the normal phototropic responses of the two animals, as well as the influence of different inhibitive reactions, and other factors, will not allow the drawing of very close comparisons between the relative capacity of image-formation in the eyes of Vanessa and Ranatra. Other things being equal, the foregoing results would indicate a more precise condition in Vanessa, but the uncertain elements which have been mentioned must be taken into account before conclusions can be drawn with any degree of confidence.

Mention has been made of individuals which at times showed much hesitancy in these reactions, and having turned toward one light seemed disturbed by the other, which was now behind them. At such times the head was usually held high, in the manner described by Holmes (:05^a, p. 312), when the light was moved to a position behind the animal. It was found that by painting over the posterior half of the eyes, so that the light was excluded on that face, the influence of the light behind the insect could be prevented. After having their eyes treated in this manner, even individuals which had previously shown great hesitancy held their heads low, and once oriented toward either light, crawled straight on in that direction without turning. This was not resorted to, however, in any of the trials summarized in Table VII. If the eyes were painted on their anterior halves, exactly the opposite state of affairs was brought about. In this way individuals which had previously shown no hesitation when once oriented toward either light, were caused to turn first to one and then to the other, the reason being that, as soon as the animal was facing one light, that one could no longer be seen on account of the paint, while the other, shining on the unpainted posterior half of the eyes, caused the insect to turn from the apparent darkness ahead to the light behind. In this way it was kept turning about and about, and making no definite progress in any direction.

8. *Pomace Fly* (*Drosophila ampelophila* Loew).

Upon this form and the succeeding one, only negative results were obtained as regards the question of inferred image-formation. It was expected from their general behavior toward light that these animals would be eminently suitable for this investigation, and the failure to obtain definite results with them was a disappointment. Long series of experiments were tried both with these flies and with snails, involving much time and labor; a brief account of them is given here merely to illustrate how certain unforeseen conditions may influence the reactions of an animal in ways entirely unexpected.

In his investigations upon the reactions of Drosophila, Carpenter (:05) found that this little fly is always positive in its reactions to light, at least in its locomotor responses.[7] It, like Vanessa, often comes to rest with the head directed away from the light, but Carpenter does not consider this an indication of negative phototropism. His explanation is that "The fatigued insects remain quiet in this position because it is the one in which the least light enters the eyes, and in which, as a consequence, the kinetic stimulus is least" (p. 170). The desirability in the present instance of having animals which were actively phototropic and in which the character of the response, whether positive or negative, remained constant, has already been mentioned. The pomace fly seemed to fulfil these conditions remarkably well, since it is always, when in motion, positive to light, and can usually be put into motion by slight mechanical agitation whenever it shows a tendency to come to rest.

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[7] There are undeniably times when Drosophila may crawl away from the source of light; but this is probably in response to other stimuli, such as food,

All this works out very well with a single light, but in using both lights the difficulty arises of getting the flies properly oriented at the beginning of the trial. If they chance at the beginning to be headed toward either light, they appear to continue crawling in that direction, heedless of the other light and indifferent as to whether the one toward which they are headed is the larger or the smaller. The plan was first tried of liberating a number of individuals in a box through a hole in its bottom, the box being provided with glass sides, one directed toward either of the lights. When once they alighted on either glass, they crawled upward, in consequence of their natural negative geotropism, and so passed into a trap device at the top, where they could be counted at leisure. It was expected that when they were placed in a small dark box beneath the hole in the bottom of the one with the glass sides, they would at once fly upward from the dark into the light of the upper box, and so would become suddenly exposed (as they flew upward, and oriented at random) to the influence of the large and small illuminated areas. It was found that on the contrary few of the insects flew out; the most of them crawled up over the edges of the opening and proceeded to crawl on in the direction in which they chanced to be oriented. Furthermore, a large proportion of them evinced a decided tendency to settle down and remain quiet soon after getting out into the light, coming to rest here and there all over the bottom, top, ends, and the glass sides of the box, and it took more than moderate jarring of the box to get them started to crawling again.

Next, the plan was tried of placing backwardly projecting strips of paper around the opening, making it more difficult for the flies to crawl out at the edge of the opening. At the same time there was placed

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which for the time inhibit or overcome the natural phototropic reaction, and is not to be considered an active negative reaction to light.

in the middle of the opening a small wooden cone, up which the flies could easily climb. Nevertheless many still persisted in crawling out at the edge of the opening in spite of the obstructions, while those that climbed the cone usually came to rest on its sides instead of climbing up to its apex and flying thence as it was hoped they would do. Obviously, too, those which climbed the sides of the cone were often shaded from one or the other of the lights by the cone itself, and this introduced another chance for error. Finally, an outer paper cone was so arranged as to make it impossible for the flies to come out anywhere except through an opening at its apex, from which there led a narrow Y-shaped paper, one arm extending toward the large light, the other toward the small, with strings leading upward to the traps on either side. But even with this apparatus the tendency of the flies to continue crawling up the edge of the Y upon which they happened to emerge appeared to overcome in large part their reaction to light. With a single light — the large one — only about twice as many individuals went toward the light as in the opposite direction; and when both lights were used the readings were almost equal — 222 to 226.

From these results it will be seen that, although the eyes of Drosophila would, from their structure, appear to be as well adapted to image-formation as those of either of the other adult insects employed, the experiments with the two luminous areas of different size furnish no data whatever to aid in the determination of this point.

9. *Garden Snail of Europe* (*Helix pomatia* Linn.).

These snails, during the winter months when they have withdrawn into their shells for hibernation, are imported into this country by certain French restaurants in the larger cities, and may in this way be readily obtained in the living condition. Under the influence of warmth and moisture they emerge and live well, so that they may be kept in the laboratory for a considerable period. A few preliminary experiments tried with them seemed to indicate that they were for the most part decidedly positive in their reaction to light, often turning to it at once at a sharp angle. This led to the hope that they might be suitable for experimentation in regard to the effect of the two lights. The snails were allowed to crawl on the ground-glass plate in the same manner as described for the earthworm and land planarian. They are easily handled by the shell and can be placed in any position desired. As in the case of the other forms, they were first oriented in the "normal" position in the shaded area between the screens and then allowed to crawl out into the light. Trials were made alternately with the animals headed in one direction and then in the opposite.

It soon became apparent that these snails were by no means uniformly positive to light, so it was necessary each time to test them first with a light at one side only in order to be certain of the state of the animal. It was furthermore found that they appeared to become fatigued very quickly, and apparently this was often accompanied by a change in the character of the phototropic response. Finally, the method was adopted of making only a few trials with each individual each day. The first trials were made with both lights, one trial in each of the two orientations, and then trial was made with a single light to ascertain whether the response was positive or negative to that. The records for animals that were positive were kept separate from those of animals that were negative. The responses were so irregular and apparently so dependent upon the physiological state of the animal that no very definite conclusions could be drawn. As far as they showed anything, however, they seemed to indicate a fairly indifferent reaction to the two lights in the case of animals that were negative as well as in those that were positive. This leads to the inference that the eyes of the snail do not aid greatly, if at all, in the discrimination of two lights differing in area as the two used. If, as is maintained, the general integument is sensitive to light-stimulation, a result similar to that obtained in the reactions of the earthworm to the influence of light on the skin would be expected, since, without the eyes, the condition would be comparable to that of the earthworm. These experiments on the snail, as far as they go, point to the conclusion that the ability to discriminate differences in the size of luminous areas is aided but little, if at all, by the eyes.

10. *European Garden Slug* (*Limax maximus* Linn.).

A few experiments were also made with the slug *Limax maximus*, as an example of a mollusk normally negative in its reaction to light. As in the case of Helix, however, the animals were found to be so inconstant in their responses that it was exceedingly difficult to obtain consistent results from them. Frandsen (: **01**), in his work on the reactions of Limax to direction stimuli, found much individual difference in the reactions to light, as well as to gravity. Furthermore, he found that the reactions of the same individual vary in intensity, and even in character, with variations in the intensity of the light, and probably also in response to undeterminable physiological states. These variable conditions introduce so many elements of uncertainty that Limax, like Helix, is unsuitable for use in experiments where constancy in the character of the reactions is necessary.

11. *Cricket Frog* (*Acris gryllus* Le Conte).

In the search for animals with the type of eye often known as the "camera eye" which could be employed in these experiments, the choice appeared to be rather limited. Of the vertebrates which could be used out of water most appear to be either indifferent to the action of directive light, or else their reactions are of such a complex nature that the natural responses to light are inhibited by other external stimuli or by fear. Certain of the amphibia appeared most promising, especially the frog, this animal having been experimented upon by Graber ('84) more than twenty years ago, and by a number of observers since that time. Two recent papers, one by Parker (:03^a) and the other by Torelle (:03), treat rather fully of the reactions of frogs to light. Parker's observations relate exclusively to the leopard frog (*Rana pipiens* Gmelin), while Torelle used this species and the green frog (*R. clamata* Daudin) indiscriminately.

In previous years, at Ann Arbor, Mich., I had incidentally noted the marked positive phototropism of the little cricket frog, which is abundant about ponds and lakes and in the marshes of that region. It is an active little animal, capable of making leaps to a distance of a meter or more. Specimens placed on a large table top above which swung an electric light at a height of a half-meter or so were observed to make long leaps toward the light. They usually missed striking the globe, seldom going so high, and landed on the other side of the table headed away from the light. Ordinarily they would remain in this position for a short time, then turn around so as to face the light again, and then, after another interval, again leap toward it; this proceeding would be repeated until a chance jump carried them completely off the table on to the floor. The decisiveness and persistence of the reaction suggested that this animal might prove even better to work with than the common species of Rana. In consequence, through the kindness of friends in Ann Arbor, several dozen specimens of Acris were obtained from that place for the purpose of these experiments. In the meantime a number of other amphibians, including two species of Plethodon and *Diemyctylus viridescens*, were tested as to their light reactions, but although these in general appeared to be negative to light, as would be suspected from their habits, their reactions to the intensity of light used did not appear sufficiently marked to make them suitable for these experiments.

In the experiments with Acris the animal was placed beneath a small glass box (approximately 4 cm. × 5 cm.), large enough to allow the frog to turn freely in any direction but not permitting it to hop any distance. Care was taken to keep the sides of this box normal to the directive axis of the apparatus, and although the intervention of the glass between the animal and the light on each side introduced reflections which could not be avoided, it appeared, from certain tests that were made, that these were sufficiently insignificant to be disregarded. The method of orienting the animals at the beginning of each trial, at right angles to the line joining the two lights, was similar to that employed in working with Ranatra. When it was desired to start with the frog headed north, for example, a small incandescent light was turned on at that side, while screens were placed between the animal and the experimental lights. Under these conditions the frogs usually turned within a short time and faced the incandescent lamp. This got them into the desired position, so that the incandescent lamp was then turned off and the screens removed as nearly simultaneously as possible, leaving the frog exposed to the influence of the large and small lights, one at either side. The next trial was made in precisely the same way except that the incandescent lamp was placed to the south instead of the north, and in consequence the frog was oriented in exactly the opposite direction to what it was in the previous trial.

Four series of experiments, comprising in all 300 trials, were made upon the reactions of Acris to the two lights. The first three of these, (Table VIII, *A*, *B*, and *C*) were made with the lights at the same distance that had been used throughout all the experiments, namely, at 2 meters from the animal, or 4 meters apart. As has already been stated (p. 345), this gave a light intensity of 1.25 to 5 C.M. falling upon the animal on each side. The combined results of these three series are given in Table VIII. Before making series *D*, which is similarly summarized in Table IX, the lights were moved nearer together until they were but 2 meters apart, thus reducing the distance between them and the animal by half and increasing their intensity at that, the median, point four times. The intensity of light striking each side of the animal was consequently then about 5 C.M. to 20 C.M. With the stronger intensity the frogs reacted much more quickly and uniformly. They sometimes acted almost immediately when the orienting light was turned off and the screens removed exposing them to the experimental lights; but usually there was an interval varying from a few seconds to half a minute, or even longer, before they turned toward one light or the other. Parker (: **03**^{a}, p. 29) found similarly that *Rana pipiens* responded much more quickly to the stronger light intensities. At the

TABLE VIII.

Reactions of *Acris gryllus* to Both Lights.

Series.	Date.	Individual.	Turnings to *Lg.*	Turnings to *Sm.*	Total Trials.	Remarks.
A	May 14	1	10	0	10	
	" "	2	7	3	10	
	" "	3	9	1	10	
	" "	4	7	3	10	Inactive. Apparently not strongly +phototropic.
	" "	5	3	7	10	
	" "	6	7	3	10	
	" "	7	5	5	10	Turned in all cases to its left.
	" 15	8	4	6	10	
	" "	9	4	6	10	
	" "	10	7	3	10	
B	" 25	1	3	1	4	
	" "	2	2	0	2	
	" "	3	1	5	6	
	" "	4	2	2	4	
	" "	5	3	1	4	
	" "	6	3	1	4	
	" "	7	4	0		
	" "	8	4	0	4	Very inactive. Not strongly +. Turned always to its left.
	" "	9	2	2	4	
	" "	10	2	2	4	
C	" 26	1	4	0	4	
	" "	2	1	3	4	Seemed quite strongly +.
	" "	3	4	0	4	
	" "	4	2	2	4	Turned always to its left.
	" "	5	2	2	4	
	" "	6	2	2	4	Turned always to its left. Strongly +.
	" "	7	3	1	4	
	" "	8	3	1	4	
	" "	9	3	1	4	
	" "	10	4	2	6	
	" "	11	4	0	4	Reactions prompt.
	" "	12	3	1	4	
	" "	13	3	1	4	
	" "	14	2	2	4	
	" "	15	1	1	2	
Totals			130	70	200	

lower intensities used by him (in the neighborhood of 1 C.M.) "the animals often did not react for from five to ten minutes or even longer." It was only an exceptional specimen of Acris that did not react within less than about a minute, and as the intensity of light employed was not far from the lower intensities used by Parker, it is obvious that, as judged by the time required for response, Acris is much more sensitive to this form of photic stimulation than the leopard frog.

TABLE IX.

Reactions of *Acris gryllus* to Both Lights at Half the Customary Distance.

Series.	Date.	Individual.	Turnings to *Lg.*	Turnings to *Sm.*	Total Trials.	Remarks.
D	May 29	1	4	0	4	
	" "	2	8	0	8	Turned toward *Lg*, even if facing at first more toward *Sm.*
	" "	3	7	1	8	
	" "	4	3	5	8	
	" "	5	8	0	8	
	" "	6	7	1	8	
	" "	7	8	0	8	Strongly +; reactions very definite.
	" "	8	8	0	8	
	" "	9	8	0	8	
	" "	10	7	1	8	
	" "	11	5	3	8	Did not orient well to single light. Apparently — part of time.
	" "	12	4	4	8	At first +; then became —.
	" "	13	6	2	8	
Totals . .			83	17	100	

An inspection of Table VIII will show that out of 200 trials 130 (65 per cent) were toward the larger light, an excess of 30 per cent in that direction over those in the opposite direction. The proportion of turnings to the large light is undoubtedly considerably lowered by the inclusion of several individuals which appeared to be indifferent or defective in their reactions, such as No. 7 in Series *A*, and Nos. 4 and 6 in Series *C*, which turned in all cases to their left. This may have been due to indifference to light, the animals turning to their left possibly for some structural reasons, or there may have been some defect of sight in the right eye, which would also result in a turning always to the left. It should be mentioned that the same individuals, in part at any rate, were used in the different series, but that the numbers by which they are designated in the various sets do not correspond. For this reason No. 4 or No. 6 of Series *C* may have been the same individual as No. 7 of Series *A*, but it is impossible to say definitely. It was found, too, that sluggish or inactive individuals, or those which did not orient readily to the light used for that purpose before making the trial, gave in general a much smaller proportion of reactions toward the large light than those frogs which reacted promptly and oriented readily to the incandescent lamp. Individuals No. 5, Series *A*, and No. 9, Series *B*, are examples of the inactive sort.

Series *D* comprised 100 trials, on 13 different individuals, with the lights at half the usual distance. The results are shown in Table IX, and are so decided as to admit of no doubt as to the character and meaning of the reactions. Of the 100 trials, 83 were toward the large light and only 17 toward the small one. This means an excess of 66 per cent of the turnings in the direction of the large light. This is considerably larger than the corresponding value found for Ranatra (45.8 per cent; cf. p. 387), but smaller than that found in the reaction of Vanessa (75 per cent; cf. p. 382). It will be noticed that with the exception of individuals 4, 11, and 12, all the animals turned toward the large light in at least three-fourths of the trials with them, that is, in at least 6 out of 8 trials; and if the records of these three had been omitted, it would have brought the total number of reactions toward the large light up to 93 per cent, or an excess of 86 per cent over those in the opposite direction. No memorandum appears to have been made at the time with regard to the behavior of individual 4 in other respects; but No. 11 "did not orient well to single [incandescent] light" and was "apparently negative," at least part of the time; while No. 12 was "first positive, then became negative." In correspondence with this change of behavior to a single light in this last individual, it first gave 4 turnings to the large light and then 4 to the small.

It will thus be seen that the proportion of reactions to the large light by Acris was considerably lowered by the inclusion of the records of these three animals, which were either in a negative, or at least an inconstant or indifferent, state as regarded the character of their responses to one-sided illumination. As with Ranatra, this inconstant and not accurately determinable factor precludes making direct quantitative comparison with the results obtained upon Vanessa or other forms. If such a comparison were to be attempted, it would probably be justifiable to leave out of account entirely the results on the three specimens of Acris which appeared to be negative, since Vanessa appears to be always positive, and the comparison would then be between animals which were in similar constant phototropic states. If such were done, the proportion of turnings to the larger light would be greater for Acris (as based on Series *D*) than for Vanessa, and in so far might perhaps be interpreted as indicating the formation of more distinct and better images of the lights on the retinae of the former than the latter.

Parker (:**03**[a]) proved that *R. pipiens* is as a rule positively phototropic to stimuli received by the skin, as well as to those received by the eyes. The idea suggested itself of using Parker's methods and testing in Acris the effect of the large and small lights when falling (a) upon the eyes alone, the skin being protected from the light, and (b) upon the skin after the optic nerves had been cut. If the persistent turning of the normal frog toward the large area of light was to be attributed to the ability of the eyes to form images of the lights, it would be expected that frogs with the skin protected from the light would still react as before, so long as vision with the eyes was unimpaired. On the other hand, an eyeless frog — or what amounts to the same thing, one in which the optic nerves had been severed — might be compared directly with an earthworm, so far as its light-perceiving abilities are concerned. It has only the general integument for the reception of photic stimuli, and while, as in the earthworm, this is undoubtedly sensitive to differences of intensity, there is no more reason to suppose that it would enable the animal to discriminate between different areas of like intensity but unequal size than in the case of the worm.

In making the test with the eyes exposed and the skin covered the method employed by Parker was followed in detail. The skin was removed in one piece from a large, dark-colored individual, turned inside out, and drawn over a slightly smaller specimen which was known to be active and strongly positive in its reaction to light. In addition to the eyes, the snout and fore and hind feet were exposed to the light. Only a single individual was used for this test, but the nature of its responses was so decided that further experiments did not seem necessary. When exposed as the normal frogs had been, to the influence of the two lights, it turned toward the larger in 11 out of 14 trials, about as large a proportion as was obtained with normal animals. The frog was much slower in its reactions than it had been previous to having the skin of the other specimen slipped over it, not responding within two minutes in more than half of the trials, so that it had to be started by mechanical stimulation. As a check experiment, the enveloping skin was now drawn forward so that it covered the eyes as well as the rest of the body. The frog no longer turned toward the large light, but, on the contrary, an unexpected result was obtained; for it now turned in nearly all cases toward the smaller light, when it would have been expected to be indifferent, since both skin and eyes were protected from the light. A larger series of experiments might have explained this apparent anomaly,[8] but by this time the specimen had become exceedingly sluggish and inactive, and it was believed that the results obtained sufficiently demonstrated that Acris discriminated between the two areas by means of the light from them that entered the eyes.

TABLE X.

Reactions of *Acris* after the Optic Nerves had been cut.

Both Lights.				Small Light.			
Individual.	Direction of Reaction.			Individual.	Direction of Reaction.		
	Lg.	0	*Sm.*		—	0	+
1	2	6	2	1	1	5	4
2	3	4	3	2	1	4	15
3	. .	6	. .	3	1	2	1
4	3	4	3	4	2	2	6
Totals	8	20	8	. .	5	13	26

It now remained to test animals by exposing their skin to the full influence of the lights, after their eyesight had been destroyed. This was accomplished by cutting the optic nerves, a simple operation, performed by inserting the points of a pair of fine scissors just back of the eyeball, through the membrane lining the roof of the mouth, and severing the nerve where it leaves the eye. The frogs did not seem to be greatly inconvenienced by the operation, and three of the four individuals so treated appeared to be as active as before, except that they did not respond so quickly to light. The fourth animal (No. 3 in Table X) was more sluggish and behaved much as frogs do that have had the cerebral hemispheres removed. All the specimens were so slow in turning toward the incandescent light which had been used in the previous experiments for orienting the animals into the normal position, that they were placed in the desired orientation by moving the glass plate upon which they were sitting. This could not be rotated directly to bring the frogs into position on account of the compensating circus reflex which resulted, but the plate was moved backward and forward in line with the longitudinal axis of the frog's body a distance of about 15 cm., and each time was turned slightly until the animal was finally oriented. In this way the greater part of the motion was lengthwise of the frog, and the rotating was so slight in comparison with the backward and forward movement that there seemed to be no tendency to give the compensating reflex.

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[8] Two possible explanations of what may have caused the results obtained suggest themselves: (1) The portion of skin covering the eye toward the small light may have been more pervious to light than that over the other eye, due to its having been stretched more tightly, to a difference in pigmentation, or possibly to injury, such as an abrasion or a small perforation; or (2) it may be that the light from a small area such as was used may have a greater penetrating power through a membrane of this character than the more diffuse light from a larger area.

Individuals 1 and 2 usually turned in one direction or the other, or hopped or walked ahead — in other words, gave some locomotor response — within 2 minutes of the time they were exposed to the lights. If not, they were stimulated by touching them from behind. As has been mentioned, No. 3 was inactive and slow to react, while No. 4 was intermediate between No. 3 and the other two. In cases where the animal hopped or walked ahead without turning at least 45° in one direction or the other the record was taken as indifferent, and is included in the column headed 0 in the table.

The left half of Table X summarizes the reactions to both lights of the four individuals with the optic nerves cut. The larger part of the records (20 out of 36) were indifferent; but it is a curious fact that each individual turned exactly as many times toward one light as toward the other. In all there were 8 turnings toward each lamp. This shows a perfectly indifferent condition, and demonstrates that Acris is unable to discriminate by means of the skin alone between luminous fields of different size where the intensity of the light from each falling upon the animal is the same.

In order to make certain that the skins of these animals were sensitive to light, the same individuals were finally tested with the small light alone. The results of this test are recorded in the right half of Table X. It will be seen that each individual gave a preponderance of reactions toward the light, except No. 3, which turned once in each direction, and twice went ahead without turning. Three of the frogs were thus undoubtedly positively phototropic to light stimuli received by the skin, and the inherent inactiveness of No. 3 has already been remarked. From the totals it is seen that out of 44 trials 26, or more than half, were positive, or toward the light, 5 negative and 13 indifferent.

From the foregoing experiments it seems that the following conclusions may legitimately be drawn:

1. *Acris gryllus* is preponderatingly positively phototropic, though there is some inconstancy in the reactions of the same and of different individuals, depending upon unascertained factors.
2. Exposed to luminous areas of different size but equal intensity, it turns in by far the greater number of trials toward the larger of the two areas.
3. The result is substantially the same when the skin of the animal is protected from the light, but the eyes are exposed.
4. Acris deprived of sight by severing the optic nerves, but having the skin exposed to the light, is indifferent to the size of the luminous field.
5. From this it follows that the discriminating power as to the different areas lies in the eyes, and is the result, it is safe to infer, of the ability of these organs to form comparatively clear images of external objects.
6. Acris with the optic nerve severed is still positively phototropic, but in this case, where the light must be perceived by the skin, its condition as regards image-formation and the consequent ability to discriminate between luminous areas of different sizes is directly comparable to that of an eyeless animal, such as the earthworm.

12. *Green Frog* (*Rana clamata* Daudin).

A number of experiments of a similar nature to those last described were made with the green frog for the purpose of comparing its reactions with those of Acris. The general method employed was the same, but differed slightly in details. The frogs were placed beneath a glass box 10 cm. × 12.5 cm. × 10 cm. high, and an attempt was made to get them to orient in the normal position by means of the incandescent lamp, as was done with Acris. They were so slow in responding, however, that it was found more expedient to raise the box and orient them with the hand. Care had to be taken to avoid a one-sided turning due to circus compensation or thigmotactic stimuli received by this act, and in order to guard against such results the animal was left sitting screened from the lights for a period of 15 seconds to 30 seconds after it had been placed in orientation; if it had not moved by the end of that time the screens were removed, exposing it to the lights from the two fields.

Rana clamata was found to be much slower in its reactions than Acris. It was usually given 5 minutes in which to react, and if at the end of that period it had not moved, it was induced to do so by means of an electrical stimulus applied to its lower side. This was accomplished by having the board on which the frog was placed crossed by fine parellel copper wires 1 cm. apart, alternate wires being connected respectively to the opposite poles of a single "Columbia Dry Cell No. 6." By means of a simple "key" placed in the circuit the frog sitting on the wires was stimulated by the "make" and "break" of the current whenever the key was pressed down and released. This method was used rather than simply touching the animal from behind, as it was believed that there was less chance of the stimulus being one-sided, and because no movement of the operator was necessary in the field of vision of the animal. On the whole it was found to work very satisfactorily.

The results need not be given in detail, but may be stated briefly as follows:

1. As stated by Torelle (:03), *Rana clamata* is positively phototropic in nearly all cases at the ordinary temperature of the room (about 20° C.).

2. Such positive individuals, when exposed to the two light areas of different size, turned in the great majority of cases toward the larger light. This result agrees with that obtained with Acris, but as would be expected from an animal which responds less quickly and definitely to a single light, the proportion of the turnings to the large light was not so large.

3. At low temperatures (6° C. to 10° C.) the frogs were usually negative in their responses to a single light, or at least were indifferent or inconstant in their reactions; and in accordance with this

4. Such individuals turned toward the smaller of the two lights, or were inconstant and gave indifferent results under these conditions as with the single light. This last result was first obtained from a specimen which was brought into the experimenting room from a tank in which the water registered 8° C. and was at once tested with the two lights. Its first 8 reactions were all toward the small light, after which

it turned in most cases toward the larger area. Out of 32 trials following the first 8, 22 were in this direction. By the time the first 8 trials had been made the frog had been in the warm room for a half hour or more, and it seemed reasonable to assume that the change in its reactions was due to its having become warmed up to the room temperature. Further observations tended to confirm this conclusion; and this agrees with the results obtained by Torelle (:03), who says, (p. 487): "A rise in the temperature to 30° C. accelerates the rate of the positive response. A lowering of the temperature to 10° C. produces movements away from the light."

IV. General Considerations and Discussion.

Conclusions as to the degree of the ability possessed by the various animals studied to form images, as inferred from the results of the experiments, have been stated in connection with the description of the experiments upon each animal employed. It still remains to consider certain general phases of the subject: to examine the relationship existing between the natural habits of the animals and their reactions to the two lights; to determine, in so far as possible, what part their phototropism — especially in its relation to luminous areas of different sizes — plays in their ordinary activities; and to attempt to trace, as well as can be done with the limited data at hand, the probable steps which have led to the development of highly specialized organs capable of forming richly detailed images of external objects.

It is a noticeable fact that those forms which showed by their reactions the most evidence of discrimination between the two lights (viz., Vanessa, Ranatra, and the two species of frogs) are all positive in their ordinary phototropic responses. On the other hand, the earthworm, the land planarian, the mealworm, the sow bug, and the cockroach — all of which showed little or no ability to discriminate between the different areas, but responded almost entirely according to the relative intensities of the lights — are all ordinarily negative. When one remembers that all the animals in the second group live either in the ground, or beneath stones, logs, or similar objects, or in other dark places, the nature of their responses is not surprising. For, whatever view may be held as to the way in which adaptation may have come about, it is undeniable that organisms are adapted to their ordinary conditions of life; consequently it would not be expected that those which live in darkness, or in very dim light, would be so likely to be provided with image-forming organs, since for these they would have little or no use. The ability to distinguish light from darkness is, however, important to them, since it aids them in remaining concealed, and is thus of service in protecting them from enemies to which they would be exposed if they came into the light, and in preventing them from going into situations that would be unsuitable in other respects (lack of moisture, etc.). The negative reaction to light is not, however, the only form of response to change in environmental conditions which operates to keep these animals in their accustomed habitats; responses to contact stimuli, to moisture, temperature, and other variable conditions, are often undoubtedly of as much, if not, indeed, of greater importance. Such, for example, must be the case, as pointed out by Loeb ('**90**, p. 51), in the larva of the willow-borer, which lives in a dark situation, but is, nevertheless, positive in its reaction to light when exposed to its influence. It is not necessary to enlarge upon the advantages which the ability to perceive distant objects (i. e., those with which the oganism does not come in direct contact) in their proper spacial relation gives to animals living in the light. An organism, by means of what we ordinarily speak of as "sight," is brought into close relation with a part of its environment which organisms wanting this faculty have no means of appreciating. This obviously is of distinct advantage. As the converse of what has just been said about dark-inhabiting forms, it may be expected that those normally living in the light are as a rule positively phototropic, and this seems in general to hold true. Where a change in the character of the phototropic response takes place, — as is the case in many animals which are under certain conditions positive and under others negative, — it is probable that this change can in most cases be explained by the natural habits of the animal. Different factors may operate to bring about this change of reaction; it may be mechanical stimulation, a variation of temperature, or a change in the intensity of the light itself. The result, in many cases at least, is to bring the organism into an optimum condition, either as to light intensity or some other stimulus. The frog (Rana), for example, becomes negative to light at temperatures below 10° C., and this response is probably an important one under natural conditions in inducing the animal to swim downward to the bottom of the pond and bury itself on the approach of cold weather.

We do not need to concern ourselves at this point with the theories as to how this adaptation has come about. It is sufficient to show that such adaptation exists. It may be asked whether certain animals have become negatively phototropic because they live in dark places, or whether they live in dark situations because they are negative in their reactions to light. The two probably have developed together.

An animal is dependent upon certain reactions to its environment to keep it in the proper conditions for its best welfare, and in many cases the response to light is without doubt of great importance in this respect. It is questionable whether a Bipalium, for example, were it positively phototropic, could long survive unless it had developed in addition to its phototropism some other form of response that would prevent its going, under the influence of the light, directly into the most unfavorable conditions. Loeb ('90, p. 51) stated that, so far as he had found, all larvae of Lepidoptera were positively phototropic, even including the willow-borer, which, as previously mentioned, like most negative animals, lives naturally in a situation with little or no light. It would seem that we have here a case of secondary adaptation; the willow-borer, we may infer, has become adapted to living in a dark situation *in spite* of its positive phototropism. It comes from a group of animals uniformly positive in their reactions to light, and has probably inherited this character from its positively phototropic ancestors. We have seen that as a rule animals like the earthworm and Bipalium, which live in dark situations, lack image-forming eyes; at least such is the inference from the reponses of these forms to equal illumination from areas of different size. It has been pointed out that the possession of such eyes could be of no use to them in the dark; and it is likewise obvious that the simpler organs are equally effective in preventing the animals from straying out into the light. All that is necessary for them is to avoid light altogether. This can be done, however, with greater precision when the organism is able to appreciate more exactly the *direction* from which the light comes. The earthworm probably has little power of discrimination in this respect beyond recognizing upon which side the source of light lies, — on its right or on its left, — according to whether its right or its left side is illuminated. Bipalium, on the other hand, by the arrangement of many of its eyes around the anterior margin of the semicircular head, is able to determine the direction of the light more accurately, and consequently shows a quicker and more precise response by moving directly away from it. Similarly, in the mealworm, the sow bug, and the cockroach, the eyes are probably little more than "direction eyes."

The experiments on Helix failed to give any decisive evidence as to the ability of these animals to discriminate between the two areas. Nor do the observations of Mitsukuri (:01) and Bohn (:05) help us much in this respect, since neither of these authors discriminated between the size of the areas and the relative intensity of the light received from them. Bohn studied the reactions of Littorina when vertical screens, both black and white, were placed in the illuminated field, and speaks of these animals as being "attracted" by the one and "repulsed" by the other. This terminology is rather misleading. It should be remembered that every object upon which the light falls and from which it is reflected becomes thereby a secondary source of light and must be so treated; its size is therefore of importance as one of the factors in determining the total amount of light which it reflects. If the surface is what we ordinarily term black, it reflects very little light, and its position may consequently represent the region of lowest light intensity in the whole illuminated field. A white screen, on the other hand, may reflect practically all the light that falls upon it; and if of large size, it is reasonable to predict that upon such animals as Vanessa, Ranatra, and the frog it would exert a greater influence than the source from which the light primarily came, provided the latter were of relatively small area. Thus, if the light from a projection lantern were thrown upon a large white screen and one of the above-mentioned animals were placed midway between the screen and the lantern, one would expect, from the results of the experiments performed on these species, to see the animal turn toward the large area of light, viz., the screen.

Loeb appears not to have considered in his experiments light reflected from surrounding objects, but in speaking of light coming from more than one source, he says (Loeb :05, pp. 61, 62): "When the diffused daylight which struck the [fly] larvae came from two windows the planes of which were at an angle of 90° with each other, the paths taken by the larvae lay diagonally between the two planes;" but in other places (*cf.* :05, p. 2, footnote) he states specifically that "if there are several sources of light of unequal intensity, the light with the strongest intensity determines the orientation and direction of motion of the animal;" in this case apparently the animal ignores the influence of the other lights altogether.

Miss Towle (:00, p. 365), in her work on Cypridopsis, found that these animals took a diagonal course due to light coming from other directions beside the main source, and decided that "the resultant direction [which the animal would take] could be found by compounding all these forces if their direction and relative value were known." The same question was later examined carefully, critically, and in an able manner, by Holt and Lee (:01); but in both these papers only the *intensity* of the various light components was considered, no account being taken, even in the theoretical discussion, of the possible influence of the *size* of the areas of the various light-sources.

In his study of Littorina, Bohn (:05, pp. 28, 29) puts the matter the other way about and uses the direction taken by the animals (their "trajectories") as an indication of the resultant of the light forces;[9] their paths he takes as indicating "veritable lines of luminous force." This comes very close to reasoning in a circle, and is at best merely qualitative, since he has not attempted to make a physical determination to verify his contention that the direction taken by the animals does really coincide with the resultant of light intensities. If Littorina responds only to the intensity of the light, we should expect that its course, in an illuminated field of this nature, would coincide with the resultant of the light intensities, could such be determined by physical measurements; but if, like Vanessa, it reacts differently to different areas, not in proportion to the intensity of the light received from them, but according to their extent, the direction taken by the snail would probably deviate accordingly from the resultant of intensities. It seems impossible to decide from the results either of Bohn or of Mitsukuri (:**01**), whether Littorina reacts to differences of intensity only, — the size of the screens or other objects in the field being of importance merely in determining the amount of light reflected or absorbed, and thus influencing the direction of the resultant "lines of luminous force," — or whether, like Vanessa, it may react to the areas according to their size, more or less irrespective of the intensity of light received from them. Whichever may be the case, Bohn (:**05**, p. 30) has pointed out that the shaded surfaces of rocks act in the same way as black screens, and that when a Littorina is in the vicinity of two such shaded areas it does not go directly toward either, but strikes a course which is a mean and eventually leads it into the crevice between the rocks.[10] This is apparently another case where the reactions of the animal to light are well adapted to its needs, since the snails are undoubtedly much better protected in the crevices than they would be upon the exposed surfaces of the rocks.

To animals commonly living in the light the possession of eyes capable of forming images must be of distinct advantage. Nevertheless, there are numerous examples of positively phototropic forms whose reactions to light tend to carry them always into regions of greater illumination, but whose eyes are probably of little use beyond determining the direction and the relative intensities of lights. Here, again, there appears to be a correlation between the habits of the animals and the conditions under which they live, for an inspection will show that these are usually creeping forms whose movements toward the light take them in the direction of their food, or else that other conditions prevent their phototropism from taking them into unfavorable surroundings. The caterpillars of Porthesia (and probably of most other Lepidoptera) may be taken as an example of the former. Although no tests have been made on these forms to determine their ability to discriminate between luminous areas of different size, the rudimentary condition of their eyes, and the experiments with beetle larvae (*cf.* Tenebrio) make reasonable the assumption that they respond only to the intensity of the light reaching them, and not to the size of the area whence it immediately comes. Under ordinary conditions, in the sunlight, the largest patches of light are on the ground; but the strongest intensity is skyward, and responding to this, as soon as hatched, the young caterpillars of Porthesia crawl upward and outward on the branches until the reaction to food overcomes their phototropism, or they are prevented from going farther by reaching the tips of the branches. In these two ways their progress skyward is checked at the proper time. In a similar manner positively phototropic snails or other crawling forms are restrained by the natural conditions from continuing indefinitely their migrations in the direction of the greatest illumination. Parker (: **03**, p. 462) has called attention to the fact that the surface of the water forms a similar barrier to certain marine organisms (such as the copepod Labidocera) which swim upward through the water toward the light.

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[9] "En un point donné d'un champ lumineaux la direction du champ n'est que la direction de la résultante de toutes les forces attractives et répulsives exercées par les surfaces éclairantes, surtout par les surfaces les plus étendues, les plus hautes (fenêtres, murs)."

[10] "Chaque surface d'ombre exerce une attraction proportionellement à son étendue, et l'animal suit une direction qui est celle de la résultante des forces attractives. En particulier, quand une littorine se trouve dans le voisinage de deux rochers présentant des surfaces d'ombre d'étendue par trop inégale, elle prend une direction qui est celle de la diagonale du parallélogramme des forces attractives, et ainsi il lui arrive de se mouvoir vers un espace compris entre les deux rochers et de ne rencontrer ni l'un ni l'autre, bien qu'attirée par l'un et par l'autre."

With forms which fly, and so, being independent of solid objects upon which to crawl, are not limited in the distance which they might move upward, the case is entirely different. A query which Romanes ('**83**, p. 279) found among Darwin's manuscript notes shows careful observation and puts the question very clearly. It is as follows: "Query. Why do moths and certain gnats fly into candles, and why are they not all on their way to the moon — at least when the moon is in the horizon? I formerly observed that they fly very much less at candles on a moonlight night. Let a cloud pass over and they are again attracted to the candle." Romanes thinks the answer is that "the moon is a familiar object, the insects regard it as a matter of course, and so have no desire to examine it." As a result of Parker's work on Vanessa and those of the present investigation, however, we are able to give another and, it would seem, a more reasonable explanation. The moths and gnats referred to react, like Vanessa, to large areas of light rather than to a point of more intense light. As a consequence they remain near the ground, on account of the bright patches of moonlight, instead of flying toward the moon itself. If, however, they come close to a candle, its relatively great intensity at so short a distance may overcome the reactions to the moonlit areas, and the insects accordingly fly into the flame. This is especially the case, as recorded in the note, when the moon is obscured by a cloud and the patches of moonlight disappear.

Parker (:**03**, p. 461) has pointed out, furthermore, that the alighting of Vanessa in sunny spots with the wings expanded is probably an adaptation which serves to bring the sexes together; for it is in this position, headed as it always is away from the sun, that the colors and markings of the upper surfaces of the wings are most conspicuous. He adds, "I am sure from direct observations that females, as well as males, will circle around an oriented and expanded individual of either sex, till both fly off together." The recognition by one butterfly of another as a definite object in its field of vision indicates a much finer perception than that which distinguishes merely the difference of size in illuminated areas, and approaches to the highest type of vision. Although no experiments were made which bear directly upon this point, it is, nevertheless, worthy of brief consideration.

Nuel (:**04**) speaks of "la perception simultanée des fins détails des objets visuels" as "iconoperception" (p. 82), and of the resulting reactions as "icono-réactions."[11] He says (p. 83): "Chez les animaux, nous ne parlerons pas d'iconopsie, mais d'icono-réactions, dans la cas où, comme chez l'homme, des mouvements sont suscités ou guidés, réglés par les fins détails visuels des objets." As the perception of the details of the objects in the general image of the visual field becomes more refined, the reactions to these objects, or to certain particular ones of them, appear largely to inhibit the reactions to light in general, including that to large areas as contrasted with small. Besides the case of Vanessa, mentioned above, it seems certain that many Lepidoptera and bees react to certain flowers, while many predacious insects, such as the dragonflies, are remarkably quick in detecting their prey. That color is not, in all cases at least, the determining factor in these responses, is indicated by such observations as that of Latter (:**04**, p. 88), who "once observed a brimstone butterfly visiting flowers of the Dog Violet scattered along a bank, and picking out these flowers to the exclusion of all others with great precision, not approaching even other blue flowers that were present." The description of the feeding of Ranatra by Holmes (:**05**ª, p. 325) furnishes an excellent illustration of the inhibition of the ordinary phototropic responses by attention to particular objects in the visual field. Holmes's description is as follows: "The phototactic response may also be inhibited by efforts to obtain food. Ranatra which are swimming towards the light can often be caused to discontinue their phototactic efforts if several small insects are placed near them. If the phototactic activities are very lively and vigorous, it is more difficult to divert the attention of the insect to the capture of prey. When attention is once directed to seizing the smaller insects, the light is disregarded. When the prey has once been captured and the Ranatra is engaged in sucking out its juices little attention is paid to the light. The repast being finished the insect may resume its positive response."

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[11] The words *iconotropism* and *iconotaxis* would correspond with the terms more generally employed, such as *phototropism* and *phototaxis*, and might be employed to designate specifically the reactions of animals to areas of light and to objects in the field of vision, in contradistinction to reactions to the intensity of the light.

Attention to moving objects would appear to be more general than, and probably precedes, the response to stationary objects. This is well illustrated in the feeding habits of frogs, toads, and many lizards, which seldom or never notice an insect so long as it is quiet, but are attracted by it at once if it moves. An object moving in the field of vision may, however, affect the elements of the retina in quite another way than does an object which is stationary; for, unless its color intensity is uniform with that of its background, its movement must produce a change in the intensity of the light reaching certain of the visual elements in the retina, and the response may be to this change in intensity rather than to a definite and clear-cut perception of the object as such. The question of how far animals are able to, and do, distinguish stationary objects is rather a difficult one to solve. As has just been said, it is well known that frogs react to small moving objects, such as insects, which constitute their food. It is also a matter of common observation that if a frog sitting on the bank of a stream or pool is disturbed, it ordinarily jumps at once in the direction of the water, even if it is approached from a direction parallel to the shore line. The question arises as to whether the frog recognizes the water by its appearance or whether the response is merely a reaction to a larger area of illumination. For it seems quite certain that the open water and sky on the one hand must in general form a larger area of illumination than the bank, with usually tall grass, bushes, or similar dark objects, on the other. We have seen (p. 393) that, although a frog may sit for a considerable time in a position without apparent orientation to the light, if it is disturbed it usually turns at once and jumps toward the light; and if there are two areas of illumination of different size, it turns toward the larger. Is, then, its ordinary response of jumping toward a pool or stream to be considered as a simple reaction of this kind? It will be seen that this is similar to what, in Mitsukuri's (:**01**) opinion, determined the shoreward migration of Littorinas. A series of observations was started in connection with the present investigation in an attempt to determine the question for the frog, but as yet a sufficient number of experiments has not been made to settle the matter definitely. It need only be said that so far as they have gone, the results appear to indicate that the reaction is not so simple as has been suggested — that apparently the objects in the visual field exert an influence beyond that merely of the amount of light received from any direction, or of the size of the area from which the light is received.

In those higher animals whose actions correspond still less to simple reflexes, as acute vision (the perception of details in the visual field) becomes more perfectly developed, simple phototropic responses become more and more a secondary matter, until they appear to be entirely absent, or at least are not recognizable as such. Birds give evidence of possessing especially acute vision, and under ordinary circumstances certainly show no evidence of simple phototropic responses; but the way in which migrating birds often, on stormy nights, gather about lighthouses and dash into the glass only to be killed, recalls strongly the flying of moths into a flame, and it seems possible that this is an expression of phototropism in birds which is ordinarily inhibited by other responses.

Finally, in recapitulation, we may distinguish roughly the following four stages or types of reactions of animals to stimuli received by the photo-receptive organs.

Type A. Response of eyeless forms.

These are in general given by animals which live in dark situations and are negatively phototropic to light of ordinary intensity, though they may (e. g., earthworm) be positive to lower intensities. Some, such as Hydra (Wilson, '**91**), are positive to light of ordinary intensity. The reactions of a frog with optic nerves cut are essentially those of a positive eyeless form. Animals in this group respond only to light *intensities.*

Type B. Response of forms with "direction eyes."

Animals with eyes of this type also react to light *intensity* only, and are more commonly negative; but some of them, such as many Copepods, larvae of Lepidoptera, etc., are positive; these are commonly limited, however, in their movements toward the light by physical conditions, which either bring them into favorable relations with their surroundings (e. g., to their food supply), or at least prevent their coming into unfavorable conditions.

Type C. Response to size of luminous field.

The animals which give this type of reaction appear to be usually positive to light of ordinary intensity, in which case, other things being equal, they turn toward the larger of two areas of illumination. Some of them (e. g., Ranatra and the frog) under certain conditions change the character of their response, and probably turn oftener to the smaller light than to the larger one, though not enough observations have been made to settle this point conclusively. It is probably only when the animals are positive that the reaction to the size of the illuminated area is adaptive.

Type D. Response to definite objects in the visual field.

This form of response includes what we ordinarily mean by the term "vision." As Nuel (: **04**, p. 10) says, "Le mot 'voir' supposant généralement une distinction visuelle et une représentation visuelle et psychique des objets." The responses to definite objects are not often in the nature of simple reflexes, but are complicated by psychical processes. In most cases they inhibit direct phototropic response, which may be in evidence at certain times only (as in Ranatra and the frog, and possibly in birds), or may apparently be absent altogether.

In the lower forms we speak of these responses as phototropic reactions, photo-reactions, or simply reactions to light; in the higher forms, whose reactions give evidence of being governed, or at least influenced, by definite objects in the visual field, they are usually termed vision. These stages, at least in the examples adduced, must not be taken as representing a genetic series. They shade insensibly into one another, and we have seen how some forms, such as the frog, for example, may give responses which fall into two of the classes. The ordinary responses of the frog to light which enters its eyes fall under what is termed Type *C*, and to an undetermined extent under Type *D*. But when the optic nerves are cut, so that the eyes no longer function, the frog still reacts to light, its responses falling under Type *A*. Can we say that the frog's "sight" has been destroyed then by cutting the optic nerves? It still is responsive to directive light much as it was before. Certainly the commonly accepted usage of the terms "sight" and "vision" would not cover the responses to light perceived by the integument, and it would perhaps be better to employ for the whole series the terms "photo-reception" and "photo-reaction," which have recently been proposed (see Beer, Bethe u. Uexküll, '**99**, and Nuel, :**04**).

Although, as has been said, this classification cannot be taken as representing a genetic series, nevertheless it does indicate in a rough way the steps which have probably led up to the possession of the highest type of vision. The classes as here established are far from distinct, and especially is this true of the last two, which necessarily must depend in part upon each other. However, the form of response outlined under Type *C* is in general more primitive than that under *D*. It depends upon a definite phototropic reaction, and as more acute vision is gained phototropism becomes inhibited more and more until it apparently disappears.

No experiments were made to test color perception and its relation to image-formation. This is an extremely difficult field, since it is well-nigh impossible to get an objective criterion as to whether animals perceive color as it is interpreted by the human eye, or whether colors represent to them merely differences in light intensity.

V. SUMMARY.

In the study of the reactions of certain animals to two lights of different areas, the one 41 cm. square, the other only one ten thousandth as much (for all practical purposes a point), the two lights were always adjusted so that they gave an equal intensity of light at a plane midway between them — the plane of experimentation. This intensity varied from about 5 candle meters to 1.25 candle meters. A summary of the principal results obtained is as follows :

1. The earthworm (*Allolobophora foetida*) was negative in its response to either of the two lights used separately ; to the two lights used simultaneously it was indifferent, turning almost exactly as many times in one direction as in the other. Allolobophora therefore apparently responds only to the *intensity* of the light.

2. *Bipalium kewense* was similarly negative to either light used singly. To the two lights operating at the same time it was nearly indifferent, but showed a slightly larger number of turnings away from the larger light ; this may indicate a slight ability to discriminate between the lights, and if so, it is probably owing to the arrangement of the eyes around the periphery of the semicircular head.

3. The mealworm (larva of *Tenebrio molitor*) was decidedly negative in its reactions to the lights employed separately. As inferred from its reactions to the two lights acting simultaneously, — the turnings to

and from the large light being equal in number, — its eyes have no ability to form distinguishable images of objects which differ from each other in size no more than did the two lights used.

4. *Oniscus asellus* was found to be negative to light of the intensities mentioned. Its responses to light are not so definite as those of the larva of Tenebrio, but there is some evidence that it has a somewhat greater power of discriminating between the two lights acting simultaneously.

5. The cockroach (*Periplaneta americana*) is decidedly negative to light from one side, but the reactions to the two lights used simultaneously showed an almost indifferent condition. The excess of 4.5 per cent of the reactions was on the side of motion toward the larger light, whereas one would have expected that if there were any difference there would have been a predominance of reactions away from the larger light. This is possibly to be explained as due to other disturbing factors.

6. The mourning-cloak butterfly (*Vanessa antiopa*) is uniformly positive in its locomotor reactions to unilateral illumination. Exposed to the simultaneous influence of both lights, it went in 87.2 per cent of its responses toward the larger light, thus confirming Parker's conclusion that this insect responds to the size of the illuminated area rather than to the intensity of the light received from it.

7. *Ranatra fusca* varied in the character of its response to unilateral illumination; but, as far as possible, only positive individuals were employed in the experiments with the two lights used simultaneously. It was found that Ranatra gave a somewhat smaller proportion of turnings to the larger light (70.6 per cent) than did Vanessa; but it is believed that this may be due to the inconstancy of the phototropic states of Ranatra, rather than to less efficiency of the eyes in forming images of the two lights.

Individuals with the posterior half of the eyes blackened went straight ahead, without hesitating, toward whichever light they chanced to face; when the anterior half of the eyes was blackened, they kept turning from one light to the other, since the only one from which light could enter the eyes was always the one behind the insect.

8. The pomace fly (*Drosophila ampelophila*) gave only negative results, owing, apparently, to technical difficulties in the experiments.

9. *Helix pomatia* did not prove to be a good form for the purpose of these experiments, on account of the inconstancy of its phototropic state. So far as the experiments went, they pointed to the conclusion that the eyes are of little or no use in enabling this animal to discriminate between the sizes of the luminous areas employed.

10. *Limax maximus*, on account of the inconstancy of its phototropic responses, was found to be unsuitable for these experiments.

11. The results of the experiments upon the cricket frog (*Acris gryllus*) will be found concisely enumerated on p. 400.

12. The results of the experiments upon the green frog (*Rana clamata*) are enumerated on p. 401.

13. The animals which showed by their reactions the most evidence of discrimination between the two lights (viz., Vanessa, Ranatra, and the two species of frogs) are all positive in their ordinary phototropic responses.

14. The negative animals experimented upon (earthworm, land planarian, mealworm, sow bug, cockroach) showed little or no ability to discriminate between the two areas of illumination, but appeared to respond almost entirely to the intensity of the lights.

15. These reactions are correlated with the natural habits of the animals.

16. There are certain positive forms (especially larvae of Lepidoptera and certain marine Copepoda) which apparently respond to light *intensity* only; but these are prevented by the conditions under which they live from being brought into unfavorable circumstances by their movements toward the light.

17. The responses of animals to light may be divided roughly into the following types:

Type A. Response of eyeless forms. Usually negative; sometimes positive, and then usually to very weak light. Respond to *intensity* of light only (e. g., earthworm).

Type B. Response of forms with "direction eyes." Usually negative (e. g., Bipalium, Periplaneta, Tenebrio larva); sometimes positive (e. g., larva of wood-borer), in which case special adaptation prevents their following the light until it brings them into unfavorable conditions. Response almost wholly to *intensity* of light.

Type C. Response to size of luminous field. Animals usually positive, though they may be temporarily negative, as was seen to be the case with the frog, for example. Probably the response to the size of the field is adaptive only when they are positive.

Type D. Response to definite objects in the visual field. Not simple reactions; responses usually involve psychic phenomena. Respond (1) to moving objects, (2) to stationary objects. This form of response usually inhibits ordinary phototropic reactions.

Types *C* and *D* are developed together.

The types outlined above do not necessarily represent a genetic series.

VI. Bibliography.

Adams, G. P.

:03. On the Negative and Positive Phototropism of the Earthworm Allolobophora foetida (Sav.) as determined by Light of Different Intensities. Amer. Jour. Physiol., Vol. 9, No. 1, pp. 26–34.

Beer, T., Bethe, A., und Uexküll, J. v.

'99. Vorschläge zu einer objectivirenden Nomenclatur in der Physiologie des Nervensystems. Centralbl. f. Physiol., Bd. 13, No. 6, pp. 137–141.

Bell. F. J.

'86. Note on Bipalium kewense, and the Generic Characters of Land-planarians. Proc. Zoöl. Soc. London, 1886, pp. 166–168, pl. 18.

Bergendal, B.

'87. Contribution to the Knowledge of the Land-Planariae. Ann. Mag. Nat. Hist., Ser. 5, Vol. 20, No. 115, pp. 44–50. (*Translated from the* Zoologischer Anzeiger, Bd. 10, No. 249, April 18, 1887, pp. 218–224.)

Bohn, G.

:05. Attráctions et oscillations des animaux marines sous l'influence de la luminère. Mem. Inst. général physiologique, Paris, No. 1, 110 pp.

Carpenter, F. W.

:05. The Reactions of the Pomace Fly (Drosophila ampelophila Loew) to Light, Gravity, and Mechanical Stimulation. Amer. Nat., Vol. 39, No. 456, pp.157–171.

Cole, L. J.

:01. Notes on the Habits of Pycnogonids. Biol. Bull., Vol. 2, No. 5, pp. 195–207.

Darwin, C.

'44. Brief Descriptions of Several Terrestrial Planariae, and of some Remarkable Marine Species, with an Account of their Habits. Ann. Mag. Nat. Hist., Ser. 1, Vol. 14, No. 91, pp. 241–251, pl. 5, figs. 1–4.

Darwin, C.

'81. The Formation of Vegetable Mould through the Action of Worms, with Observations on their Habits. New York, 8vo, viii + 326 pp.

Exner, S.

'91. Die Physiologie der facettirten Augen von Krebsen und Insecten. Leipzig und Wien, 8vo, viii + 207 pp., 1 Lichtdruck und 7 Tafeln.

Frandsen, P.

:01. Studies on the Reactions of Limax maximus to Directive Stimuli. Proc. Amer. Acad. Arts and Sci., Vol. 37, No. 8, pp. 185–227.

Graber, V.

'84. Grundlinien zur Erforschung des Helligkeits- und Farbensinnes der Tiere. Prag, Leipzig, 8vo, viii + 322 pp.

Harper, E. H.

:05. Reactions to Light and Mechanical Stimuli in the Earthworm Perichaeta bermudensis (Beddard). Biol. Bull., Vol. 10, No. 1, pp. 17–34.

Hesse, R.

'96. Untersuchungen über die Organe der Lichtempfindung bei niederen Thieren. I. Die Organe der Lichtempfindung bei den Lumbriciden. Zeitschr. f. wiss. Zool., Bd. 61, Heft 3, pp. 393–419, Taf. 20.

Hesse, R.

'97. Untersuchungen, u. s. w. II. Die Augen der Plathelminthen, insonderheit der tricladen Turbellarien. Zeitschr. f. wiss. Zool., Bd. 62, Heft 4, pp. 527–582, Taf. 27, 28.

Hoffmeister, W.

'45. Die bis jetzt bekannten Arten aus der Familie der Regenwürmer. Braunschweig, 43 pp.

Holmes, S. J.

:05. The Selection of Random Movements as a Factor in Phototaxis. Jour. Comp. Neurol. and Psychol., Vol. 15, No. 2, pp. 98–112.

Holmes, S. J.

:05^a. The Reactions of Ranatra to Light. Jour. Comp. Neurol. and Psychol., Vol. 15, No. 4, pp. 305–349.

Holt, E. B., and Lee, F. S.

:01. The Theory of Phototactic Response. Amer. Jour. Physiol., Vol. 4, No. 9, pp. 460–481.

Jänichen, E.

'96. Beiträge zur Kenntnis des Turbellarienauges. Zeitschr. f. wiss. Zool., Bd. 62, Heft 2, pp. 250–288, Taf. 10, 11.

Kühne, W.

'79. Chemische Vorgänge in der Netzhaut. Hermann's Handbuch der Physiologie, Bd. 3, Theil 1, pp. 235–342.

Latter, O. H.

:04. The Natural History of some Common Animals. Cambridge [Eng.], 8vo, x + 331 pp.

Loeb, J.

'90. Der Heliotropismus der Thiere und seine Uebereinstimmung mit dem Heliotropismus der Pflanzen. Würzburg, 8vo, 118 pp.

Loeb, J.

:05. Studies in General Physiology. Univ. of Chicago Press, Decennial Publications, Ser. 2, Vol. 15, Part I, pp. i–xvi + 1–424, Part II, pp. i–xiv + 425–782.

Mitsukuri, K.

:01. Negative Phototaxis and other Properties of Littorina as Factors in determining its Habitat. Annotationes Zoologicae Japonenses, Vol. 4, Part 1, pp. 1–19.

Moseley, H. N.

'74. On the Anatomy and Histology of the Land-Planarians of Ceylon, with some Account of their Habits, and a Description of two New Species, and with Notes on the Anatomy of some European Aquatic Species. Phil. Trans. Roy. Soc. Lond. for 1874, Vol. 164, pp. 105–171, pls. 10–15.

Moseley, H. N.

'78. Description of a New Species of Land-Planarian from the Hothouses at Kew Gardens. Ann. Mag. Nat. Hist., Ser. 5, Vol. 1, No. 3, pp. 237–239.

Nuel, J. P.

:04. La vision. Bibliothèque internationale de Psychologie expérimentale normal et pathologique. Paris, 376 pp.

Parker, G. H.

'95. The Retina and Optic Ganglia in Decapods, especially in Astacus. Mitth. Zool. Sta. Neapel, Bd. 12, Heft 1, pp. 1–73, Taf. 1–3.

Parker, G. H.

:03. The Phototropism of the Mourning-cloak Butterfly, Vanessa antiopa Linn. Mark Anniversary Volume, No. 23, pp. 453–469, pl. 33.

Parker, G. H.

:03ª. The Skin and the Eyes as Receptive Organs in the Reactions of Frogs to Light. Amer. Jour. Physiol., Vol. 10, No. 1, pp. 28–36.

Parker, G H., and Arkin, L.

:01. The Directive Influence of Light on the Earthworm Allolobophora foetida (Sav.). Amer. Jour. Physiol., Vol. 5, pp. 151–157.

Romanes, G. J.

'83. Mental Evolution in Animals. With a Posthumous Essay on Instinct by Charles Darwin. London, 8vo, 411 pp.

Sharp, [B.]

'91. On a Probable New Species of Bipalium. Proc. Acad. Nat. Sci. Phil., 1891, pp. 120–122.

Stern, R.

:05. Ueber Sehpurpurfixation. Graefe's Arch. f. Opthal., Bd. 61, Heft 3, pp. 561–563. *Abstract in* Zentralbl. f. Physiol., Bd. 19, No. 20, p. 757.

Torelle, E.

:03. The Response of the Frog to Light. Amer. Jour. Physiol., Vol. 9, No. 6, pp. 466–488.

Towle, E W.

:00. A Study in the Heliotropism of Cypridopsis. Amer. Jour. Physiol., Vol. 3, No. 8, pp. 345–365.

Wilson, E. B.

'91. The Heliotropism of Hydra. Amer. Nat., Vol. 25, No. 293, pp. 413–433.

Woodworth, W. McM.

'96. Notes on Turbellaria. 1. On the Occurrence of Bipalium kewense (Moseley) in the United States. Amer. Nat., Vol. 30, No. 360, pp. 1046, 1047.

Woodworth, W. McM.

'98. On the Occurrence of Placocephalus (Bipalium) kewense in the Sandwich Islands. Science, N. S., Vol. 8, No. 192, p. 302.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

EXPANSION AND COMPRESSIBILITY OF ETHER AND OF ALCOHOL IN THE NEIGHBORHOOD OF THEIR BOILING POINTS.

BY ALPHEUS W. SMITH.

INVESTIGATIONS ON LIGHT AND HEAT MADE AND PUBLISHED, WHOLLY OR IN PART, WITH APPROPRIATION FROM THE RUMFORD FUND.

EXPANSION AND COMPRESSIBILITY OF ETHER AND OF ALCOHOL IN THE NEIGHBORHOOD OF THEIR BOILING POINTS.

By Alpheus W. Smith.

Presented by E. H. Hall, May 9, 1906. Received October 30, 1906.

[For a summary of the results reached in this paper, see p. 458.]

Introduction.

This subject was suggested to the writer by Professor Hall, to whom it became of interest in his work on the van der Waals a in ethyl ether and in ethyl alcohol. In obtaining values of the van der Waals a by one of the methods employed in that paper[1] Professor Hall found it desirable to know both the coefficient of expansion and the coefficient of compressibility of these liquids just before they change into the vapor state.

The investigators who have concerned themselves with the coefficients of expansion and of compressibility of liquids have for the most part used large differences of temperature and of pressure in producing the change of volume in the liquid, so that every coefficient is the average coefficient over a considerable range of temperature and of pressure. Such data do not give us with certainty the coefficient of expansion and of compressibility at a particular temperature and at a pressure differing but little from the vapor pressure of the liquid at that temperature.

I, therefore, undertook to find by means of new measurements both the coefficient of expansion and the coefficient of compressibility of the two liquids in question at pressures which were only slightly greater and at those which were somewhat less than the vapor pressure of the liquid at the temperature used, — i. e., the coefficient of expansion and the coefficient of compressibility of these liquids just before they reach their boiling conditions, and these same coefficients when the liquids are in the superheated state.

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[1] Boltzmann-Festschrift, 1904.

If T is the absolute temperature, p the external pressure, v the specific volume, k the coefficient of compressibility at constant temperature, and e the coefficient of expansion at constant pressure, then k and e have by definition the following meanings.

$$k = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right),$$

$$e = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right).$$

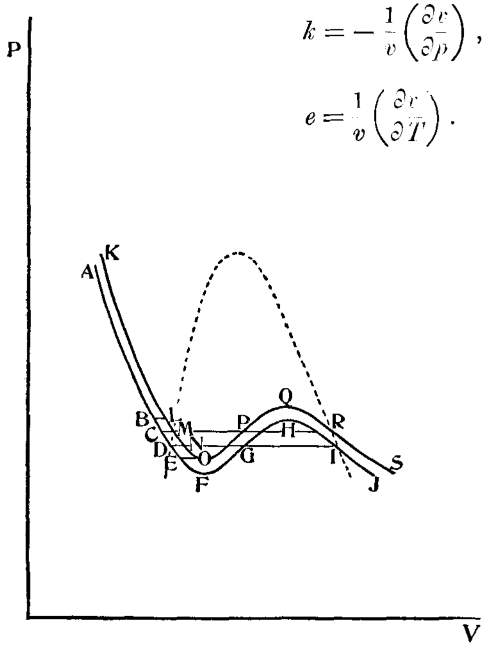

FIGURE 1.

If, according to the van der Waals equation, we plot on the pv-plane, as in Figure 1, the isothermals of the liquid, k and e each have a geometric interpretation; k is the reciprocal of the slope of the isothermal divided by v; and e is the distance between two isothermals measured parallel to the v-axis, divided by the difference of temperature between the isothermals and divided also by v.

The purpose of this research may now be made somewhat clearer by an examination of Figure 1, where we will suppose that we have plotted two of the isothermals of either ethyl ether or ethyl alcohol. Suppose the temperatures for which these isothermals are plotted differ from each other only by two or three degrees. To be specific, consider the isothermal A F H J. On this isothermal the point D represents the liquid when it is under a pressure equal to its vapor-pressure. The part D F of this isothermal represents the liquid in the superheated state. We are to determine the slope of such an isothermal, (1) at B and C, where the external pressure exceeds the vapor-pressure by about one atmosphere, (2) at D, where the external pressure is equal to the vapor-pressure, (3) at E, where the external pressure is less than the vapor-pressure. Furthermore, the distance B L between any two neighboring isothermals, when p is equal to one atmosphere, is to be compared, (1) with the distance C M between the same isothermals when p is equal to the vapor-pressure of the liquid at the higher temperature, (2) with the distance E O between these isothermals when p is less than the vapor-pressure of the liquid at either of the temperatures. B L, C M, and E O are parallel to each other and to the v-axis.

PART I: LIQUIDS UNDER PRESSURES GREATER THAN THEIR VAPOR-PRESSURES.

Thermostat.

In order to study the compressibility of liquids at low pressures, one must use small differences of pressure, not greater than 10 or 15 cm. of mercury, in getting the change of volume in the liquid. The changes in volume which result from such slight changes of pressure are very small and, since the coefficient of expansion of each of these liquids is large in comparison with its coefficient of compressibility, small changes in the temperature would introduce large errors in the determination of the compressibility. So small a change of temperature as 0.001° C., in either of these liquids, would introduce an error of about 3.5 per cent in the compressibility, if the change of volume were produced by increasing or by decreasing the pressure 20 cm. of mercury. If, therefore, the data were to have an accuracy of one per cent, the constancy of the temperature would become an important question. Moreover, the apparatus to be introduced into the thermostat was of such a form that the ordinary types of thermostats did not lend themselves to the purposes of this experiment. After some difficulty the following type was devised, which has met very well the requirements of this experiment.

The thermostat consisted in the main of a rectangular box, which was 150 cm. long, 60 cm. high, and 46 cm. wide. This box for brevity will be referred to as the *air-bath.* The top of the air-bath could be removed at will to allow the introduction of apparatus. Extending the whole length of the air-bath at mid-height of each side was an opening 2 cm. in width. These openings were covered with plate glass and served as windows through which observations could be made in a manner to be described later. In the interior of the air-bath, along its sides and bottom at a distance of about 5 cm. from the adjacent walls, were strung spirals of No. 18 german-silver wire, which in the aggregate had a resistance of about 20 ohms. These coils of wire, when connected in series with the city alternating circuit of 110 volts, served as heating coils by means of which the temperature of the air-bath could be raised to any desired value less than 60° C. In order to keep the air in the air-bath homogeneous as to temperature, a fan-motor which ran at fairly constant speed was placed at each of its ends. These motors stood just outside of the air-bath, and their shafts, each carrying a fan, projected into the interior.

For the regulation of the temperature of the air in the air-bath a modified form of the usual device, depending upon the expansion of alcohol, was used. A brass tube, with thin walls and an internal diameter of about 0.7 cm., was wound in the form of a spiral 50 cm. long and 6 cm. in diameter. One end being closed, the other was connected by means of a short piece of stout rubber tubing to the larger end of the glass tube, GADB (Figure 2). It was found better to make the spiral of brass than of glass. The brass, having a lower specific heat and a higher thermal conductivity than the glass, allowed the liquid which it contained to assume more rapidly the temperature of the air which surrounded it. The large surface which the spiral presented, in comparison with the volume of liquid which it contained, about 150 (cm.)^3, increased the rapidity with which the liquid assumed the temperature of the surrounding air. The spiral stood in a vertical position in the air-bath, and the glass tube B D (Figure 2) was also vertical. The thermo-regulator was filled with well-boiled alcohol. From the line E F around to the platinum point at B, it contained mercury. By taking out, or by adding mercury or alcohol, one fixed the temperature of the air-bath at any desired value. The platinum wires A and B (Figure 2) were con-

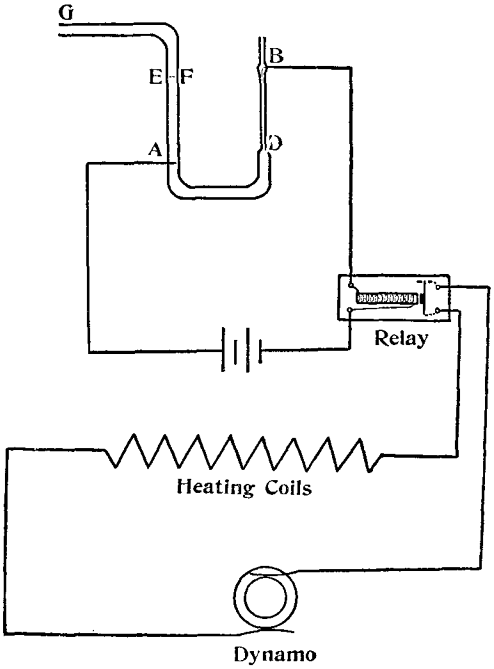

FIGURE 2.

nected in series with a relay of 1700 ohms and a battery of 8 volts. The other terminals of the relay were connected in series with the heating coils of the air-bath. These connections are shown diagrammatically in Figure 2. The oscillations which occurred in the air-bath were of the order of 0.2° C., and took place in about one minute.

I thought it well to isolate a portion of the space inside of the air-bath from the remainder by means of non-conducting materials, hoping thus to get an inner region in which these oscillations of temperature would have little effect. Since the apparatus to be kept at constant temperature was about 70 cm. long, and was to lie in a horizontal position, I chose as the region to be thus isolated the space inside of a cylindrical shell, 120 cm. in length, 15 cm. in internal and 25 cm. in external diameter. The walls of this cylindrical shell were made of concentric sheets of asbestos, the space between these sheets being packed with asbestos wool. The openings in the ends were filled with cotton wool. For the purposes of observation, it was necessary to have two narrow windows in the shell. These windows were diametrically opposite each other, and the glass plates closing them were at the inner surface of the shell. The temperature of the space thus enclosed was found to remain very nearly constant.

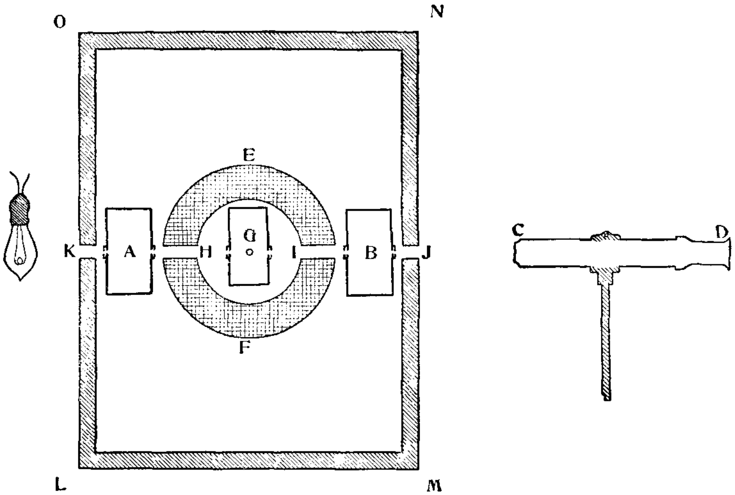

FIGURE 3.

An incandescent lamp was used as a source of illumination, and at first the heat from it, passing into the thermostat, caused a slight drift of temperature. To prevent this drift, and to prevent, as far as possible, the flow of heat from the thermostat into the room, water-windows were placed just inside of the air-bath and opposite the small openings in the cylindrical shell.

Figure 3 shows a cross section of the thermostat, made through the water-windows, the telescope, and the source of illumination. L M N O is the air-bath; E and F are the halves of the cylindrical asbestos shell; A and B, the two water-window s. The openings at K and J and those at H and I, were closed by plate-glass windows.

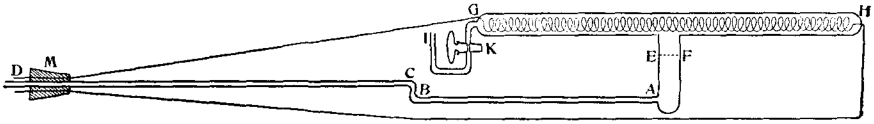

Figure 4.

Dilatometer and Piezometer.

The dilatometer (Figure 4) consisted of a cylindrical glass bulb H G, 30 cm. long and 1.7 cm. in diameter. For purposes of filling, which operation will be described later, one end of this bulb was closed by means of the stopcock K. At right angles to the axis of the bulb and midway between its ends was sealed another glass tube 5 cm. long, and of the same internal diameter as the bulb. The lower end of this short tube was closed, and to it was sealed the capillary tube A B C D, which from A to B was parallel to the bulb, from B to C at right angles to it, and from C to D again parallel to it. Near the end D this tube carried the rubber stopper M. The purpose of this stopper will be explained later. Inside the bulb of the dilatometer there was a spiral of fine platinum wire, to be used as a platinum thermometer. The description of the use of this platinum thermometer will be postponed.

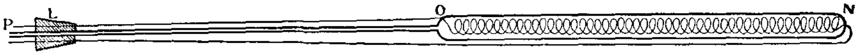

Figure 5.

The piezometer (Figure 5) consisted of the cylindrical bulb O N, to one end of which was sealed the capillary tube O P. The piezometer also contained a platinum thermometer, the lead wires of which passed out through the rubber stopper L. The capacity of the piezometer was determined by weighing it empty and then weighing it filled with distilled water at a known temperature. The capacity of the dilatometer was determined in precisely the same way. The former had a capacity of 64.04 (cm.)^3, the latter a capacity of 70.32 (cm.)^3.

The capillary stem of the piezometer, and also that of the dilatometer, was calibrated by use of a mercury column, so that the cross section of each of them was accurately known at all points. The capillary tube of the dilatometer, which had a mean cross section of 0.01918 (cm.)^2, happened to have a very uniform bore, which showed a variation of only about one half of one per cent over a distance of 20 cm. The capillary tube of the piezometer, with a mean cross section of 0.01536 (cm.)^2, showed a variation of about 2 per cent over a distance of 20 cm.

Brass Jacket.

In order to have the pressure on the inside of the piezometer the same as that on its outside it was inserted in a rectangular brass vessel which for the sake of brevity will be referred to as the *brass jacket*. In the course of the whole work two slightly different types of brass jackets were used. As their differences were of minor importance, only one of them will be described. The brass jacket consisted essentially of a rectangular brass box 71 cm. long, 11 cm. high, and 5 cm. wide, the walls of which were made of strips of brass 0.6 cm. in thickness. In one end of the brass jacket there was a rectangular opening 7 cm. long and 3 cm. wide, through which the piezometer or dilatometer could be introduced. This opening was then closed by the means of two castings, which were separated from each other and from the end of the brass jacket by means of rubber gaskets. These castings, after being firmly joined together and to the end of the brass jacket by means of screws, afforded a conical shaped orifice into which M (Figure 4) or L (Figure 5) fitted snugly. In order to prevent the stopper from coming out under pressure greater than one atmosphere, it was pressed into the conical orifice by means of a disc which, having at its centre an aperture through which passed the capillary stem of the piezometer or the dilatometer and the lead wires of the platinum thermometer, was firmly fastened to the castings by means of four screws. In either side of the brass jacket was a narrow window, closed by thick plate glass used with a rubber gasket, through which observations could be made. Into the top of the brass jacket and near each of its ends was inserted a short cylindrical brass tube. One of these tubes was connected to the pressure gauge so as to allow any desired pressure to be applied to the liquid in the brass jacket. The other was for purposes of filling, and was closed with a cylindrical brass cap after the filling.

Although the brass jacket was designed especially for experiments on compressibility, it was found convenient to insert the dilatometer in it. Figure 6 shows the brass jacket with the dilatometer in position. Part of one side is removed to show the dilatometer in the interior.

Pressure Gauge.

For the measurement of the pressure on the liquid an open manometer of the form shown in Figure 7 was used. In this figure parts of the vertical tubes are omitted. The manometer was made of heavy glass tubing with an internal diameter of 1.2 cm. The U-shaped part of the tube was filled with clean mercury to a height of 120 cm. Between the branches A B and C D of the U-shaped part of the tube were placed meter rods, end to end, on which one read off the difference in height of the mercury in the two parts of the tube. One had, of course, to read the barometer each time in order to get the absolute pressure on the liquid. From H and G the pressure gauge was connected by means of pressure tubing to the capillary tube of the piezometer or the dilatometer and to the small tube in the top of the brass jacket. Hence the internal and the external pressure on the walls of the piezometer or of the dilatometer were the same. By means of the stopcock K_1 the pressure gauge could be made to communicate with the external atmosphere. The large tube I, which was 42 cm. long and 5.5 cm. in diameter, served as an air-chamber into which air could be forced or from which it could be exhausted. By turning the stopcock K_2 the pressure on the liquid could be increased or decreased by as small an amount as one wished. The other end of I was closed with the stopcock K_3.

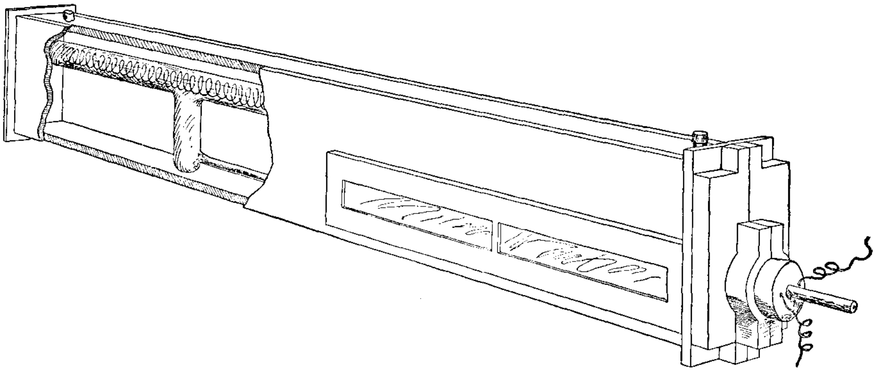

FIGURE 6.

The stopcock K_4 was for convenience in case the mercury had for any reason to be removed from the pressure gauge.

Filling the Dilatometer and the Piezometer.

The greater part of the capillary stem and the short vertical tube of the dilatometer up to the line E F (Figure 4) were filled with a known amount of clean, dry mercury. The mercury was poured into the dilatometer, care being taken to avoid, as far as possible, any air bubbles. One, however, could never be perfectly sure that slight traces of air were not left either in the mercury or along the sides of the tube which contained the mercury. Since the coefficient of expansion of air is only about twice the coefficient of expansion of either ethyl ether or of ethyl alcohol, these very slight traces of air would introduce no appreciable error in the data on the expansion of these liquids. The end of the capillary tube being closed, I (Figure 4) was joined by means of a short piece of rubber tubing to a glass tube bent so as to dip into a flask containing the liquid to be investigated. This liquid had been previously boiled to remove traces of air. By alternately heating the dilatometer above the boiling point of the liquid and then cooling it, the bulb of the dilatometer was finally filled with liquid which was free from air. When the liquid had assumed a temperature nearly equal to the temperature at which observations were to be made, the stopcock K was closed.

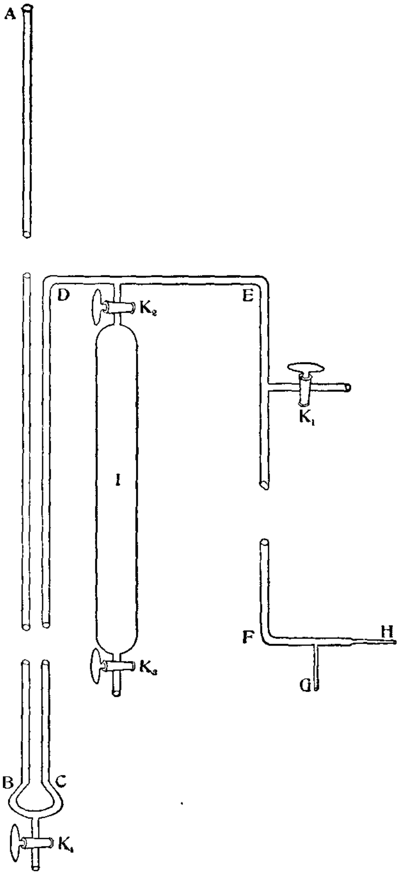

FIGURE 7.

It was desirable to have the mercury in the dilatometer for two reasons. At low pressures the otherwise free surface of the ether or the alcohol in the capillary tube afforded an opportunity for evaporation, and as considerable time elapsed between observations on the coefficient of expansion, the loss due to evaporation would introduce some error in the measurement of the change of volume. Furthermore, the liquid retreating down the capillary tube with decreasing temperature, leaves a film on the walls of the tube. This makes the observed change of volume greater than the true change of volume. This film will not disappear for a long time, and a similar cause of error enters when the temperature is again increased. Both of these sources of error were avoided by having mercury in the lower part of the dilatometer.

It was hoped that a piezometer of the same form as the dilatometer might be used in the work on compressibility; for the error due to the adhesion of the liquid on the walls of the capillary tube enters into the data on compressibility as it did into the data on expansion. The error due to the evaporation from the surface of the liquid was in the work on compressibility not large enough to be of importance; for the observations on compressibility were made very rapidly. It was, however, found that since it was impracticable to boil the mercury into such a piezometer, the slight traces of air from which the bulb and stem could not be entirely free, introduced serious error in the compressibility at low pressures. It was therefore decided to dispense with the mercury in the piezometer, and to use an instrument of the kind shown in Figure 5. The liquid was boiled into this piezometer in the same way in which it was boiled into the dilatometer. After the filling, the liquid in the bulb was heated 2° or 3° C. above the temperature at which observations were to be made. When the liquid again cooled down to the desired temperature, the exposed surface was somewhere near the middle of the capillary tube O P.

It was possible to get a rough estimate of the error due to the adhesion of the liquid on the walls of the tube. A glass tube of about the same bore as the capillary tube of the piezometer was filled with ether or alcohol and the liquid was then allowed to flow out of it, leaving the inner walls of the tube wet. The weight of the film of liquid adhering to the tube was determined at once. Knowing the capacity of the tube, one found that about 2.5 per cent of the liquid was left on the walls of the tube. In all the work on compressibility the necessary correction was made for this source of error.

Measurement of Change of Volume.

Since the cross section of the capillary tube of the piezometer and that of the dilatometer were accurately known, in order to measure the change of volume corresponding to a definite change of pressure or of temperature one need only observe the distance which the liquid meniscus in the capillary tube moved. For this purpose a telescope mounted with its axis horizontal on the screw of a dividing-engine was used. The position of the telescope of the dividing-engine is indicated in Figure 3. The screw of the dividing-engine had a pitch of 0.05 cm. and was graduated to read to 0.000125 cm.

Measurement of Temperature.

It has been already pointed out that both the piezometer and the dilatometer contained a platinum thermometer for the measurement of the temperature of the liquids. Both of these thermometers were taken from the same piece of fine platinum wire, which was about 0.015 cm. in diameter. This wire was in each case annealed by passing through it a current of 2 amperes for about 6 seconds. The wire was then wound in the form of a helix, 30 cm. long and 1.3 cm. in diameter. To each end of the helix was hard-soldered a short piece of rather stout platinum wire. This was found necessary in order to prevent the wire from breaking off where it was sealed into the ends of the glass bulbs, G and H (Figure 4) and O and N (Figure 5). Each of the platinum spirals, when inside the bulb, extended nearly the whole length of the bulb, and the axis of the spiral coincided nearly with the axis of the bulb. To the terminals of the spirals were soldered copper lead wires, which, after being carried along parallel to the capillary tubes of the dilatometer and the piezometer respectively, passed out through the respective rubber stoppers M (Figure 4) and L (Figure 5). It was found necessary to enclose these lead wires in fine rubber tubing, in order to prevent battery-action between them and the liquid in which they were subsequently immersed. The total resistance of the lead wires on the dilatometer was 0.114 ohm, while those on the piezometer had a resistance of 0.123 ohm.

Since both platinum thermometers were taken from the same piece of wire and had been treated in precisely the same way, it seemed sufficient to calibrate only one of them. I calibrated the platinum thermometer which was in the piezometer and used the temperature coefficient thus obtained in the work with the one which was in the dilatometer. In the calibration of the platinum thermometer the bulb of the piezometer was immersed in a mixture of ice and water or in the vapor of one of the following liquids, boiling under atmospheric pressure, — water, benzol, chloroform, and carbon disulphide. The temperatures, except those of melting ice and boiling water, were determined by means of a well-tested Baudin thermometer.

Table I shows the resistance of this platinum thermometer at the five temperatures previously referred to, and the change per degree of its resistance over the four intervals of temperature.

TABLE I.

t° C.	Resistance.	Range of Temp.	Change per Degree.
0°	25.581 ohms	0 – 45.9	.0838 ohms
45°.9	29.432 "	45.9– 60.4	.0841 "
60°.4	30.640 "	60.4– 80.3	.0844 "
80°.25	32.323 "	80.3–100.2	.0895 "
100°.2	34.111 "		

It will be observed from this table that between 0° C. and 80° C. the change per degree in the resistance of the platinum wire is very nearly constant, and over this range of temperature shows a variation of about 0.7 per cent. Between 80° C. and 100° C. it increases about 1.2 per cent. As all the temperatures at which the platinum thermometer was used were below 80° C. the temperature of the liquid is obtained with sufficient accuracy from a knowledge of the change in the resistance of the platinum thermometer.

An attempt was made to fit the data into the formula of Callendar and Griffiths.[2]

$$T - p\,t = \delta\left[\left(\frac{T}{100}\right)^2 - \frac{T}{100}\right]$$

where

T = absolute temperature,
δ = constant,

$$p\,t = \frac{(R_t - R_0)\,100}{R_{100} - R_0}.$$

R_0 = Resistance of thermometer at 0° C.
R_t = " " " " t° C.
R_{100} = " " " " 100° C.

This attempt was, however, unsuccessful, and it appears that the law given by Callendar and Griffiths does not hold for the specimen of platinum wire here used.

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[2] Phil. Trans., **182**, A. (1892), p. 119.

The resistance at 0° C. of the platinum thermometer which was in the dilatometer was found to be 24.528 ohms.

For the measurement of the change in the resistance of the platinum thermometer during the experiments on compression and expansion a Carey-Foster bridge was used. The bridge wire, which was of manganin, was calibrated in the usual way and had a resistance of 0.01317 ohm per cm. The platinum thermometer was connected to the bridge by means of manganin lead wires with a resistance of 2.359 ohms and a temperature coefficient of 0.000015 per degree C. The coils on the other side of the bridge were also made of manganin wire, which had the same temperature coefficient as the lead wires. The resistance of each of these coils was accurately known. The change in the resistance of these coils due to change in their temperature was very small, but as it introduced in the coefficient of expansion an error of about 0.2 per cent for a change of 1° C., the temperature of the room in the neighborhood of the coils was always noted and the necessary correction applied. The temperature of the room never changed more than 2° C. or 3° C. between observations on expansion.

For a battery one Leclanché cell was used. A sensitive form of Thomson galvanometer was found satisfactory for the resistance measurements.

The heating of the platinum thermometer by the current used in measuring its resistance had, of course, to be considered. By putting a resistance of 1 ohm in the battery circuit and by using coils with a resistance of 0.1 ohm for balancing coils in the Carey-Foster bridge, it was practicable to reduce the current through the platinum thermometer to about 0.005 ampere. This current flowed for not more than one second. The amount of heat thus generated was small, and would, it seems, for the most part be communicated to the liquid which surrounded the platinum wire. Moreover, as this heating effect was always about the same, it could not at worst introduce more than a small constant error in the determination of the temperature, and in getting the change of temperature this small error would be eliminated. It may therefore be neglected here.

This method of measuring the change of temperature in the liquid was found quite satisfactory. In the work on expansion the temperature of the liquid was changed only 2° C. or 3° C. The corresponding change in the resistance of the platinum thermometer was never greater than the resistance of the bridge wire, and care was taken to measure the change of resistance in terms of the bridge wire alone. The work was thus free from any error that might have been made in determining the resistance of the standard coils. It was practicable to determine the change in the temperature of the liquid with certainty to 0.01° C., but the absolute temperature of the liquid was not known closer than 0.1° C.

Compressibility of Glass.

Since the liquid under examination was enclosed in a glass bulb which changed its capacity with change of pressure on the liquid, the change of volume in the liquid was partly hidden. As the piezometer contained a platinum thermometer, the compressibility of the glass of which it was made could not be determined directly in the usual way, by use of mercury. Accordingly another piezometer was made which differed from the first only in this, — that it contained no platinum thermometer. Its bulb was made from the same piece of glass tube from which the bulb of the other piezometer was taken. Its capillary stem had a mean cross section of 0.00541 $(\text{cm.})^2$ and had been calibrated in the usual way. The capacity of this piezometer, determined by weighing it empty and then weighing it filled with mercury, was found to be 63.21 $(\text{cm.})^3$. The mercury being at 20° C., observations were made in the usual way on the apparent change of volume of the mercury in glass for a change of pressure, external and internal, of three atmospheres. Before the measurement of the change of volume a wait of ten minutes was allowed for the heat of compression to disappear. Experiment showed, $k_1 - k_2 = 1.77 \times 10^{-6}$, where

k_1 = compressibility of mercury at 20° C.
k_2 = " " glass " " "

This value is the mean of twenty observations, — ten made by increasing and ten by decreasing the pressure. According to Amagat [3] $k_1 = 3.92 \times 10^{-6}$, whence $k_2 = 2.15 \times 10^{-6}$. This value is in good agreement with the value given by Amagat [4] and with that given by Richards and Stull.[5]

Expansion of Glass.

Not only does the capacity of the piezometer change when the pressure on it is changed, but also the capacity of the dilatometer changes when its temperature is increased or decreased. One must therefore know the coefficient of expansion of glass in order to make the necessary correction in the work on expansion of liquids. By the ordinary method with mercury, the coefficient of expansion of the glass in the dilatometer was found to be 0.00023. One cannot be sure about the last figure in this value, but an error of two or three units in that place is not of importance in the present work.

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[3] Comp. Rend., **108**, 228 (1889).

[4] Loc. cit.

[5] Publication No. 7 of Carnegie Institution of Washington.

Preparation of Liquids.

The ether used in these experiments was obtained from Kahlbaum and had been distilled over sodium. Not all the observations were made on the same specimen of ether, but they were all made on ether which had been treated in precisely the same way. The alcohol was purified in the usual way by distillation in the presence of lime. The distillation took place behind suitable drying tubes to prevent the alcohol from taking up water from the air, and during all subsequent work the flask containing the alcohol was not allowed to communicate with the external atmosphere except through a drying tube. The purity of the alcohol was determined by the ordinary methods of specific gravity. The purity corresponding to a particular density was found from the tables of Landolt and Börnstein.

Expansion.

After the dilatometer had been filled with liquid in the manner already described and had been placed in the thermostat and allowed to assume a constant temperature, the pressure on the liquid was fixed at the desired value and observations were made in the usual way on the change of volume corresponding to a change of 2° C. or 3° C. in the temperature of the liquid. Before the change of volume, or the change of temperature, was finally observed, about seven hours was allowed for the temperature to equalize. Sometimes the observations were made by increasing, and sometimes by decreasing, the temperature. The change of volume due to a change of 2° C. in the temperature of the liquid was rather large, causing a movement of about 15 cm. along the capillary stem, and no difficulty was experienced in measuring it. The observed change of volume due to a certain change of temperature was, of course, due to the change of volume of the liquid and of the mercury and of the glass bulb. Knowing the volume of the mercury in the dilatometer and its coefficient of expansion, one gets at once the change of volume due to this source. The error due to the expansion of the glass bulb was corrected for in the way already pointed out.

Tables II and III show the results of these observations. In these tables p is external pressure on the liquid, t_1 is the initial temperature, and t_2 is the final temperature; P is the vapor-pressure of the liquid at the corresponding temperature t_2. The values of the vapor-pressure at the different temperatures were obtained from vapor-pressure curves plotted from the data of Ramsay and Young,[6] and of Regnault.[7] The mean of the two values thus obtained is given in the first column of these tables. The values of the vapor-pressure given are certainly correct to within a few millimeters of mercury, an accuracy which is quite sufficient for our present purposes. Both the vapor-pressure and the external pressure are expressed in centimeters of mercury.

TABLE II.

Expansion of Ethyl Ether.

P.	p.	t_1.	t_2.	Δt.	$e \times 10^5$.	Mean $e \times 10^5$.
	73.5	22°.1	24°.6	2°.45	169	
	73.3	22.4	24.7	2.34	170	170
	51.5	22.3	24.7	2.40	170	
51.0	51.5	22.1	24.2	2.07	170	170
	73.5	26.5	28.8	2.29	172	
	73.4	26.3	28.7	2.42	171	
	73.3	26.4	28.7	2.29	172	172
59.0	59.3	25.9	28.3	2.35	171	171
	116.6	30.3	32.9	2.63	173	173
	73.2	30.1	33.5	3.41	175	
	73.3	30.5	33.5	2.99	176	
	73.5	30.2	32.8	2.62	174	
	73.4	30.5	33.0	2.51	173	
	73.5	30.7	33.3	2.61	173	174
	69.5	30.4	33.0	2.61	174	
69.0	69.5	29.8	32.6	2.81	174	174

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[6] Phil. Trans., **177**, I, 123 (1886).
[7] Mém. de l'Acad., **26**, 336 (1862).

TABLE III.

Expansion of Ethyl Alcohol — 99.5 Per Cent.

P.	p.	t_1.	t_2.	Δt.	$e \times 10^5$.	Mean $e \times 10^5$.
	73.5	20.2°	23.5°	3.27°	112	
	73.5	20.2	24.0	3.79	112	112
	6.5	20.4	24.3	3.89	111	
5.3	6.2	20.5	23.6	3.14	112	112
	74.2	39.0	42.4	3.39	115	
	74.1	39.4	42.9	3.52	116	116
	16.9	39.1	42.9	3.84	114	
16.0	17.0	39.6	43.5	3.92	116	115
	123.0	74.6	84.8	10.2	144	
	123.0	73.5	84.8	11.3	142	
99.0	123.0	73.5	84.4	10.9	142	143

Compressibility.

In compression the liquid is heated. One must therefore proceed in one of two ways. One must either wait for the heat of compression to disappear, or read the change of volume at once and correct for the error due to the heat of compression. The first method depends upon the operation being as nearly as possible isothermal; the latter, on the condition that the compression be very nearly adiabatic. Since it was practicable to read the change of volume in less than 30 seconds after the change of pressure had been made, it seemed possible to get very nearly adiabatic compression. This method had the further advantage that, by reading the change of volume rapidly, any drift of temperature, to which we have already referred as a source of error, would be much less serious than if we were obliged to wait for the heat of compression to equalize. To prevent as far as possible any loss of heat during compression, the brass jacket was filled with the same kind of liquid which was in the piezometer. The temperature of the liquid which surrounded the piezometer was therefore raised the same amount by compression as was the temperature of the liquid in the piezometer. There was, of course, some loss due to the heat capacity of the piezometer.

To calculate the heat of compression, we made use of the well-known formula,[8]

$$\varDelta t = \frac{1033}{42350} \frac{e}{C_p} T$$

where e is the coefficient of expansion at constant pressure; T is the absolute temperature of the liquid; Δt is the change in the temperature of the liquid due to an adiabatic compression of one atmosphere; and C_p is the specific heat of the liquid at constant pressure. The values of C_p for different temperatures for both ether and alcohol were calculated from the equations given by Regnault,[9]

$$q = 0.529t + 0.000296t^2 \text{ for ether,}$$
$$q = 0.5475t + 0.00112t^2 + 0.00000221t^3 \text{ for alcohol,}$$

where t is temperature of the liquid on the ordinary centigrade scale, and q is the amount of heat in calories required to raise the temperature of the liquid from 0° C. to t° C.

In order, however, to make sure that the method of adiabatic compression would give trustworthy results, the compressibility of alcohol was determined at 20° C. by the first method, — i. e. by waiting for the heat of compression to equalize before the change of volume was read. The compressibility of the same specimen of alcohol was then determined by the second method, in which the change of volume is read rapidly and corrected for the error due to the heat of compression. Ether was examined in a similar way. The values obtained by the first method differing from those obtained by the second method by only one per cent, the method of adiabatic compression was regarded as justified, and was used in all subsequent work on compressibility.

After the piezometer had been filled and had been in the thermostat long enough to assume a constant temperature, observations were made in the way just indicated, on the apparent change of volume of the liquid in glass. The change of volume produced by a change of pressure of 15 cm. of mercury was very small. In such a case, the total movement of the liquid meniscus in the capillary tube amounted to about 0.08 cm., and it was difficult to observe this change of volume accurately. Moreover, the liquid meniscus did not preserve a perfectly symmetrical form, and this caused some error. By taking the mean of twenty observations, — ten made by increasing and ten by decreasing the pressure, — the probable error was reduced to one per cent or less.

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[8] Sir William Thomson, Mathematical and Physical Papers, **3**, 238.

[9] Mém. de l'Acad., **26**, 262 (1862).

Tables IV and V contain the results of these observations on ether and on alcohol. In these tables, k is the compressibility when the pressure is expressed in atmospheres, and P is the vapor-pressure, in centimeters of mercury, of the liquid at the corresponding temperature t.

TABLE IV.

Compressibility of Ethyl Ether.

P.	t.	Range of Pressure in Cm. of Mercury.	$k \times 10^6$.
46.5	21.6	152–76	184
		76–47	185
		76–61	184
		61–47	185
56.0	26.5	152–77	195
		152–77	193
		77–56	194
		77–56	193
67.6	31.5	152–77	204
		77–68	206

Part II: Liquids under Pressures less than their Vapor-pressures.

In order to have a liquid exist under a pressure less than that corresponding to the vapor-pressure at the respective temperature, the gaseous and liquid phases must, of course, not be in contact. In the dilatometer used in the work on the coefficient of expansion described in Part I of this paper, the liquid under investigation was in contact with mercury in such a way that its gaseous phase was not allowed to form. The boundary condition for superheating being thus fulfilled,

TABLE V.

COMPRESSIBILITY OF ETHYL ALCOHOL.

P.	*t.*	Purity.	Range of Pressure in Cm. of Mercury.	$k \times 10^6$.
4.4	20°.0	% 99.0	150–76	103
			76–56	103
			56–36	103
			36–16	104
			16–6	104
			75–38	103
			40–6	103
4.4	20.1	99.6	154–76	108
			76–56	109
			56–36	109
			56–36	108
			36–16	108
			16–7	108
4.2	19.2	99.5	152–76	106
			76–40	107
			40–6	106
13.3	40.0	99.5	152–76	122
			76–40	122
			40–14	121
32.0	58.3	99.5	153–76	140
			76–36	139

it was at first expected to use this dilatometer for determining the coefficient of expansion and a similar piezometer for determining the coefficient of compressibility in the superheated condition. After some efforts, in which this method was in part successful, it was abandoned for another which has proved more satisfactory.

Apparatus.

The dilatometer used in this part of the work consisted of the cylindrical glass bulb A B, Figure 8, to one end of which was sealed the capillary tube B C D E F, which was bent in the form indicated in that figure. The bulb was about 30 cm. long and 1.7 cm. in diameter and had a capacity of 60.73 (cm.)^3. The capillary tube had an internal diameter of about 0.15 cm., except the part from E to F, which had a mean cross section of 0.1506 (cm.)^2. In order to keep the temperature constant, the dilatometer was placed in a water-bath, which was heated by means of a Bunsen burner. Since the sides and top of the water-bath were covered with asbestos and the space between the bottom of the water-bath and the table on which it stood was enclosed by asbestos walls, a small flame was sufficient to keep the water-bath at the required temperature. After some attempts it was found possible, by means of an ordinary pinch-cock, to regulate the supply of gas so that the temperature of the water-bath remained sufficiently constant. The capillary tube B C passed out through a rubber stopper in the end of the water-bath, and the U-shaped part C D E F dipped into a large beaker filled with ice and water. For this beaker there was later substituted

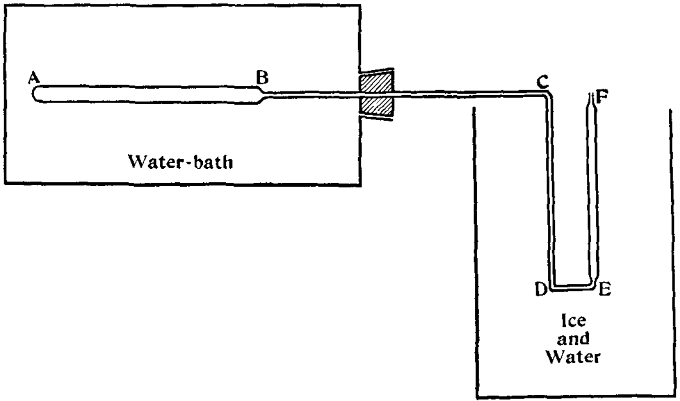

FIGURE 8.

a rectangular brass box with glass windows. In the experiments on the expansion of ether between 0° C. and 16° C. a freezing mixture containing ice, water, and salt, at —2° C., was used instead of ice and water. The liquid in the U-tube was therefore kept near 0° C., while the remainder of the liquid was at a higher temperature. One has here, then, the boundary condition for superheating all the liquid except that contained in the U-shaped part of the capillary tube which dipped into the beaker of ice and water, for it was in contact only with glass or with some of the same sort of liquid at a lower temperature.

The piezometer used in determining the compressibilities in the superheated state was of the same form as the dilatometer. For the vertical tube E F was substituted a smaller tube of cross section 0.0184 $(\text{cm.})^2$. The bulb of the piezometer, which had a capacity of 61.11 $(\text{cm.})^3$, was inserted in a cylindrical brass tube, one end of which was closed with a rubber stopper, through the centre of which passed the capillary tube B C. The other end of this brass tube was closed by means of a disk, in the centre of which was a small tube, through which the interior of the brass tube could be made to communicate with the pressure gauge, so that the pressure on the inside and that on the outside of the bulb of the piezometer were the same. The pressure on the inside of the capillary tube was not equal to that on its outside. On account of this inequality of pressure the internal volume of the capillary tube varied somewhat, and the magnitude of the error introduced into the data on compressibility from this source had to be considered. By means of the formula given on page 117 of Poynting and Thomson's "Properties of Matter," the change in internal volume for a typical case was calculated, and the error introduced from this source was found not to exceed one fifth of one per cent, and may be neglected.

Since in the work on expansion that part of the liquid which was in the U-shaped tube remained at 0° C., it introduced no error into the results. In the work on compressibility, however, the observed change of volume, due to a certain change of pressure on the liquid, was the sum of the change of volume of the liquid in the bulb, which was superheated, and the change of volume of the liquid in the U-shaped tube, which was not superheated, less, of course, the change in the volume of the glass bulb. Knowing the compressibility and the volume of the liquid in the U-shaped tube, one easily made the necessary correction for the error arising from this source, which was about one per cent. There was another correction which had to be applied to the observed change of volume both in compressibility and in expansion. In either of these cases the observed change of volume is supposed to be measured at the temperature of the liquid under investigation. In the present method, however, the change of volume due to a change of temperature or pressure was practically transferred either from the bulb A B (Figure 8), where it had a temperature greater than 0° C., to the tube E F (Figure 8), where its temperature was about 0° C., or from the tube E F back to the bulb A B. In either case it is obvious that the observed change of volume is less than the true change of volume. Taking an approximate value of the coefficient of expansion of the liquid, one at once finds what value the observed change of volume would have if it had been measured at the temperature of the liquid in the bulb instead of at 0° C. The magnitude of this correction varied with the temperature of the liquid in the bulb, but was never greater than seven per cent of the total change of volume.

Expansion.

If observations were to be made on the coefficient of expansion, the dilatometer was filled in the manner already described on page 429 and then placed in position in the water-bath, the temperature of which was raised to the desired value and allowed to become constant. Enough of the liquid was removed from the tube E F (Figure 8) to allow the free surface to stand about 2 cm. above the point E. The end of the tube E F was connected to the pressure gauge described on page 429 and the external pressure on the liquid was made slightly less than the vapor-pressure corresponding to the temperature of the liquid in the bulb of the dilatometer. This temperature was determined by means of a well-tested Baudin thermometer which passed down through the cover of the water-bath and had its bulb near the middle of the cylindrical bulb of the dilatometer. The pressure remaining constant, observations were then made on the change of volume corresponding to a definite change of temperature. Before observing the change of volume a wait of about one hour was allowed for the temperature to attain equilibrium. After observations on the change of volume and on the change of temperature had been made, the external pressure on the liquid was increased, until the liquid was no longer superheated.[10] The liquid was then allowed to cool down to

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[10] This method of procedure was adopted after the first four attempts to obtain values of the coefficient of expansion of ether in the superheated state. In each of these attempts the following peculiarity was noticed.

I had the ether at a temperature of 35° C. and under a pressure of 75 cm. of mercury. Leaving the pressure unchanged, I increased the temperature up to 45° C., and when the temperature had become constant, noted the change of temperature and the corresponding change of volume. Without changing the

TABLE VI.

EXPANSION OF ETHYL ETHER IN THE SUPERHEATED STATE.

P.	p.	t_1.	t_2.	Δt.	$e \times 10^5$.	Mean $e \times 10$.
18.5	18.0	0°	16°.2	16°.2	152	
18.5	17.8	0	19.1	19.1	151	152
43.2	42.0	19.8	34.0	14.2	170	
43.6	42.0	20.2	35.3	15.1	171	
42.0	41.7	19.1	35.8	16.7	170	
42.5	39.2	19.5	35.9	16.4	172	171
64.9	54.1	30.5	40.9	10.4	174	
65.1	51.6	30.6	41.0	10.4	178	
66.3	51.4	31.1	41.2	10.1	177	
64.0	51.9	30.2	41.4	11.2	179	177
80.2	75.0	35.9	44.9	9.0	185	
80 5	75.0	36.1	45.1	9.0	186	
77.9	75.4	35.3	46.9	11.6	184	
77.9	75.4	35.3	45.5	10.2	183	185

pressure, I removed the flame from beneath the water-bath in order to allow the temperature of the ether to return to 35° C., expecting to observe the decrease in volume corresponding to this decrease in temperature. But, instead of cooling quietly down to its former temperature, the ether suddenly changed into vapor shortly after the flame had been removed. It occurred to me that the reversing of the temperature was the cause of this sudden vaporization. I then adopted the method described above, and no further difficulty was encountered from these sudden vaporizations. It is seen that by this method I returned to the unsuperheated state by an increase of pressure instead of by a decrease of temperature.

I do not consider that the four observations which I made justify me in saying that the reversal of the temperature was the true cause of these sudden vaporizations, but as we do not fully understand the conditions of equilibrium of the superheated liquid, it seems best to record these unexplained observations.

its former temperature and a new series of observations commenced. The cross section of the tube E F (Figure 8) being large, no correction was made for the surface film which adhered to its sides.

Tables VI and VII show the results for ethyl ether and ethyl alcohol respectively. In these tables P is the vapor-pressure, in centimeters of mercury, of the liquid at the corresponding temperature *t*, and *p* is the external pressure, in centimeters of mercury, on the liquid.

TABLE VII.

EXPANSION OF ETHYL ALCOHOL IN THE SUPERHEATED STATE.

P.	p.	t_1.	t_2.	Δt.	$e \times 10^5$.	Mean $e \times 10^5$.
13.3	12.5	40°.3	55°.6	15°.3	119	
13.4	10.3	40.5	55.4	14.9	121	
13.4	10.8	40.6	55.2	14.6	121	
13.4	10.4	40.7	55.1	14.4	120	120
35.8	29.5	60.5	71.1	10.6	132	
35.2	30.2	60.1	70.1	10.0	135	
34.3	27.9	59.5	70.2	10.7	133	
32.0	28.4	58.0	69.3	11.3	131	133

It will be observed from an examination of these tables that the mean of the results obtained under the same conditions has in all cases a probable error of less than one per cent.

Compressibility.

The piezometer, after having been filled, was inserted into the cylindrical brass tube which had been previously filled with water. Both were then placed in position in the water-bath. The connections were made with the pressure-gauge in the usual way. After the temperature of the water-bath had been raised to the desired value and allowed to become constant, it was determined by means of the same Baudin thermometer used in the work on expansion. From an initial value somewhat less than the vapor-pressure of the liquid in the bulb of the piezometer, the pressure was decreased about 40 cm. of mercury and the corresponding change of volume was read. The pressure was then increased to its original value and the change of volume noted. These changes of volume were small and were measured with a cathetometer. The observations were made rapidly, and the apparent change of volume had, therefore, to be corrected for the error due to the heat of compression. This correction was made in the same way in which it was made in Part I of this paper. Here one has, however, to assume that the specific heat of the liquid is about the same in the superheated state as it is when the liquid is not superheated. As it would require a change of ten per cent in the specific heat to produce an error of about one per cent in the compressibility, this assumption seems justified.

TABLE VIII.

COMPRESSIBILITY OF ETHYL ETHER IN THE SUPERHEATED STATE.

P.	*t.*	Range of Pressure in Cm. of Mercury.	$k \times 10^6$.	Mean $k \times 10^6$.
77.4	35°.1	76–27	216	
77.2	35.0	76–27	214	
77.7	35.2	76–27	218	
77.9	35.3	76–27	212	215
88.5	39.1	76–28	226	
88.5	39.1	76–28	225	
88.8	39.2	76–28	224	
89.1	39.3	76–28	224	225

In Tables VIII and IX are given the results for ethyl ether and for ethyl alcohol. Each of the values tabulated in the fourth column of these tables is the mean of twenty observations, ten made by decreasing and ten by increasing the pressure. The mean values in the fifth column have a probable error of about one per cent. In these tables *P* is the vapor-pressure in centimeters of mercury at the corresponding temperature *t*. In finding the values of *k*, pressures are reckoned in atmospheres.

Application to the van der Waals a.

[A considerable part of the following discussion was written by Professor Hall.]

It has already been pointed out that the determination of the coefficients e and k in the neighborhood of the boiling condition was undertaken on account of the importance which is attached to them in the study of the van der Waals a. Professor Hall, in the discussion referred to at the beginning of this paper, dealt with two assumptions relating to the liquid and the vapor conditions :

TABLE IX.

COMPRESSIBILITY OF ETHYL ALCOHOL IN THE SUPERHEATED STATE.

P.	t.	Range of Pressure in Cm. of Mercury.	$k \times 10^6$.	Mean $k \times 10^6$.
75.6	78.0	76–36	158	
75.9	78.2	76–36	156	
75.8	78.1	76–36	156	
76.0	78.3	76–36	157	157

"1, That the pressure due to molecular attraction within a fluid is $\frac{a}{v^2}$, where v is the specific volume and a is some constant;

"2, That the energy per molecule, aside from the potential energy due to the attraction just mentioned, is a function of temperature only, so that it remains constant during any isothermal change of state."

Using the data of Amagat for the expansibility and the compressibility of ethyl ether and ethyl alcohol, he found that, *if assumption 2 held good*,[11] assumption 1 could not be true for either of these substances in the liquid state, and that, on the contrary,

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[11] I do not commit myself to the opinion that assumption 2 is necessarily true. It is quite possible that we have in both liquid ether and liquid alcohol some peculiar state of aggregation of the molecules in pairs or triplets or other small groups such as Sutherland has shown the probability of in the case of liquid water. If such groups do not exist in a given liquid, we should, according to the van der Waals hypotheses, expect both assumption 1 and assumption 2 to hold. If such groups do exist in a given liquid, and if their number is independent

"I, a is not a constant but, in each of the liquids examined, a function of both p and T.

"II, In each liquid at constant temperature a increases with increase of volume," that is, with decrease of pressure.

Amagat's data do not, in the case of either liquid, serve for the determination of a with certainty very near the boiling condition at either high or low temperatures. On the other hand Professor Hall, taking the data furnished by Regnault for the evaporation of ether and of alcohol, and taking assumption 2 as holding through the process of vaporization, found by means of a well-known formula the value of a for this process at a number of temperatures in the case of each liquid. The value of a obtained from any set of evaporation data he called a', and the following propositions were established with relation to the values of a and a':

"III, In alcohol a is much less than a' at low temperatures, but with rise of temperature the difference diminishes, a growing larger and a' smaller.

"IV, In ether a, at moderate pressures, is somewhat larger than a'; and both a and a' diminish slowly with rise of temperature, apparently approaching equality."

The formula by means of which a is calculated is

$$a = \left(\frac{e}{k} T - p\right) v^2,$$

in which

e = the coefficient of expansion,
k = " " " compressibility,
T = " absolute temperature,
p = " pressure on the liquid,
v = " specific volume of the liquid.

The various quantities entering into this formula are expressed in terms of the c. g. s. system.

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of temperature and pressure, we should expect both assumptions to hold. Perhaps the natural interpretation of the fact that both cannot hold for ether or for alcohol is the hypothesis that such groups do exist in each liquid, but that their number is a function of both temperature and pressure. From this point of view the magnitude of the variations which we find in a, when assumption 2 is held, may be taken as a measure of the discordance of the two assumptions and therefore some indication of the rate of variation of the number of groups with variations of temperature and pressure. It might be better to keep assumption 1, taking a as really constant, and see what change would be necessary in the proposition given as assumption 2.

It appears to be common opinion (see pp. 270 and 271 of the 1904 English edition of Nernst's *Theoretical Chemistry*) that so-called polymerization exists in liquid alcohol but not in liquid ether. E. H. H.

The equation used in finding the values of a' is

$$\rho = a' \left(\frac{1}{v_2} - \frac{1}{v_1} \right),$$

in which, ρ = the internal work of vaporization, in ergs,
v_2 = " specific volume of the liquid state, in cu. cm.,
v_1 = " " " " " saturated vapor " "

The Value of a *in Ether.*

In Table X are given five values of *a* in ether, calculated by means of my data on compressibility and expansibility. Of these values the first three are for pressures slightly greater than the vapor-pressure, and the last two are at pressures less than the vapor-pressure.

TABLE X.

ETHER.

$t°C.$	$a \times 10^{-7}$.
22°	543
27	541
32	536
36	533
39	537

We have, I believe, no definite information as to the method of purification of the ether used by Amagat. The way in which the ether used by me was purified has already been pointed out. In comparing the values of *a* obtained from my data with those obtained from the data of Amagat, we must neglect any differences due to difference in purity of the ether used in the two cases. It is doubtful if this would be a source of any considerable error.

For purposes of comparison the values found by Professor Hall for *a* and *a'* in ether are given in the following table. The numbers in brackets to the right of each *a*-column, are, as stated by Professor Hall, "an attempt to establish a regular gradient of *a* with varying pressure, corresponding in a general way to the indications of the *a*-column."

TABLE XI.

ETHER.

P (atm.).	20° C. $a' = 495 \times 10^7$.		60° C $a' = 491 \times 10^7$.		100° C. $a' = 482 \times 10^7$.	
	v.	$a \times 10^{-7}$.	v.	$a \times 10^{-7}$.	v.	$a \times 10^{-7}$.
50	1.411	545 (538)	. . .	. . . . . .	. . .	. . . . . .
100	1.400	532 (535)	1.493	515 (527)	1.603	504 (511)
200	1.380	533 (528)	1.462	522 (522)	1.552	506 (508)
300	1.362	509 (521)	1.435	519 (517)	1.514	504 (504)
400	1.347	514 (514)	1.413	518 (512)	1.486	511 (501)
500	1.333	507 (507)	1.393	512 (507)	1.462	503 (498)
600	1.321	504 (500)	1.377	506 (506)	1.439	487 (494)
700	1.309	499 (493)	1.365	501 (497)	1.419	485 (491)
800	1.299	487 (486)	1.352	500 (492)	1.403	485 (488)
900	1.289	475 (497)	1.340	475 (487)	1.387	486 (485)
1000	1.280	472 (473)	1.330	481 (483)	1.374	490 (482)

P (atm.).	138° C. $a' = ?$		198° C. $a' = ?$	
	v.	$a \times 10^{-7}$.	v.	$a \times 10^{-7}$.
50	. . .	. . . . . .	. . .	. . . . . .
100	. . .	. . . . . .	. . .	. . . . . .
200	1.656	510 (509)	. . .	. . . . . .
300	1.600	506 (505)	1.766	491 (500)
400	1.560	502 (501)	1.699	484 (496)
500	1.528	493 (497)	1.648	497 (493)
600	1.500	490 (492)	1.607	506 (490)
700	1.476	484 (488)	1.574	511 (486)
800	1.455	484 (484)	1.545	493 (483)
900	1.435	481 (480)	1.521	465 (480)
1000	1.419	480 (476)	1.500	457 (477)

Examination of Tables X and XI shows that the values of *a* obtained from my data are in very close accord with those calculated by Professor Hall from the data of Amagat. Indeed, if the bracketed estimates of *a* given in Table XI were continued by extrapolation to the state of no external pressure, or an external pressure less than one atmosphere, the 20° column would begin with the number 542, as against 543 for 22° in Table X, and the 60° column would begin with the number 532. Interpolation between 542 for 20° and 532 for 60° would give 537 for 40°, whereas Table X gives 537 for 39°. This closeness of agreement is highly satisfactory from one point of view, though it fails to reveal any change of especial interest occurring at or near the point of passage into the superheated condition.

In Professor Hall's paper the values of a' which were calculated from Regnault's evaporation data are not given for temperatures above 100° C. In Table XII below are given a number of values of a' which I have calculated from the probably more accurate evaporation data found by Ramsay and Young.[12] These values are considerably less than the corresponding values obtained from the data of Regnault. This discrepancy is accounted for by the fact that the latent heat of vaporization of ether as given by Ramsay and Young is less than the corresponding value given by Regnault. Now each value of a' given in Table XI is smaller than the value which we have found for a under like conditions of temperature and pressure. The reduction now made in the values of a' by the adoption of Ramsay and Young's data will make the difference between a and a' larger than before. Moreover, a comparison of Table XII with the 198° part of Table XI may seem to indicate that the two values do not approach the same

TABLE XII.

ETHER.

$t°$ C.	$a' \times 10^{-7}$.	$t°$ C.	$a' \times 10^{-7}$.
40°	468	150°	409
60	457	160	398
80	450	170	385
100	434	180	372
110	433	185	362
120	428	190	378
130	422	192	392
140	418	193	395

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[12] Phil. Trans. (1886), p. 123.

limit, as we should expect them to do, near the critical temperature, which is about 194° C. But in explanation it is to be noted that the difference between the bracketed and the unbracketed a-columns is much greater in the 198° part of Table XI than it is elsewhere. Indeed, it now seems quite possible that in the unbracketed a-column at 198° the appearance of a maximum value for a in the neighborhood of 700 atmospheres external pressure is really significant. If the Amagat data enabled us to calculate the value of a at 198° through descending stages of pressure to 50 atmospheres, we might find a rapid fall to a value not very different from that which the Ramsay and Young data give for a' near the critical temperature. On the other hand, Table XII seems to indicate a minimum value of a' near 185° C. with a rise from that point on as the critical temperature is approached; but it is doubtful if the data from which the values of a' have been calculated have sufficient accuracy in the neighborhood of the critical temperature and pressure to allow any importance to be attached to the minimum here suggested.

The Value of a *in Alcohol.*

The values of a given in Table XIII were calculated in the usual way from my data on the compressibility and the expansibility of alcohol. Of these values the first two are for pressures slightly greater than the vapor-pressure, and the last is obtained by using a value of the coefficient of compressibility in the superheated state with a value of the coefficient of expansion in the unsuperheated state. As I have shown that neither of these coefficients is much changed in passing from unsuperheated to superheated state, the above method of getting the value of a at 78° C. causes no serious error.

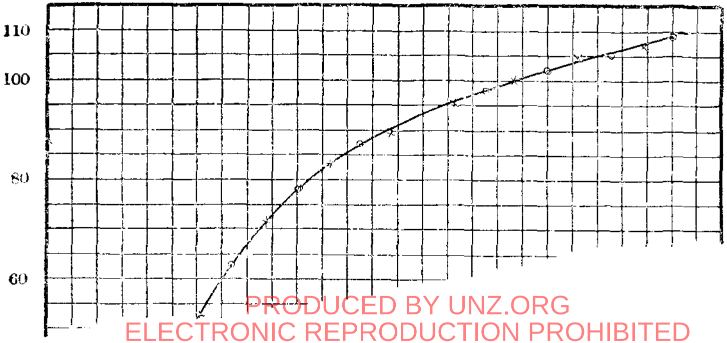
TABLE XIII.

ALCOHOL.

t.° C.	$a \times 10^{-7}$.
20°	481
40	505
78	570

The alcohol examined by me contained 0.5 per cent water. So far as the writer is aware, Amagat does not state the purity of the alcohol which he used, though it is fair to assume that it contained little water.

This lack of information makes it difficult to compare my values of *a* with those obtained from his data, and it becomes necessary to take up the question of the effect of a small amount of water on the compressibility and on the expansibility of alcohol. The latter part of the problem has been examined by Dupré and Page,[13] and moreover, the necessary information is to be had from tables [14] on the specific gravity of mixtures of alcohol and water. Each of these sources of information indicates that the difference between the coefficients of expansion of absolute alcohol and that of alcohol which contains 0.5 per cent water

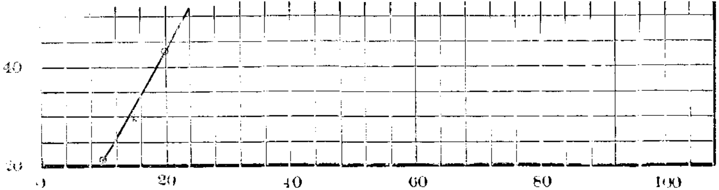


examined the compressibility of mixtures of alcohol and water, but his mixtures never contained more than 38 per cent of alcohol and his data do not, therefore, throw sufficient light on the question which is here being raised. Dupré and Page have also dealt with this question, but the value for the compressibility of absolute alcohol given by them seems to be in error, as is seen by comparing it with the value given by Röntgen[16] or with that given by Amagat, and this throws doubt on their other data. Moreover, they pass by a single step from 100 per cent to 90 per cent alcohol, and this leaves unexplored the precise

TABLE XIV.

Compressibility of Mixtures of Alcohol and Water.

Per cent of Alcohol by Weight.	Temp. C.°	$k \times 10^6$.
0.0	24°.5	45.9
7.5	24.1	44.1
	25.3	43.2



Curve I.

would not be so much as one per cent of either coefficient and may be neglected here. These facts are brought out clearly by Curve I, in which $e \times 10^5$ is plotted as ordinate and per cent of alcohol as abscissa. The points surrounded by circles are from the data of Dupré and Page and the points marked with crosses are from the tables of densities of mixtures of alcohol and water. It is seen that each set of values gives essentially the same curve; and for points between 99 per cent and 100 per cent alcohol the tangent to the curve makes only a small angle with the axis of abscissae.

The information heretofore available on the change in compressibility with the presence of water is by no means satisfactory. Pagliani[15] has

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[13] Phil. Trans. **159**, 591.

[14] Landolt and Börnstein.

[15] Jour. de Phys. (2) **10**, 589 (1891).

45.0	25.0	51.8
59.0	23.8	59.8
77.0	22.1	76.0
84.0	24.3	83.0
91.0	24.6	94.0
99.0	24.8	106.0
99.7	23.9	111.0

region which is of importance here. I have, therefore, examined the compressibility of certain mixtures of alcohol and water with the results given in Table XIV. The last two values, which are of chief importance to this work, completely confirm values obtained earlier in this paper. (See p. 440.) The relation between the compressibility and the per cent of alcohol in these mixtures is brought out in Curve II, in which $k \times 10^6$ is plotted for ordinate and per cent of alcohol for abscissa.

The points on the curve marked with crosses are from the data of

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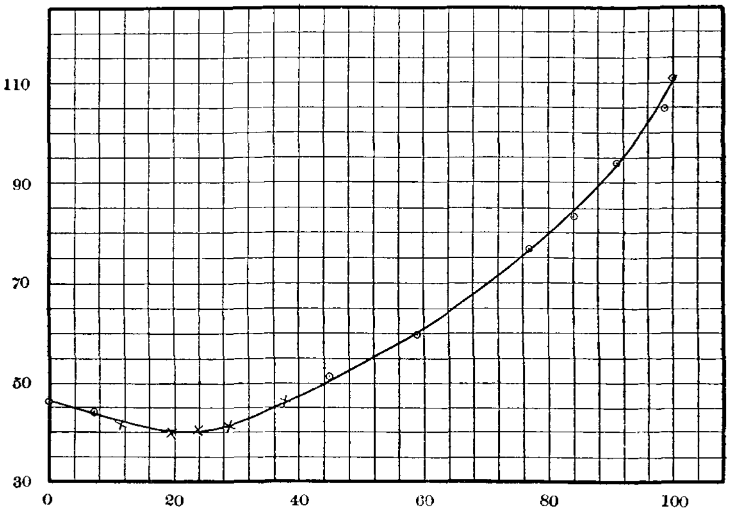
[16] W. A., **44**, 1 (1891).

Pagliani, because my observations did not cover that region. This curve, after passing its well-known minimum, is seen to become steeper as the amount of alcohol in the mixture is increased; and it is seen at once that the compressibility of absolute alcohol decreases rapidly with the introduction of a small amount of water. This decrease in the compressibility would mean an increase in a. It is therefore probable that the values of a given in Table XIII, which are for alcohol containing 0.5 per cent water, may be as much as 3 per cent greater than the corresponding values for absolute alcohol. Allowing for this possible discrepancy, we may still consider the values here found as being of interest in connection with those obtained from the data of Amagat.



Curve II.

We may perhaps see most clearly the contribution which these recent values make to our knowledge of the van der Waals a in alcohol by inserting them in their proper places in the table given by Professor Hall. As given there, they have been corrected for the possible error due to the impurity of the alcohol. That is, I have reduced by 3 per cent the values of a given in Table XIII, after making an estimate of the value at 80° from the value at 78°, assuming at the same time that Amagat used absolute alcohol. The values of a calculated from my data are enclosed in brackets in order to distinguish them from the values obtained from Amagat's data. The numbers enclosed in parentheses have the same meaning which was attached to the corresponding numbers in the analogous table for ether.

TABLE XV.

ALCOHOL.

P (atm.).	20° C. $a' = 1197 \times 10^7$.		40° C. $a' = 1197 \times 10^7$.		60° C. $a' = 1164 \times 10^7$.	
	v.	$a \times 10^{-7}$.	v.	$a \times 10^{-7}$.	v.	$a \times 10^{-7}$.
		[467]		[490]		
50	1.259	453 (465)	1.285	478 (479)	1.315	483 (496)
100	1.253	458 (463)	1.276	471 (477)	1.306	486 (494)
200	1.242	460 (458)	1.262	476 (472)	1.291	491 (491)
300	1.231	457 (453)	1.250	470 (467)	1.276	494 (487)
400	1.222	449 (448)	1.239	465 (463)	1.265	492 (484)
500	1.214	448 (443)	1.230	459 (459)	1.253	486 (481)
600	1.205	439 (438)	1.222	455 (454)	1.245	479 (477)
700	1.197	422 (433)	1.213	450 (450)	1.236	476 (474)
800	1.189	422 (429)	1.205	444 (446)	1.227	472 (471)
900	1.181	426 (424)	1.197	445 (442)	1.219	464 (468)
1000	1.175	434 (420)	1.189	438 (438)	1.211	466 (465)

P (atm.).	80° C. $a' = 1112 \times 10^7$.		100° C. $a' = 1072 \times 10^7$.	
	v.	$a \times 10^{-7}$.	v.	$a \times 10^{-7}$.
		[556]		
50	. . .	. . . .	. . .	. . . .
100	1.340	531 (533)	1.371	567 (570)
200	1.321	524 (527)	1.350	560 (564)
300	1.306	519 (522)	1.330	556 (559)
400	1.291	521 (517)	1.315	563 (553)
500	1.276	518 (512)	1.300	551 (548)
600	1.265	513 (507)	1.288	538 (542)
700	1.256	506 (502)	1.276	526 (537)
800	1.247	490 (497)	1.268	533 (532)
900	1.239	486 (493)	1.259	521 (527)
1000	1.230	494 (488)	1.250	541 (522)

It will be seen that in the case of alcohol, as in the case of ether, the values of a calculated from the data of this paper are about what one would get by extrapolating from the values previously obtained from the data of Amagat. It is true that the [490] now placed at the head of the 40° column is slightly larger than the numbers beneath it would lead one to expect, and a still greater divergence in the same direction is to be observed at the head of the 80° column. Moreover, these discrepancies would have been considerably greater if a possibly too great allowance (3 per cent) had not been made for the greater impurity of the alcohol used in the research of this paper as compared with the alcohol used by Amagat. But at the most they are hardly great enough to give certain evidence of any noteworthy change in the condition of the alcohol in approaching and passing into the superheated condition.

TABLE XVI.

ALCOHOL.

Temp. C°.	$a' \times 10^7$.	Temp. C°.	$a \times 10^7$.
110°	1083	180°	937
120	1069	190	904
130	1050	200	875
140	1035	210	843
150	1019	220	812
160	994	230	769
170	964	240	718

It appears in Table XV that a' decreases and that a increases with rising temperature. Here, as in the case of ether, we find it interesting to follow the change of each of these quantities as far as we can, in order to see how nearly they approach each other in the neighborhood of the critical temperature, which for alcohol is about 243° C. We should of course expect them to approach the same limit in approaching the critical temperature and pressure. The values of a' given in Table XVI were calculated from the evaporation data of alcohol as given by Ramsay and Young. It appears from this table that a' continues to decrease until the critical temperature is approximately reached.

I have made an attempt to obtain values of a at temperatures not very far from the critical temperature. Ramsay and Young have with considerable care plotted some of the high isothermals of alcohol. In obtaining from these curves values of the coefficient of compressibility and of the coefficient of expansion considerable uncertainty was introduced, but at the three temperatures given in the following table it seemed possible to find a value of each of these coefficients having sufficient accuracy to be useful in this paper. The three values of *a* calculated from the coefficients thus obtained are given in Table XVII.

TABLE XVII.

ALCOHOL.

Pressure (atm.).	Temp. C°.	$a \times 10^{-7}$.
67	200°	737
67	210	787
67	220	832

It would appear from Tables XVI and XVII, as they here stand, that the two quantities *a* and *a′* meet and cross in magnitude in the neighborhood of 215° C. This is unlikely. It is more probable that the data for the calculation of *a* and *a′* in the neighborhood of the critical temperature and pressure are somewhat unreliable.

SUMMARY.

1. The coefficient of compressibility, *k*, and the coefficient of expansion, *e*, of ether and of alcohol have been measured at various temperatures at pressures near the saturation pressures of the respective vapors. In some cases the liquids have been examined in the superheated condition. Within the error of my observations these coefficients have been found to be constant over the range of pressure employed in this paper. By reference to Figure 1 the meaning of this result may be more clearly brought out. It states that the slope of any one of the isothermals examined in this paper, — e. g. the isothermal A F H J (Figure 1) — has the same value at C, where the external pressure is somewhat greater than the vapor-pressure, that it has at E, where the external pressure is less than the vapor-pressure; i. e., the direction of the isothermal does not change when the saturation curve is crossed. It has been recently stated by Amagat[17] that nothing is known of the

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[17] Comp. Rend., May 21, 1906.

exact course of the Thomson isothermal where it cuts the saturation curve. It is seen that the present investigation has been able to supply some information on this point.

2. From the data given in Tables II, III, IV, V, VI, and VII, values of the van der Waals a have been calculated, for the two liquids in the various states examined, by the method used by Professor Hall in his paper in the Boltzmann *Festschrift*, that is, by use of the formula $a = \left(\frac{e}{k}T - p\right)v^2$, which assumes, tentatively, that *the energy per molecule, aside from the potential energy due to the attraction* $\frac{a}{v^2}$, *is a function of the temperature only, so that it remains constant during any isothermal change of state.* (But see footnote to p. 447.)

3. The values of a thus found prove in the case of ether to be almost exactly those which would be obtained by extrapolation from the values calculated by Professor Hall from the data of Amagat. There is, therefore, no indication of any significant change of the internal state of ether as it approaches and enters the superheated condition.

A like conclusion is probably to be drawn in the case of alcohol, though the evidence against such a change of internal state upon entering the superheated condition is less satisfactory in the case of alcohol than in the case of ether.

4. From the data of Ramsay and Young on the vaporization of ether the value which Professor Hall calls a' has been calculated for this substance at various temperatures, ranging from 40° C. to 193° C., by means of the formula $a' = \rho \div \left(\frac{1}{v_2} - \frac{1}{v_1}\right)$, in which

ρ = the internal latent heat of vaporization,
v_2 = the specific volume of the liquid state,
v_1 = the specific volume of the saturated vapor state.

These values are smaller than those which Professor Hall found for a' from the Regnault vaporization data for ether, and these latter were smaller than the values found for a at corresponding temperatures. We should expect a and a' to approach the same limit at the critical temperature and pressure. The lack of satisfactory data for determining a in the immediate neighborhood of this condition makes it impossible at present to put this expectation to the proof. The last stage of approach reached in the calculations leaves

$a = 491 \times 10^7$ at 198° C. and 300 atmospheres
$a' = 395 \times 10^7$ at 193° C. and 34.5 atmospheres.

The critical condition is about 194.4° C. and 35.6 atmospheres. There is some indication of a maximum value of a for the temperature 198° C., in the neighborhood of 700 atmospheres.

From the data of Ramsay and Young values of a and of a' have been found for alcohol, supplementary to those calculated by Professor Hall from the data of Amagat and of Regnault. At low temperatures the value of a' in alcohol is more than twice as great as the value of a. At the last stage, at highest temperatures reached in the calculations, the following values were found:

$a = 832 \times 10^7$ at 220° and 67 atmospheres,
$a' = 718 \times 10^7$ at 240° and 60 atmospheres.

The critical condition is about 243° and 63 atmospheres. It would appear, according to the data now available, that the values of a and a' meet and cross in magnitude in the neighborhood of 215° C.; but this seems improbable. Data for the neighborhood of the temperatures here considered are rather dubious.

5. Partly from the data of Pagliani and partly from new data a table (XIV) of the compressibility of mixtures of alcohol and water has been made, ranging from 0 per cent to 99.7 per cent of alcohol. It is found that the effect of a small amount of water is very marked. Thus the compressibility of 99.7 per cent alcohol is about 4 per cent greater than that of 99 per cent alcohol.

6. In four successive attempts to return ether from the superheated state to the unsuperheated state by cooling at constant pressure sudden vaporization occurred, after which this method of procedure was abandoned. No difficulty was experienced in effecting the return by increasing pressure at constant temperature. The reason for the apparent upsetting effect of the cooling process is not evident.

Grateful acknowledgment is made by the writer to Professor Hall for constant advice and personal aid in the course of this work.

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY, SYRACUSE UNIVERSITY; ALSO CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION FOR RESEARCH.—No. 9.

THE HYDROIDS OF BERMUDA.

BY EDGAR DAVIDSON CONGDON.

THE HYDROIDS OF BERMUDA.[1]

EDGAR DAVIDSON CONGDON.

Presented by E. L. Mark, November 14, 1906. Received November 2, 1906.

THIS paper has for its purpose the description of certain hydroids which I collected in the summer of 1903, while an attendant at the Bermuda Biological Station. They were investigated under the guidance of Dr. C. W. Hargitt in the Zoölogical Laboratory of Syracuse University. I wish to express my great indebtedness to Dr. Hargitt for his suggestions and criticisms, and to thank the Bermuda Biological Station for courtesies extended to me during the season.

Verrill ('99, p. 571) has stated in the Proceedings of the Connecticut Academy of Science that eleven hydroids occur at Bermuda, but he does not name or describe them. I know of no other zoölogist who has occupied himself with the subject.

Of the eighteen species that were found, eight were previously undescribed. Each species which had been previously described varied in some small degree from the type individuals. The various common hydroid families are quite equally represented. *Eudendrium hargitti*, a new species, is especially interesting because of phenomena of oögenesis which have been elsewhere described (Congdon, :06). Two new species of Halecium present female gonophores whose structures are significant when compared with the gonophores of other species of the genus.

Few hydroids are found on the exposed southern shore of the Bermudas. The coves, inlets, and reefs of the opposite shore are well supplied with individuals and species. The Sargassum, which floats in after a prolonged south wind, often is the home of an abundance of *Aglaophenia minuta*, *Halecium*, and *Clytia simplex*.

Pennaria tiarella, *Eudendrium ramosum*, *Sertularia humilis*, and *Sertularella brevicyathus* are the most common species. *E. hargitti*, *Sertularella speciosa*, and *Thyroscyphus intermedius* are each confined to some single very restricted locality. In the few places especially favorable to hydroid life the strife for foothold is so marked that seven of the small species may be found growing on the larger ones.

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[1] Contributions from The Zoölogical Laboratory, Syracuse University; also Contributions from the Bermuda Biological Station for Research, No. 9.

The Bermuda hydroids show a close relationship to those of the West Indies and the Gulf of Mexico. All the genera of the new species and all but one of the species previously described are there represented. The remaining species, *Bimeria humilis*, has not to my knowledge been found south of New England.

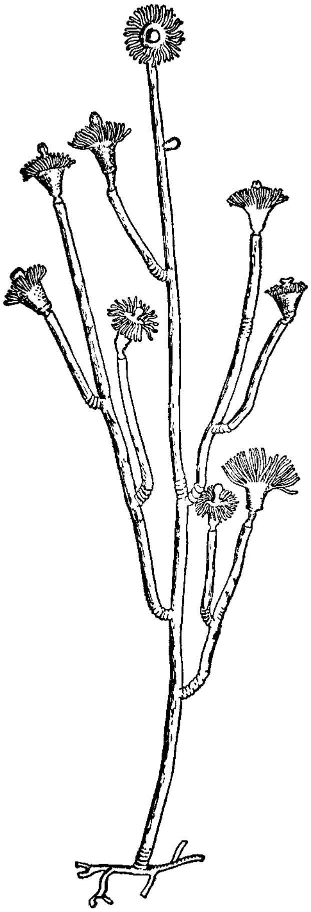


Figure 1. Colony of *Eudendrium hargitti* (×10).

Genus PENNARIA McCrady.

Pennaria tiarella McCrady.

The *Pennaria tiarella* of Bermuda has on the average three more filiform tentacles than that of Wood's Hole, Mass. Clarke ('79) described a member of this genus from Cuba, under the name of *P. symmetrica*, in which the gonosome was lacking. The characters which he considers specifically distinctive are: the exact form of the hydranth, the origin of the tentacles from a little above the base, and the presence of eighteen filiform tentacles. The first two characters vary greatly with age and the amount of food in the hydranth. The number of tentacles does not seem to me of specific importance, because it varies considerably, owing only in part, I think, to the degree of maturity. It seems probable that *P. symmetrica*, like the Bermuda form, is a geographical variety of *P. tiarella*.

Genus EUDENDRIUM Ehrenberg (in part), 1832.

Eudendrium ramosum Linnaeus.

E. ramosum differs in three respects from the individuals of Wood's Hole, Mass. There is a slightly larger average number of tentacles; there may be one more lobe to the male gonophore; the hydranth to which the clusters are attached is often entirely aborted.

Eudendrium hargitti (new species). Figures 1–5.

This hydroid was found at only one place, a shallow inlet on the south shore of Bermuda (lat. 32° 16′ 50″, long. 64° 45′ 5″). It is a handsome little form with bright reddish brown hydranths and horny brown perisarc, which contrasts with the usual substratum of white coral sand (Figure 1).

Trophosome. Stem unfascicled; colony twenty to fifty millimeters long, becoming nearly transparent toward the extremities. Branches straight, few, nearly parallel to the main stem, distributed irregularly, joining stem by an abrupt bend. Annulations at bases of colony and branches, occasionally elsewhere. Hydranth most deeply colored at base of hypostome; tentacles from thirty-five to forty-five, in contraction forming two closely appressed rows; hypostome very mobile, contracting into a shallow cup or extending to a length greater than that of the hydranth body. Some hydranths provided with a groove near the base containing gland cells and thread cells.

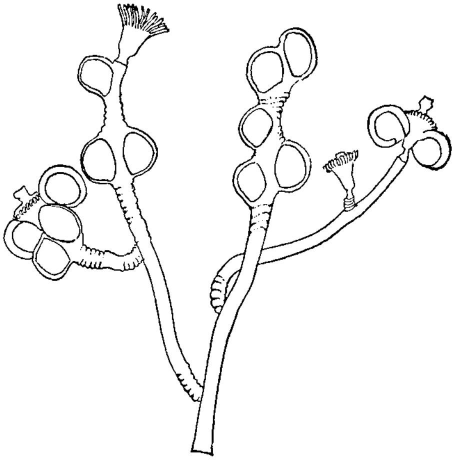
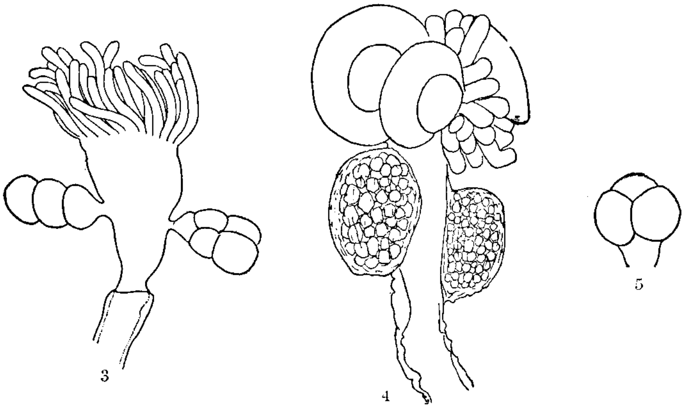


FIGURE 2. *Eudendrium hargitti.* Orthospadiceous and streptospadiceous gonophores (× 11).

Female Gonosome. Colonies dioecius. Two types of orange-colored gonophores (Figures 2, 4). One begins its development before the other, has an undivided spadix, consisting of a tube passing from the attachment upward and around the egg, and forms in conjunction with not more than four others a circle around the base of the hydranth body. The gonosomes of the second type are associated in clusters of two to seven closely and rather irregularly around a thick finely annulated pedicel, which may or may not have a terminal hydranth. They are partly confluent with the stem, ovoid, completely invested on the exposed side by a spadix, often indistinctly separated into a proximal and a distal group. A dozen clusters may occur close together on a basal branch. Found toward the base of the colony always below the first type, whether occupying the same stem or not. A pedicel bearing this type may support a hydranth upon which the other type occurs.

Male Gonosome. Gonophores on an aborted hydranth and much annulated pedicel (Figures 3, 5). Three-chambered in a moniliform arrangement (Figure 3, left side) with but slight constriction between the lobes and their relative diameter variable. Four lobes may be arranged so as to form a diamond-shaped cluster, or another may be added proximally upon one side (Figure 3, right side).



Figures 3–5. *Eudendrium hargitti.*

Figure 3. Hydranth with common type of male gonophore to the left and a less common form to the right (× 26).

Figure 4. Distal streptospadiceous and proximal orthospadiceous gonophores (× 12).

Figure 5. An unusual form of male gonophore (× 22).

The variability and simplicity of these gonophores in comparison with other Eudendria is suggestive of degeneration. This hydroid has the distinction of being the only member of the genus whose egg is known to grow by the absorption of other cells.

It is a pleasure to name this species after Professor Charles W. Hargitt. To those who, as students of the Hydromedusae, are acquainted with his various contributions relating to problems of the group, the reason for so doing is apparent.

GENUS BIMERIA S. Wright, 1859.

Bimeria humilis Allman. Figure 6.

Dense growths of the colonies of this small animal are to be found on Eudendrium, Pennaria, sponges, and the like. The stem of Pennaria often forms the centre of a cylindrical mass a centimeter in diameter. No gonosome is present.

The genus Bimeria was established in 1859 by Strethill Wright for a hydroid of the Firth of Forth, characterized by a covering of perisarc on the hydranth body and around the bases of the tentacles (Allman, '**71**, p. 297). Allman ('**77**, p. 8) added to the genus the species *B. humilis* from the Tortugas, which differs but slightly from the Bermuda Bimeria.

Gonosome is lacking. In two respects the hydroids differ. The Tortugas form has a very opaque perisarc as described by Allman; in the Bermuda form it is transparent; but descriptions of other species added to the genus later by different authors suggest by their dissimilarity that in regard to this character individuals may vary in appearance. Secondly, Allman states that he was not able to detect the ends of the tentacle tubes and his figures do not show them. By focussing on the end of an expanded hydranth, I had no difficulty in finding them. There is thus no warrant for constituting a new species unless the gonosomes reveal differences.

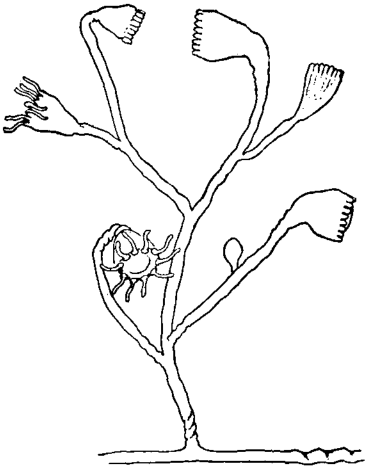


FIGURE 6. Colony of *Bimeria humilis* (× 18).

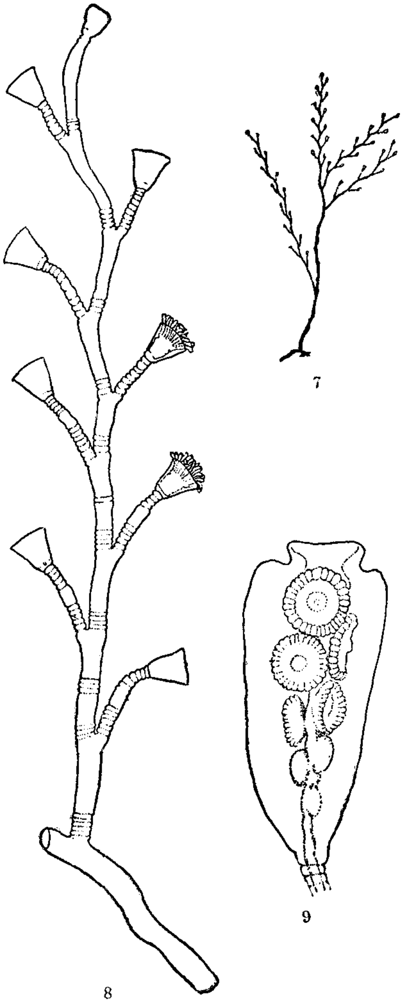
GENUS LAFOEA Lamouroux, 1812.

Lafoea calcarata A. Agassiz.

A small number of colonies were found growing around the base of a hydroid. Unfortunately the collection was lost before it could be examined at length.

GENUS OBELIA Peron and Leseur, 1809.

Obelia hyalina S. F. Clarke. Figures 7–9.



FIGURES 7–9. *Obelia hyalina.*

FIGURE 7. Colony (× 3).
FIGURE 8. Colony (× 30).
FIGURE 9. Gonotheca (× 60).

Obelia hyalina grows on sponges, algae, and large hydroids which inhabit the shallow waters of Bermuda (Figures 7, 8). Stems and branches may develop short stolons. The branches are described by Clarke as arising from the axils of hydrothecae. I found them more commonly resulting from the extension of pedicels. The gonothecae which Clarke found were evidently immature, since they are figured with a truncate end. Later these develop a small aperture surrounded by a flaring rim (Figure 9).

GENUS CAMPANULARIA
Lamouroux (in part) 1816.

Campanularia insignis
Allman. Figures 10–12.

The animal lives in all the localities along the shores of Bermuda where hydroids are plentiful. The trophosome (Figure 10) was described by Allman from a dredging of the Challenger expedition in thirty fathoms of water off the coast of Bermuda. I have to add a description of the gonosome.

An interesting character not to my knowledge elsewhere found in the genus Campanularia, is the vegetative reproduction through the agency of stolons, much as takes place in plants by creepers

(Figure 11). The outgrowths arise from the ends of the stems and branches or may replace the latter. They are present in colonies of all sizes and are most abundant at the distal end. By their elongation, sometimes aided by a bending of the colony, their tips come in contact with the substratum. A clump of rhizoids forms and a new colony rises from them. Only a small proportion of the stolons were engaged in this process. Hincks ('68, p. 170) figures similar stolons on *C. angulata*, but they were not seen to give rise to colonies.

Nutting (:00, p. 44) has seen a species of Aglaophenia in the act of conjugating by means of stolons. A hooked stolon of one colony catches a similar structure of another. Fusion occurs, and after a resting period of three months a colony arises at the point of contact.

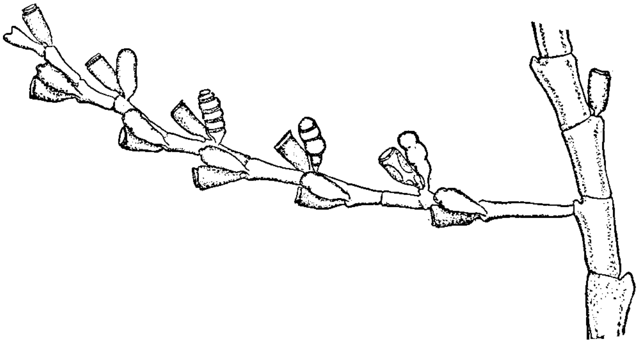


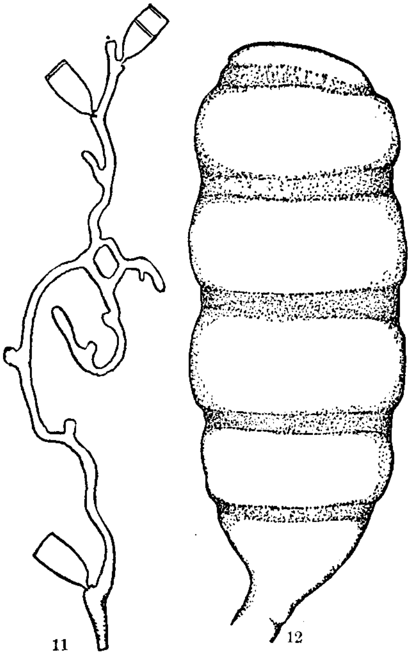
Figure 10. *Campanularia insignis.* Branch and part of main stem, showing two types of gonophores (× 4).

Though I found no case of fusion in many hundred colonies, the possibility of its occurrence is suggested by the hooked ends of the young stolons. Sometimes a small hooked stolon arises perpendicularly from near the end of another. As the branches grow larger, the hook gradually straightens, until there is but a slight curve, as found in *C. angulata*, *S. pumila*, and other species.

Gonosome. I found two types of gonothecae. The one was cylindrical and divided into about five lobes by regular and broad furrows (Figures 10, 12). It was not apparent from their structure whether they were female or immature male gonothecae, and I was not able to investigate their histology. The other form was ovoid with a constricted opening and a single male gonophore (Figure 10).

The spermatozoa develop in gonothecae instead of in medusae. A thick layer of sperm mother-cells lies next the mesogloea in the entoderm.

Mature gonothecae within this layer have a zone of degenerating, ill defined, columnar cells, whose fate I have not followed. The ectoderm was also found undergoing retrogressive change while the sperms were nearing maturity. It would seem probable that the entoderm furnishes nourishment for the spermatic layer from its own substance at this stage, while at an earlier period of growth it passes food onward from the central cavity to it.



FIGURES 11, 12. *Campanularia insignis.*

FIGURE 11. Young colony growing from a stolon (× 4).

FIGURE 12. One of the types of gonotheca, sex undetermined (× 40).

There are minor points of structure which require to be reconciled with Allman's description of *Campanularia insignis.* He gives the height of large colonies as nine inches. Although I found none having more than half that height, this is so variable a feature that it should have little weight. The rim of the hydroid is described by Allman as narrow and more transparent than the rest. In his plate it is represented by a line parallel to and just below the edge. My collections show the extreme border of the hydranth set slightly inward from the general surface. Two or three grooves may appear encircling the hydrotheca, but the top one may occasionally be lacking.

Allman places his species only provisionally in the genus Campanularia because of lack of gonosome. The character of the gonosome as described above settles the question of genus, placing the hydroid among the Campanularia.

GENUS CLYTIA Lamouroux (in part), 1812.

Clytia fragilis (new species). Figure 13.

The hyaline colonies of *Clytia fragilis* occur in company with *Halecium bermudense* on *Pennaria tiarella.*

Trophosome. Colonies twelve to eighteen millimeters long, the small diameter of the stem and the hyalinity of the perisarc giving the appearance of fragility. Short, strongly marked nodes, ending below with an abrupt curve and attached to the side of the next lower node; ending above, apparently in the pedicel of a hydranth. Stem geniculate, somewhat curved in each node. Hydranths alternate. The branches, which arise from the growth of a pedicel, given off irregularly, duplicating the structure of the stem. Annulation at lower end of node, sometimes extending well up or occurring midway in its length (Figure 13).

Hydrothecae campanulate, elongated, with nearly straight sides, tapering most abruptly close to pedicels. Rim with twelve to fourteen pointed teeth separated by rounded edges. In old individuals the walls often folding and cracking longitudinally, causing the hydrothecae to collapse; the teeth in part breaking off, producing an irregular edge. Diaphragm with a small opening sometimes quite far from base of hydranth. Pedicels often as long as a node. If entirely annulated there are from ten to twenty rings.

Gonosome. Gonothecae attached closely to base of a hydranth pedicel, or carried away from the stem by its elongation, nearly twice as long as hydrothecae, flattened, ovoid, truncate above, with a flaring ring and tapering to the short annulated pedicel. There may be some suggestion of annulation on the wavy surface. About six medusa buds can be found on the blastostyle. Their bells are deep and their manubria large. The presence of four tentacles indicates the genus Clytia.

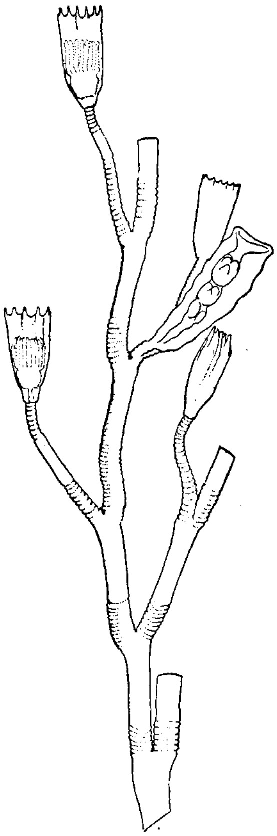
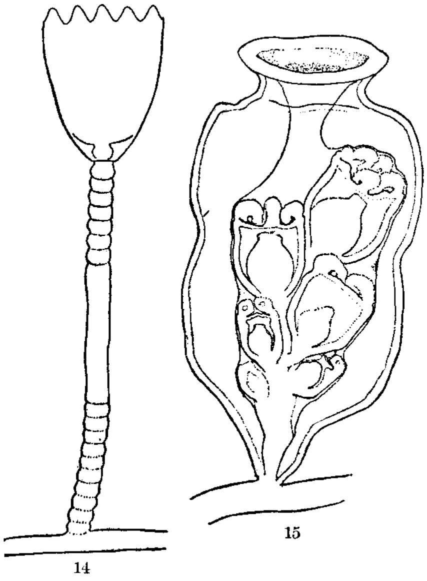


FIGURE 13. *Clytia fragilis.* Half of a colony bearing a gonotheca (× 18).

Clytia simplex (new species). Figures 14, 15.

This hydroid is only a little less abundant on Sargassum than *Aglaophenia minuta.* It grows on some of the larger Bermuda hydroids.

Trophosome. Minute white colonies, seldom ten millimeters long, arising rather sparsely from a creeping stolon, bearing a deeply campanulate hydrotheca, pedicel annulated most commonly at both ends, (Figure 14), hydrothecae only one and one-third to twice as long as wide, the proportion depending on the age and condition of expansion of hydranth. Ten to twelve triangular teeth, more or less rounded and of varying proportions. Wall of hydrotheca thickened below hydranth to form a spheroidal cavity. Hydranth of usual type with twenty to twenty-four tentacles.



Figures 14, 15. *Clytia simplex.*
Figure 14. An individual (× 18).
Figure 15. Gonotheca (× 60).

Gonosome. Gonothecae arising from stolons, sessile, twice as long as hydrothecae, ovoid and flattened, distal end truncate and flaring, and of one half to two thirds the diameter of the gonothecae (Figure 15). About eight medusa buds are attached along the blastostyle, of which the most mature are campanulate in form with four tentacles and four other rudimentary organs interradial in position. The manubrium may occupy half the space within the bell, is nearly spherical, and apparently not provided with any terminal lobes.

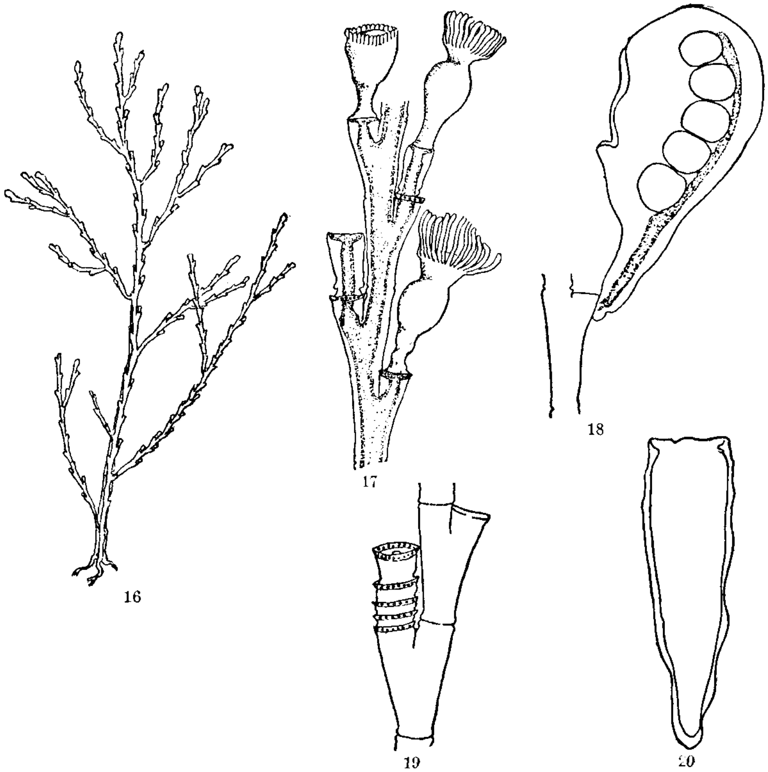
This Clytia combines the unbranched habit of *C. bicophora* and *C. grayi* with the non-annulated or weakly annulated gonosome of *C. cylindrica.* The teeth of the hydrothecae are less pointed than those of *C. bicophora*, and, though as rounded as those of *C. grayi*, are much more deeply cut. The specific name "simplex" is suggested by the marked simplicity in the form of trophosome and gonosome.

Genus HALECIUM Oken, 1815.

Halecium bermudense (new species). Figures 16–20.

The hydroid occurs in those places along the shores of Bermuda which are most frequented by related species.

Trophosome. Colonies delicate and graceful, with hyaline or brownish perisarc, usually twenty to thirty-five millimeters long, main stem and large branches fascicled in part, elsewhere slightly geniculate (Figures 16, 17). Branches irregularly placed, arising from below hydrophores, when young, by an abrupt curve, which later is less apparent; occasionally, toward the end of larger branches, arising midway between hydrophores. Branches may twice subdivide. They tend to form a cluster at the tip of the colony.



Figures 16–20. *Halecium bermudense.*

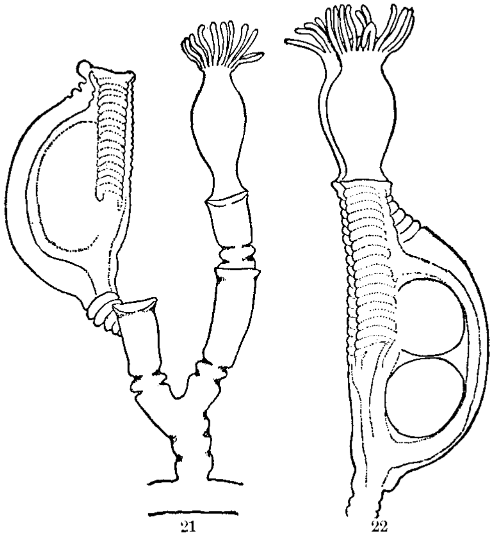
Figure 16. Colony (× 4).
Figure 17. Part of a colony (× 24).
Figure 18. Female gonotheca (× 27).
Figure 19. Compound hydrophore (× 30).
Figure 20. Male gonotheca (× 27).

Hydrophores alternate, very shallow, with usual ring of refractile bodies; in the older part of the colony quite largely double, occasionally seen with seven divisions (Figure 19). Annulations above each hydrophore divide the stem into nodes, others at base of branches and occasionally elsewhere. Hydranths large with rounded hypostome and twenty to thirty tentacles.

Gonosome. Colonies dioecius. Gonothecae sessile at the axils of hydrophores, sometimes found arising from hydrophores.

Female gonotheca ovoid, flattened, with a short pedicel-like base, one side open for two thirds its length, the edges of the opening forming similar compound curves (Figure 18). The blastostyle extends up around the opposite side, curving toward the opening. The development of the eggs is accompanied by the breaking down of the tissue between them and the opening.

Male gonotheca (Figure 20) cylindrical and unusually slender, truncate, and tapering toward base, often marked by an irregular encircling groove somewhat wavy in outline, one third the way from the base.



Figures 21, 22. *Halecium marki.*

Figure 21. Colony (× 60).
Figure 22. Female gonotheca and hydranths (× 75).

Halecium marki (new species). Figures 21–23.

The creeping stolons of this species form a network over the Sargassum and to a less extent cover the bases of some of the large Bermuda hydroids.

Trophosome. Colonies are commonly one and a half to three millimeters high (Figures 21, 23). A thick layer of spherical, unicellular algae extending through the coenosarc of the entire colony colors it green. A colony begins its growth as a single hydrophore arising from a stolon. Its structure, like other hydrophores, is typically that of a cylinder; it is three times as long as broad, with a flaring end and one or more deep annulations at the base. Growth in a straight line may result from the addition of successive hydrophores. Their rims are outside the walls of the next in the series and so may be lost off. Pairs of branches arise as hydrophores, attached by short abrupt curves to the opposite sides of a hydrophore, or, less frequently, but one occurs. There is no especial order in the arrangement of hydrophores. Small colonies result, whose units are partly branching and partly linear. Seldom are there more than four hydrophores in a series; stolons in a few instances were found terminating branches.

Large and slightly retractile hydranths terminate each branch. There are sixteen to twenty-five tentacles and the hypostome is short and rounded.

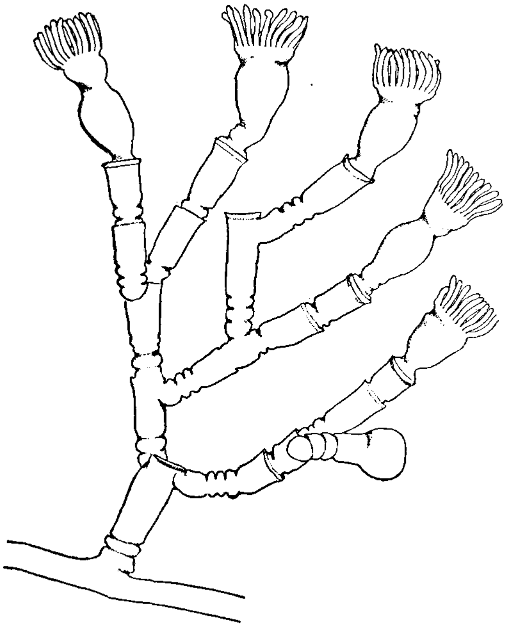


FIGURE 23. *Halecium marki.* Large colony (× 60).

Gonosome. Ovoid gonothecae take the place of branches (Figures 21, 22). Most colonies examined had from one to three of them. Male and female gonothecae occur on colonies attached near to each other on the stolon. The constricted opening and the short deeply annulated pedicel make a straight line down one side. Midway upon it two delicately annulated tubes arise, which are closely confluent with each other and with the gonotheca. At the curiously annulated mouth of the latter they end as hydrophores which bear hydranths.

The blastostyle in either sex is joined with the coenosarcal column, which branches into the two tubes. The single female gonophore contains two large eggs, one above the other. The male gonophore is also single and ovoid.

Halecium beanii is the only species of the genus which has a gonophore in any way similar to this. It falls in a natural group with the other species here described, *H. beanii*, *H. dichotomum*, *H. sessile*, and *H. bermudense*, because all have large hydranths, shallow hydrophores, and slipper-shaped female gonothecae. It is of interest for purposes of comparison to arrange the salient characters of the female gonophores of these forms in an order suggested by their structure.

H. marki. Female gonotheca has two tubes, each bearing a hydranth, which reach from the centre of the gonophore to its summit. The opening is at the summit.

H. beanii. Tubes much shortened and mouth at top on a level with the tubes.

H. dichotomum. Only one tube springing from the side half-way down.

H. sessile. Like *H. bermudense*, but the opening at top and the remains of tube gone.

Though recognizing that caution should be used in formulating theories as to the development of animal structures from such comparisons as are suggested above, it seems to me that the facts point forcibly to the origin of the slipper-shaped gonotheca in the following way. The gonotheca structure in the present species suggests that a hydrophore bearing a gonotheca also gave rise to a branch on either side, the latter a common condition in the species. The two branches with their terminal hydranths then became fused with the gonotheca. In this species the fusion is complete half-way up the gonotheca. Next the tubes shortened further by a contraction of their upper part until they became mere openings and the aperture of the gonotheca remained near to them, as in *H. macrocephalum.* In *H. bermudense* the opening is elongated toward the top. *H. sessile* and others have the opening at the top, but retain the slipper-like form.

GENUS SERTULARELLA Gray (modified), 1847.

Sertularella speciosa (new species). Figures 24-28.

This form was found only at the opening of an underground passage connecting Harrington Sound and Castle Harbor (lat. 32°20′ 30″, long. 64°42′ 10″).

Trophosome. Colonies fascicled, truncate, usually five to eight inches high (Figures 24, 28). Occasionally a large branch occurs resembling the main colony. Pinnae alternate, making an angle of eighty degrees with the stem, divided into nodes at intervals of some five hydrothecae by grooves slanting alternately in opposite directions. Stolons occur at ends of stems and branches.

Hydrothecae alternate, embedded in stem or pinna nearly to the opening, separated by a considerable space from each other, cylindrical and flattened laterally, the lower end tapering slightly, with no line marking its union below and no diaphragm, the upper end bending out at an angle of sixty degrees. The hydrothecae of the branches grow from a pair of tubes on opposite sides of the stem. There are usually three between successive pinnae on one side, one of which is just above the origin.

The opercula of young hydrothecae have four, or occasionally five, ridges, terminating in corresponding projections of the edge; one abcauline, and the others symmetrically placed. Operculum usually first ruptured at centre. Four ragged lobes or a rim may remain until maturity, but they are more commonly lost. Projections usually persist, becoming less distinct. Two or three rings usually found below the edge of old hydrothecae.

Expanded hydranths trumpet shaped, hypostome conical, tentacles twenty to thirty. A blind sack is attached to abcauline wall of hydrotheca, from which projects a transverse edge.

Gonosome. Colonies dioecious. Male and female gonothecae, arising from pinnae, externally similar, equal to five hydrothecae in length (Figure 26). Theca hyaline, cylindrical, distally truncate, proximally constricted for one fourth its length into a pedicel-like base. Axis slightly curving, markings of surface often very faint, consisting of eleven longitudinal ridges terminating in lobes along the distal margin, separated by more faint parallel ridges and six or eight broad shallow circular

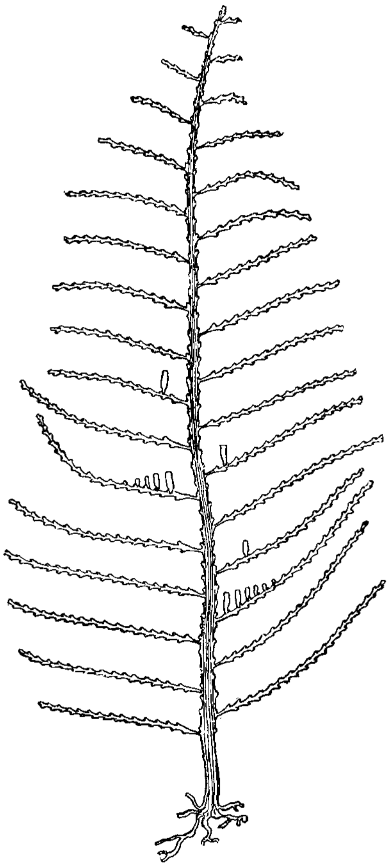
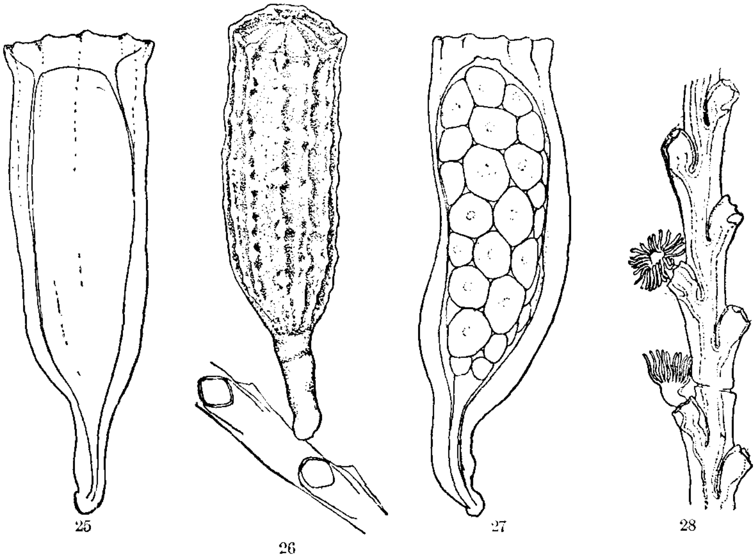


FIGURE 24. *Sertularella speciosa.* Colony with gonothecae (× 1½).

grooves. Top has radial ridges terminating in the lobes. Gonothecae attached below hydrothecae, one or two near base of branches, or a series decreasing in size distally or otherwise arranged. Female gonothecae with gonophores filling the cavity containing about forty eggs arranged around a lumen (Figure 27). The male gonothecae similar (Figure 25).

I examined the structure of the stem by macerating it in caustic potash. Twenty tubes occur near the base of a small colony and four near the top. They extend for various distances upward to terminate by joining others. The rhizoids consist of tubes which themselves unite near the base of the colony much as do the former. In each colony there is one large hydrothecae-bearing tube which gives rise to the pinnae of one side near the base and further up for both sides. It arises from a union of other tubes near the base of the colony and traverses the entire stem. Pinnae on the side opposite the large tube arise from tubes which become hydrothecae-bearing for a short distance, give off two or three other branches, and then terminate in a tapering

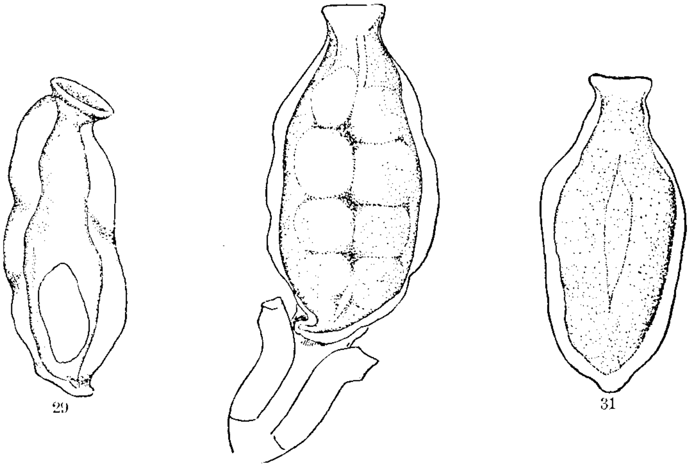


Figures 25–28. *Sertularella speciosa.*

Figure 25. Male gonotheca (× 30).
Figure 26. Female gonotheca (× 30).
Figure 27. Female gonotheca (× 30).
Figure 28. Part of branch (× 22).

process which joins another tube of similar structure. The large tube and all hydrothecae-bearing tubes are segmented, as well as the others, just before uniting with them.

The present species is an illustration of the difficulty which is found in delimiting the genus Sertularia and some other genera of the same family. The genus is confined by Nutting (:**04**, p. 37) to those species in which the operculum is well defined or stretched tightly over the top of the hydrotheca. Though neither condition occurs here, the characters of hypostome and gonosome place the species unmistakably in this genus. *Thuiaria pinnata*, described by Allman from the Double-headed Shot Key, is similar in trophosome, but the hydrothecae are less confluent (Allman, '**77**, p. 28). The gonosome is not known. *Sertularella distans* is also much like this species (Nutting, :**04**, p. 88). The stem is, however, unfascicled and the hydrothecae provided with a constant border.



FIGURES 29–31. *Sertularella humilis.*

FIGURE 29. Female gonotheca, one egg remaining (× 25).
FIGURE 30. Female gonotheca, eggs not discharged (× 25).
FIGURE 31. Male gonotheca (× 25).

GENUS SERTULARIA Linnaeus (in part), 1767.

Sertularia humilis (new species). Figures 29–32.

This is one of the hydroids most common at Bermuda. It grows so abundantly between tide marks as to make thick brown mats, which frequently contain coral mud and diatoms. Doubtless it is protected at low tide by its deneness, which mitigates desiccation. The name refers to its humble appearance.

Trophosome. Colonies are a deep horn color, pinnate, unfascicled, thirty to forty millimeters long. The sometimes geniculate stem is constricted below each of the ten or more alternate pinnae (Figure 32).

Three sessile hydrothecae usually found on each joint of stem, most frequently two on the same side as the pinna, the other sub-opposite the more distal. Variations in arrangement are found, such as two pairs of opposite hydrothecae on a joint. The flattened pinnae with two or three pairs of opposite, or slightly sub-opposite, hydrothecae to a segment. Base of stem and pinnae devoid of hydrothecae.

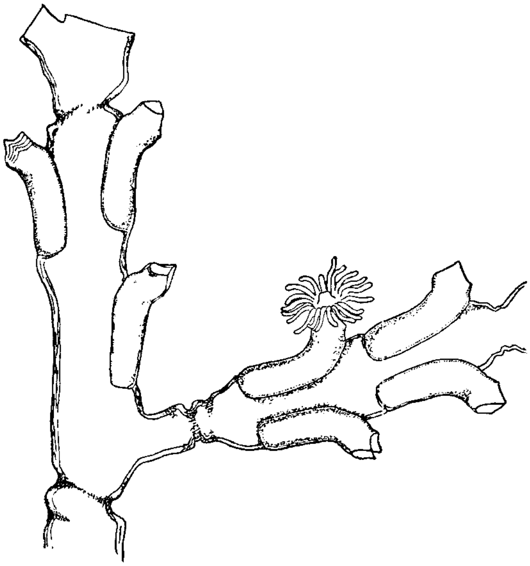


FIGURE 32. *Sertularella humilis.* Part of main stem and branch (× 25).

Hydrothecae of stem and pinnae tubular and two or three times as long as wide for the lower two thirds of their length, and at the base confluent with the stem. Above, bending outward at an angle of thirty-five to forty-five degrees, and also slightly toward each other. On this side the intervening space usually not so wide as a hydrotheca, though width increases with age. Outer border of hydrotheca continuous with border of stem or pinna, which extends proximally for a short distance to the next hydrotheca or constriction. The confluent portion of the hydrotheca diverges from the axis to a small degree, distally. There is a proximal diaphragm.

Edge of hydrotheca provided with a single adcauline and abcauline scallop, each of which is marked by a slight projection at its centre. In older hydrothecae several parallel ridges near the edge and operculum may be largely torn away; in younger consisting of three parts, meeting in ridges extending from the centre to the three projections. A lateral view of the colony presents the edge of many opercula, but shows the upper valve of some.

Gonosome. Colonies dioecious. Form of gonotheca the same in both sexes (Figures 29–31), more than twice as long as hydrothecae, ovoid, with a flaring mouth. They are most abundant in proximal part of colony, attached to hydrothecae just below diaphragm. A gonotheca was once observed rising from the interior of a hydrotheca. Characteristic markings may have been lost as the gonothecae were not well preserved. About sixteen eggs in the single large gonophore (Figure 30). Male gonothecae of similar appearance.

Sertularia brevicyathus Versluys.

So prolific is this little animal in some places that one can hardly pick up a piece of sponge, seaweed, or large hydroid without finding a dense growth attached.

Versluys ('**99**, p. 40) first described it from the Cape Verde Islands, and later Nutting (:**04**, p. 60) from the Bahamas. Neither author came upon the rather infrequent branches, which are attached at an acute angle just below the hydrothecae and differ in no respect from colonies.

Stolons are not uncommon. They may be given off like branches or from the tip of the colony. In the latter case they are often as long as a colony. If a number occur in one colony, they are arranged in pairs or are irregularly distributed.

The gonosome has not been previously described. Gonothecae are usually solitary on a colony, and attached just below a basal hydrotheca. Both male and female gonothecae are sessile, with a constricted opening, a low flaring lip, and wavy surface. They are closely similar to the gonothecae of *S. pumila.* These have been so completely described by Weismann as to render further account unnecessary.

Sertularia versluysi Nutting.

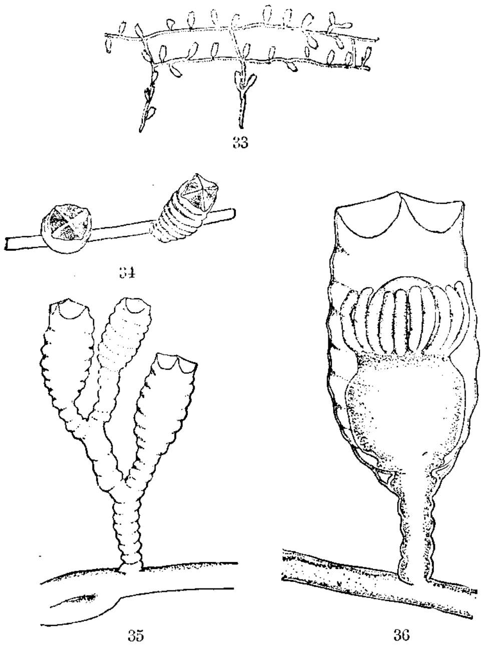
Sertularia versluysi was first described by Allman from dredgings off the coast of Bermuda in thirty fathoms. I have only a few colonies, which I found on floating Sargassum.

Versluys believed the animal had an operculum with a single abcauline flap. Nutting (:04, p. 53) was inclined to think there were two flaps on his badly ruptured specimen. Material in good condition shows that there are two large lateral projections from the edge of the hydrotheca and a smaller adcauline one. The three parts of the operculum meet in ridges along lines passing from the centre of the opening to the projecting points.

GENUS THYROSCYPHUS Allman, 1877.

Thyrocyphus intermedius (new species). Figures 33–36.

My only collection of this new form is from the eel grass of a shallow, muddy cove of Mangrove Bay (lat. 32° 18′ 10″, long. 64° 51′ 30″). Hydrothecae usually single, with pedicels one and a half millimeters long (Figure 36). A small proportion have two or three hydrothecae, whose annulated, geniculate, erect stems resemble elongated stolons (Figure 35). A terminal hydrotheca is found at one end of the axis and one or two others are attached at geniculations. The annulated pedicels are shorter than hydrothecae; when single, not more than a third as long.



FIGURES 33–36. *Thyroscyphus intermedius.*

FIGURE 33. Colony (× 1¾).
FIGURE 34. Hydranths, showing operculum (×7½)
FIGURE 35. Colony (× 10).
FIGURE 36. Hydranth (× 60).

Hydrothecae (Figure 36) are more than twice as long as wide, nearly cylindrical, marked by eight or more annulations, and tapering slightly toward the top. An operculum is formed by the bending of the hydrotheca wall to form four deep symmetrical scallops. The edges meet in two deep intersecting ridges, which terminate at the four points between the scallops (Figure 34). The narrow diaphragm consists of a fold which encloses a considerable space.

Colonies arise from creeping stolons, which are as large as a pedicel and not annulated (Figure 33). They grow parallel to each other along the blades of eel grass, sending frequent connecting branches across to neighboring stolons.

The few well-preserved hydranths are contracted into the lower half of the hydrotheca. They are attached to the diaphragm only, possibly because of unsatisfactory killing. There are about twenty tentacles and the hypostome is rounded.

The characters of the trophosome are allied to both Campanularidae and Sertularidae. The occurrence of diaphragm, pedicel, and an ovato-cylindrical annulated hydrotheca suggest the former. Yet we do not find a four-part operculum in that family except in the genus Thyroscyphus, erected by Allman in 1877 for the single species *T. ramosus*, which is of doubtful affinity (Nutting, :**04**, p. 10). The jointed stem, shortness of pedicel, and four-part operculum suggest its close relation with the genus Sertularella. If the gonosome is of the Sertularella type, the existence of a pedicel would be the only character separating it from that genus.

The genus Sertularella as revised by Hartlaub contains species with a hydrotheca and operculum closely similar to the Bermuda hydroid. The one character in which it differs from the genus is the presence of a pedicel. This difference Allman judged so important in the case of *T. ramosus* as to demand a new genus. I have accepted his view of the matter.

The chief differences between the two species, as far as we now know, are these: *T. ramosus* is larger, has a bordered hydrotheca, and a jointed stem bearing many hydrothecae instead of a short, annulated, unjointed one bearing at most three hydrothecae.

GENUS AGLAOPHENIA Lamouroux, 1816.

Aglaophenia minuta Fewkes. Figure 37.

This form is rather widely distributed. It occurs on the Sargassum, which drifts to Bermuda from the south. Though I did not find the gonosome, the complex trophosome was sufficient for identification. A small nematophore is always present on each side of the axil of a branch. The fact is not mentioned by Fewkes nor in the fuller description by Nutting.

GENUS **LYTOCARPUS** Kirchenpauer, 1872. Figure 37.

Lytocarpus philippinus Kirchenpauer.

Lytocarpus philippinus is found at the places where hydroids most abound along the shores of Bermuda. It was also dredged from the Challenger banks, south of Bermuda. It was first described by Kirchenpauer from the Philippine Islands, and since has been noted from many stations in the Pacific and from Jamaica.

The following slight variations from the description and plate of Nutting are to be found: the mesial nematophore is longer; the intrathecal ridge nearer the base, and the colony shorter.

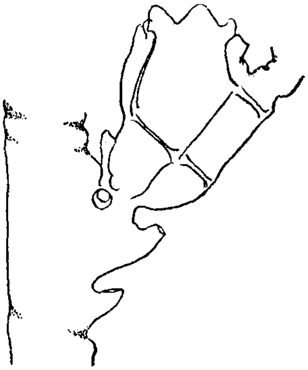


FIGURE 37.

Lytocarpus philippinus. Base of pinna (× 105).

GENUS **PLUMULARIA** Lamarck, 1815, modified by Nutting, 1900.

Plumularia alternata Nutting.

Nutting has described this form from a collection made by Agassiz at Barracuda Rocks during the cruise of the "Wild Duck" in 1893. I found it to be one of the less common hydroids growing upon the floating Sargassum in the vicinity of Bermuda. The gonosome is not known.

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July, 1906.

BIBLIOGRAPHY.

Allman, G. J.

'71. A Monograph of the Gymnoblastic or Tubularian Hydroids. Ray Soc. publ. for 1869 and 1870. 450 pp., 83 text figs., and 23 pls.

'77. Report on the Hydroida collected during the Exploration of the Gulf Stream by L. F. de Pourtalès, Assistant United States Coast Survey. Mem. Mus. Comp. Zoöl. Harvard Coll., Vol. 5, No. 2, 66 pp., 34 pls.

Clarke, S. F.

'79. Report on the Hydroida collected during the Exploration of the Gulf Stream and of the Gulf of Mexico by Alexander Agassiz, 1877–1878. Bull. Mus. Comp. Zoöl. Harvard Coll., Vol. 5, No. 10, pp. 241–252, pls. 1–5.

Congdon, E. D.

:**06**. Notes on the Morphology and Development of Two Species of Eudendrium. Biol. Bull., Vol. 11, No. 1, pp. 27–46, 11 text figs.

Hincks, T.

'**68**. A History of the British Hydroid Zoöphytes. London, John Van Voorst. 338 pp., 42 text figs., and 67 pls.

Nutting, C. C.

:**00**. American Hydroids. Part I, The Plumularidae. Bull. Smithsonian Inst., Washington, 285 pp., 124 text figs., and 34 pls.

:**04**. American Hydroids. Part II, The Sertularidae. Bull. Smithsonian Inst., Washington, 325 pp., 139 text figs., and 41 pls.

Verrill, A. E.

'**99**. Additions to the Anthozoa and Hydrozoa of the Bermudas. Trans. Conn. Acad. Arts and Sci., Vol. 10, Art. 14, pp. 551–572, 1 text fig., and pls. 47–49.

Versluys, J., Jr.

'**99**. Hydraires calyptoblastes recueillis dans la mer des Antilles pendant l'une des croisières accomplies par le comte R. de Dalmas sur son Yacht "Chazalie." Mém. soc. zool. France, Tom. 12, Pt. 1, pp. 29–58, 24 figs.

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CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION
FOR RESEARCH, No. 10; AND FROM THE LABORATORY
OF HISTOLOGY AND EMBRYOLOGY, WESTERN
RESERVE UNIVERSITY.

THE SPERMATOGENESIS OF THE MYRIAPODS.

V.—*ON THE SPERMATOCYTES OF LITHOBIUS.*

By Maulsby W. Blackman.

With Two Plates.

THE SPERMATOGENESIS OF THE MYRIAPODS.

V.—ON THE SPERMATOCYTES OF LITHOBIUS.[1]

By Maulsby W. Blackman.

Presented by E. L. Mark, December 12, 1906. Received November 22, 1906.

For a number of years the author has been collecting material for a comparative study of the spermatogenesis of the principal groups of the Chilopoda. Five papers based upon this material have already been published; a series of four papers (Blackman, **:01**, **:03**, **:05**, **:05^a**) upon two species of Scolopendra (S. heros and S. subspinipes) and one paper upon Scutigera forceps by Medes (**:05**). In the present article it is my purpose to carry this comparative study further.

The present paper is based upon a study of the spermatocytes in three species of the genus Lithobius and is from material collected in three widely separated localities. The principal observations were made on the testes of Lithobius mordax, collected in the vicinity of Lawrence, Kansas, during the spring and summer of 1902 and 1903. Using as a basis the knowledge gained from the study of this species, additional observations have been made upon Lithobius sp.? and Lithobius multidentatus, with a view to comparison. The material of Lithobius sp.? was collected by the author near Flatts, Bermuda, during July, 1903, while working at the Bermuda Biological Station for Research; that of Lithobius multidentatus was collected near Cambridge, Mass., during October, 1904.

At the time this comparative study of myriapod spermatogenesis was begun, no detailed work upon the germ cells of chilopods had been attempted under modern conditions of technique. The only observations upon this material then published were those of Carnoy (**'85**) in his classical Cytodierèse chez les arthropodes, and those of Prenant (**'87**, **'92**). Since then, however, a number of papers based upon the sperm cells of myriapods have appeared. Of those dealing with problems touched upon in this article the chief are those by Meves and von Korff (Lithobius forficatus), :01; P. Bouin, :00, :01, :03, :03^a, :04; P. et M. Bouin, :02, Lithobius forficatus and Scolopendra morsitans; P. Bouin et R. Collin, :01, Geophilus linearis.

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[1] Contributions from the Bermuda Biological Station for Research, No. 10; and from the Laboratory of Histology and Embryology, Western Reserve University.

The results obtained by these authors differ so markedly in several important particulars from those obtained by myself (Blackman, :01, :03, :05, :05^a) on Scolopendra and by Medes (:05) on Scutigera that it seemed advisable to extend the comparative study to the genera upon which they had worked. What I consider the important discrepancies between the observations upon American and those upon European forms have to do with the condition of the chromatin during the growth period and its later behavior in the prophase. According to my observations upon the two species of Scolopendra, the chromatin during the growth period is not arranged in the form of a spireme, or reticulum, throughout the nucleus, but the threads are aggregated into a very dense mass, the karyosphere, and at the beginning of the prophase the chromosomes arise from this mass directly. The observations of Medes (:05) show that an essentially similar condition exists in Scutigera. All the observations upon the European forms, however, except the early ones of Carnoy ('85) upon Lithobius and Scutigera, seem to lead to the conclusion that the large intra-nuclear bodies seen in all chilopod spermatocytes are true nucleoli and have no genetic relation to the prophase chromosomes. As will be seen later in this paper, my present observations on Lithobius confirm my earlier ones on Scolopendra.

TECHNIQUE.

In the preparation of the material much difficulty was at first experienced in obtaining a good fixation of all the cell structures, owing to the peculiar character of the cytoplasm. Thus, in the early study, a large number of fixing reagents were used, including the following: Flemming's strong solution, Flemming's weak solution, Gilson's fluid, Perenyi's fluid, Merkel's fluid, and various picro-acetic and sublimate-acetic mixtures. Of these, much the best results were obtained with Gilson's nitric-acetic-sublimate mixture. While the results of this fixation were not always entirely satisfactory, the majority of preparations thus obtained apparently left nothing to be desired. In later studies Bouin's picric-acetic-formol mixture was tried, but the results were much inferior to those obtained with Gilson's fluid.

The sections were cut with the Minot rotary microtome, the thickness varying from 2 to 6 micra, and were affixed to the slide by means of very dilute albumen water. The sections were then stained by one or another of a number of methods, the best cytological results being obtained with Heidenhain's haemotoxylin used either alone or with Congo red as a counterstain. Sections thus stained were used for all detailed studies of the cell structures. For micro-chemical tests, Flemming's three-color stain and the Ehrlich-Biondi mixture were used, the results obtained with the latter being especially satisfactory.

OBSERVATIONS.

The testis of Lithobius consists of a single unpaired sac tapering at the anterior end, where it ends blindly, and also tapering slightly at the posterior end, where it is directly continuous with the vas deferens. In adults, which were used exclusively in this study, the spermatogonia are found only at the periphery of the testes. They are small spindle-shaped cells containing oval nuclei (Plate 1, Figure 1), and are similar in all respects to the spermatogonia of Scolopendra, with the exception that each nucleus contains two karyospheres instead of one. These karyospheres consist largely of chromatin, as is shown by their reaction to stains. They are stained black in iron-haemotoxylin, and after treatment with the Ehrlich-Biondi mixture they are colored green, while all other cell structures assume a red tint. The remaining portion of the nucleus is occupied by a network of linin fibres, which stains as lightly as does the cytoplasm outside of the nucleus.

The constancy with which two karyospheres, and never more than two, occur in the spermatogonia of Lithobius mordax has led me to ask what significance this may have. When we consider that there are always two karyospheres in the spermatogonia, and never more than one in the spermatocytes of this species, the explanation suggests itself that one of the spermatogonial karyospheres may represent the chromosome group derived from the mother, while the other one is made up of paternal chromosomes. However, it would seem impossible to obtain proof of this, and the suggestion is made merely as an interesting possibility. Furthermore, this condition is constant only for Lithobius mordax, while in Lithobius sp.? the number of karyospheres in the spermatocytes is often as great as four. At the beginning of the active prophase of the spermatogonial mitoses in Lithobius, the chromatin leaves the karyospheres and becomes arranged throughout the nucleus in the form of a large number of fine granular threads (Figure 2), each one of which represents a chromosome. The number of these segments at this stage is so great, and the nucleus is so small, that it has been found impossible to count them; but from counts of the chromosomes during the spermatocyte stages, it may be inferred that the number of these granular threads found in the spermatogonia is forty-eight, so that with the accessory chromosome the total spermatogonial number is forty-nine.

After the origin of the chromatin segments from the karyospheres the residue consists of two sorts of material: a small, deeply staining body, the accessory chromosome, and two lightly staining plasmosomes (Figure 2). The plasmosomes remain in the nucleus during the prophase, and are dissolved at the time of the breaking down of the nuclear membrane. The accessory chromosome is usually closely applied to one of these nucleoli, although it may be entirely free from both.

Thus the conditions at this time are quite similar to those found in Scolopendra subspinipes (Blackman, :05[a]), where a portion of the residue of the karyosphere is nucleolar material, while the other portion is a modified chromosome. In Scolopendra heros, on the contrary, all the products of the karyosphere are chromatic. One point of difference, however, between the spermatogonial prophases in Lithobius on the one hand, and in the two species of Scolopendra (Blackman, :03, :05[a]) and in Scutigera forceps (Medes, :05) on the other, is the shape and appearance of the chromosomes. In Lithobius these elements are slender granular segments, similar to the corresponding structures in insect spermatogonia, while in the other three species mentioned the chromosomes are in the form of small rounded granular masses, which never lengthen out to form threads. The later stages of the spermatogonial mitosis offer no points of especial interest, and will not be described in this paper.

The telophase stages of the last spermatogonial division are similar in all essential respects to those in Scolopendra and Scutigera. The daughter chromosomes, as they move toward opposite poles of the spindle, are aggregated into dense masses. At this stage the chromosomes are rounded bodies possessing clear-cut outlines. Soon, however, just as in Scolopendra, they begin to lengthen out and become granular, and thus in place of a mass of closely aggregated homogeneous bodies there soon arises a more loosely arranged group of granular segments. At first this is enclosed in no membrane, but lies within a vacuole of hyaloplasm. Eventually, however, a definite nuclear membrane appears and the chromatin segments become distributed throughout the space enclosed by it (Figures 3, 4). It may now be readily seen that the number of these chromatin segments is much smaller than the number of chromosomes seen during the spermatogonial prophase. Careful counts give the number of segments at this stage as 24, which, together with the accessory chromosome, produces a total of 25, the number of chromosomes characteristic of the following mitosis. Thus here, just as in Scolopendra, pseudo-reduction has already occurred as the result of the union in pairs of the spermatogonial chromosomes during the telophase of the last mitosis of the division period.

Evidences of the nature of this union are to be seen in many nuclei, although, owing to the close grouping of the chromosomes in the telophase, actual stages in the union are very difficult to find. In the two species of Scolopendra (Blackman, :03, :05, :05[a]) it has been shown that this union of the chromosomes is accomplished by an end to end fusion of the elements, similar to that described by Montgomery (:00, :01) for Peripatus, and by Sutton (:02) for Brachystola. In the three species of Lithobius studied, evidences of the origin of the spermatocyte chromosomes by a similar union are seen in Figures 3 and 4. The chromatin segments are often bent at a sharp angle at about their middle point, the two equal limbs representing spermatogonial chromosomes which have conjugated during the preceding telophase. Thus in Lithobius, just as in Scolopendra and in Scutigera, there is no stage in which all of the chromatin is arranged in a continuous spireme, and at no time is the number of chromatin segments less than the number of chromosomes characteristic of the succeeding metaphase.

In the meantime, even before the reconstruction of the daughter nuclei in the last spermatogonial telophase, the cells have entered upon that period of remarkable growth characteristic of the spermatocytes of chilopods. This growth is so marked that at the time of the appearance of the nuclear membrane (Figures 3, 4) each of the daughter cells has attained a size about equal to that of the parent cell. In succeeding stages this increase in size continues (Figures 3–8), until finally in the vesicle stage (Figure 10) the diameter of the spermatocytes is more than ten times that of cells in the spermatogonial telophase. Thus the ultimate size of the spermatocyte in the vesicle stage is even greater than in Scolopendra.

At the time of the reconstruction of the nuclear membrane, as has already been mentioned, chromatin segments of the reduced number become scattered throughout the entire nuclear cavity (Figure 3). As the cell continues to grow these chromatin threads become more diffuse and the granules composing them become finer, and are now in a condition apparently similar in all respects to the segmented spireme stage of the spermatocytes of other animals. Now, however, a rearrangement of the chromatin begins to take place, which seems to be characteristic of chilopod spermatocytes and is seldom met with in the male cells of other animals. These changes have been studied in detail in Scolopendra (Blackman, :03, :05, :05[a]) and in Scutigera (Medes, :05), and while they are essentially similar in these two genera, they still present several interesting minor differences.

In Scolopendra heros (Blackman, :03, :05) their changes are as follows: "The chromatin segments begin slowly to break down, and their substance becomes aggregated about the accessory chromosome, thus apparently increasing the size of this body enormously. This process continues until finally all of the chromatin of the cell is aggregated in one large intensely staining body." (Blackman, :05, p. 14.) This body I have called the karyosphere. It is not a simple homogeneous body of chromatin, but a very complex mass of chromatin fibres closely enveloping a solid chromatin body, the accessory chromosome. It is made up of chromatin, linin, and karyolymph, but contains no nucleolar material.

In Scolopendra subspinipes (Blackman, :05^a) the conditions are slightly different, owing to the presence of a true nucleolus. Here the karyosphere is formed by the massing together of all of the chromatin of the cell, but it also contains a nucleolus in addition to the material derived from the chromosomes. The chromosomic material is deposited in a dense granular mass over the outside of the nucleolus, forming a mantle around it.

In Scutigera forceps (Medes, :05) the origin of the karyosphere is quite similar to that in Scolopendra heros. The material composing it in these early stages is derived entirely from the chromosomes. Later, however, during the growth period, the karyosphere acquires in addition a large quantity of nucleolar material, as is shown both by its staining reaction in the vesicle stage and by the nucleolar residue remaining after the origin of the chromosomes in the prophase following. Thus, while the component materials of the karyosphere in Scutigera are identical with those of the same body in Scolopendra subspinipes, there is in the former no such definite localization of chromatic and nucleolar material as is found in the latter.

In the three species of Lithobius which I have now studied, two conditions exist as to the origin of the karyosphere. In Lithobius mordax the origin of the karyosphere is more like that in Scolopendra heros and Scutigera forceps, while in Lithobius multidentatus and Lithobius sp. the stages observed resemble more closely those found in Scolopendra subspinipes.

Stages in the formation of the karyosphere in Lithobius mordax are shown in Figures 3, 5, 6, 7, and 8. In Figure 3 the outlines of the accessory chromosome can still be distinguished. Closely apposed to one side of this element are several masses, which are more granular. These represent chromatin segments which have become shortened and thickened by a rearrangement of their granules, and have taken up a position upon one side of the accessory chromosome. Other chromosomes, still in the form of granular segments, are in contact with the forming karyosphere and very plainly are about to undergo a similar process. This disposition of the chromosomes around the accessory chromosome continues (Figure 5) until all of them are aggregated in one irregular densely staining mass, the karyosphere (Figures 6, 7).

In Scolopendra heros it was never possible to establish conclusively that the chromatin enters the karyosphere as definite individual chromosomes. That this is the case was, however, always believed by the author, and indeed there was considerable evidence supporting this view. In Scolopendra subspinipes more convincing evidence was found, as the nucleolar material there present serves as a background against which the chromosomes can be seen in favorable cases. However, some doubt might still remain.

The stages in the formation of the karyosphere in Lithobius mordax furnish better evidence of the individuality of the chromosomes during the early spermatocytic stages than those of any other myriapod yet studied. It can be very plainly seen that *the chromosomes enter into the formation of the karyosphere as distinct elements without losing their individuality.* Evidences of this fact are found at all stages in the formation of the karyosphere in the irregular lobulated structure of this body (Figures 5–8). Each one of these lobules undoubtedly represents a chromatin segment which has become shortened and thickened into an irregularly rounded body. In Figure 5 several of these chromosomes still remain detached from the rest and their identity is easily recognized. In Figure 6 the individuality of many of the elements may still be readily seen, while in Figure 7 the grouping of the chromosomes is but little closer.

Shortly after the stage shown in Figure 7, the karyosphere begins to undergo a change by which the dense mass of chromosomes is converted into the completely formed karyosphere of the vesicle stage. This change consists in the acquisition of a large amount of nucleolar material. This results in three changes in the appearance of the karyosphere. The first two of these are changes in size and shape, by which the irregular lobulated karyosphere of the young spermatocytes is converted into the larger spherical body characteristic of the vesicle stage. The presence of nucleolar material in the karyosphere also brings about a change in its staining reactions.

In Figure 8 is seen an early stage in this metamorphosis. By comparing the karyosphere in this cell with that shown in Figure 7, the changes mentioned above are immediately apparent. The karyosphere has undergone a considerable increase in size and a portion of the surface has assumed a regular rounded outline. The portion of the karyosphere possessing the regular outline has also changed in its reaction to the stains, showing that it is in this part that the nucleolar substance lies. Here certain rounded areas, which represent the chromosomes, still retain the haematoxylin strongly, while surrounding these chromosomes are irregular areas which show little or no affinity for the chromatin stains. Thus the rounding off of the karyosphere is due to the filling of the interstices between the chromosomes with nucleolar material, which thus acts as the matrix in which the chromatin is embedded. The acquisition of achromatic material continues during succeeding stages until, in the vesicle stage, the fully formed karyosphere is considerably larger, possesses a clearly cut outline, and is spherical in shape.

The origin of the karyosphere in Lithobius multidentatus and Lithobius sp.? differs from that in Lithobius mordax in several respects, and at this stage the cells resemble those of Scolopendra subspinipes more than they do those of Scolopendra heros. This is due to the facts that the young spermatocytes of these two species of Lithobius contain one or more true nucleoli and that this nucleolar material also enters into the formation of the karyosphere. In Figure 4 is represented a young spermatocyte of Lithobius sp.? showing an early stage in the formation of the karyosphere. The spermatocytes of this species are characterized by the presence of two or more nucleoli during the growth period. Figure 4 shows a cell possessing two nucleoli. Chromatin in the form of diffuse segments is already being deposited upon the surface of one of these, and the accessory chromosome may be seen embedded in one side of it. The other nucleolus (Figure 4) is still free of chromatin, but would later be involved in these changes also. Each nucleolus usually forms the basis for a separate karyosphere, as may be seen by referring to Figures 14, 16, 17. In Figure 15 one of the nucleoli contains no chromatin deposits whatever. This, however, is exceptional.

In Lithobius multidentatus the origin of the karyosphere is similar to that just described for Lithobius sp?, except that here only one nucleolus occurs, and only one karyosphere is formed (Figures 11–13).

After all of the chromosomes of the cell have become massed together to form the karyosphere, the remaining portion of the nucleus no longer stains deeply with chromatin stains (Figures 5, 6) and is no more dense in its structure than the cytoplasm. Soon irregular amorphous deposits appear in the nucleus, and these form a mantle surrounding the karyosphere. This mantle, however, does not envelope the karyosphere closely, but is separated from it by an area of transparent substance so that this body seems to lie in a vacuole of hyaloplasm (Figures 5, 6, and 7). The remaining portion of the nucleus is occupied by an irregular reticulum of linin fibres.

As the cell continues to grow this amorphous deposit increases in amount (Figures 8, 9, 10), until finally, in the vesicle stage, it forms quite a conspicuous portion of the nucleus (Figures 9, 10). Just what the significance of this material may be I am at a loss to say. From its straining reactions, however, it is seen that it is chemically similar to the nucleolar substance. In Lithobias mordax it is much more abundant than in either of the other species studied, while in Lithobius sp. ? (Figures 14, 17) very little if any occurs. Thus it varies inversely with the amount of nucleolar material contained in definite nucleoli. In Lithobius multidentatus less is present than in L. mordax, but in the former species (Figures 11, 13) it occurs in much more definite bodies, and these often approach nucleoli in form and structure. So the conclusion would seem justified that these deposits represent nucleolar material which has never become arranged in the form of distinct nucleoli.

We will now return to the early spermatocyte and consider the evolution of the extra-nuclear structures. First, as regards the centrosome. In Scolopendra heros (Blackman, :**05**) I was so fortunate as to be able to find the centrosome during all the stages from the spermatogonium to the mature spermatozoön. Here it was readily found and recognized, owing to the fact that during the growth period it is enclosed in a specialized portion of the archoplasm. In Lithobius, however, such aggregations of archoplasm do not occur, and the centrosome has not been observed during the growth period. In Figure 7 two small, densely stained bodies enclosed in a denser portion of the cytoplasm are shown. It is, however, impossible to state with any certainty that these are centrosomes, owing to the fact that such structures have not been observed at other stages of the growth period. However, it does not necessarily follow that no centrosome exists during the growth period, as no determined effort has been made to find it and study it, as was done in Scolopendra.

The structure of the cytoplasm in Lithobius has been studied by P. et M. Bouin ('**99**), and by Meves und von Korff (:**01**). The brothers Bouin report the presence within the cytoplasm of Lithobius forficatus of certain irregularly formed bodies, which they have designated "formations ergastoplasmiques." These are believed to arise by the disintegration of the astral rays. Just before the prophase these deposits undergo what is described as a gelatinous metamorphosis and are not seen later than the early prophase. Meves und von Korff (:**01**), working on the same species, find these bodies in all phases of the first spermatocyte mitosis, although in later stages they are reduced to smaller granules. In Scutigera (Medes, :05) structures which are probably homologous are formed both in the nucleus and in the cytoplasm. These bodies, which are classed as metaplasm, stain densely and vary widely in number in different cells. As the prophase approaches, they begin to disintegrate, and soon disappear entirely.

In Scolopendra (Blackman, :05) there are two sorts of formations occurring in the cytoplasm. The first of these is the archoplasm, which is similar in structure to the cytoplasm and appears as a thickened and condensed part of it. The second consists of rather small, dark staining granules, which in appearance and behavior are similar to those in Scutigera. These undoubtedly represent metaplasm — either ingested food particles, or by-products of the cell activity.

In well-preserved cells of the three species of Lithobius studied I have been able to find no structures corresponding to the "formations ergastroplasmiques" of P. et M. Bouin, although in sections of poorly fixed material such deposits are often seen. The cytoplasm in Lithobius is very fine meshed and seems to be very difficult to fix properly. When wrongly treated, the fine reticulum becomes distorted and irregularly massed together in such a manner as to give the appearance of definite structures, and these I believe have been incorrectly regarded as normal structures. However, a few small, distinct *granules* do seem to be present at all stages of the first spermatocyte, and these I consider to be metaplasm, i. e., food material or by-products of metabolism.

In Lithobius no such aggregations of archoplasm exist as are characteristic of the growth period and vesicle stage of Scolopendra. On the contrary, all of the cytoplasm of the cell seems to be of about the same consistency. Indeed, both in structure and staining reactions the cytoplasm in the vesicle stage is quite similar to that in the prophase of the first spermatocyte of Scolopendra, after the archoplasm has been dissolved in the hyaloplasm and dispersed throughout the entire cell. This similarity and the fact that there are no special aggregations of archoplasm have led me to conclude that during the vesicle stage in Lithobius the archoplasm is dispersed throughout the cell. This causes the cytoplasm at this stage and during the early phases of mitosis to stain more strongly than at other times.

At the completion of the growth period the spermatocyte of Lithobius mordax in the vesicle stage presents the appearance shown in Figure 10. The size of the cells is remarkable, as they often reach an average diameter of more than 100 micra, thus being even larger than the spermatocytes of Scolopendra and Scutigera. In the various species of Scolopendra (Blackman, :01, :05, :05^a; Bouin, :03, :03^a) the spermatocytes in the vesicle stage are of two distinct types as to size. In Lithobius, however, while there is considerable variation in the sizes of the cells during the vesicle stage, no such great differences exist, and the cells cannot be divided into two types on this basis.

The cells in the vesicle stage of the three species of Lithobius are quite similar to one another as regards the extra-nuclear portion, but differ considerably in the structure of the nucleus, so much so that the species may be readily identified by this alone. The differences existing between Lithobius mordax and L. multidentatus are less apparent than those between either of these and the Bermuda species, yet very little study is necessary for identification. The greatest of these specific differences are seen in the structure of the karyosphere, but the condition of the achromatic structures also varies in the three species.

What may be taken as the typical appearance of the nucleus in Lithobius mordax is shown in Figure 10. The karyosphere is stained an intense black with haematoxylin and green with the Ehrlich-Biondi mixture, showing that it is rich in chromatin. However, in many sections in which the decolorizing has been carried further, the entire karyosphere does not take the chromatin stain, thus showing that this body is not made up entirely of chromatin, as is that of Scolopendra heros, but contains nucleolar material also, as do the similar structures in Scolopendra subspinipes and Scutigera forceps. A karyosphere of this sort is shown in Figure 9. Here the chromatin is arranged in the form of a network of anastomosing strands which are embedded in the nucleolar substances. No portion of the nucleus except the karyosphere contains any chromatic material. The conspicuous deposits which form an irregular mantle around the karyosphere are very evidently achromatic material, as is shown by their staining reactions.

In Lithobius multidentatus the karyosphere is much like that just described. The chief difference lies in the fact that here the amount of chromatin to each cell is considerably less. Thus the karyosphere is not usually stained so black and the nucleolar portion is more evident (Figures 11, 12, 13). It is also true that the chromatin is usually found only on the periphery of the karyosphere outside the nucleolus, resembling in this respect Scolopendra subspinipes. The deposits of achromatic substance in the nucleus are much less abundant and are in more definite bodies, thus allowing more of the linin network to be seen.

The nucleus in Lithobius sp. ? differs from that of either of the two species described in several particulars. There are no indefinite masses of achromatic material, this all being incorporated in the nucleoli, of which there are several, the amount of nucleolar material being greater. Usually each nucleolus contains one or more chromosomes and thus constitutes a karyosphere (Figures 14–17). The number of karyospheres in a nucleus varies from one (Figure 15) to four (Figure 18) or even six, the most common number being two (Figure 16), while three are often seen (Figures 14, 17). When the number of karyospheres is greater, the relative amount of chromatin in each is smaller and the chromatin occurs in more definite masses. Often it is arranged in the form of a single thread embedded in the nucleolar material (Figure 14), while in other cases it occurs in several irregular masses (Figures 14, 15, 17). It is usually enveloped in the nucleolus and not deposited upon the periphery, as in Lithobius multidentatus.

The origin and behavior of the chromosomes during the prophase of the first spermatocyte of myriapods has been described differently by different workers. The first account is that by Carnoy ('**85**), which deals with the spermatocyte mitoses of Lithobius forficatus, Scutigera arachnoides, Geophilus, and Scolopendra dalmatica. In Lithobius, Scutigera, and Geophilus the nucleole-noyau (karyosphere) gives rise to the chromatin thread, from which the chromosomes are derived. The process of tetrad formation was not studied at all. In Scolopendra dalmatica the nucleolus is said to take no part whatever in the origin of the chromosomes.

Later investigators upon European myriapods have arrived at results quite different from those of Carnoy. Meves und von Korff (:**01**) confined their observations almost entirely to the peculiar modification in the achromatic division figure of the first spermatocyte of Lithobius forficatus. The chromosomes were but little studied, and those figured are already in the form of distinct elements. The description, however, gives the impression that they arise from the network of the nucleus.

In several short papers from the University of Nancy, P. Bouin, M. Bouin, and R. Collin have dealt briefly with the chromosomes, although most of their work has been upon achromatic structures. In Geophilus linearis P. Bouin et R. Collin (:**01**, page 101) find no connection between the large "nucleolus" and the origin of the chromosomes, but on the contrary state that these elements arise from a chromatic reticulum: "Le réticulum chromatique est constitué par de très fins microsomes distribués sur les mailles du réseau de linine; au niveau des points d'entrecroisement de ce réseau, ils sont rassemblés en amas plus ou moins volumineux. Ces microsomes chromatiques se réunissent bientôt en petites masses qui se condensent rapidement en sphérules homogènes ; celles ci paraissent subir tout de suite une segmentation transversale. Peu de temps après le début de ces processus on peut voir, disséminés dans l'aire nucléaire, et aussi bien contre la face interne de la membrane du noyau que sur les travées du réseau linien, des chromosomes segmentés transversalement et qui figurent, comme l'un de nous l'a déjà signalé à propos du Lithobius forficatus, des sortes de diplosomes chromatiques minuscules. Ces diplosomes, chez Geophilus linearis, sont au nombre de huit. Ils nous a été impossible, à cause sans doute de l'extrême petitesse de ces formations, de nous rendre compte si les indices de la deuxième division de maturation apparaissaient sur ces chromosomes dès ce stade de la caryocinèse." The authors conclude that each of the succeeding mitoses results in a transverse division of the chromosomes.

The criticism by Grégoire (**:05**) of these results of Bouin et Collin seem to me to be just. He says (p. 284): "Cette interprétation appelle plusieurs remarques. D'abord, si la description des auteurs est exacte, il faut reconnaître que cet objet est détestable pour l'étude du subjet actuel et qu'on n'en peut tirer de conclusion d'aucune sorte. En effet, il semble bien impossible de savoir si la division en deux d'une 'sphérule' constitue une division transversale à mettre sur le pied de ce qu'on appelle ailleurs de ce même nom ; de plus, à la seconde cinèse, comment fixer la valeur de la bipartition d'une 'granulation minuscule'? L'objet, tel qu'il est décrit, serait donc à condamner au point de vue actuel."

In a later paper P. Bouin et M. Bouin (**:02**) have published the results of their studies on the chromatin reduction in Lithobius forficatus. Here the chromosomes are larger than in Geophilus, although still somewhat smaller than those described in this paper. The authors believe that in Lithobius also the chromatic elements have no connection with the large "nucleolus." But in the figures accompanying this and other papers on the same species, it is seen that at the beginning of the prophase the nucleolus is large and stains like chromatin, while later, after the chromosomes are formed, it is much smaller and more lightly stained. However, the authors state that the chromosomes arise from the nuclear reticulum. The chromatin granules, while still a part of this network, undergo cleavage, so that each thread of the reticulum becomes double. Later the chromatin granules collect and fuse to form bipartite chromosomes or "diplosomes." No true tetrads were observed, but the authors conclude that reduction is accomplished by a longitudinal division followed by a transverse division.

In the American myriapods the origin and behavior of the chromosomes of the first spermatocytes have been studied in three species: in Scolopendra heros (Blackman, :01, :03, :05), in Scolopendra subspinipes (Blackman, :05^a), and in Scutigera forceps (Medes, :05). In these species the essential phenomena are similar, although the details of the processes vary considerably. In all three forms the chromosomes arise from chromatin stored in the karyosphere. The chromosomes appear first as segments equal in number to the elements later taking part in mitosis. These granular segments undergo first a longitudinal and later a transverse cleavage, and soon take on the well-known cruciform shape characteristic of the heterotypical chromosomes of invertebrates. During the late prophase the chromatin granules fuse and the tetrad becomes a homogeneous four-lobed chromosome.

The formation of the tetrads in Scutigera varies in one interesting particular from the typical process as observed in Scolopendra, as has been shown by Medes (:05, p. 163): "Immediately after leaving the karyosphere the chromosomes shorten into dense granular cords. These now undergo a longitudinal cleavage and the double thread of chromatin breaks up into a number of short portions of irregular size." These portions are spherical in shape and of homogeneous consistency; they appear to be lobules of chromatin in a liquid condition.

In describing the origin and behavior of the chromosomes of the first spermatocyte in the forms used in the present paper, the observations upon Lithobius sp. ? will be given first, as here the phenomena are more like those observed in other animals. Indeed, as regards the changes in the chromosomes themselves, the nuclear processes are nearly identical with those in Scolopendra. The first change in the nucleus during the prophase has to do with the appearance of the chromosomes, which, as we have already seen, unite with the nucleoli during the early part of the growth period to form several karyospheres. During the growth period and vesicle stage this chromatin may be seen in the karyospheres as dense masses or threads embedded in the nucleolar material. The first change in the nucleus is in this chromatin. The dense masses become more granular and more diffuse and move to the surface, thus coming to lie outside the nucleolar portion. Strands of the chromatin leave the karyosphere and lie free in the nucleus. This process continues until all of the chromatin has left the karyospheres. When this is accomplished, the number of chromatic segments in the nucleus is the same as the number of chromosomes participating in the succeeding division. Figure 18 (Plate 1) represents an advanced stage in the disintegration of the karyospheres. Many of the chromosomes are already free, while those which still remain in the karyosphere are spread out as diffuse flocculent deposits upon the surface of the nucleolar portions.

In later stages (Figures 19–21), when all of the chromosomes are free in the nucleus, the nucleoli, much reduced in size, may still be seen. These nucleoli, although at first spherical (Figure 21), soon become irregular in outline and rapidly disintegrate (Figures 19^c, 20). At least a portion thus set free in the nucleus is used in the evolution of the chromosomes.

The chromatin segments of the reduced number as they arise from the karyosphere are diffuse granular threads (Figure 18). These soon elongate, and the granules composing them become coarser. The segments assume a variety of shapes, but most often they are roughly U-shaped or V-shaped, just as they were in the early spermatocytes before they entered the karyosphere. At about their middle point, — i. e., at the angle of the V, or at the loop of the U, — there is very often a small gap in the chromatin thread (Figure 18), as has been observed by several investigators on other material (Montgomery, :**00**, :**05**; Sutton, :**02**; Blackman, :**03**, :**05**, **05**^{a}; Foot and Strobell, :**05**). This undoubtedly represents the point at which the conjugating chromosomes united during synapsis.

Soon each segment undergoes a longitudinal splitting, resulting in two more slender segments (Figures 19^a–21). The halves thus resulting typically lie parallel to each other, but very often they are twisted about each other, as is shown by several chromosomes in Figures 19^a and 20. Less often the ends of the halves may be separated and drawn in opposite directions, as is the case with several chromosomes in Figures 19^a and 19^c. Typically, however, the two threads lie near each other and are approximately parallel.

The next change in this double segment consists in the cleavage of each of the halves at about their middle point, resulting in a four-parted chromosome or tetrad. This is a transverse division of the chromosome and occurs at the point where in preceding stages an interruption in the chromatin was observed. Thus, when this division is completed, it results in the separation of entire spermatogonial chromosomes.

As soon as the transverse cleavage is established the chromatids revolve upon each other in one of several ways, producing tetrads varying in shape, but all referable to one type. The more common method is as follows: At the point of transverse cleavage the threads show a tendency to bend at a more or less acute angle, and in each half-segment the adjacent ends produced by the cross division become drawn out in the same direction, but perpendicular to the axis of the original thread, while the corresponding portion of the other half is drawn out in the same plane, but in an opposite direction. This results in the typical cross-shaped figures shown by several tetrads in Figures 19^b and 20. The four arms of the cross are approximately equal in length, and each arm is split longitudinally. Thus the chromosome is formed of four chromatids, each of which consists of a continuous chromatin thread extending the length of the two adjacent arms and forming one half of each (Figures 19^b, 20). Another common shape assumed by the tetrad is illustrated by the double-V figures (Figure 20), similar to those observed in other animals by Paulmier ('98), McClung (:00, :02), Sutton (:02), Blackman (:01, :03, :05, :05^a), Foot and Strobell (:05), and others. These are obtained by a longitudinal and transverse division of the V-shaped segments, no rotation of the chromatids occurring. A third form of the tetrad is shown by chromosomes in Figure 22. Here the straight segment undergoes the two cleavages, as in the more common type. It is not converted into the typical cross-shaped figure, however, but remains as a straight band of chromatin consisting of four parts — the chromatids.

In later stages of the prophase the changes in the tetrads consist principally in a contraction of the arms and a condensation of the chromatin. Thus, in a fairly late prophase (Figure 22) the more common form of the chromosomes is still that of a cross, but the arms of the cross are much shorter and thicker, and in the more advanced tetrads the condensation of the chromatin granules has proceeded so far as to mask the planes of cleavage, these being seen only in the most favorable cases. In still later stages the chromatin becomes more condensed, and the tetrad goes to the equatorial plate as an apparently homogeneous body typically of a four-lobed shape.

Some of the chromosomes in the process of condensation pass through stages similar to those described by Medes (:05) in Scutigera. The chromatin, instead of massing together into one body, becomes condensed into a number of homogeneous globules, which lie embedded in a more lightly staining substance (Figure 22). These globules later fuse in such a manner as to form a metaphase chromosome of typical appearance. This process will be followed more in detail in the other species of Lithobius, where it is the common method.

The nuclear changes during the prophase in Lithobius mordax and Lithobius multidentatus will be treated at one time, as they are essentially similar in the two forms. In both of these forms all of the chromatin of the nucleus is massed in a single karyosphere, which also contains a considerable portion of nucleolar material. In addition to this there are plentiful deposits of achromatic material in the nucleus, it being more abundant in Lithobius mordax than in the eastern species. This material, although purely achromatic in nature, plays a part in the evolution of the chromosomes.

Just as in Lithobius sp.?, the first nuclear change in the prophase of the first spermatocyte concerns the chromatin in the karyosphere. This leaves the dense masses and collects near the periphery either in the form of coarse granular flakes or loose aggregations of small globules. At the same time the achromatic deposits in the nucleus break up into a number of diffuse, lightly staining masses (Figure 13), which come into relation with the chromosomes as they arise from the karyosphere and become a part of them.

The chromatin arises from the karyosphere (Figure 23) in the form of small globules, which, as they leave the karyosphere, become embedded in the diffuse masses of the achromatic substance. These masses of achromatin lengthen out into short segments, and in each of these the chromatin becomes arranged in the form of distinct darkly stained globules (Figure 23). Thus each of these composite masses at this time consists of a diffuse, lightly stained band of achromatic material, in which is embedded a single row of intensely black globules of chromatin. These globules at this stage are always of about the same size, and are always spherical, presenting the appearance of droplets of liquid chromatin. It is not always easy to observe the typical arrangement of the chromatin globules, as the chromosome is often so bent, or in such a manner distorted, as to obscure their linear arrangement.

The changes immediately following the origin of the chromosomes are similar in results to those in other animals. Each globule is divided into two equal parts, so that the chromosome, instead of being composed of a single row of chromatin particles, now consists of two such rows (Figure 23), each of which contains the same number of globules. This undoubtedly represents a longitudinal division of the chromosome, as the linear arrangement of the globules doubtless has the same significance as the similar arrangement of the granules in the ordinary chromatin segment of the prophase. Following the longitudinal splitting, the transverse cleavage is accomplished in the same manner as in Lithobius sp.? by a division of each row of globules at its middle. The four chromatids thus produced may rotate upon each other in such a way as to result in the typical cruciform tetrad (Figure 23), or may remain a double row of granules with a gap between the two pairs nearest the middle wider than that between any other two pairs. In Figure 23 two tetrads showing the typical cross-shaped form are shown; these are redrawn at the side of the figure with the achromatic portions omitted and the globules comprising each chromatid connected by lines, in order to show diagrammatically the author's interpretation of their structure.

Owing to the distortions of the arms of the tetrad, it is often difficult to correlate the chromosomes at this stage with the type forms described. In later stages this difficulty is rendered still greater by the changes which next take place. These have to do with the condensation of the chromatin by which the tetrads of this stage are converted into the homogeneous chromosomes of the metaphase. The condensation is accomplished by the fusing together of the chromatin globules composing each chromatid of the tetrad. It eventually results in the production of a tetrad made up of four large, deeply stained chromatin globules, each one of which represents a chromatid (Figure 25).

Tetrads when they have reached this stage are very readily understood and correlated with those of an earlier period. The intervening stages (Figures 24, 25), however, proved rather puzzling. These are much more numerous, and were observed before either the foregoing or succeeding ones were found. After all stages were studied, however, the interpretation was simple. All of the chromatin globules belonging to a chromatid do not fuse at once, but this is a process involving considerable time. Thus the chromosomes may contain a variable number of globules, and these often are of very different sizes (Plate 1, Figures 24, 25 ; Plate 2, Figure 26). In other cases chromosomes were found containing from six to ten spherules of approximately equal size (Figures 24, 25, 27). The latter fact at first inclined me to the belief that the chromosomes here were multiple chromosomes similar to those found by Sinéty (:01) and McClung (:05) in insects, where some chromosomes possess as many as ten chromatids. However, a study of the chromosomes in the late prophase and in the metaphase (Plate 2, Figures 28, 29, 30) rendered this plausible view untenable. The fusing of the chromatic globules continues until all belonging to one chromatid have united to form one large spherule of apparently pure chromatin.

The structure of the chromosome when it has reached this stage is shown by several examples in Figures 25 and 26. The four chromatids, of about equal size and arranged symmetrically, are enclosed in an irregular homogeneous or granular mass of achromatic material. In the later prophase (Figures 26, 27) this achromatic mantle contracts and becomes more dense, serving to bind the four parts of the chromosome together. Thus the chromosomes at the time of the breaking down of the nuclear membrane are four-lobed structures, which in Lithobius mordax are apparently homogeneous in consistency.

In Lithobius multidentatus the formation of the tetrad is similar to that in the western species. Apparently the only difference in the chromosomes of the two species is in the smaller size of those in L. multidentatus (compare Figures 26 and 27 with Figure 28) and in the relatively larger amount of achromatic material contained in the fully formed chromosomes. In Figures 26 and 27 are seen various late stages in the evolution of the tetrads. The younger stages here shown are practically identical with those of the other species. The later stages, however, are here much clearer. The larger quantity of achromatic material causes the component parts to be more loosely bound together, and the individual chromatids may be readily distinguished. It is possible even to recognize the planes of the cleavage, although it cannot be said with certainty which is longitudinal and which is transverse.

When all of the chromosomes have left the karyosphere, the body remaining is purely nucleolar in nature and stains lightly (Plate **1**, Figures 24, 25). It is considerably smaller than the karyosphere, and in succeeding stages of the prophase gradually disintegrates. Usually it breaks up into a number of irregular granular bodies, which may persist even to the stage of the metaphase or anaphase (Figures 30, 32). In such cells these particles are usually grouped irregularly in the cytoplasm near the poles of the nuclear spindle, as has been observed in both Lithobius by P. Bouin (**:01**), by Meves und von Korff (**:01**), and in Scutigera by Medes (**:05**). In other cells, however, the nucleolus seems to disappear entirely, there being no residue at the time of the formation of the spindle (Figure 29).

Often the nucleus, instead of breaking up into several granular masses, becomes dissolved in the nuclear sap while retaining its spherical shape. In such cases its circumference often remains about the same, and sometimes even increases. An example of this is shown in Plate **1**, Figure 23, the increase in circumference being due to the presence of a large vacuole, which represents nucleolar material converted into a liquid. The contents of the vacuole remain entirely unstained. In later stages more and more of the nucleolus undergoes this change, until eventually none remains.

The changes in the achromatic structures in chilopods, and the peculiarly modified spindle occurring in these forms, have been the subject of several papers. Meves und von Korff (**:01**) confined their observations to the first spermatocyte division. The centrosomes during the early prophase are not upon the nuclear membrane, but lie at a considerable distance from it. In later stages of the prophase they diverge and pass through the cytoplasm to the cell membrane, where they take up a position at opposite sides of the nucleus. Later, the nuclear spindle arises, but the mantle fibres have no connection with the centrosomes, each fibre ending free in the ctyoplasm. The astral fibres radiating from the centrosomes have no connection with the spindle.

The observations of P. Bouin (:**00**, :**01**) upon the same species are practically identical with the foregoing, and those of Bouin et Collin (:**01**) extend these observations to Geophilus linearis. These authors (p. 106) reach the following conclusions regarding the modified spindle: "De cette étude sur le fuseau chez le Geophilus linearis nous nous croyons autorisés à conclure qu'il existe également chez ce Myriapoda une indépendence complète entre le fuseau d'une part, les sphères et les corpuscles polaires, d'autre part, indépendence evidente au stade de la métacinèse, plus difficilement observable, mais très vraisemblable cependant dès prophases de la mitose."

In Scutigera forceps (Medes, :**05**) much the same condition exists in the fully formed spindle, although earlier stages in its formation present some important differences, and at this time more closely resemble corresponding stages in the small spermatocyte of Scolopendra (Blackman, :**01**, :**05**). The centrosomes at first lie at the poles of the spindle. Later, however, they leave this position and migrate to the cell membrane, retaining no apparent connection with the nuclear spindle. During the metaphase the author (Medes, p. 168) states: "The mutual independence of the spindle-fibres and the astral rays in Scutigera is the more clearly shown by the great dissimilarity in their structure and general appearance; for while the latter are very fine and delicate, . . . the former are heavy, stain an intense black, and present the appearance of wires, the ends of which may be distinctly seen lying free in the cytoplasm."

In the present study of the achromatic structures the cells of Lithobius mordax have been used almost entirely, although observations showing that the changes are common to the genus have been made on the other two species. It will be seen that in general my observations on the achromatic structures are largely confirmatory of those on the European species of Lithobius made by Bouin (:**00**, :**01**) and by Meves und von Korff (:**01**).

In the prophase of Lithobius mordax the centrosomes were first seen in cells in which the nucleus is about in the stage represented in Plate **1**, Figure 23. At this stage there are two, one lying at each side of the nucleus and about midway between this structure and the cell membrane. Each centrosome has already begun to divide to produce the centrosomes of the second spermatocyte mitosis, and is now a small, darkly stained, dumb-bell shaped body. It lies in a small, clear area of the cytoplasm, and is surrounded by astral radiations which, though very fine and delicate, are still distinct from the cytoplasmic reticulum.

In later stages the centrosomes move through the cytoplasm toward the cell membrane, and finally take up their position on the inner face of this structure. In this migration they do not move in opposite directions, but proceed toward the side of the cell nearest to a line connecting the two centrosomes. Thus when they reach the cell membrane they usually both lie upon one side of the cell, which as yet still preserves its rectangular outline (Figures 24, 25). During this migration the nucleus also undergoes a movement in a similar direction, seemingly striving to retain its position in a line between the two centrosomes. Thus at a time when the centrosomes have reached the cell membrane, the nucleus lies very near to the same side of the cell, as is shown in Figure 25. When the centrosomes come in contact with the membrane, they immediately begin to move apart along its inner face until they have reached opposite poles of the cell, the nucleus meantime changing its position so as to lie between the two (Figure 24).

Meanwhile the centrosomes have changed considerably in appearance. By the time they have reached the cell membrane each is enclosed in quite a distinct centrosphere. This is homogeneous or finely granular, and stains slightly darker than the cytoplasm (Figures 24, 25). The astral rays have increased both in size and number, and now penetrate to nearly all regions of the cytoplasm. Those from the two poles may be distinctly seen to cross one another near the centre of the cell (Figures 24, 25), although this crossing is not so apparent as it is later (Figure 28). These fibres are entirely distinct from the cellular network, as the cytoplasmic reticulum is still evident except in the regions immediately surrounding the poles. In these regions, however, the network seems to have entirely broken down, leaving only the hyaloplasm and the penetrating astral rays. Thus each centrosphere is surrounded by a transparent area of considerable width (Figures 24, 25), which serves as an excellent background for the centrosome and centrosphere. Appearances similar to this have been observed in other myriapods by Meves und von Korff (:**01**), P. Bouin (:**00**, :**01**), and Medes (:**05**).

During the time of the divergence of the centrosomes upon the cell membrane the cell has altered considerably in shape. In the vesicle stage and early prophase the cells are typically rectangular in outline. Now, however, they undergo a change by which their form becomes approximately spherical. This I believe to be directly due to the appearance and development of the archoplasm (Blackman, :**05**), for the rounding off of the cell is only perfected when the astral systems have reached the height of their development (Plate **2**, Figures 28–30).

The achromatic changes thus far described have been entirely independent of the nucleus, but during this same period the nucleus is also active. Aside from the evolution of the chromosomes and the disintegration of the nucleolus, which have already been described, the linin reticulum has also changed. Before the origin of the tetrads, a considerable portion of the nuclear area is occupied by the diffuse masses of achromatic material, which is so abundant as to obscure the greater part of the nuclear framework (Figures, 9, 10). After the chromosomes have arisen from the karyosphere, however, this diffuse material is collected into smaller and denser masses, which closely surround the chromosomes. Thus the linin network is rendered visible and is seen to consist of a coarse, irregular reticulum (Figures 23–25). At first this is granular and stains very diffusely. As the tetrads condense to form the homogeneous chromosomes of the metaphase, the network becomes coarser and more irregular. At the same time the linin fibres begin to lose their granular appearance and to take on the form of more definite threads, which retain the stains much more strongly. Thus the nucleus in the late prophase presents the appearance shown in Figures 26 and 27 (Plate 2). The linin network is much more irregular than in preceding stages, and at many places is made up of distinct threads which have lost their granular structure. In later stages they become more and more numerous and definite, and the network becomes more and more broken up, although as yet the fibres have no definite arrangement with reference to the poles of the cells.

The breaking down of the nuclear membrane and the formation of the central spindle is a process involving considerable time, as is shown by the numerous cells found in these stages. The nuclear membrane disappears rather early, often before all of the chromosomes have attained their homogeneous structure. The distinction between nuclear and cytoplasmic material is not so early lost, however, and may indeed be seen until the following anaphase. In the late prophase the line of demarcation between nuclear material and cytoplasm becomes very irregular, and shortly before the completion of the spindle presents the appearance shown in Figure 28. The distinction between nuclear sap and cell sap is shown by the much more transparent appearance of the former. The linin at this time (Figure 28) is all in the form of definite threads, and these are being oriented so as to point toward the poles.

In the later prophase this orientation of the linin fibres continues until it eventually results in the formation of such a spindle as is represented in Figures 29, 30. The central spindle is derived from the nucleus, as is shown by numerous stages in its formation. The chromosomes lie near the centre and are arranged in nearly one plane, forming an equatorial plate. Extending from the chromosomes toward the poles are the linin fibres, which have now become mantle fibres. These at first (Figure 29) are comparatively short, but later they lengthen (Figure 30) and extend about one half of the distance from chromosome to centrosome. They are much thicker than the astral rays and stain much darker. The central spindle, or nuclear spindle, is entirely distinct from the astral systems and lies in a clear area devoid of reticulum, which represents the nuclear sap that has not yet become mixed with the cytoplasm. The entire spindle is barrel-shaped, the fibres converging slightly, but not coming to a point. At this time, and at all later stages of mitosis, the fibres if continued through the cytoplasm would converge at the centrosomes, thus showing that these small bodies, while not directly connected with the nuclear spindle, still are the directing forces which determine its orientation and toward which its fibres converge.

During the late prophase and metaphase the astral systems continue to develop, the rays becoming longer and more numerous. In the equatorial region they may often be seen to cross, although this crossing of the rays is not so pronounced as it is later during the anaphase (Figure 32), at which time the archoplasmic structures reach their highest development. In the metaphase (Figure 30) the centrosome has often already divided to form the centres of the second division, and this is always completed by the anaphase. The two centrosomes are surrounded by the centrosphere just as in the prophase. The lighter area of cytoplasm surrounding the centrosomes, however, is usually much reduced by the time the spindle is completed (Figure 29), and usually disappears entirely in the early anaphase. The cell membrane in the region of the centrosomes is always depressed during metakinesis and cytoplasmic cleavage (Figures 29–34). This depression first becomes apparent in the late prophase at the time the outline of the cell is becoming rounded. This phenomenon probably has the same significance as that ascribed to it in Scolopendra (Blackman, **:05**, p. 55).

With the formation of the nuclear spindle the chromosomes, which heretofore have been scattered throughout the nuclei, become lined up near the middle of the spindle to form a fairly definite equatorial plate (Figures 29, 30). While they do not all lie in exactly the same plane, they are still arranged much more regularly than in Scolopendra. We have seen that the characteristic form of the chromosomes in the late prophase is that of a four-lobed structure, each lobe representing a chromatid. The chromosomes in the equatorial plate are likewise typically of this form, yet there are individual variations both in the form of the tetrads and in the attachment of the mantle fibres which are interesting and instructive. A peculiarity seen in these chromosomes which I have not seen reported by any other observer is shown in Figures 29–31. The chromosomes here represented were studied in preparations in which the extraction of the haematoxylin had been carried farther than usual. This caused the greater part of the chromosome to stain not a dense black, but a dark gray. At the ends of each tetrad, at the point where the mantle fibres attach, there is a small, distinct body which stains much darker than the rest of the chromosome, and is of about the size of the centrosome (Figure 31). This spherule is embedded in the chromosome and is continuous with the mantle fibre and appears to be an enlargement or knob upon the end of this thread. Whether this body has any other significance than that of serving as an attachment to the chromosome I am unable to say, but it is a very distinct structure and is always found in the metaphase chromosome. The position of this body is very useful in determining the exact point of attachment of the mantle fibres and as an aid in determining whether the division of the chromosome is transverse or longitudinal. Usually the mantle fibres are attached at opposite ends of the four-lobed chromosome, and in such cases the fact that the division is longitudinal can be determined only by the shape assumed by the chromosome during the gliding apart of the two component dyads (McClung, :00 ; Blackman, :03, :05). Quite often, however, the mantle fibres attach to two of the adjacent lobules or chromatids of the tetrad (Figure 31). In such cases the chromosome lies with its longer axis parallel to the equatorial plate and division occurs by the separation of the chromosome along its longer axis. Such chromosomes are seen in all equatorial plates studied and are represented in Figures 29–31.

Thus the shape of the chromosomes at this time strongly indicates that the division of the first spermatocyte is a longitudinal one, but the best evidence of this is seen in the early prophase and during the anaphase of the first spermatocyte division. We have seen that in the prophase the first change in the chromatin segment consisted in the longitudinal cleavage. It is but natural, then, to believe that this longitudinal division is the one first completed in the two mitoses following. In the anaphase of the first spermatocyte, strong evidence as to the character of the chromosomic division just completed is furnished by the shape of the daughter chromosomes during their passage toward the poles. In Figure 32 is shown a mid-anaphase of the first spermatocyte. The chromosomes are already approaching their final position, but as yet are not massed together as they are at a slightly later stage (Figure 33). The chromosomes are of a distinctly dumb-bell shape, the constriction at the middle of each one representing the line of transverse cleavage at which point the chromatids separate during the following metaphase. During the succeeding stages (Figures 33, 34) and in the prophase of the following mitosis (Figure 35) the chromosomes retain this shape. In the metaphase of the second spermatocyte (Figure 36) the dumb-bell shaped elements are arranged with the constriction parallel to the equatorial plate and the separation of the component chromatids occurs at this place. This is the reduction division of the chromosomes, and by it the parts of the bivalent element are separated, doubtless at the same point at which they united during the telophase of the last spermatogonium.

In the meantime the nuclear spindle of the first spermatocyte remains entirely independent of the centrosomes and their system of astral rays. During the division and separation of the chromosomes the mantle fibres elongate considerably and their polar ends approach the centrosomes (Figure 32), but at no time do they abut upon it. Thus the chromosomes are evidently not separated by the contraction of the mantle fibres, but these seem merely to act as runways along which the daughter chromosomes move as they approach the poles. This, however, does not necessarily mean that the centrosomes do not have an important function in the separation of the chromosomes, for the influence of the chromosomes over the nuclear spindle is at all times apparent. The nuclear spindle is oriented parallel to a line connecting the two poles of the cell, and at all stages the mantle fibres converge toward the centrosomes. It seems probable that the separation of the daughter chromosomes is accomplished by an attractive force exerted upon them by the centrosomes rather than that they move apart by a force inherent in themselves. At any rate it cannot be disputed that the centrosomes determine the direction of this migration.

In a former paper (Blackman, :**05**) I have given at some length reasons for believing that the centrosomes and astral rays play an important part in the cytoplasmic cleavage. This work upon Lithobius strengthens the belief there expressed by showing that similar phenomena occur here also. The astral rays are not so conspicuous as in Scolopendra, but in all other respects they are essentially the same, and in Lithobius, just as in Scolopendra, I believe that the constriction of the cell is brought about by a force which is inherent in the centrosome and is exerted upon the cell membrane through the astral rays.

The further changes in the maturation divisions offer but few points of special interest, and will be treated very briefly here. After the separation of the chromosomes in the first spermatocyte division, the mantle fibres extend between the two masses of daughter chromosomes, presenting the peculiar appearance shown in Figure 33. The nuclear substance may still be distinguished from that of the ordinary cytoplasm by its more transparent appearance. With the constriction of the cell, however, the distinction between nuclear-sap and cell-sap is finally lost, although a disc of more transparent material still remains in the plane of constriction until cell division is nearly complete (Figure 34).

By the constriction of the cell, the interzonal filaments are grouped together in a sheaf-like bundle. This is much shorter and broader than in Scolopendra, and persists but a short time. At the plane of constriction each fibre is interrupted by a small enlargement. These bodies at first occur only in the superficial fibres, where, by their fusion, they form a darkly stained ring about the sheaf of spindle remnants (Figure 35). Later, however, as constriction proceeds still farther, the interzonal filaments entirely disappear, while the ring surrounding them at a former stage has been converted into a darkly stained plate. This is continuous with the constricted cell membrane, and closes the opening between the two daughter cells. In sections it appears as a black band, and extends through a considerable depth of focus.

The second spermatocyte mitosis follows the first very closely, there being no distinction between the telophase of the first and the prophase of the second division (Figure 35). The centrosomes do not leave their position upon the cell membrane, but in separating move along this structure until they arrive at opposite sides of the cell. The nuclear membrane then breaks down, and the spindle is formed. This spindle, just as in the first spermatocyte division, is truncate at the poles and remains entirely distinct from the astral systems.

The succeeding changes in the second mitosis are so similar to those in the first spermatocyte as to render separate description unnecessary. The dividing cells in the second spermatocyte may, however, always be readily distinguished from those of the first by several characteristics. The cells are much smaller, the spindle is more attenuated, and the chromosomes are of quite a different shape. The changes in the spermatid during its development into the spermatozoön will be the subject of a later paper.

SUMMARY.

Pseudo-reduction is accomplished by an end-to-end union in pairs of the chromosomes during the telophase of the last spermatogonial division. All of the chromosomes take part in this except the accessory chromosomes.

During the early part of the growth period *all of the chromosomes become massed together to form the karyosphere. The chromosomes enter this body as distinct elements, and the individual elements can be recognized for a considerable time.* The karyosphere also contains nucleolar material.

The karyosphere varies in structure with the species. In Lithobius mordax the amount of nucleolar material is less than in the other species studied. In Lithobius multidentatus the chromatin is peripheral in position, and surrounds the nucleolar material. In Lithobius sp. ? the amount of nucleolar substance is greater, and there are always two or more karyospheres. The chromatin in this species is usually central in position, and is surrounded on all sides by nucleolar material.

During the growth period *no special aggregations of archoplasm are present, but this substance is dispersed throughout the cytoplasm of the cell.*

In the three species of Lithobius studied *the chromosomes arise from the karyosphere* and develop into tetrads, but the manner of development of the tetrads varies with the species.

In Lithobius sp. ? the stages in the formation of the prophase chromosome are similar in all essential respects to those of the typical arthropod tetrad.

In *Lithobius mordax and Lithobius multidentatus the chromatin arises from the karyosphere in the form of globules which, as they arise, become arranged in a single row for each chromosome, and each chromosome is embedded in a separate mass of achromatic material.* In the formation of the tetrad *each globule divides into two equal halves, giving rise to two parallel rows of globules. This represents the longitudinal or equational division of the chromosomes. The transverse or reduction division is accomplished by a separation of the rows of globules at their middle point, the globules themselves remaining intact. In later stages the globules forming each chromatid fuse into one, thus giving rise to a tetrad composed of four spherules of chromatin.*

After the origin of the chromosomes, the nucleolar portion of the karyosphere remaining gradually breaks down. Portions of it may often be seen at the poles of the nucleolar spindle during the metaphase and anaphase of the following division.

In the early prophase the *centrosomes separate and move through the cytoplasm to the cell membrane.* Along the inner face of the membrane they migrate apart until they are upon opposite sides of the cell. During the migration the cell outline becomes rounded off, and by the time of the metaphase the cell is spherical in shape.

When the centrosomes are at opposite sides of the cell the nuclear membrane breaks down, and its component parts become rearranged to form a spindle. *The linin network is transformed directly into the mantle fibres.*

The nuclear spindle remains entirely distinct from the centrosomes and astral systems. Its fibres do not converge at a special point at each pole, but end free in the cytoplasm, the poles of the spindle thus being truncate. *Each mantle fibre, however, points directly toward the centrosome.*

The distinction between nuclear substance and cytoplasm may be readily seen up to the late anaphase of the first division.

The second division follows very closely on the first. The centrosomes separate and move apart upon the cell membrane, forming a spindle similar to that of the first spermatocyte.

The first maturation mitosis results in a longitudinal or equation division of the chromosomes, while the second is the reduction division. Evidences of this are seen in the sequence of cleavages in the prophase, in the attachment of the mantle fibres during the metaphase of the first mitosis, in the shape of the daughter chromosomes during the anaphase and telophase of the first division, and in the shape of the dyads and their position in the equatorial plate of the last division.

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Bibliography.

Blackman, M. W.

:01. The Spermatogenesis of the Myriapods. I. Notes on the Spermatocytes and Spermatids of Scolopendra. Kansas Univ. Quart., Ser. A., Vol. 10, No. 2, pp. 61–76, pl. 5–7.

Blackman, M. W.

:03. The Spermatogenesis of the Myriapods. II. On the Chromatin in the Spermatocytes of Scolopendra heros. Biol. Bull., Vol. 5, pp. 187–217, 22 fig.

Blackman, M. W.

:05. The Spermatogenesis of the Myriapods. III. The Spermatogenesis of Scolopendra heros. Bull. Mus. Comp. Zoöl. Harvard Col., Vol. 48, No. 1, pp. 1–138, 9 pl. and 9 fig.

Blackman, M. W.

:05^a. The Spermatogenesis of the Myriapods. IV. On the Karyosphere and Nucleolus in the Spermatocytes of Scolopendra subspinipes. Proc. Amer. Acad. Arts and Sci., Vol. 41, No. 13, pp. 331–344, 1 pl.

Bouin, P.
:00. Mitoses spermatogénétiques chez Lithobius forficatus, L. Étude sur les variations du processus mitosique. XIII^e. Congrés Internat. de Med., Paris, Section d'Histol. et d'Embryol., pp. 46–51.

Bouin, P.
:01. Sur le fuseau, le résidu fusorial et le corpuscule intermédiaire dans les cellules séminales de Lithobius forficatus. C. R. Assoc. Anat., Sess. 3, pp. 225–233, 6 fig.

Bouin, P.
:03. Centrosome et Centriole. C. R. Soc. Biol., Paris, Tom. 55, No. 17, pp. 647–649.

Bouin, P.
:03ª. Sur l'existence d'une double spermatogénèse et de deux sortes de spermatozoids chez Scolopendra morsitans. Arch. zool. expér., Sér. 4, Tom. 1, pp. 3–6.

Bouin, P.
:04. Recherches sur la figure achromatique de la cytodiérèse et sur le centrosome. Arch. zool. expér., Sér. 4, Tom. 2, Notes et Revue, pp. lxxiii–lxxxviii, 6 fig.

Bouin, P., et Bouin, M.
'99. Sur la présence et l'évolution des formations ergastoplasmiques dans les cellules séminales de Lithobius forficatus. Bibliogr. Anat., Nancy, Tom. 7, pp. 141–150.

Bouin, P., et Bouin, M.
:02. Réduction chromatique chez les Myriapodes. C. R. Assoc. Anat., Sess. 4, pp. 74–78.

Bouin, P., et Collin, R.
:01. Contribution à l'étude de la division cellulaires chez les Myriapodes. Mitoses spermatogénétiques chez le Geophilus linearis (Koch). Anat. Anz., Bd. 20, pp. 97–115, 11 fig.

Carnoy, J. B.
'85. La Cytodiérèse chez les arthropodes. La Cellule, Tom. 1, pp. 189–440, 8 pl.

Foot, K., and Strobell, E. C.
:05. Prophases and Metaphase of the First Maturation Spindle of Allolobophora foetida. Amer. Jour. Anat., Vol. 4, No. 2, pp. 199–243, 9 pl.

Grégoire, V.
:05. Les résultats acquis sur les cinèses de maturation dans les deux règnes. Mém. I, Revue critique de la littérature. La Cellule, Tom. 22, pp. 221–376, 147 fig.

McClung, C. E.
:00. The Spermatocyte Divisions of the Acrididae. Kansas Univ. Quart., Ser. A, Vol. 9, No. 1, pp. 73–100, pl. 15–17.

McClung, C. E.

:02. The Spermatocyte Divisions of the Locustidae. Kansas Univ. Sci. Bull., Vol. 1, pp. 185–231, pl. 7–10.

McClung, C. E.

:05. The Chromosome Complex of Orthopteran Spermatocytes. Biol. Bull., Vol. 9, No. 5, pp. 304–340, 21 fig.

Medes, Grace.

:05. The Spermatogenesis of Scutigera forceps. Biol. Bull., Vol. 9, No. 3, pp. 156–187, pl. 2–6.

Meves, F., und von Korff, K.

:01. Zur Kenntnis der Zelltheilung bei Myriapoden. Arch. f. mikr. Anat., Bd. 57, pp. 481–486, Taf. 21, 6 fig.

Montgomery, T. H., Jr.

:00. The Spermatogenesis of Peripatus balfouri up to the Formation of the Spermatid. Zool. Jahrb., Abth. f. Morph., Bd. 14, pp. 277–368, Taf. 19–25.

Montgomery, T. H., Jr.

:01. A Study of the Germ Cells of Metazoa. Trans. Amer. Phil. Soc., Phila., Vol. 20, pp. 154–236, pl. 4–8.

Montgomery, T. H., Jr.

:05. The Spermatogenesis of Syrbula and Lycosa, with General Considerations upon Chromosome Reduction and the Heterochromosomes. Proc. Acad. Nat. Sci. Phila., Vol. 57, pp. 162–205, pls. 9, 10.

Paulmier, F. C.

'99. The Spermatogenesis of Anasa tristis. Jour. Morph., Vol. 15, Suppl., pp. 223–272, pl. 13, 14.

Prenant, A.

'87. Observations cytologiques sur les éléments séminaux de la Scolopendre et de la Lithobie. La Cellule, Tom. 3, pp. 415–436, 2 pl.

Prenant, A.

'92. L'origine du fuseau achromatique nucléaire dans les cellules séminales de la Scolopendre. C. R. Soc. Biol., Paris, Tom. 44 (Sér. 9, Tom. 4), pp. 249–253.

Sutton, W. S.

:02. On the Morphology of the Chromosome Group in Brachystola magna. Biol. Bull., Vol. 4, pp. 24–39, 11 fig.

Explanation of Plates.

All drawings were made with the aid of a camera lucida. A Leitz $\frac{1}{12}$-inch homogeneous immersion objective and No. 4 ocular were used, producing, at the projection distance employed, a magnification of 1230 diameters. All figures were reduced $\frac{1}{3}$ in reproduction, making a final magnification of 820 diameters.

PLATE 1.

FIGURE 1. Spermatogonium of *Lithobius mordax* in the "resting stage." Each nucleus possesses two karyospheres, which contain all of the chromatin of the cell.

FIGURE 2. Prophase of spermatogonium of *L. mordax*. The chromosomes have arisen from the karyosphere, and are distributed throughout the nucleus in the form of fine granular threads. The nucleoli and the accessory chromosome are all that remain of the karyosphere.

FIGURE 3. Early spermatocyte of *L. mordax*. The chromosomes have undergone pseudo-reduction. The massing of the chromosomes about the accessory chromosome to form the karyosphere has already begun.

FIGURE 4. *Lithobius sp.*? at about the same stage as is shown in Figure 3. Here the nucleoli serve as the basis of the karyosphere. Two nucleoli are seen, one of which is already acquiring chromatin.

FIGURES 5, 6, 7. *First* spermatocytes of *L. mordax* in the early growth period, showing massing of the chromosomes to form the karyosphere. The outlines of many of the chromosomes can still be seen.

FIGURE 8. Later stage of growing *first* spermatocyte of *L. mordax*. The karyosphere has increased in size, is becoming more regular in outline, and stains less deeply, owing to the acquisition of nucleolar material. The nucleus also contains diffuse deposits of achromatic material. The outlines of some of the chromosomes are still to be seen.

FIGURE 9. Nucleus of *first* spermatocyte of *L. mordax* in the vesicle stage, showing the presence of both chromatin and nucleolar substance.

FIGURE 10. The vesicle stage in *L. mordax*, showing characteristic shape of cell, dense structures of the cytoplasm, and characteristic appearance of the nucleus. The karyosphere appears entirely black, and is surrounded by a mantle of non-chromatic material.

FIGURES 11, 12. Nuclei of *first* spermatocytes of *L. multidentatus* in the vesicle stage. The amount of chromatin is here less than in L. mordax. It is spread out over the surface of the nucleolus, forming a karyosphere similar to that of Scolopendra subspinipes. The achromatic deposits are confined to several dense masses.

FIGURE 13. Very early prophase of *first* spermatocyte of *L. multidentatus*. The chromatin in the karyosphere is becoming more loosely arranged. The deposits of achromatic material have broken up into a number of smaller masses.

FIGURES 14–17. Nuclei of spermatocytes of *Lithobius sp.*? in the vesicle stage. Two or more karyospheres are usually present, each containing considerable nucleolar material.

FIGURE 18. Nucleus of *first* spermatocyte of *Lithobius sp.?* in early prophase. Most of the chromosomes have left the karyosphere, and are in the form of granular segments.

FIGURES 19^a, 19^b, 19^c, 20, 21. Later prophase of *first* spermatocyte in *Lithobius sp.?*, showing various stages in formation of the tetrads and in disintegration of the nucleoli. Figures 19^a, 19^b, and 19^c represent successive sections of the nucleus of one cell.

FIGURE 22. Mid-prophase of *first* spermatocyte in *Lithobius sp.?* The tetrads have contracted to form denser bodies of chromatin.

FIGURE 23. Nucleus of *first* spermatocyte, showing early stages in the formation of the tetrads in *L. mordax*. Some of the chromosomes have undergone longitudinal splitting, resulting in double rows of globules. Others are already taking on the form of characteristic tetrads. One chromosome is just arising from the karyosphere. The nucleolar material is already becoming vacuolated.

FIGURES 24, 25. Mid-prophases of the *first* spermatocyte of *L. mordax*. The centrosomes are moving apart along the inner face of the cell membrane. The fusion of the globules forming each chromatid of the tetrad has begun. In two chromosomes in Figure 25 this fusion is complete, there being only four parts to the tetrad. The nucleolus is much reduced in size.

PLATE 2.

FIGURES 26, 27. Late prophase of first spermatocyte of *L. multidentatus*. Many of the tetrads have already become homogeneous. However, the dividing planes are still very evident. The linin fibres are being converted into definite threads.

FIGURE 28. Late prophase of *L. mordax*. The nuclear membrane has disappeared, and the linin fibres are becoming oriented to form the mantle fibres of the spindle.

FIGURES 29, 30. Division figures of the *first* spermatocyte of *L. mordax*, showing the complete independence of nuclear spindle and astral systems. The fibres of the spindle, however, converge toward the centrosomes. The distinction between nuclear sap and cytoplasm is very distinct. The chromosomes are arranged in an equatorial plate. At the point of attachment of the mantle fibres to the chromosome, a small, darkly stained body is seen.

FIGURE 31. Chromosomes seen in a single equatorial plate of *L. mordax* showing the variety of shapes and sizes and the manner of attachment of the mantle fibres.

FIGURE 32. Mid-anaphase of *first* spermatocyte of *L. mordax*. The daughter chromosomes are of a distinct dumb-bell shape, the constriction representing the plane of the second division.

FIGURES 33, 34. Later stages in division of *first* spermatocyte of *L. mordax*. Where chromosomes can still be distinguished, their form is unchanged.

FIGURE 35. Late telophase of *first*, or early prophase of *second*, spermatocyte of *L. mordax*. Chromosomes still dumb-bell shaped. Centrosomes are moving apart along inner surface of cell membranes.

FIGURE 36. Metaphase of *second* spermatocyte of *L. mordax*. Achromatic structures similar to corresponding stage of first division. The dumb-bell shaped chromosomes are arranged with the constriction in the plane of the equatorial plate.

FIGURE 37. Telophase of *second* spermatocyte of *L. multidentatus*.

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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.

NEW SERIES. — NO. XXXIII.

REVISION OF THE GENUS SPILANTHES.

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Presented by B. L. Robinson, October 10, 1906. Received November 10, 1906.

INTRODUCTION.

WHEN this revision was undertaken, the genus Spilanthes was found to be in a very confused condition, and the work was greatly hampered by lack of abundant and authentic material; a lack, however, which it was in a large measure possible to overcome through the great generosity of numerous botanists in different parts of the world. But for the purpose of further study and for the more intelligent determination of ranges a great deal of material of the genus should be collected. The genus has been so little understood that it seemed more than usually desirable to base information regarding the ranges merely on actual specimens or reproductions examined rather than on published accounts. For habitat the labels give almost no data, and except in a few cases there were not specimens enough to form satisfactory conclusions as to the dates of flowering and fruiting, so that no mention of these subjects has been made. It is hoped, however, that such attention in the field will be accorded this interesting genus that the knowledge of all these matters may be supplied.

The species have proved to be almost entirely lacking in constant technical characters, so that it has seemed artificial to maintain many of the earlier ones in an independent rank. The genus thus contains several complex groups of interrelated types, the study of a considerable amount of material of which shows them to intergrade very strongly with each other. It is certainly very undesirable to attempt to distinguish species upon the basis of mere leaf and habital variations. Therefore in treating the genus it has been the plan to make the species stand for something definite, at least, so that they may represent an aggregation of variations more like each other than like any other variations, and to relegate to the rank of varieties and forms, under species into which they converge, plants of less numerous or significant characters.

The nomenclature of the revision is strictly in accordance with the Vienna Rules.

To Dr. R. Chodat, of Geneva, Drs. C. F. Millspaugh and J. M. Greenman, of the Field Museum of Natural History, Dr. J. N. Rose, of the United States National Museum, Capt. J. D. Smith, of Baltimore, and Prof. William Trelease, of the Missouri Botanical Garden and Henry Shaw School of Botany, I desire to express my gratitude for their kind loan of valuable specimens. To Dr. N. L. Britton, of the New York Botanical Garden, and his associates, I owe special thanks for the courteous reception afforded me, and for permission to consult the fine herbaria of the Garden and of Columbia University. For photographs of type specimens important to the work, I am indebted to B. Daydon Jackson, Esq., Secretary of the Linnean Society, and Prof. F. O. Bower, of the University of Glasgow. Dr. Casimir de Candolle, of Geneva, Prof. A. Engler, of the Royal Botanical Garden in Berlin, Lieut.-Col. David Prain, F.L.S., F.R.S.E., of the Royal Gardens at Kew, and Dr. A. Zahlbruckner, of the Imperial Museum of Natural History in Vienna, were so kind as to contribute excellent tracings, drawings, and photographs. Dr. Otto Kuntze, of San Remo, Italy, and Alfred J. Swart, Esq., of the Victorian Department of Agriculture, Victoria, Australia, kindly contributed valuable authentic specimens. I wish to thank Dr. H. Pittier, of the Bureau of Plant Industry, Department of Agriculture, and Dr. H. H. Rusby, of the New York College of Pharmacy, for useful information in regard to geographical matters. I wish to express my special thanks also to the librarian of the Gray Herbarium, Miss Mary A. Day, for her untiring efforts in connection with the bibliographical work and for much other valuable assistance and advice. To Miss I. W. Anderson, of the Gray Herbarium, I am greatly indebted for assistance in preparing the material for study. Prof. M. L. Fernald and Mr. H. H. Bartlett, of the Gray Herbarium, have also given many useful suggestions, for which I wish to thank them very much. But above all I am indebted to the Curator, Dr. B. L. Robinson, for his constant advice and assistance throughout the work.

REVISIO GENERIS SPILANTHIS.

SPILANTHES Jacq. Plantae inter erectas et prostratas variantes herbaceae 6–15 dm. altae. Folia opposita inter sessilia et valde petiolata variantia. Capitula discoidea vel radiata (sed radiis saepius parvis)

inter subglobosa et oblonga variantia (regulariter conica) plerumque parva pedunculata. Involucri squamae 6–mult. obtusae vel acutae ovatae vel lanceolatae. Radii lutei aut albi vel nulli. Achaenia valde compressa raro vix triangulata nunquam columnaria inaequaliter biaristata aut inaristata (aristis rigidis aut saepius ciliis persimilibus) margine ciliata vel glabra raro superficiebus ciliata nonnunquam margine crassa sed plerumque (praeterquam in no. 1) inconstanter. — (Derivatio: σπίλος, macula + -ἀνθής, florens). — Enum. Syst. Pl. Carib. 8 (1762).

Genera praecedenti magis affinia (secundum Engl. et Prantl Natürlichen Pfl. iv, 5, 229, in clave) sunt Lipochaeta *DC.*, Flourensia *DC.*, Salmea *DC.*, Salmeopsis *Benth.* Lipochaeta distinguitur achaeniis non valde compressis, plerumque multis capitulis minimis subglobosis et saepe foliis non simplicibus; Flourensia capitulis valde aromaticis et achaeniis longis pilis tectis, foliis alternis; Salmea Salmeopsisque inflorescentia cymose disposita distinguuntur; praeter Lipochaetam, omnes habitu suffruticoso.

Aspectus Isocarphae *R. Br.* generi nostro simillima est at illius achaenia columnaria et 4–6-angulata sunt.

Generis nostri locus systematicus. — Tribus Heliantheae (*DC.*) *Cass.* incl. *Spilantheae* Cass. Dict. Sc. Nat. v, 419 (1825). Subtribus Verbesininae *O. Hoffm.* *Verbesineae* (Cass.) Lindl.

Synonymia. — Praelinnaeana:

Ceratocephalus Burm. Thes. Zeyl. 58 (1737); Ktze. Rev. Gen. Pl. i, 326 (1891).

ABCDaria Rumpf. Amb. Kruidb. (Herb. Amb. Burm. tr. latina) vi, 145, t. 65 (1750).

Postlinnaeana:

Spilanthus L. Syst. Nat. ed. XII, ii, 533 (1767).

Pyrethrum Med. Act. Acad. vel Hist. et Comment. . . . Theod. - Palat. (Phys.) iii, 237 (1775), in parte.

Spilantus R. W. Darw. Fam. Pl. ed. II, ii, 544 (1787).

Athronia Neck. Elem. Bot. i, 32 (1790).

Ceruchis Gaertn. ex L. - Schreb. Gen. Pl. ed. VIII, ii, 543 (1791).

Acmella Rich. in Pers. Syn. Pl. ii, 472 (1807).

Synonymum exclusum. — *Mendezia* DC. Prod. v, 532 (1836) = Zinnia *L.*

Distributio. — Per regiones tropicas hemisphaeriorum amborum et in Hemisphaerio Occidentali in regionibus Zonae Temperatae calidioribus a Missouri ad Louisianam, Texas, Floridam in Civitatibus Foederatis Americae.

Generis Clavis.

A. Capitula discoidea (praeterquam in no. 12). B. . . . . Sectio I. *Salivaria.*

B. Folia sessilia (nonnunquam in no. 7 minute subpetiolata) inter linearia et lanceolata variantia integra. C.

C. Achaenia margine crassa (margine ca. 1 mm. lata) uniaristata. 1. *S. chamaecaula.*

C. Achaenia margine non crassa inaequaliter biaristata. D.

D. Antherae apice nigrae; folia non spatulata; plantae Mundi Novi. E.

E. Folia conspicue venata; radices fasciculatae. . . . 2. *S. nervosa.*

E. Folia tribus nervis plus minusve prominentibus; radices non fasciculatae. F.

F. Plantae inter glabras et leviter pubescentes variantes. 3. *S. urens.*

F. Plantae laneae. . . . . 4. *S. urens* f. *lanea.*

F. Plantae hispidulae. . . . . 5. *S. urens* var. *hispidula.*

D. Antherae apice nigrae; folia non spatulata; plantae australienses. 6. *S. anactina.*

D. Antherae apice non nigrae; folia linearispatulata; plantae Mundi Novi. . . . . 7. *S. pusilla.*

B. Folia petiolata obovata valde et saepe obtuse sinuatodentata. 8. *S. insipida.*

B. Folia petiolata ovata subintegra vel crenata serrata vel incisa. G.

G. Discus ovoideocolumnaris tempore maturitatis 1–1.7 cm. latus; capitula saepe apice fusco. . . . . 9. *S. oleracea.*

G. Discus multo gracilior; capitula colore uniformi. H.

H Plantae Mundi Novi. I.

I. Achaenia inaristata glabra; involucri bracteae numerosae. 10. *S. leucantha.*

I. Achaenia aristata vel ciliata; involucri bracteae 4–8. K.

K. Folia saepius integra vel subintegra sed nonnunquam serrata (raro conspicue); non multis capitibus brevipedunculatis.

Capitula sine radiis. . . . . 11. *S. ocymifolia.*

Capitula cum radiis. . . . . 12. *S. ocymifolia* f. *radiifera.*

K. Folia serrata (plerumque conspicue); regulariter multis capitibus brevipedunculatis. . . . . . . 13. *S. ocymifolia* var. *acutiserrata.*

H. Plantae Mundi Antiqui. L.

L. Pedunculi 3–10 cm. longi (plerique ca. 6 cm.). M.

M. Achaenia glabra calvaque. . . . . 14. *S. calva.*

M. Achaenia aristata vel ciliata. N.

N. Folia nervis longitudinalibus non conspicuis.

Folia inter subintegra et serrata variantia non incisa.

Folia plerumque 3–6 cm. longa non albescentia. 15. *S. Acmella.*

Folia plerumque maiora fere semper infra albescentia. 16. *S. Acmella* var. *albescentifolia.*

Folia incisa . . . . 17. *S. Acmella* var. *lanceolata.*

N. Folia nervis longitudinalibus prominentibus. 18. *S. costata.*

L. Pedunculi 14–17.2 cm. longi (plerique ca. 15 cm.). 19. *S. callimorpha.*

A. Capitula radiata. O. . . . . Sectio II. *Acmella.*

O. Radii capituli maturi involucrum vix superantes plerumque perminuti et paulo ovati. P. . . . . Subsectio I. *Parvoradiatae.*

P. Involucri squamae inter late ovatas et suborbiculares variantes obtusae vel acutae nunquam acuminatae.

Folia 1.1–2.5 cm. longa subintegra vel integra.

Paleae apice violaceae. . . . . 20. *S. iodiscaea.*

Paleae apice non violaceae. . . . . 21. *S. iodiscaea* f. *leucaena.*

Folia 2.8–5 cm. longa plerumque serrata (raro subintegra). 22. *S. uliginosa.*

P. Involucri squamae inter ovatas et ovatolanceolatas variantes acuminatae. Q.

Q. Plantae Mundi Novi. R.

R. Capitula acute obtuseve conica. S.

S. Folia nec integra nec subintegra. T.

T. Folia non mucronuloide attenuata. U.

U. Plantae non conspicue tomentulosae aut hispidulosae. V.

V. Folia lanceolata; capitula pleraque 6 mm. longa 4 mm. lata. . . . . 23. *S. Lundii.*

V. Folia inter anguste ovata et ovatolanceolata variantia; capitula pleraque 8 mm. longa 6 mm. lata vel minora.

Plantae erectae vel ascendentes; folia inter late ovata et ovatolanceolata variantia.

Folia plerumque integra aut crenata; achaenia fere semper margine valde ciliata. (Vide supra sub no. 12.) 12. *S. ocymifolia* f. *radiifera.*

Folia saepius dentata (nonnunquam irregulariter incisa); achaenia plerumque omnino non vel leviter margine ciliata. . . . . 24. *S. ciliata.*

Plantae prostratae; folia ovata vel subrotunda quam in praecedentibus minora (ca. 2–3 cm. longa). 25. *S. ciliata* var. *diffusa.*

U. Plantae dense tomentulosae vel hispidulosae, maxime surculi recentiores. . . . . 26. *S. subhirsuta.*

T. Folia abrupte mucronuloide attenuata. . . . 27. *S. caespitosa.*

S. Folia integra aut vix undulata.

Plantae et maxime involucri squamae canescentes. 28. *S. poliolepidica.*

Plantae non conspicue canescentes.

Folia linearilanceolata, nervo medio prominente, non ciliolata. 29. *S. disciformis.*

Folia ovatolanceolata, nervis aequaliter prominentibus tribus, margine ciliolata. . . . . 30. *S. limonica.*

R. Capitula truncata fere planoconvexa. . . . . . 31. *S. alpestris.*

Q. Plantae Mundi Antiqui.

Capitula subglobosa vel conica 5–8 mm. longa 5–9 mm. lata. 32. *S. mauritiana.*

Capitula oblonga 1.5 cm. longa 5–6 mm. lata. 33. *S. mauritiana* f. *madagascariensis.*

O. Radii capituli involucrum conspicue superantes plerumque magni atque liguliformes. X. . . . . Subsectio II. *Magnoradiatae.*

X. Folia angustissima 2-2.5 cm. longa 1.5–2 mm. lata. . 34. *S. stenophylla.*

X. Folia non angustissima vel si angustissima multo breviora. Y.
Y. Capitula 3.5–5.5 mm. longa (pleraque 4.7 mm.). . . . 35. *S. filipes.*
Y. Capitula non minus quam 6 mm. longa ac plerumque multo magis. Z.
Z. Radii longitudine conspicue minus quam dimidia pars disci.
36. *S. iabadicensis.*
Z. Radii longitudine non minus quam dimidia pars disci. a.
a. Caules colore rubro tincti crassiores. . . . . 37. *S. phaneractis.*
a. Caules non colore rubro tincti tenuiores. b.
b. Plantae non radicibus rigidis aut crassissimis et nunquam radicum fasciculis reptantes; radii saepius non latissimi. c.
c. Folia integra undulata vel distanter sinuatodentata ovatolanceolata.
Folia longiacuminata . . . . 38. *S. macrophylla.*
Folia non longiacuminata . . . . 39. *S. papposa.*
c. Folia inter integra et valde serrata variantia; si integra tum late linearia vel lanceolata. d.
d. Plantae Insularum Malayensium.
Folia inter ovata et lanceolata variantia 3–7 cm. longa .7–2.5 cm. lata . . . . 40. *S. grandiflora.*
Folia linearia 2.5–5.5 cm. longa .5–.8 cm. lata.
Ligulae involucrum multo superantes.
41. *S. grandiflora* var. *brachyglossa.*
Ligulae involucrum vix aut omnino non superantes.
42. *S. grandiflora* var. *calva.*
d. Plantae Abyssiniae vel Mundi Novi. e.
e. Folia minutissima 7–15 mm. longa (pleraque 8 mm.).
43. *S. pammicrophylla.*
e. Folia non minus quam 2 cm. longa et plerumque multo magis (nri. 46, 47, 48, 49 nonnunquam minus quam 2 cm. longa sunt sed dentata non subintegra ut in no. 43). f.
f. Radii colore albo puro; plantae abyssinicae.
44. *S. abyssinica.*
f. Radii luteo saltem tincti; plantae Mundi Novi. g.
g. Folia inter elliptica et ovatolanceolata variantia. h.
h. Folia saepius crenata quam serrata; plantae Mexico, et Americae Centralis Australisque. i.
i. Plantae erectae aut deorsum repentes sursum ascendentes; folia maiora (6–18 cm. longa).
45. *S. americana.*
i. Plantae magis prostratae plerumque omnino; folia minora (1.5–4.5 cm. longa). k.
k. Discus parvus gracilisque (saepius .5 cm. longus 1 mm. crassus). 46. *S. americana* var. *ramosa.*
k. Discus maior. l.
l. Plantae glabrae aut vix pubescentes.
47. *S. americana* var. *parvula.*
l. Plantae pubescentes.
48. *S. americana* var. *parvula* f. *parvifolia.*
l. Plantae dense lanuginosae.
49. *S. americana* var. *parvula* f. *lanitecta.*
h. Folia saepius serrata quam crenata; plantae Americae Borealis. . . . 50. *S. americana* var. *repens.*

g. Folia linearia. m.

m. Folia non margine ciliata. n.

n. Plantae non longissimis internodiis; folia integra vel distanter subserrata.
51. *S. americana* var. *stolonifera.*

n. Plantae longissimis internodiis; folia distanter serrata.
52. *S. americana* var. *stolonifera* f. *longiinternodiata.*

m. Folia margine ciliata. o.

o. Folia obtusa; cilia minima non distantia.
53. *S. americana* var. *stolonifera* f. *ciliatifolia.*

o. Folia acuta; cilia conspicua distantia.
54. *S. blepharicarpa.*

b. Plantae radicibus rigidis aut crassissimis, saepius radicum fasciculis reptantes; radii saepius latissimi. p.

p. Capitula non latissima; receptacula non crassissima. q.

q. Folia caulina inconspicue vel omnino non serrata.

Folia caulina 5–9 cm. longa lanceolata; folia basalia saepe foliis caulinis valde dissimilia semperque in rosulae forma disposita. . . . . 55. *S. decumbens.*

Folia caulina 2.5–6 cm. longa foliis basalibus non valde dissimilia.

Folia duarum formarum fieri inclinata (aliqua late lanceolata aliqua anguste lanceolata vel linearia) vel omnia late lanceolata.
56. *S. decumbens* var. *macropoda.*

Folia unius formae (linearia).
57. *S. decumbens* var. *leptophylla.*

q. Folia caulina conspicue sed non omnia valde serrata. r.

r. Folia dense hispidulosa sed margines non valde ciliatae; folia basalia caulinis non similia. . . . . 58. *S. grisea.*

r. Folia sparse vel omnino non hispidulosa sed margines valde ciliatae; folia basalia caulinis similia. s.

s. Folia lanceolata; pedunculi longissimi (1–2.8 dm.). t.

t. Folia 3–7 cm. longa 1.7–2 cm. lata.
59. *S. grisea* var. *intermedia.*

t. Folia 7–10 cm. longa 1–2.2 cm. lata.
60. *S. grisea* var. *setosa.*

t. Folia 4.8–8 cm. longa, .5–.8 cm. lata; serrationes quam in praecedentibus prominentiores.
61. *S. grisea* var. *Chodatana.*

s. Folia ovata; pedunculi 5.5–7 cm. longi.
62. *S. grisea* var. *micra.*

p. Capitula latissima (1.1–2 cm.); receptacula crassissima (plerumque 3.5 mm.) . . . . 63. *S. eurycarena.*

Sectio I. SALIVARIA DC. capitulis discoideis. — Prod. v, 624 (1836). Distributio. — In India, Indiis Orientalibus Nederlandicis, Insulis Philippinis, Australia, Mexico, America Centrali Australique, Indiis Occidentalibus.

1. **S. chamaecaula** A. H. Moore spec. nov. glabra vel subglabra; longissimis caulibus diffusis prostratis; foliis late linearibus supra infraque paulum scabris plerumque ca. 3 cm. longis .4–.6 cm. latis margine non revolutis, nervo medio; capitulis globosis vel subglobosis; receptaculis ca. 4–5 mm. latis ca. 7 mm. longis; achaeniis crassis uniaristatis. Haec species a ceteris achaeniis uniaristatis margine constanter crassissimis longe abest.
Distributio. — In Borneo boreali.
Specimen typicum. — **Borneo**: *F. W. Burbidge*, ann. 1877–1878 (in Herb. Gray.).

2. S. NERVOSA Chod. 1.8–3 dm. alta sparse ciliata; caulibus erectis vel ascendentibus; foliis lanceolatis vel ovatolanceolatis obtusis sine squamulis valde pinninervatis, petiolis valde pilosis vel subtomentosis; capitulis semiglobosis, involucri squamis oblongis; radicibus fasciculatis. — In Bull. Herb. Boiss. ser. 2, iii, 724 (1903).
Distributio. — In regione cursus superioris fluminis Apa in Paraguay.
Specimen examinatum. — **Paraguay**: *E. Hassler*, 8274, in regione cursus superioris fluminis Apa.

3. S. URENS Jacq. 2–5 dm. alta glabra aut plus minusve ciliata; caulibus decumbentibus; foliis anguste vel late lanceolatis raro linearibus acutis acuminatisve 3.5–5.5 cm. longis, supra squamulis albis scabris infra glabris, margine saepe perminute revolutis saepius trinerviis; pedunculis plerumque 1.4–2 dm. longis; capitulis globosis vel subglobosis, involucri squamis ovatis; receptaculis ca. 2 mm. crassis et 5 mm. longis; achaeniis tenuissimis plerumque aristis duabus ciliis persimilibus et inaequalibus. — Enum. Syst. Pl. Carib. 28 (1762), et Select. Stirp. Am. Hist. 214, t. 126, f. 1 (1763); DC. Prod. v, 625 (1836): *Santolina pyrethri sapore humi fusa*, Plum. Cat. Pl. Am. 10 (1703).
Synonymia. —

Cotula Spilanthus L. Mant. Pl. 116 (1767).
Bidens angustifolia Lam. Encycl. Meth. (Bot.) i, 416 (1783).
Bidens angustifolia Lam. var. *minor* Poir. Lam.-Poir. Illustr. Genres iii, 244, t. 668, f. 2 (1823).
Spilanthes repens Spreng. ex DC. Prod. v, 625 (1836).
Ceratocephalus urens (Jacq.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Distributio. — In Jamaica, Martinica, litoribus colombianis borealibus, Brasilia, et adventive (?) Altatae in Mexico.
Specimina examinata. — **Mexico**: SINALOA: *T. S. Brandegee*, Altatae, Sept. 2, 1904. **Jamaica**: *A. S. Hitchcock*, Luceae, Jan. 3, 1891, Port Antonio, Dec. 26, 1890, Port Morant, Dec. 22, 1890; *J. Moulton-Barrett*, Jackson Tower, Iun. 15, 1902; *C. F. Millspaugh*,

2028, Bowden. **Martinica:** *L. Hahn*, 126, Fort de France, et 726, in valle St. Pierre; *P. Duss*, 1733 et 4077, Fort de France. **Brasilia:** *G. Gardner*, 4252. **Colombia:** MAGDALENA: *H. H. Smith*, 553, Santa Marta.

4. S. URENS Jacq. f. **lanea** A. H. Moore f. nov. praecedenti omnino similis sed caules lanei et nonnunquam folia.

Distributio. — Lagoa Santa in Brasilia.

Specimen typicum. — **Brasilia:** MINAS GERAËS: *E. Warming*, Lagoa Santa, Oct. 27, 1863 (in Herb. Hort. Bot. Novebor.).

5. S. URENS Jacq. var. HISPIDULA DC. S. urenti persimilis sed conspicue hispidula; foliis plerumque 2.5 cm. longis; pedunculis 1.5–2.5 cm. longis. — *S. urens* Jacq. *β. hispidula* DC. Prod. v, 625 (1836).

Synonymia. —

Spilanthes leiocarpa DC. Prod. v, 626 (1836).
Spilanthes Macraei H. et A. in Hook. Jour. Bot. iii, 317 (1841).
Spilanthes Macrali Ktze. Rev. Gen. Pl. i, 326 (1891).
Ceratocephalus leiocarpus (DC.) Ktze. l. c.

Distributio. — Per oram peruvianam et chilensem borealem.

Specimina examinata. — **Perua:** CALLAO: U. S. So. Pac. Explor. Exped., Callao. **Chile:** TACNA: *W. Lechler*, 1532, prope Aricam. CONCEPCIÓN: *J. D. Hooker*, Concepción (photographa).

6. S. ANACTINA F. Muell. glabra erecta vel raro decumbens; foliis linearibus aut linearilanceolatis non revolutis; nervo medio prominente; pedunculis 1.1–2 dm. longis; achaeniis inaequaliter biaristatis. — Fragm. Phyt. Austr. v, 63 (1865–1866).

Synonymia. —

Ceratocephalus anactinus (F. Muell.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Distributio. — In Australia.

Specimina examinata. — **Australia:** QUEENSLAND: In insula Sweer in sinu Carpentariae (sine collectoris nomine). WALLIA NOVA AUSTRALIS: *S. Mossman*, 509, ad flumen Brisbane (planta per chartam delineata).

7. S. PUSILLA H. et A. 1.5–12 cm. alta; caulibus decumbentibus; foliis angustissime lanceolatis ad linearispatulata vergentibus ca. 2 cm. longis 3–5 mm. latis aliquando leviter petiolatis; capitulis globosis vel subglobosis ca. 5–7 mm. diametro; receptaculis ca. 5 mm. longis 1 mm. crassis; alioqui S. urenti *Jacq.* similis. — In Hook. Jour. Bot. iii, 317 (1841).

Synonymia. —

Spilanthes stolonifera DC. var. *pusilla* (H. et A.) Bak. in Mart. Fl. Bras. vi, **3**, 235 (Maio 1, 1884).

Distributio. — Buenos Aires in Argentina.

Specimen examinatum. — **Argentina**: BUENOS AIRES: *J. Tweedie*, 223, Buenos Aires.

8. S. INSIPIDA Jacq. ad 3 dm. alta; foliis supra infraque hispidulis vel subtomentosis valde aut saepe obtuse sinuatodentatis petiola (quae ca. 1 cm. longa) decurrentibus; pedunculis ca. 10–15 cm. longis, textura crassa vel raro tenui; capitulis vel globosis subglobosis aut raro conicis in pedunculo singulo singulis vel plurimis; achaeniis fere vel omnino glabris calvisque. — Enum. Syst. Pl. Carib. 28 (1762), et Select. Stirp. Am. Hist. 215, t. 126, f. 2 (1763); DC. Prod. v, 626 (1836).

Synonymia. —

Bidens insipida (Jacq.) Lam. Encycl. Meth. (Bot.) i, 416 (1783).

Ceratocephalus insipidus (Jacq.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Distributio. — In Cuba.

Specimina examinata. — **Cuba**: *C. Wright*, 3610; ? *E. Otto*, 94. MATANZAS: *J. W. Robbins*, prope Matanzas, Apr. 1864; *F. Rugel*, 6, Matanzas; *N. L. Britton, E. G. Britton, J. A. Shafer*, 206, in faucibus Sumuri prope Matanzas.

9. S. OLERACEA L. ad 4 vel 5 dm. alta; foliis deltoideis, basi 1.5–5.5 cm. lata inter cuneatam et subtruncatam variante, 2.4–7.5 cm. longis conspicue crenatis vel dentatis, petiolis quam in praecedentibus plerumque multo longioribus (1–4 cm. longis); capitulis plerumque ovoideocolumnaribus vel oblongis raro subglobosis 1–3 cm. longis 1–1.7 cm. latis, saepe apice fusco; receptaculis crassissimis ovoideocolumnaribus .8–2.3 cm. longis .5–1 cm. latis; achaeniis plerumque inaequaliter biaristatis conspicue ciliatis (ciliis saepe basi tuberculoidea) et margine crassa. — *Spilanthus oleracea* L. Syst. Nat. ed. XII, ii, 534 (1767); Regel Gartenfl. ii, t. 42 (1853), *Spilanthes oleracea* Jacq. secundum DC., et β. ? *fusca* (Hort. Par.) DC. Prod. v, 624, nris. 30 et 30β, cf. Jacq. Hort. Bot. Vind. ii, 63, t. 135 (1772).

Synonymia. —

Bidens acmelloides Berg in Kongl. Vetensk. Acad. Handl. xxix, **2**, 245 (Iul.–Sept. 1768).

Pyrethrum Spilanthus Med. in Act. Acad. vel Hist. et Comment. . . . Theod.-Palat. (Phys.) iii, 242, t. 18 (1775).

Bidens fervida Lam. Encycl. Meth. (Bot.) i, 415 (1783); Lam.-Poir. Illustr. Genres iii, 244, t. 668, f. 1 (1823).

Bidens fusca Lam. l. c.

Spilanthus fusca Hort. Par. ex Lam. l. c.

Buphthalmum strigosum Spreng. N. Entd. ii, 140 (1821).

Spilanthes brasiliensis Spreng. L.-Spreng. Syst. Veg. iii, 444 (1826).

Acmella brasiliensis Spreng. l. c.

Buphthalmum heterophyllum Willd. ex L.-Spreng. l. c. 592.

Spilanthes oleraceus var. Bl. Bijdr. Fl. Nederl. Ind. xiv, 912 (1826).
Spilanthes oleracea L. β. ? *fusca* (Hort. Par.) DC. Prod. v, 624 (1836).
Bidens oleracea (L.) Cav. ex Steud. Nom. Bot. ed. II, i, 203 (1840).
Spilanthes Acmella (L.) Murr. var. *oleracea* (L.) Zoll. Syst. Verz. Ind. Arch. 123 (1854).
Bidens fixa Hook. f. Fl. Brit. Ind. iii, 307 (1882). Forsan etiam Spilanthes oleracea *L.* β. rufa *Ott.* ex Ind. Sem. Hort. Berol., an. 1858, collect. 10 (1858), nom. nud.

Distributio. — In India, Antillibus Minoribus, Brasilia orientali.

Specimina examinata. — Ex Mus. Bot. Berol.; *J. E. Teijsmann*, "ex horto bogoriensi Javae misit, 1869." **Guadeloupe:** *P. Duss*, 2498. **Martinica:** *P. Duss*, 1449, Trois Ponts in St. Pierre. **Brasilia:** *W. J. Burchell*, 4734. Minas Geraës: *S. E. Henschen*, ser. I, 269, Caldas. Rio de Janeiro: *C. Gaudichaud-Beaupré*, 681, Rio de Janeiro. **India:** Behar in Bengalia: *H. v. Schlagintweit-Sakünlünski*, sub catal. no. 12959, in alveo sicco fluminis Gandak prope Patnam. Konkan in Bombay: *J. D. Hooker*, ex "Herb. Ind. Or. Hook. fil. & Thomson" Bombay. **Specimina culta:** Ex Hort. Bot. Univ. Harv., an. 1874; ex Herb. I. A. Lapham, e seminibus e Cuba missis cult.; *J. Gay*, Parisiis, Sept. 1835. *C. J. Maximowicz*, in itinere secundo, Yokohamae, in Japonia, an. 1862; *G. Engelmann*, Berkeley in California in Civitatibus Foederatis Americae, Sept. 6, 1880.

10. S. leucantha HBK. foliis ovatolanceol
dentatis 4.5–7.5 cm

crassiore), colore viridi aut alboviridi; pedunculis 1.5–10 cm. longis; capitulis 2–20 ovoideis (plerisque albis vel albidovirescentibus); achaeniis plerumque insigniter ciliatis.

Synonymia. —

Bidens ocymifolia Lam. Encycl. Meth. (Bot.) i, 416 (1783); Lam.-Poir. Illustr. Genres iii, 244, t. 668, f. 3 (1823).

Spilanthus albus L'Hér. Stirp. Nov. i, 7, t. 4 (1784), *Spilanthes alba* Willd. secundum DC. Prod. v, 625 (1836), cf. Willd. iii, **3**, 1714 (1804).

Spilanthus Salivaria Domb. ex L'Hér. l. c.

Spilanthus radicans Jacq. Collect. Bot. Chem. Hist. Nat. iii, 229 (1789).

Spilanthus exasperata Jacq. Ic. Pl. Rar. iii, 15, t. 584 (1781–1793) ("Spilanthus radicans mihi dicta fuit in Collectaneis vol. 3"); DC. Prod. v, 626 (1836).

Spilanthes radicans Schrad. ex DC. l. c. 624.

Spilanthes exasperata Jacq. *β. cayennensis* DC. l. c. 626.

Spilanthes leucophaea Hort. Berol. ? ex Klatt Leopold. xxiii, 5 (1887).

Spilanthes Botterii Wats. in Proc. Am. Acad. xxvi, 141 (1891).

Ceratocephalus Salivaria (Domb.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Ceratocephalus exasparatus (Jacq.) Ktze. l. c.

diroi Hieron. ex Sod. in Engl. Bot. Jahrb. xxix, 842 inter subcuneatam et subcordatam variante, petiolis inferiorum brevioribus superiorum saepius nullis; pedunculis 1.7–9 cm. longis; capitulis inter globosa et ovatoconica variantibus .7–1.5 cm. diametro, bracteis numerosis; achaeniis glabris calvisque; sequenti valde affinis. — Nov. Gen. et Sp. Pl. iv, 210, t. 370 (1820); DC. Prod. v, 625 (1836).

Synonymia. —

Isocarpha Kunthii Cass. in Dict. Sc. Nat. xxiv, 19 (1822).

Ceratocephalus leucanthus (HBK.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Distributio. — In Andibus ecuadorensibus.

Specimen examinatum. — **Ecuador**: BOLÍVAR: *E. André*, Guarandae, Iul. 8, 1876.

11. **S. ocymifolia** (Lam.) A. H. Moore comb. nov. magnopere variabilis erecta vel prostrata fere semper pubescens; foliis regulariter ovatis inter subintegra et dentata vel crenata variantibus 1.5–9 cm. longis 5 mm.–5 cm. latis (saepius 5.5–6 cm. longis ca. 3.4 cm. latis) plerumque non valde acuminatis, textura tenui vel crassa (plerumque

(Maio 22, 1906).

Distributio. — Per Mexico et Americam Centralem, in Venezuela, Colombia, Ecuadore, Perua. In Mexico haec species perabundans et constantior est.

Specimina examinata. — **Mexico**: *Friedr. Mueller*, 267 et 274. Sinaloa: *T. S. Brandegee*, Cafradiae prope Culiacán, Oct. 20, 1904; *E. Palmer*, 1565, Lodiego. Jalisco: *C. G. Pringle*, 2946 et 4341, prope Guadalajaram. Colima: *E. Palmer*, 1192, Colimae. Michoacán: *C. G. Pringle*, 4340, Monte León. Morelos: *C. G. Pringle*, 11572 et *C. C. Deam*, Ian. 3, 1899, Cuernavacae. Oaxaca: *C. Conzatti* et *V. González*, 647, Oaxacae. Vera Cruz: *E. Bourgeau*, 3098, *M. Botteri*, 825, *Friedr. Mueller*, 4072, Orizabae. **Guatemala**: Zacapa: *C. C. Deam*, 239, Gualán. Amatitlán: *H. v. Türckheim*, 8705, Amatitlán. Santa Rosa: *H. T. Heyde* et *E. Lux*, 3812, prope Rio de Las Cañas. **Salvador**: San Salvador: *L. V. Velasco*, 8851, San Salvador. **Costa Rica**: Guanacaste: *A. Tonduz*, 13628, Nicoyae. Alajuela: *A. Alfaro*, 5807 B, Alajuelae. **Venezuela**: Mérida: *A. Fendler*, 691 et 691B, prope Tovar. Carácas: *J. R. Johnston*, 102, ad flumen

Asunción in insula Margarita; . . . *Birschel*, Carácas. **Panama**: *B. Seemann*. **Colombia**: MAGDALENA: *H. H. Smith*, 591, prope Mamatocam prope Santa Marta. BOLÍVAR: *J. B. Boussingault*, Cartagenae, an. 1833 (planta per chartam delineata). **Brasilia**: RIO DE JANEIRO: *R. Spruce*, Rio de Janeiro, an. 1864. **Ecuador**: *A. Sodiro*, 39/1 (specimen per chartam delineatum). GUAYAS: *T. Hartweg*, 867, Guayaquil. **Perua**: Herb. Hook.; Plantae Schottianae. LIMA: U. S. So. Pac. Explor. Exped., infra Obrajillo. CUZCO: ? *J. Gay*, Oct. 1839–Feb. 1840. **Specimen cultum**: *M. S. Bebb*, Fountaindale in Illinois in Civitatibus Foederatis Americae.

12. S. OCYMIFOLIA (Lam.) A. H. Moore f. **radiifera** A. H. Moore f. nov. praecedenti omnino similis sed radiifera.

Distributio. — In Panama, Colombia, ora Americae Australis occidentali ab Ecuadore ad Chile, et in ora Brasiliae orientali.

Specimen typicum. — **Colombia**: MAGDALENA: *H. H. Smith*, 513, Santa Marta (in Herb. Hort. Bot. Novebor.) (specimina cum eodem lecta in Herbb. Gray., Mus. Hort. Bot. Mo., Mus. Hist. Nat. Field).

Specimina alia examinata. — **Panama**: *B. Seemann*; *A. Fendler*, 166, Chagres. **Surinam**: *W. H. De Vriese*. **Colombia**: BOLÍVAR: *E. André*, Armadae, Maio 22, 1876. **Brasilia**: RIO DE JANEIRO: *C. Gaudichaud-Beaupré*, 682, et *E. Warming*, Maio 16, 1863, Rio de Janeiro. **Ecuador**: MANABI: *H. F. A. v. Eggers*, 15646, Hacienda del Recreo. **Bolivia**: *M. Bang*, 2024. **Chile**: *J. Gay*. **Specimen cultum**: Hort. Bot. Univ. Harv., an. 1874.

13. S. OCYMIFOLIA (Lam.) A. H. Moore var. **acutiserrata** A. H. Moore var. nov. foliis valde acuminatis et subinaequaliter serratis; capitulis ovoideis aut saepius subglobosis fere semper numerosissimis (25–100).

Distributio. — In Mexico et per Costa Rica.

Specimen typicum. — **Costa Rica**: CARTAGO: *J. J. Cooper*, 5807, Cartago (in Herb. J. D. Sm.) (specimen cum eodem lectum in Herb. A. H. Moore).

Specimina alia examinata. — **Mexico**: TEPIC: *E. Palmer*, Ian. 5–Feb. 6, 1892. **Costa Rica**: SAN JOSÉ: *A. Tonduz*, 1429, circum San José, 8493, San Francisco de Guadalupe.

14. S. CALVA DC. decumbens vel repens, nodis saepius radicans, pubescens vel subtomentosa; foliis 1.5–3.5 cm. longis plerumque 1 cm. latis inter subintegra et dentata vel crenata variantibus brevipetiolatis; pedunculis 2.5–9 cm. longis; capitulis ovoideis vel subglobosis .5–1.3 cm. diametro; achaeniis glabris calvisque. — DC. ex Wight Contr. Bot. Ind. 19 (1834); DC. Prod. v, 625 (1836).

Synonymia. —

Spilanthes Acmella Bl. Bijdr. Fl. Nederl. Ind. (1826), non (L.) Murr.

Spilanthes rugosa Bl. ex DC. Prod. v, 625 (1836).
Spilanthes Acmella Wall. ex Steud. Nom. Bot. ed. II, ii, 622 (1841), non (L.) Murr.; nomen ex Wall. Cat. 3185/295 (Dec. 1, 1828).
Spilanthes rugosa Bl. var. *truncata* Miq. Fl. Ind. Bat. ii, 81 (1856–1859).
Spilanthes Acmella (L.) Murr. var. *calva* (DC.) Clarke ex Hook. f. Fl. Brit. Ind. iii, 307 (1882).
Spilanthes Pseudo-Acmella Wall. ex Hook. f. l. c.; nomen ex Wall. Cat. 3185/295 (Dec. 1, 1828).
Cotula conica Wall. ex Hook. f. l. c.; nomen ex Wall. l. c.

Distributio. — In India australi, Ceylonia, et ? Java.

Specimina examinata. — **India**: *R. Wight*, 1456. ORISSA in BENGALIA: Edidit R. F. Hohenacker, 1017, et *G. S. Perrottet*, 27, in collibus Nilgiri. MADRAS: "Herb. Ind. Or. Hook. fil. & Thomson," Madras. **Ceylonia**: *G. H. K. Thwaites*, 684. **Java?**: *F. W. Junghuhn*, 307.

15. S. ACMELLA (L.) Murr. erecta ascendens decumbensve plerumque non nodis radicans subpubescens vel pubescens vel subhispida nonnunquam glabra; foliis 1.5–5 cm. longis .7–3 cm. latis obtusis acutis acuminatisve subintegris aut acute vel obtuse plerumque non valde serratis brevipetiolatis; pedunculis 3–10 cm. longis; achaeniis inaequaliter biaristatis plerumque margine ciliatis. —*Spilanthus Acmella* (L.) Murr. L.-Murr. Syst. Veg. ed. XIII, 610 (1774).

Synonymia. —

Verbesina Acmella L. Sp. Pl. ed. I, ii, 901 (1753), et Mat. Med. 142 (1749); DC. Prod. v, 623 (1836); Wight Ic. Pl. Ind. Or. iii, 1109 (1843–1850); Trim. Hand Bk. Fl. Ceyl. iii, 40 (1895): *Chrysanthemum bidens zeylanicum Acmella dictum*, Breyn. f. Dissert. Bot.-Med. (1700); *Senecio ind. orient. ocymi majoris folio profunde crenato*, Pluk. Alm. Bot. Mant. 343 (1700) t. 315, f. 2 (1696).
Pyrethrum Acmella (L.) Med. in Act. Acad. vel Hist. et Comment. . . . Theod.-Palat. (Phys.) iii, 243, t. 19 (1775).
Bidens Acmella (L.) Lam. Encycl. Meth. (Bot.) i, 415 (1783).
Spilantus Aemella R. W. Darw. Fam. Pl. ed. II, ii, 544 (1787).
Spilanthus melissaefolius Salisb. Prod. Stirp. Hort. Chap. Allert. 186 (1796).
Acmella Linnaei Cass. in Dict. Sc. Nat. xxiv, 330 (1822), *A. Linnaea* Cass. secundum Hook. f. Fl. Brit. Ind. iii, 307 (1882).
Spilanthus Amella Roxb. Fl. Ind. iii, 410 (1832).
Verbisina Amella Roxb. l. c.

Spilanthes paniculata Wall. ex DC. Prod. v, 624 (1836); Hook. f. Fl. Brit. Ind. iii, 307 (1882); nomen ex Wall. Cat. 3186/296 (Dec. 1, 1828).

Spilanthus peregrina Blanc. Fl. Filip. ed. I, 622 (1837).

Spilanthus lobata Blanc. l. c.

Spilanthes Acmella (L.) Murr. var. *paniculata* (Wall.) Clarke ex Hook. f. Fl. Brit. Ind. iii, 307 (1882).

Ceratocephalus Acmella (L.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Distributio. — In India orientali, China, Formosa, Insulis Philippinis, Australia.

Specimina examinata. — *J. E. Teijsmann*, "ex horto bogoriensi misit, 1869"; **India**: *R. Wight*, 449 et 1607. BENGALIA: *J. D. Hooker*, Sikkim. MADRAS: "Herb. Ind. Or. Hook. fil. & Thomson," sine collectoris nomine, Madras. TENASSERIM in BURMA aut INSULAE ANDAMAN: *J. W. Helfer*, 3186. **Formosa**: *R. Oldham*, an. 1864; *A. Henry*, 812, sine localitate, et 219, in Garampi (qui locus etiam Promunturium Australe appellatus est). **Insulae Philippinae**: *H. Cuming*, 2361; Herb. U. S. So. Pac. Explor. Exped., Manilae. **China**: YUNNAN: *A. Henry*, 12706, Sze-mao.

16. S. ACMELLA (L.) Murr. var. **albescentifolia** A. H. Moore var. nov. foliis quam in praecedente plerumque maioribus (4.5–9.5 cm. longis 2–3.5 cm. latis) fere semper infra albescentibus.

Distributio. — Incognita sed sine dubio in Mundo Antiquo.

Specimen typicum. — "The Bernhardi Herbarium" nullis notis adiectis (in Herb. Hort. Bot. Mo.).

Specimina alia examinata. — Tria specimina alia "The Bernhardi Herbarium" nullis notis adiectis.

17. S. ACMELLA (L.) Murr. var. **lanceolata** (Lk.) A. H. Moore comb. nov. erecta vel basi decumbente; foliis ovatolanceolatis longiacuminatis incisis 3–7 cm. longis 1–4.5 cm. latis, petiolis 6.5–10.5 cm. longis numerosis; capitulis .8–1.3 cm. longis .7–1 cm. latis.

Synonymia. —

Spilanthes Pseudacmella Spreng. L. - Spreng. Syst. Veg. iii, 444 (1826), non (L.) Murr.

Verbesina Pseudacmella Spreng. l. c., non L.

Acmella lanceolata Lk. var. ex Spreng. l. c.

Distributio. — Incognita sed sine dubio in Mundo Antiquo.

Specimina examinata. — Ex Mus. Bot. Berol.; ex Hort. Bot. Petropol.

18. S. COSTATA Benth. ascendens nodis radicans; foliis 4–6.2 cm. longis 2–3 cm. latis saepius obtusis brevipetiolatis subintegris fere integris; pedunculis ca. 9 cm. longis. — Ex Hook. et Benth. Fl. Nigr. 436 (1849).

Distributio. — In promunturio Palmas in Africa.
Specimen examinatum. — **Liberia**: *J. B. T. Vogel*, in promunturio Palmas.

19. **S. callimorpha** A. H. Moore spec. nov. laxa longe decumbens vel prostrata, longis internodiis, nodis distantibus radicans, subpubescens vel glabra; foliis plerumque 4.7 cm. longis 1 cm. latis longe acuminatis, apice obtuse mucronuloideo, acute serratis aut saepius subincisis, petiolis plerisque 1 cm. longis; pedunculis 14–17 cm. longis; capitulis ovoideoconicis .9–1.1 cm. longis 6–8 mm. latis, involucri squamis 4–8 subacutis subciliatis; achaeniis inaequaliter biaristatis subciliatis vel glabris calvisque. Nostra species a ceteris sectionis longe distat foliis valde serratis, pedunculis internodiisque longissimis.
Distributio. — Sze-mao, China.
Specimen typicum. — **China**: YUN-NAN: *A. Henry*, 12260A, Sze-mao (in Herb. Hort. Bot. Novebor.) (specimen cum eodem lectum in Herb. Gray.).

Sectio II. ACMELLA (Rich.) DC. capitulis radiatis. — Prod. v, 620 (1836).
Synonymia. —

Genus *Acmella* Rich. in Pers. Syn. Pl. ii, 473 (1807).
Genus *Athronia* Neck. Elem. Bot. i, 32 (1790).
Subgenus Acmellae *Rich. Erpota* Raf. New Fl. N. Am. i, 51 (1836).
Sectio Spilanthis *Jacq. Megaglottis* F. Muell. Fragm. Phyt. Austr. v, 63 (1865–1866).

Distributio. — Per Indiam, Indias Orientales Nederlandicas, in Insulis Philippinis, in Abyssinia aliisque Africae regionibus, in Civitatibus Foederatis Americae a Carolina Boreali ad Floridam, in Texas et adventive in Nova Caesarea, per Mexico et Americam Centralem, in Indiis Occidentalibus, et in variis locis per totam oram Americae Australis.

Subsectio I. **Parvoradiatae** A. H. Moore subsect. nov. Radii involucrum vix superantes plerumque perminuti et paulo ovati.
Distributio. — In Indiis Occidentalibus, in Mexico, Costa Rica, et per oram Americae Australis occidentalem a Colombia ad Chile et Argentinam, et in Africa orientali occidentalique.

20. **S. iodiscaea** A. H. Moore spec. nov. decumbens vel prostrata sparse pubescens; caulibus tenuioribus. Folia 2.4–3.6 cm. longa .9–1.2 cm. lata, textura tenui, minute et sparse dentata, petiolis brevissimis vel nonnunquam nullis. Pedunculi 1.5–5 cm. longi (plerique ca.

2.5 cm.). Capitula ovoidea saepius acuta parva (5.7 mm. longa 4–5 mm. lata), paleis sursum violaceis deorsum luteis ad virides vergentibus, nihil praeter partem violaceam manifestum; radii minutissimi albi vel paulo luteotincti, apices ovate expansi; involucri squamae ovatae, margine ciliata. Receptacula tenuissima acutissimaque ca. 4–5 mm. longa 1 mm. minusve lata breve inaequaliterque biaristata. Radices tenues fasciculatique.

Haec species a ceteris longe distat palearum apicibus valde violaceis.

Distributio. — In insula Porto Rico.

Specimen typicum. — **Porto Rico**: MAYAGUEZ: *P. E. E. Sintenis*, 718, Cabo Rojo, locis cultis (in Herb. Gray.) (specimen cum eodem lectum in Herb. Mus. Hist. Nat. Field).

Specimina alia examinata. — **Porto Rico**: BAYAMÓN: *P. E. E. Sintenis*, 1149, "ad vias locis cultis" Palo-seco, Bayamón. PONCE aut GUAYAMA: *A. A. Heller* et *uxor*, 548, inter Aibonito et Cayey.

21. S. IODISCAEA A. H. Moore f. **leucaena** A. H. Moore f. nov. praecedenti omnino similis sed paleis albidovirescentibus.

Specimen typicum. — **Porto Rico**: PONCE aut GUAYAMA: *A. A. Heller* et *uxor*, 550a, inter Aibonito et Cayey. Annotatio in titulo dicit "unique" (in Herb. Hort. Bot. Novebor.).

22. S. ULIGINOSA Sw. pervariabilis ascendens laxa vel prostrata; foliis lanceolatis distanter irregulariterque serratis crenatisve 1.5–4.5 cm. longis .3–1.6 cm. latis, textura inter tenuem et crassam variante; pedunculis 1.1–5.2 cm. longis, discorum radiorumque colore inter luteum fere album et aureum variante; receptaculis tenuibus acutisque. — *Spilanthus uliginosa* Sw. Nov. Gen. Pl. seu Prod. Descr. Veg. Ind. Occ. 110 (1788); DC. Prod. v, 624 (1836).

Synonymia. —

Acmella uliginosa (Sw.) Cass. in Dict. Sc. Nat. xxiv, 331 (1822).
Jaegeria uliginosa (Sw.) Spreng. L.-Spreng. Syst. Veg. iii, 590 (1826).
Spilanthes Acmella (L.) Murr. var. *uliginosa* (Sw.) Bak. in Mart. Fl. Bras. vi, **3**, 233 (Maio 1, 1884).
Ceratocephalus Acmella (L.) Ktze. var. *uliginosa* (Sw.) Ktze. Rev. Gen. Pl. i, 326 (1891), var. *uliginosus* l. c. iii, 140 (1898).
Ceratocephalus Acmella (L.) Ktze. var. *depauperata* Ktze. Rev. Gen. Pl. i, 326 (1891).

Distributio. — In Jamaica, per Antilles Minores a St. Christopher ad Grenadam, Tobago, Trinidad, et in Panama, et in Sierra Leone in Africa occidentali.

Specimina examinata. — **Jamaica**: *J. R. Churchill*, St. Ann's Bay, Mart. 19, 1897; *A. S. Hitchcock*, Luceae, Ian. 3, 1891, et Port Morant, Dec. 20, 1890; *N. L. Britton*, 830, inter Constant Spring et Annotta Bay; *A. Fredholm*, 3059, prope Port Antonio; *A. E. Wight*, 35, Port Antonio; *W. Fawcett*, 8005, Castleton; *C. F. Millspaugh*, 1888, in paeninsula Tichfield. **St. Christopher**. *N. L. Britton* et *J. F. Cowell*, 678, in fundo Molyneaux. **Guadeloupe**: *P. Duss*, 2521, Basse-terre, 2822, apud castra Jacob, et 893 et 4447, Adonis prope castra Balata. **Dominica**: *H. F. A. v. Eggers*, 74, Goodville: *F. E. Lloyd*, 473, Petite Soufrière. **Martinica**: Herb. Rich, in arvis campestribus; *C. P. Belanger*, 179, prope St. Pierre; *P. Duss*, 930, St. Pierre; *L. Hahn*, 1107, in palude du Lamantin. **St. Vincent**: *H. H.* et *G. W. Smith*, 96. **Grenada**: *W. E. Broadway*, in fossa ad montem Parnassum, St. George's, Ian. 16, 1905; *H. F. A. v. Eggers*, 6063, Belvidere. **Tobago**: *H. F. A. v. Eggers*, 5760, ad flumen Great Dog. **Trinidad**: *O. Kuntze*, Apr. 1874. **Panama**: *J. F. Cowell*, 392, inter Ahorca Lagarto et Culebram. **Sierra Leone**: *W. H.* et *A. H. Brown*, 32a, Freetown.

23. S. LUNDII DC. erecta; foliis lanceolatis obtuse vel plerumque acute et saepius aequaliter serratis acuminatis brevipetiolatis, petiolis non ciliatis; capitulis saepius in planta multis (8 vel pluribus) plerisque 6 mm. longis 4 mm. latis, ligulis minimis. Plantae precedenti aspectu non dissimiles sed involucri squamis lanceolatis non late ovatis. — Prod. v, 622 (1836).

Synonymia. —

Spilanthes Salzmanni DC. Prod. v, 623 (1836).

Distributio. — In Brasilia.

Specimina examinata. — **Brasilia**: *L. Riedel*; *W. J. Burchell*, 7956–2. BAHIA: *K. F. P. v. Martius*, 438, Bahiae; *I. Blanchet*, circa Bahiam, an. 1831. RIO DE JANEIRO: *G. Gardner*, ser. I, 70, prope Rio de Janeiro.

24. S. CILIATA HBK. pervariabilis erecta vel ascendens; foliis lanceolatis vel plerumque ovatolanceolatis crenatis vel serratis (saepe inaequaliter) acutis vel breviacuminatis, petiolis plus minusve brevibus nonnunquam fimbriatis vel ciliatis; capitulis .7–1.9 cm. longis 6–9 mm. latis, ligulis quam in praecedente plerumque paulo maioribus. Plantae aspectu S. uliginosae *Sw.* dissimiles. — Nov. Gen. et Sp. Pl. iv, 208 (1820); DC. Prod. v, 621 (1836).

Synonymia. —

Spilanthes fimbriata HBK. Nov. Gen. et Sp. Pl. iv, 208 (1820); DC. Prod. v, 621 (1836).

Spilanthes debilis HBK. l. c.; DC. l. c. 624.

Spilanthes tenella HBK. l. c.; DC. l. c.

Acmella ciliata (HBK.) Cass. in Dict. Sc. Nat. xxiv, 331 (1822).

Acmella fimbriata (HBK.) Cass. l. c.

Acmella debilis (HBK.) Cass. l. c.
Acmella tenella (HBK.) Cass. l. c.
Acmella brachyglossa Cass. l. c. l, 258 (1827).
Spilanthes grandis DC. Prod. v, 622 (1836).
Spilanthes Poeppigii DC. l. c.
Spilanthes Mariannae DC. l. c. 623.
Spilanthes arrayana Gardn. in Hook. Lond. Jour. Bot. vii, 408 (1848).
Spilanthes Mandonii Sch. Bip. in Linnaea xxxiv, 529 (1866).
Ceratocephalus ciliatus (HBK.) Ktze. Rev. Gen. Pl. i, 326 (1891).
Ceratocephalus fimbriatus (HBK.) Ktze. l. c.
Ceratocephalus debilis (HBK.) Ktze. l. c.
Ceratocephalus tenellus (HBK.) Ktze. l. c.
Ceratocephalus Poeppigii (DC.) Ktze. l. c.
Spilanthes Eggersii Hieron. in Engl. Bot. Jahrb. xxviii, 608 (Ian. 11, 1901).

Distributio. — In Brasilia et Mexico, et per oram Americae Australis occidentalem a Colombia ad Boliviam.

Specimina examinata. — "The Bernhardi Herbarium," 229. **Mexico**: Herb. Humboldt, apud montem Jurullo (planta per chartam delineata). **Brasilia**: *W. J. Burchell*, 1025 et 9236; *F. Sellow*, 848 et 1078; Herb. U. S. So. Pac. Explor. Exped., ad basim montium Organ. GOYAZ: *G. Gardner*, ser. VIII, 3866, Arrayas. **Colombia** aut **Ecuador**: *F. C. Lehmann*, 7987. **Colombia**: CUNDINAMARCA: ? *I. F. Holton*, Fusagasugae, Dec. 21, 1852. **Ecuador**: MANABI: *H. F. A. v. Eggers*, 14931, Hacienda el Recreo (specimen per chartam delineatum). **Perua**: LIMA: U. S. So. Pac. Explor. Exped., Limae et Obrajillo. **Bolivia**: LA PAZ: *G. Mandon*, 63, prope Soratam. YUNGAS: *H. H. Rusby*, 919.

25. S. CILIATA HBK. var. **diffusa** (Poepp.) A. H. Moore comb. nov. praecedenti persimilis sed prostrata; foliis ovatis, basi subcuneata, margine regulariter acute dentata, ca. 2.3 cm. longis 1.7 cm. latis; pedunculis singulis.

Synonymia. —

Spilanthes diffusa Poepp. Nov. Gen. et Sp. Pl. iii, 50 (1845).
Ceratocephalus diffusus (Poepp.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Distributio. — In Andibus ecuadorensibus peruvianisque.

Specimina examinata. — **Ecuador**: *J. P. Couthouy*, in Andibus quitensibus, an. 1855. **Perua**: *W. E. Stafford*, ad flumen Rimac, Aug. 29, 1887.

26. S. SUBHIRSUTA DC. decumbens; surculis maxime recentioribus subhirsutis vel dense tomentulosis aut saltem valde pubescentibus;

foliis ovatis irregulariter sed saepius acute serratis acutis non mucronuloide attenuatis brevipetiolatis, petiolis subhirsutis; capitulis radiis minimis. — Prod. v, 622 (1836).

Synonymia. —

Ceratocephalus subhirsutus (DC.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Spilanthes popayanensis Hieron. in Engl. Bot. Jahrb. xxviii, 608 (1901).

Distributio. — In Mexico et Colombia.

Specimina examinata. — **Mexico**: Tamaulipas: *J. D. Berlandier*, Tampico (pars speciminis typici et eiusdem habitus per chartam delineatus). **Colombia**: Cauca: *F. C. Lehmann*, 8010, Popayán.

27. S. caespitosa DC. laxa prostrata vel subglabra; foliis anguste ovatis parvis, apice conspicue mucronuloide attenuato; capitulis radiis minimis. — Prod. v, 622 (1836).

Synonymia. —

Ceratocephalus caespitosus (DC.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Distributio. — In Minas Geraës in Brasilia.

Specimen examinatum. — **Brasilia**: Minas Geraës: . . . *Vauthier*, 308 (pars speciminis typici et habitus photographa).

28. **S. poliolepidica** A. H. Moore spec. nov. erecta canescens pilosula. Folia lanceolata 3–5 cm. longa .7–1.4 cm. lata brevipetiolata vel subsessilia, supra obscuriora infra pallidiora canescentia saltem colore, margine integra nisi vero minutissime undulata subrevoluta. Capitula ovoidea 6–8 mm. longa; radii subaurei; involucri squamae valde pilosulae; paleae membranaceae deorsum albae sursum luteae vel subaureae. Achaenia ciliata fere aequaliter biaristata.

Distributio. — Chilamate prope flumen Sarapiqui in Costa Rica.

Specimen typicum. — **Costa Rica**: Heredia: *P. Biolley*, 7420, Chilamate prope flumen Sarapiqui (in Herb. Gray.) (specimen cum eodem lectum in Herb. J. D. Sm.).

29. S. disciformis Rob. procumbens cum ramis ascendentibus (parte erecta 1.8–2.5 dm. alta), caulibus maxime prostratis lignosis maxime erectis rubrotinctis, nodis radicantibus; radicibus crassis in fasciculis dispositis; foliis integris subacutis subsessilibus aut brevipetiolatis, nervo medio prominente albicante, internodiis non longissimis et foliis quam internodiis plerumque longioribus. Pedunculi 5–7 cm. longi (plerumque 5.5 cm.). Capitula elongatoconica; radii quam capitula multo breviores minimi. Achaenia parum ciliata inaristata. — In Proc. Am. Acad. xxvii, 176 (Nov. 2, 1892).

Distributio. — Prope Guadalajaram in Mexico.

Specimen examinatum. — **Mexico**: Jalisco: *C. G. Pringle*, 3489, prope Guadalajaram (specimen typicum).

30. **S. limonica** A. H. Moore spec. nov. laxa sed erecta 3–5 dm. alta, caulibus nunquam rubrotinctis sparse pubescentibus; foliis ovatolanceolatis integris vel subintegris (vix undulatis vel dentatis) obtusis aut vix acutis tenuibus, margine ciliolata; pedunculis 8–16 cm. longis (plerumque 10 cm.); capitulis elongatoconicis, radiis quam capitulis multo brevioribus minimis.

Distributio. — Limones in Santa Clara, Cuba.

Specimen typicum. — **Cuba**: SANTA CLARA: *C. G. Pringle*, 75, Limones (in Herb. Gray.) (specimina cum eodem lecta in Herbb. Hort. Bot. Mo., Hort. Bot. Novebor., Mus. Hist. Nat. Field).

Appellatio. — Ex nomine localitatis in qua specimen typicum lectum est.

31. S. ALPESTRIS Griseb. erecta ad 7.3 dm. alta; foliis late lanceolatis 1.5–6.6 cm. longis 1.5–2.6 cm. latis acute distanterque serratis, petiolis nullis; pedunculis ca. 11 cm. longis; capitulis subhemisphaericis supra planioribus fere planoconvexis. — In Gött. Gel. Abh. xix, 185 (1874).

Distributio. — In Argentina.

Specimen examinatum. — **Argentina**: TUCUMAN: *P. G. Lorentz* et *G. Hieronymus*, 903, Siambon (regione typica) (specimen per chartam delineatum).

32. S. MAURITIANA (Rich.) DC. erecta aut saepius decumbens procumbensque nodis saepe radicans puberula; foliis deltoideis vel ovatis serratis 2.3–3.2 cm. longis 1.5–2.8 cm. latis; pedunculis 1–1.5 cm. longis, bracteis late ovatis; capitulis subglobosis vel conicis 5 mm. –1.5 cm. longis 5–9 mm. latis, radiis latitudine saepe eis subsectionis sequentis aequantibus; involucri squamis angustioribus discos superantibus vel aequantibus. — Prod. v, 625 (1836).

Synonymia. —

Acmella mauritiana Rich. in Pers. Syn. Pl. ii, 472 (1807).
Acmella caulirhiza Del. Cent. Pl. d'Afr. du Voy. à Meroé, 45, t. 3, f. 7 (1826).
Spilanthes caulirhiza (Del.) DC. Prod. v, 623 (1836).
Spilanthes africana DC. l. c.
Spilanthes caulorrhiza Benth. Fl. Austr. iii, 541 (1866).
Acmella caulorrhiza (Benth.) Hook. f. et Jack. Ind. Kew. i, 29 (1895).
Acmella mauritanica Hook. f. et Jack. l. c.

Distributio. — In Pondoland in Africa, in Madagascare, et in Mauritio.

Specimina examinata. — **Africa?**: "The Bernhardi Herbarium," 303. **Pondoland**: *W. Tyson*, 1057, in cultis, Enshlewzi. **Madagascar**: . . . *Blackburn*, "comm. Admiral W. Bowles," Iul. 17, 1863. **Mauritius**: Sine collectoris nomine (planta per chartam delineata).

33. S. MAURITIANA (Rich.) DC. f. **madagascariensis** (DC.) A. H. Moore comb. nov. capitulis maturis oblongis quam in praecedente magis elongatis 1.5 cm. longis 5–6 mm. latis.
Synonymia. —
Spilanthes caulirhiza (Del.) DC. *β. madagascariensis* DC. Prod. v, 623 (1836).
Distributio. — In Madagascare.
Specimen examinatum. — **Madagascar**: *M. A. Shufeldt*, 105.

Subsectio II. **Magnoradiatae** A. H. Moore subsect. nov. Radii capituli maturi involucrum conspicue superantes plerumque magni atque liguliformes.
Synonymia. —
Sectio Spilanthis *Jacq. Megaglottis* F. Muell. Fragm. Phyt. Austr. v, 63 (1865–1866).
Distributio. — In Civitatibus Foederatis Americae a Missouri ad Floridam et Texas, per Mexico et Americam Centralem, et in America Australi a Colombia ad Patagoniam borealem in Argentina, etiam in Mundo Antiquo in Java, Insulis Philippinis, Australia, et in Abyssinia in Africa.

34. S. STENOPHYLLA H. et A. procumbens; foliis angustissimis 2–2.5 cm. longis 1.5–2 mm. latis densissimis; pedunculis ca. 5.5 cm. longis; radiis discum aequantibus vel paulo superantibus. — In Hook. Jour. Bot. iii, 317 (1841).
Distributio. — Apud Buenos Aires in Argentina.
Specimen examinatum. — **Argentina**: BUENOS AIRES: *J. Tweedie*, 861, Buenos Aires.

35. S. FILIPES Greenm. erecta; foliis ovatis, basi in petiolum cuneato-attenuato, conspicue et valde aequaliter crenatoserratis, apice vix acuto aut vix mucronuloideo, petiolis plerisque ca. 2.5 cm. longis; pedunculis gracillimis; capitulis minimis (3.5–5.5 mm. longis plerisque 4.7 mm. latis), regulariter multis, radiis conspicuis quam capitulis longioribus; achaeniis ciliatis inaequaliter biaristatis. — In Proc. Am. Acad. xxxv, 314 (Mart. 16, 1900).
Distributio. — In civitate Yucatan in Mexico.
Specimina examinata. — **Mexico**: YUCATAN: "Allison V. Armour Expedition," 43; *G. F. Gaumer*, 2502, Izamal, 1122, Buena Vista (specimen typicum), 1257, 1465, 2185, Chichankanab, 3835, in insula Cozumel; *C. F. Millspaugh*, 1494, San Miguel in insula dicta.

36. **S. iabadicensis** A. H. Moore spec. nov. erecta saltem 2.8 dm. alta; foliis lanceolatis distanter sed acute et saepius inaequaliter

serratis, basi acuta, apice valde acuto nunquam mucronuloideo, petiolis brevissimis; pedunculis non gracillimis; capitulis .8–1.4 mm. longis, radiis quam capitulis brevioribus ac quam in praecedente multo minus abundantibus; achaeniis ciliatis inaequaliter biaristatis.
Distributio. — In Java.
Specimen typicum. — **Java**: *J. E. Teijsmann*, "ex horto bogoriensi Javae misit, 1869." (in Herb. Gray.).

37. **S. phaneractis** (Greenm.) A. H. Moore comb. nov. prostrata decumbensve nodis radicantibus, radicibus tenuioribus; caulibus crassis rubrotinctis sed nunquam lignosis; foliis late linearibus aut lineari-lanceolatis, nervo medio prominente plerumque non albicante, internodiis plerumque longis quam foliis multo longioribus; pedunculis 8.2–14 cm. longis (plerumque ca. 11 cm.); capitulis radiis magnis (cf. no. 29).
Synonymia. —

Spilanthes disciformis Rob. var. *phaneractis* Greenm. in Proc. Am. Acad. xxxix, 108 (Sept. 25, 1903).

Distributio. — In civitatibus Jalisco et Michoacán in Mexico.
Specimina examinata. — **Mexico**: Jalisco: *C. G. Pringle*, 11312, apud aquae deiectum Juanacatlán prope Guadalajaram. Michoacán: *C. G. Pringle*, 8637 et 9530, Zamorae.

38. S. macrophylla Greenm. foliis ovatolanceolatis inter integra et distanter inaequaliterque dentata vel undulata variantibus, basi cuneata, apice acuminato, margine ciliolata; involucri squamis valde pubescentibus; radiis capitula aequantibus vel superantibus. — In Proc. Am. Acad. xxxix, 109 (Sept. 25, 1903).
Distributio. — In Honduras et Costa Rica.
Specimina examinata. — **Honduras**: Santa Barbara: *C. Thieme*, 5309, San Pedro. **Costa Rica**: San José: *H. Pittier*, 3717, in ripis fluminis Volcan, 10546, Alto del Zacatal (specimen typicum) (planta per chartam delineata). San Marcos: *A. Tonduz*, 7831, San Marcos.

39. S. papposa Hemsl. foliis ovatolanceolatis inter integra et paulum undulata variantibus, basi subcuneata, apice longe acuto sed non acuminato, margine non ciliolata; involucri squamis pilosulis; radiis capitula fere vel omnino aequantibus. — Biol. Centr.-Am. Bot. ii, 193 (Oct. 1881)
Distributio.— In Nicaragua.
Specimen examinatum: **Nicaragua**: Chontales: *R. Tate*, 1867–1868.
Synonymia. —

Ceratocephalus papposus (Hemsl.) Ktze. Rev. Gen. Pl. i, 326 (1891).

40. S. grandiflora Turcz. foliis inter ovata et ovatolanceolata

variantibus plerumque irregulariter serratis; radiis involucrum vix superantibus. — In Bull. Soc. Nat. Mosc. xxiv, 1, 185 (1851).

Synonymia. —

Ceratocephalus grandiflora (Turcz.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Distributio. — In insula Luzon in Insulis Philippinis et in Queensland in Australia.

Specimina examinata. — **Insulae Philippinae**: *H. Cuming*, 1154, in insula Luzon (cum specimine typico lectum). **Australia**: QUEENSLAND: *E. M. Bowman*, Rockhampton.

41. S. GRANDIFLORA Turcz. var. BRACHYGLOSSA Benth. praecedenti similis sed ligulis involucrum vix superantibus et foliis 2.5–5.5 cm. longis .5–.8 cm. latis. — Fl. Austr. iii, 541 (1866).

Synonymia. —

Spilanthes Acmella F. Muell. Fragm. Phyt. Austr. v, 63 (1865–1866), non L.

Distributio. — Ad rivum Sturt in Australia Occidentali.

Specimen examinatum. — **Australia**: AUSTRALIA OCCIDENTALIS: *Ferd. Mueller*, ad rivum Sturt (planta per chartam delineata).

42. S. GRANDIFLORA Turcz. var. CALVA Benth. praecedenti similis sed ligulis involucrum multo superantibus. — Fl. Austr. iii, 541 (1866).

Synonymia. —

Spilanthes macroglossa F. Muell. Fragm. Phyt. Austr. v, 63 (1865–1866).

Distributio. — Apud sinum Moreton in Queensland in Australia.

Specimen examinatum. — **Australia**: QUEENSLAND: Sine collectoris nomine, apud sinum Moreton.

43. **S. pammicrophylla** A. H. Moore spec. nov. erecta 15.5–18.5 cm. alta; caulibus glabris, surculis foliosis brevibus, radicibus fasciculatis; foliis minutissimis 7–15 mm. longis integris aut vix et sparse serratis sparse pubescentibus densis, internodiis inferioribus plerumque 1.5 cm. longis superioribus plerumque .4–1.3 cm. longis (regulariter ca. 8 cm.), petiolis 2–5 mm. longis; capitulis ovoideis .7–1 cm. longis, pedunculis 2.5–4.5 cm. longis tenuioribus; achaeniis margine ciliolatis inaequaliter biaristatis.

Distributio. — In Nicaragua.

Specimen typicum. — **Nicaragua**: *C. Wright*, Herb. U. S. No. Pac. Explor. Exped. (in Herb. Gray.) (specimen cum eodem lectum in Herb. Mus. Nat. Civ. Foeder. Am.).

44. S. ABYSSINICA Sch. Bip. prostrata vel ascendens dense sed minute pubescens vel subglabra, radicibus fasciculatis; foliis ovatis 1.5–4.5 cm. longis .8–3 cm. latis subsessilibus vel petiolatis usque ad 1.5 cm. longis;

radiis fere colore albo puro, pedunculis 2–8 cm. longis; achaeniis margine plerumque crassa. — In Théoph. Lefèbvre Voy. en Abyss. iv (Rich. Tent. Fl. Abyss. i), 415 (1847); nomen ex Hochst. in Flora xxiv, 1, "Intellbl." 26 (1841).

Distributio. — Prope Adowa in Abyssinia.

Specimen examinatum. — **Abyssinia**: TIGRÉ: *W. Schimper*, 134, prope Adowa (cum specimine typico lectum).

45. S. AMERICANA (Mut.) Hieron. pervariabilis erecta, radicibus non fasciculatis; foliis inter ovata et ovatolanceolata variantibus saepius crenatis quam serratis nonnunquam subintegris vel paulo serratis, apice inter obtusum et acutum variante (saepius obtuso), nunquam tam longis quam in no. 50 sed quam in nris. 46, 47, 48, 49 maioribus, petiolis brevibus vel longis. — Ex Sod. in Engl. Bot. Jahrb. xxix, 42 (Maio 22, 1900).

Synonymia. —

Anthemis americana Mut. ex L. f. Suppl. Syst. Veg. 378 (1781).
Anthemis oppositifolia Lam. Encycl. Meth. (Bot.) i, 576 (1783).
Anthemis occidentalis Willd. L. - Willd. Sp. Pl. iii, 2185 (1804).
Acmella occidentalis (Willd.) Rich. in Pers. Syn. Pl. ii, 473 (1807).
Spilanthes Mutisii HBK. Nov. Gen. et Sp. Pl. iv, 209 (1820); DC. Prod. v, 622 (1836).
Acmella ? *Mutisii* (HBK.) Cass. in Dict. Sc. Nat. xxiv, 331 (1822).
Spilanthes Beccabunga DC. Prod. v, 622 (1836).
Spilanthes Lehmanniana Klatt in Engl. Bot. Jahrb. viii, 43 (1886).
Spilanthes orizabaensis Sch. Bip. ex Klatt Leopold. xxiii, 5 (1887).
Spilanthes uliginosa var. Sch. Bip. ex Klatt l. c.
Spilanthes Sartorii Sch. Bip. ex Klatt l. c.
Ceratocephalus americanus (Mut.) Ktze. Rev. Gen. Pl. i, 326 (1891).
Ceratocephalus Beccabunga (DC.) Ktze. l. c.

Distributio. — In Cuba, Mexico, America Centrali, Colombia.

Specimina examinata. — **Mexico**: *T. Coulter*, 324; *C. J. W. Schiede*, 229; comm. C. Schaffner, det. Sch. Bip. SAN LUIS POTOSÍ: *J. G. Schaffner*, 338, "ex convalli San Luis Potosí," et 763, "San Luis Potosí"; *C. C. Parry* et *E. Palmer*, 464, maxime in regione San Luis Potosí. JALISCO: *C. G. Pringle*, 1821, Guadalajarae. REGIO FOEDERALIS: *C. G. Pringle*, 9964, prope Tlalpam; *E. Bourgeau*, 154, Chapultepec. MORELOS: *C. G. Pringle*, 9546, prope Cuautlam. VERA CRUZ: *M. B. Halsted*, 32, et *C. R. Barnes*, *C. J. Chamberlain*, *W. J. G. Land*, 9, prope Jalapam; *J. M. Greenman*, 25, La Laguna prope Vera Cruz, et 134, prope Cordobam; *E. Bourgeau*, 603 et 603 bis, prope Santa Fé, et 1630, in valle Cordobae; *A. Gray*, "Journey to Mexico and California, Feb., May, 1885," Cordobae; *C. Sartorius*, Mirador prope Huatucam; *H. E. Seaton*, 73, Orizabae. OAXACA: *E. W. Nelson*, 1032, prope La Parada; *L. C. Smith*, 131, Rancha de Calderas; *J. N. Rose, J. H. Painter, J. S. Rose*, 10055, prope Tomellin; *C. Conzatti* et *V. González*, 1241, Oaxacae. CHIAPAS: *A. Ghiesbreght*, 557. **Guatemala**: ALTA VERA PAZ: *H. v. Türckheim*, 758, Cobán, et 124 et *J. D. Smith*, 1604, prope Cobán. QUICHÉ: *H. T. Heyde* et *E. Lux*, 4501, Nebáj. QUEZELTENANGO: *F. C. Lehmann*, 1596, Quezeltenango. **Costa Rica**: SAN JOSÉ: *A. Tonduz*, 3030, San José. **Cuba**: HABANA: *A. H. Curtiss*, 684, et *J. W. Robbins*, Mart. 1864, prope Habanam; *J. A. Shafer*, 420, Habanae. **Colombia**: *J. Triana*, 1391. CAUCA: *F. C. Lehmann*, 3487, circum Puracé, et 3599, Chapae. **Colombia** aut **Ecuador**: *F. C. Lehmann*, 6446.

46. S. AMERICANA (Mut.) Hieron. var. **ramosa** (Hemsl.) A. H. Moore comb. nov. prostrata laxa vel procumbens; foliis 1.5–2.2 cm. longis ca. 7 mm. latis denticulatis; capitulis subglobosis (ca. 5 mm. diametro), discis gracillimis.

Synonymia. —

Spilanthes ramosa Hemsl. Biol. Centr.-Am. ii, 193 (Oct. 1881).

Ceratocephalus ramosus (Hemsl.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Distributio. — In valle Cordobae in Mexico.

Specimina examinata. — **Mexico**: VERA CRUZ: *E. Bourgeau*, 2284, in valle Cordobae; *H. E. Seaton*, 445, Cordobae.

47. S. AMERICANA (Mut.) Hieron. var. **parvula** (Rob.) A. H. Moore comb. nov. prostrata vel laxa vel procumbens glabra aut vix pubescens; foliis ovatis raro ellipticis crenatulis vel serratulis plerumque 1.8 cm. longis 1 cm. latis minoribus; capitulis plerumque 7–14 mm. longis.

Synonymia. —

Spilanthes Beccabunga DC. var. *parvula* Rob. in Proc. Am. Acad. xxvii, 176 (Nov. 2, 1892).

Distributio. — In Mexico, Guatemala, Costa Rica.

Specimina examinata. — **Mexico**: MEXICO: *C. G. Pringle*, 3643, Flor de Maria. **Guatemala**: ZACATEPÉQUEZ: *J. D. Smith*, 2125, Dueñas. SANTA ROSA: *H. T. Heyde* et *E. Lux*, 4209, Naranjo. **Costa Rica**: SAN JOSÉ: *A. Tonduz*, 449, San José.

48. S. AMERICANA (Mut.) Hieron. var. PARVULA (Rob.) A. H. Moore f. **parvifolia** (Benth.) A. H. Moore comb. nov. praecedenti omnino similis sed plus minusve conspicue pubescens.

Synonymia. —

Spilanthes parvifolia Benth. ex Oerst. in Vidensk. Meddel. Nat. For. Kjoeb. 100 (1852).

Spilanthes lateraliflora Klatt in Engl. Bot. Jahrb. viii, 43 (1886).

Ceratocephalus parvifolius (Benth.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Distributio. — In Guatemala et Costa Rica.

Specimina examinata. — **Guatemala**: ALTA VERA PAZ: *F. C. Lehmann*, 1319, circum Cobán; *H. v. Türckheim*, 8418, Cobán. **Costa Rica**: SAN JOSÉ: *A. Tonduz*, 12241, in pascuis Santa Rosa du Copey.

49. S. AMERICANA (Mut.) Hieron. var. PARVULA (Rob.) A. H. Moore f. **lanitecta** A. H. Moore f. nov. S. americanae (*Mut.*) *Hieron.* var. parvulae (*Rob.*) *A. H. Moore* omnino similis sed dense lanuginosa, maxime caules.

Distributio. — San Siguán in Guatemala.

Specimen typicum. — **Guatemala**: QUICHÉ: *H. T. Heyde* et *E. Lux*, 3381, San Siguán, alt. ca. 1770 m. (in Herb. Gray.) (specimen cum eodem lectum in Herb. J. D. Sm.).

50. S. AMERICANA (Mut.) Hieron. var. **repens** (Walt.) A. H. Moore comb. nov. magnopere variabilis pubescens vel glabra erecta vel laxa ascendens decumbens raro prostrata; foliis plerumque inter elliptica et lanceolata variantibus saepius serratis quam crenatis (inter subintegra et valde sed aequaliter serrata variantibus), apice inter obtusum et acuminatum variante saepius acuto vel acuminato, 2–12 cm. longis .7–4.5 cm. latis subsessilibus vel petiolis usque ad 4 cm. longis; pedunculis 3–12 cm. longis; capitulis plerumque 9–16 mm. longis. Haec varietas S. americanae (*Mut.*) *Hieron.* valde affinis est.

Synonymia. —

Anthemis repens Walt. Fl. Car. 211 (1788).
Spilanthes repens (Walt.) Michx. Fl. Bor.-Am. ed. I, ii, 131 (1803); DC. Prod. v, 623 (1836).
Acmella repens (Walt.) Rich. in Pers. Syn. Pl. ii, 473 (1807).
Acmella occidentalis? Nutt. Gen. N. Am. Pl. ii, 171 (1818).
Acmella Nuttalliana Raf. New Fl. N. Am. i, 52 (1836).
Spilanthes Nuttallii T. et G. Fl. N. Am. ii, 356 (Apr. 1842).
Ceratocephalus repens (Walt.) Ktze. Rev. Gen. Pl. i, 326 (1891).

Distributio. — In civitatibus australibus Civitatum Foederatorum Americae, Missouri, Arkansas, Louisiana, Mississippi, Texas, Carolina Australi, Florida.

Specimina examinata. — **Civitates Foederatae Americae**: *A. Gray*, in Civitatibus Australibus, 1846; Mexican Boundary Survey, 587 (Civ. Foeder. Am. ?). MISSOURI: *G. W. Letterman*, Aug. 15, 1875, et *C. Russell*, Sept. 1897, Poplarbluff; *F. W. Dewart*, Poplarbluff, Aug. 14, 1892, et Neeleysville, Oct. 2, 1892; *H. Eggert*, prope Neeleysville, Aug. 8, 1893, et prope Asheville, Aug. 19, 1892; *B. F. Bush*, 176, Malden, et Campbell, Sept. 18, 1893, 178, Kennett, et Iunct. Paw Paw, Sept. 15, 1893, et 82, in comitatu New Madrid; *W. Trelease*, 544, Bertig. ARKANSAS: *F. V. Coville*, 99; *B. F. Bush*, 248, Marked Tree; *W. Tre-*

lease, in insula Rattlesnake in flumine St. Francis, Aug. 29, 1897; *G. W. Letterman*, Little Rock, Aug. 29, 1879. Louisiana: *M. C. Leavenworth; J. Gregg*, prope Shreveport, Sept. 14, 1847; *E. L. Moseley*, in parochia West Carroll, Iul. 17, 1903; *J. D. Smith*, in parochia Concordia, Sept. 9, 1885; *C. R. Ball*, 639, prope Alexandriam; Baton Rouge, Sept. 14, 1867, comm. W. R. Dodson, an. 1896; *K. K. Mackenzie*, 477, prope lacum Charles; *S. M. Tracy*, 2485, 8586, et "ex Herb. J. F. Joor," Nov. 1870, New Orleans. Mississippi: *J. D. Smith; T. Nuttall*, comm. E. Durand, an. 1866. Texas: *J. Reverchon*, 161, "W. Texas"; *S. M. Tracy*, 7329, Pierce; *B. F. Bush*, 118, 295, 712, 941, 1287, Columbiae; *E. Hall*, 348, *B. F. Bush*, 252, *L. F. Ward*, Sept. 12, 1877, *F. Lindheimer*, Aug., et Sept. 1842, Houston; *J. Reverchon*, Sept. 6, 1903, et 1535, in comitatu Ellis; *J. M. Bigelow*, Las Moros. Carolina Australis: *W. Ravenel*, ad flumen Santee (regionem typicam), Sept. 1846. Florida: *M. C. Leavenworth*, "Florida," et "E. Florida"; *A. S. Hitchcock*, in comitatu Columbia, Iun.–Iul. 1898; *F. Rugel*, prope St. Marks; *M. C. Reynolds*, St. Augustine, Sept.–Oct. 1875; *A. H. Curtiss*, 6000, prope Chattahoochee, et prope Daytonam, Feb. 1881; *A. P. Garber*, in comitatu Levy, Nov. 1877, et Tampae, Maio 1876 et Oct. 1877; *S. M. Tracy*, 7148, Braidentown (forma extrema); *J. H. Simpson*, Manatee, an. 1889; *A. W. Chapman*, ad flumen Caloosahatchee.

51. S. americana (Mut.) Hieron. var. **stolonifera** (DC.) A. H. Moore comb. nov. S. americanae (*Mut.*) *Hieron.* var. repenti (*Walt.*) *A. H. Moore* valde persimilis sed foliis linearibus integris aut vix denticulatis (saepius integris) 1.7–5.5 cm. longis .2–1 cm. latis subsessilibus vel sessilibus (petiolis quam in varietate dicta plerumque brevioribus); pedunculis 4.5–17.5 cm. longis. Per varietatem dictam S. americanae (*Mut.*) *Hieron.* affinis.

Synonymia. —

Spilanthes stolonifera DC. Prod. v, 621 (1836).

Distributio. — In Paraguay et in Civitatibus Foederatis Americae adventive in Nova Caesarea et probabiliter adventive in Carolina Boreali Australique et Florida.

Specimina examinata. — **Civitates Foederatae Americae**: Nova Caesarea: *G. A. Gross*, adventive Camden, Iul. 3, 1891. Carolina Borealis: *G. McCarthy* 5, et *F. V. Coville*, 166, Wilmington. Carolina Australis: G. McCarthy, xxxi, "in oriente Carolina Australis." Florida: *A. H. Curtiss*, 5882, in parte, Carrabelle. **Paraguay**: *E. Hassler*, 1639, et 3370, San Bernardino; *T. Morong*, 89, Asunción; *K. Fiebrig*, 345, in Cordillera de Altos. **Specimen cultum**: *J. Gay*, Parisiis, Sept. 1839.

52. S. AMERICANA (Mut.) Hieron. var. STOLONIFERA (DC.) A. H. Moore f. **longiinternodiata** A. H. Moore f. nov. glabra; internodiis longis (plerumque 3.5 cm.); foliis conspicue acute et distanter dentata (spatia interdentalia .5–1 cm. longa).

Distributio. — Apud Carrabelle in Florida in Civitatibus Foederatis Americae.

Specimen typicum. — **Civitates Foederatae Americae**: FLORIDA: *A. H. Curtiss*, 5882, in parte, Carrabelle (in Herb. Gray.) (specimina cum eodem lecta in Herbb. Hort. Bot. Mo., Mus. Nat. Civ. Foeder Am., Hort. Bot. Novebor., Mus. Hist. Nat. Field).

53. S. AMERICANA (Mut.) Hieron. var. STOLONIFERA (DC.) A. H. Moore f. **ciliatifolia** A. H. Moore f. nov. pubescens; foliis margine plus minusve ciliata.

Distributio. — In Argentina.

Specimen typicum. — **Argentina**: *P. G. Lorentz*, 76 (in Herb. Gray.).

54. S. BLEPHARICARPA DC. foliis linearilanceolatis angustioribus distanter margine ciliatis, apice acuto; achaeniis valde ciliatis. — DC. Prod. v, 621 (1836).

Distributio. — In provincia Rio Grande in Brasilia.

Specimen examinatum. — **Brasilia**: RIO GRANDE: Herb. Mus. Imp. Bras., 1030 (planta per chartam delineata).

55. **S. decumbens** (Sm.) A. H. Moore comb. nov. sparse pubescens, maxime in foliis, radicum fasciculis reptans; foliis basalibus saepe quam caulinis maioribus ovatis crenatis, caulinis ovatis vel lanceolatis vel linearispatulatis angustioribus et subintegris vel integris, basi saepe attenuata; capitulis radiis latis, pedunculis 14.5–23 cm. longis.

Synonymia. —

Rudbeckia decumbens Sm. in Rees Cycl. vel Univ. Dict. Art. Sc. Litt. ed. anglica, xxx, sect. ii, parte 60, no. 11 (1815).
Rudbeckia bellioides Sm. l. c. no. 12.
Verbesina buphthalmoides Lk. et Ott. Ic. Pl. Select. Hort. Reg. Bot. Berol. 105, t. 49 (1828).
Spilanthes arnicoides DC. Prod. v, 620 (1836).
Spilanthes doronicoides DC. l. c.
Spilanthes helenioides H. et A. in Hook. Journ. Bot. iii, 317 (1841).
Ceratocephalus decumbens (Sm.) Ktze. Rev. Gen. Pl. i, 326 (1891).
Ceratocephalus arnicoides (DC.) Ktze. l. c.
Ceratocephalus decumbens (Sm.) Ktze. var. *doronicoides* (DC.) Ktze. l. c. iii, 140 (1898).

Distributio. — In Brasilia et Uruguay.

Specimina examinata. — **Brasilia**: *F. Sellow*, 1969. **Uruguay**: MONTEVIDEO: *A. Isabelle*, Montevideo, an. 1838.

56. S. DECUMBENS (Sm.) A. H. Moore var. **macropoda** (DC.) A. H. Moore comb. nov. foliis quam in praecedente plerumque paulo angustioribus duarum formarum fieri inclinatis, linearibus (4–8 cm. latis), lanceolatis (plerumque 1.5 cm. latis), ambabus 4.5–6 cm. longis; sine foliis basalibus formae peculiaris; pedunculis 11–17 cm. longis. Plantae nonnunquam folia latiora solum habent.

Synonymia. —

Spilanthes macropoda DC. Prod. v, 621 (1836).

Spilanthes arnicoides DC. var. *macropoda* (DC.) Bak. in Mart. Fl. Bras. vi, **3**, 234 (Maio 1, 1884).

Ceratocephalus decumbens (DC.) Ktze. *β. macropodus* (DC.) Ktze. Rev. Gen. Pl. iii, 140 (1898).

Distributio. — In Brasilia.

Specimina examinata. — **Brasilia**: *F. Sellow*, 1764 et 3522; *A. F. Regnell*, ser. II, 168.

57. S. DECUMBENS (Sm.) A. H. Moore var. **leptophylla** (DC.) A. H. Moore comb. nov. repens; foliis densis anguste linearibus (2–4 mm. latis 1.5–2.5 cm. longis) sessilibus subsessilibusve; pedunculis 5.4–8.5 cm. longis.

Synonymia. —

Spilanthes leptophylla DC. Prod. v, 621 (1836).

Spilanthes arnicoides DC. var. *leptophylla* (DC.) Bak. in Mart. Fl. Bras. vi, **3**, 234 (Maio 1, 1884).

Distributio. — In Brasilia.

Specimen examinatum. — **Brasilia**: *F. Sellow*, 2796.

58. **S. grisea** (Chod.) A. H. Moore comb. nov. erecta vel ascendens valde subpubescens, maxime folia et involucri squamae. Folia inter lanceolata et ovata variantia obtusa vel subacuta dentata dense hispidulosa. Capitula maxima; radii maximi (cf. no. 62).

Synonymia. —

Spilanthes arnicoides DC. f. *grisea* Chod. in Bull. Herb. Boiss. ser. 2, i, 417 (1901).

Spilanthes arnicoides DC. var. *grisea* Chod. l. c. iii, 725 (1903).

Distributio. — In Paraguay.

Specimina examinata. — **Paraguay**: *E. Hassler*, 1211, prope Tacuaral, et 4659, prope flumen Jejui Guazu.

59. S. GRISEA (Chod.) A. H. Moore var. **intermedia** (Chod.) A. H. Moore comb. nov. sparse ciliata; foliis quam in praecedente angustioribus 3–7 cm. longis .7–2 cm. latis.

Synonymia. —

S. arnicoides DC. var. *intermedia* Chod. in Bull. Herb. Boiss. ser. 2, iii, 725 (1903).

Distributio. — In regione cursus superioris fluminis Apa in Paraguay.

Specimen examinatum. — **Paraguay**: *E. Hassler*, 8273, in regione cursus superioris fluminis Apa.

60. S. GRISEA (Chod.) A. H. Moore var. **setosa** (Chod.) A. H. Moore comb. nov. maxime foliis margine conspicue ciliatis; foliis quam in no. 58 longioribus 7–10 cm. longis 1–2.2 cm. latis.

Synonymia. —

S. arnicoides DC. var. *setosa* Chod. in Bull. Herb. Boiss. ser. 2, iii, 725 (1903).

Distributio. — Prope flumen Capibary in Paraguay.

Specimen examinatum. — **Paraguay**: *E. Hassler*, 4475, prope flumen Capibary.

61. S. GRISEA (Chod.) A. H. Moore var. **Chodatana** A. H. Moore var. nov. foliis minute ciliatis distanter serratis quam in no. 59 angustioribus 4.8–8 cm. longis .5–8 cm. latis, serrationibus quam in duabus praecedentibus prominentioribus.

Distributio. — In regione cursus superioris fluminis Apa in Paraguay.

Specimen typicum. — **Paraguay**: *E. Hassler*, 7651, in regione cursus superioris fluminis Apa (in Herb. Chod.).

Appellatio. — Ex nomine doctoris R. Chodat, qui hanc novam esse varietatem iudicavit, derivata. Nota in titulo dicit "var. acced. ad var. macropoda."

62. S. GRISEA (Chod.) A. H. Moore var. **micra** A. H. Moore var. nov. Caules pendunculique pubescentes. Folia margine ciliata et praeterquam nervum medium glabriuscula ad basin subconserta. Pedunculi 5.5–7 cm. longi. Capitula maiora; radii maiores (cf. no. 58).

Distributio. — Prope Tacuaral in Paraguay.

Specimen typicum. — **Paraguay**: *E. Hassler*, 925, prope Tacuaral (in Herb. Hort. Bot. Novebor.).

63. **S. eurycarena** A. H. Moore spec. nov. erecta vel ascendens, basi procumbente; foliis anguste linearibus (2–5 mm. latis 2–3.8 cm. longis) sessilibus vel brevipedunculatis densioribus; capitulis latissimis (1.5–2.5 cm.) 1–1.8 cm. longis, discis crassissimis (plerumque 4 mm. latis 1 cm. longis), radiis brevibus latisque ca. 7 mm. longis 4–5 mm. latis; pedunculis 12–19.5 cm. longis; achaeniis margine ciliatis inaequaliter biaristatis.

Distributio. — Rio Negro in Patagonia boreali in Argentina et circum Buenos Aires.

Specimen typicum. — **Argentina**: DEL RIO NEGRO: U. S. So. Pac. Explor. Exped., Rio Negro (in Herb. Gray.) (specimen cum eodem lectum in Herb. Hort. Bot. Novebor.).

Specimen alium examinatum. — **Argentina**: BUENOS AIRES: "*Miss Parker*," Buenos Aires.

ANNOTATIONES.

SPILANTHES Jacq. Nomen in forma Spilanthes primo a Jacquin usurpatum est, sed a Linnaeo in editione duodecima Systemae Naturae sub forma Spilanthus datum est et postea hac forma Jacquin ipse usus est. Descriptio autem originalis, quamquam brevis, formam Spilanthes certam facit. S. urens et S. insipida primo a Jacquin descriptae sunt, quamobrem S. urens generis species typica est. DeCandolle duas sectiones recognovit, viz. Salivariam et Acmellam, quarum posterior imprimis a Richard pro genere descripta erat.

Sectio I. SALIVARIA DC.

1. S. CHAMAECAULA A. H. Moore. Haec species inter insignissimas generis nostri est, non solum habitu prostrato, quem paucae species aliae saltem in parte habent, sed etiam quod achaenia aristam singulam ferunt. Species generis ceterae aut biaristatae aut inaristatae sunt. Specimen typicum sub nomine S. anactinae *F. Muell.* distributum est, a qua tamen characteribus technicis habituque valde differt.

2. S. NERVOSA Chod. Haec species foliis magis ovatis et nervis anastomosantibus prominentibus a sequente differt.

3. S. URENS Jacq. Huius speciei tabula optima, *Jacq. Select. Stirp. Am. Hist. t. 126, f. 1,* hanc plantam esse quam Jacquin indicaverit sine dubio ostendit. Variationes duae notatae sunt: —

4. S. URENS Jacq. f. LANEA A. H. Moore forma lanea et

5. S. URENS Jacq. var. HISPIDULA DC. imprimis natura plus hispidulosa sed etiam habitus differentiis paucis, foliorum formis, etc., discriminatur.

6. S. ANACTINA F. Muell. Haec species S. urenti *Jacq.* antheris apice nigris foliisque integris similis est, sed foliis plus acuminatis plerumque angustioribus nunquam revolutis differt. Iudicare licet hanc speciem, cuius natura specimine a F. Mueller authenticato certa fiat, S. urenti *Jacq.*, speciei maxime Indiarum Occidentalium, respondere.

7. S. PUSILLA H. et A. Haec species parva gracilisque a praecedentibus omnibus caulibus tenuibus parvisque et foliis angustis linearispatulatis differt. Figura speciminis originalis per chartam delineata huius speciei characteres noti sunt.

8. S. INSIPIDA Jacq. Haec species foliis valde et maxime obtuse sinuatodentatis in sectione unica est. E tabula, *Jacq. Select. Stirp. Am. Hist. t. 126, f. 2,* quamquam imperfectissima, cum descriptione bona, *l. c. 215,* S. insipidae natura certa est.

9. S. OLERACEA L., saepe sed errore cum auctoritate Jacq. citata, species a ceteris discis magnis oblongis (vero in genere maximis) foliis-

que deltoideis longe abest. Figura prima bona, quod sciam, est *Jacq. Hort. Bot. Vind. ii, t. 135.* Recenter nomen S. oleracea pro synonymo S. Acmellae (*L.*) *Murr.* habitum est, potius, crediderim, quia haec ignoscebatur quam illa.

10. S. LEUCANTHA HBK. sequenti valde affinis est et charactere solum technico differt, eo nempe quod multas bracteas habet.

11. S. OCYMIFOLIA (Lam.) A. H. Moore. Haec species communiter *S. alba* Willd. (errore incertae originis pro L'Hér.) appellata est. *Spilanthus Salivaria* Domb., re vera a L'Héritier una cum *Spilanthe alba* anno 1874 editus, falso dicitur iam antea publicatus esse. Anno 1873 Lamarck in encyclopaedia sua *Bidentem ocymifoliam* pro synonymo *S. albae* edidit. Apud *Lam.-Poir. Illustr. Genres iii, t. 668* tabula invenitur quae sine dubio *S. albam* typicam repraesentat. Inde *Bidens ocymifolia* combinatio prima speciei nostri est. S. ocymifolia habitu pervariabilis praecedentibus multo latius distributa est.

12. S. OCYMIFOLIA (Lam.) A. H. Moore f. RADIIFERA A. H. Moore inter sectiones Salivariam et Acmellam radiis intermedia est. Haec forma, cum specie coextensa, multas eiusdem variationes participat.

13. S. OCYMIFOLIA (Lam.) A. H. Moore var. ACUTISERRATA A. H. Moore. Haec varietas a specie plerumque capitibus abundantioribus et foliorum serrationibus prominentioribus magis acutis differt, sensim in eandem transgreditur.

14. S. CALVA DC. a sequente achaeniis inaristatis habitu tenuiori et foliis minoribus differt. Nulla species alia inaristata descripta esse videtur. S. calvae natura e figura per chartam delineata ex Herbario DeCandolle mihi transmissa certa fit. (Vide etiam infra sub no. 15.)

15. S. ACMELLA (L.) Murr. Huius speciei naturam excogitare difficillimum erat. Figurae a Linnaeo citatae rudes sunt ac speciei status ab auctoribus disputatur. Nonnunquam etiam in sectione specierum radiatarum inclusa est. Specimina multa e regione typica examinata nullas species radiatas ibi inveniri ostendunt. Quod Hooker in *Flora of British India* de natura S. Acmellae radiorum scripsit contrarium non probat, nam infra sub titulo "var. Acmella proper," *Wight Ic. Pl. Ind. Or. iii, 1109*, tabulam citat quae sine dubio ligulas nullas exhibet. Quod ad discrimen inter S. Acmellam et S. calvam attinet, figurae a Linnaeo citatae plantam habitu crasso repraesentare videntur. (Vide etiam sub no. 14.)

16. S. ACMELLA (L.) Murr. var. ALBESCENTIFOLIA A. H. Moore a praecedente foliis plerumque maioribus infra pallescentibus vel albescentibus differt.

17. S. ACMELLA (L.) Murr. var. LANCEOLATA (Lk.) A. H. Moore. Haec varietas a specie foliis magnis valde et irregulariter incisis

differt. Descriptio *S. Pseudacmellae* Spreng., non (L.) Murr., huic varietati S. Acmellae simillima est. Nomen *S. Pseudo-Acmella* aut *Pseudacmella* ad tam multas species diversas pertinet ut usurpari non opporteat. Nomen *Acmella lanceolata* Lk. var. a Sprengel pro synonymo superioris editum est et nomen primum est quae usurpari potest.

18. S. COSTATA Benth. a S. Acmella foliis integris vix margine undulatis facile distinguitur.

19. S. CALLIMORPHA A. H. Moore species nova pulcherrima chinensis est. Internodiis longis laxis sed gracilibus et foliis serratis vel subincisis (quam in S. Acmella (*L.*) *Murr.* var. lanceolata (*Lk.*) *A. H. Moore* multo angustioribus) a speciebus sectionis ceteris longe abest.

Sectio II. ACMELLA (Rich.) DC.

Subsectio I. PARVORADIATAE A. H. Moore.

20. S. IODISCAEA A. H. Moore. Haec species lepidissima capitulis minimis, radiis minimis et foliis parvis tenuibus minute et sparse denticulatis necnon (praeter formam leucaenam) paleis sursum violaceis, disco toto visum violaceum praebentibus, a speciebus generis ceteris valde differt.

21. S. IODISCAEA A. H. Moore f. LEUCAENA A. H. Moore. Haec forma variationem probabiliter raram et certe insignificantem exhibet.

22. S. ULIGINOSA Sw. habitu pervariabilis est, sed folia et plerumque capitula crassiora quam in duabus praecedentibus habet. Una cum eisdem characterem habet qui turmam totam distinguit, viz. quod involucri squamae inter late ovatas et suborbiculares variant. Sicut paucae species aliae Indiarum Occidentalium et Americae Centralis, S. uliginosa distributionem miram habet, viz. in Indiis Occidentalibus, America Centrali, Africa occidentali.

23. S. LUNDII DC. praecedenti simillima est, sed involucri squamae magis lanceolatae sunt.

24. S. CILIATA HBK. Haec species inter totum genus maxime variabilis et forsan dificillima est, et in futuro studium diligentissimum exiget.

25. S. CILIATA HBK. var. DIFFUSA (Poepp.) A. H. Moore a praecedente forma prostrata et foliis ovatis, basi paulum subcuneata, differt.

26. S. SUBHIRSUTA DC. Haec species a praecedente charactere subtomentoso praesertim differt.

27. S. CAESPITOSA DC. a S. subhirsuta natura leviori, et a S. ciliata foliis parvis, apice mucronuloideo, differt.

28. S. POLIOLEPIDICA A. H. Moore a praecedentibus involucro canescente ligulisque subaureis facile distinguitur.

29. S. DISCIFORMIS Rob. species procumbens; caulibus ascendentibus, maxime caulibus erectis rubrotinctis caulibus prostratis nodis radicantibus; foliis nervo medio prominente albicante.

30. S. LIMONICA A. H. Moore a praecedente habitu laxo sed erecto et foliis latioribus margine ciliolatis differt.

31. S. ALPESTRIS Griseb. Haec species capitula fere planoconvexa et folia sine ciliolis in margine habet. Eiusdem speciei natura tantummodo a specimine, cum descriptione originali congruente, a P. G. Lorentz et G. Hieronymo in regione typica lecto determinabilis erat.

32. S. MAURITIANA (Rich.) DC. a praecedente facile distinguitur foliis non integris vel fere integris. Nec illi speciei nec ulli coniuncte affinis est.

33. S. MAURITIANA (Rich.) DC. f. MADAGASCARIENSIS (DC.) A. H. Moore a specie solum capitulis oblongis nec globosis truncatis nec conicis differt.

Subsectio II. MAGNORADIATAE A. H. Moore.

34. S. STENOPHYLLA H. et A. foliis linearibus angustissimis facile dinoscitur.

35. S. FILIPES Greenm. capitula parva pedunculos tenuissimos et foliorum bases cuneatoattenuatas habet, sed radii ad magnitudinem disci relati longissimi sunt. Species pulcherrima est.

36. S. IABADICENSIS A. H. Moore. In hac specie capitula bis tantum longiora quam in praecedente sed radii ad magnitudinem disci relati breviores quam in generis speciebus ceteris sunt.

37. S. PHANERACTIS (Greenm.) A. H. Moore S. disciformi *Rob.* habitu simillima est, sed praesertim radiis multo maioribus Magnoradiatis magis affinis esse videtur.

38. S. MACROPHYLLA Greenm. foliorum basibus longicuneatis apicibusque acuminatis facile distinguitur.

39. S. PAPPOSA Hemsl. radios maximos habet, sed a sequente involucri natura pilosulata longe abest.

40. S. GRANDIFLORA Turcz. Haec species radios maximos habet, sed a turmis S. decumbentis (*Sm.*) *A. H. Moore* et S. griseae (*Chod.*) *A. H. Moore* radicibus tenuioribus non fasciculatis distinguitur. A varietatibus sequentibus foliis non linearibus differt. (Vide etiam sub no. 39.)

41. S. GRANDIFLORA Turcz. var. BRACHYGLOSSA Benth. a specie et

varietate sequente radiis involucrum vix superantibus differt. (Vide etiam sub no. 40.)

42. S. GRANDIFLORA Turcz. var. CALVA Benth. a praecedente radiis involucrum multo superantibus differt. Character calvus probabiliter inconstans est, sed e figura singula per chartam delineata hoc factum indeterminabile erat.

43. S. PAMMICROPHYLLA A. H. Moore. Haec species unica plantas tenuissimas et folia in genere toto minima habet.

44. S. ABYSSINICA Sch. Bip. habitu S. americanae (*Mut.*) *Hieron.* var. parvulae (*Rob.*) *A. H. Moore* similis est, sed radii albissimi non lutei sunt.

45. S. AMERICANA (Mut.) Hieron. sub nomine *Anthemidis americanae* Mut. ex L. f. Supplemento primo descripta et recentius a Humboldt, Bonpland, et Kunth in honore Mutis *S. Mutisii* appellata plerumque sub nomine *S. Beccabungae* DC. nota est. In hac turma *Anthemis americana* nomen vetustissimum et inde primarium est. Inter specimina plurima, quae a me examinata sunt, gradationes perspicuae inventae sunt, quae distinctiones inter species varietatesque huius turmae olim habitas, quamquam in formis extremis diversissimas, innaturales et eapropter reiiciendas esse ostendunt.

46. S. AMERICANA (Mut.) Hieron. var. RAMOSA (Hemsl.) A. H. Moore a praecedente receptaculis tenuibus habituque prostrato differt.

47. S. AMERICANA (Mut.) Hieron. var. PARVULA (Rob.) A. H. Moore. Haec varietas a specie habitu prostrato et foliis parvis differt. Descriptio prima sub nomine *S. Beccabungae* DC. var. *parvulae* Rob. edita est. Nomen mutavi nomenclaturae rationibus solum adductus.

48. S. AMERICANA (Mut.) Hieron. var. PARVULA (Rob.) A. H. Moore f. PARVIFOLIA (Benth.) A. H. Moore a praecedente modo natura paulum hispidulosa differt. Hanc esse formam primo a Bentham sub nomine *S. parvifoliae* descriptam a specimine originali per chartam delineato manifestum est. Combinatio autem infelix est, nam folia non minora sunt quam in varietate, sed nomenclaturae rationibus facta est.

49. S. AMERICANA (Mut.) Hieron. var. PARVULA (Rob.) A. H. Moore f. LANITECTA A. H. Moore. Haec forma lanuginosa est.

50. S. AMERICANA (Mut.) Hieron. var. REPENS (Walt.) A. H. Moore a specie fere foliis magis acuminatis, serrationibus magis acutis, differt. Distributio etiam maxima in parte distincta est. Varietates autem formaeque praecedentes saltem in parte eadem in regione inventae sunt. Formae extremae facile dinotae sunt, sed formae aliquae intermediae sine notitia distributionis difficile distinguuntur.

51. S. AMERICANA (Mut.) Hieron. var. STOLONIFERA (DC.) A. H. Moore. Inter hanc varietatem et praecedentem, per quam tantummodo

speciei affinis est, gradationes aequales intersunt, quamquam formae extremae duarum varietatum atque distributio earundem omnino dissimiles sunt. Forma typica folia linearia integra habet.

52. S. AMERICANA (Mut.) Hieron. var. STOLONIFERA (DC.) A. H. Moore f. LONGIINTERNODIATA A. H. Moore a varietate internodiis longis et foliorum serrationibus distantibus prominentibusque differt.

53. S. AMERICANA (Mut.) Hieron. var. STOLONIFERA (DC.) A. H. Moore f. CILIATIFOLIA A. H. Moore a no. 51 foliis ciliatis differt.

54. S. BLEPHARICARPA DC. quoque folia linearia aut linearilanceolata ciliis margine distantibus sed habitum erectum habet. (Vide etiam sub nris. 51 et 53.)

55. S. DECUMBENS (Sm.) A. H. Moore. Huius speciei et varietatum sequentium nomina solum rationibus nomenclaturae mutata sunt. Nomen *Rudbeckia decumbens* Sm. plerumque pro synonymo *S. arnicoidis* DC. ductum est. Haec opinio sine dubio vera est, nam, quamquam specimen originale inspici non potuit, descriptio originalis lucidissima est. Haec turma radiis magnis et radicibus fasciculatis distinguitur. Folia basalia, dummodo extant, in forma rosulae disposita a caulinis valde differunt.

56. S. DECUMBENS (Sm.) A. H. Moore var. MACROPODA (DC.) A. H. Moore. Haec varietas plerumque foliis diversis et basi subterranea magna distinguitur, sed ambo characteres variabiles inconstantesque sunt.

57. S. DECUMBENS (Sm.) A. H. Moore var. LEPTOPHYLLA (DC.) A. H. Moore foliis linearibus a praecedentibus dinoscitur.

58. S. GRISEA (Chod.) A. H. Moore. Haec species primo a Chodat pro varietati *S. arnicoidis* DC. habita est; Chodat etiam iuste existimat varietates sequentes *griseae* valde affines esse. Mihi autem haec species propius accedere ad varietates sequentes quam ad *S. decumbentem* (Sm.) A. H. Moore (*S. arnicoidem* DC.) varietatesque eiusdem videtur. Inde optimum puto eam speciem typicam facere turmae, affinis sane *S. decumbenti*, paulo tamen discrepantis. S. grisea a praecedentibus natura pubescente facile distinguitur.

59. S. GRISEA (Chod.) A. H. Moore var. INTERMEDIA (Chod.) A. H. Moore. In hac varietate folia angustiora quam in praecedente et sparse ciliata sunt.

60. S. GRISEA (Chod.) A. H. Moore var. SETOSA (Chod.) A. H. Moore. Folia longiora quam in specie sed latiora quam in varietate praecedente margine valde ciliata, sunt.

61. S. GRISEA (Chod.) A. H. Moore var. CHODATANA A. H. Moore. Haec varietas folia quam in ceteris varietatibus angustiora cum serrationibus distantibus habet.

62. S. GRISEA (Chod.) A. H. Moore var. MICRA A. H. Moore. In hac varietate folia foliis speciei similia sunt, sed minora et maxime in marginibus et nervo medio ciliolata.

63. S. EURYCARENA A. H. Moore capitulis latissimis longipedunculatis a ceteris generis speciebus facile distinguitur.

NOMINA NUDA QUORUM SYNONYMIA IGNOTA EST.

Spilanthes Arrabidae *Hort.* ex Teijsm. et Binnend. Cat. Pl. Hort. Bot. Bogor. 105 (1866).
S. deltoidea *Wall.* Cat. 3185 / 295 (Dec. 1, 1828).
S. leucocephala *Ott.* Ind. Sem. Hort. Bot. Berol., an. 1845, collect. 6 (1845).
S. multiflora *Ott.* ex Sweet Hort. Brit. ed. II, 306 (1830).
S. mysurensis *Wall.* Cat. 3185 / 295 (Dec. 1, 1828).
S. pallida *Sweet* Hort. Brit. ed. II, 306 (1830).
S. Pseudo-Acmella *H.* et *A.* Bot. Capt. Beech. Voy. 150 (1841), non (L.) Murr. secundum Hook. f. et Jack. Ind. Kew. iv, 963 (1895).
S. rhombifolia *Zipp.* ex Span. Prod. Fl. Timor. in Linnaea xv, 323 (1841).

SPECIES FORMAEQUE NON SATIS COGNITAE SPILANTHIS VEL AD SPILANTHEM ATTRIBUTAE.

Sectio I. SALIVARIA DC.

Spilanthes diffusa *Hook. f.* in Trans. Linn. Soc. xx, 214 (1847).
S. diffusa *Hook. f.* f. minor *Hook. f.* l. c.
S. grandifolia *Miq.* Fl. Ind. Bat. ii, 80 (1856).
S. intermedia (*Rich.*) *DC.* Prod. v, 624 (1836).
Acmella intermedia Rich. in Pers. Syn. Pl. ii, 472 (1807).
S. javanica *Sch. Bip.* ex Miq. Fl. Ind. Bat. (1856–1859) ; nomen Sch. Bip. in Zoll. Syst. Verz. Ind. Arch. 123 (1854–1855) ; descriptio "Spilanthes sp. n." *Zoll.* in Nat. en Geneesk. Arch. ii, 255 (1845) secundum Miq. l. c.
Ceratocephalus javanicus (Sch. Bip.) Ktze. Rev. Gen. Pl. i, 326 (1891).
S. macropoda *Turcz.* in Bull. Soc. Nat. Mosc. xxiv, 2, 71 (1851).
Ceratocephalus macropodus (Turcz.) Ktze. Rev. Gen. Pl. i, 326 (1891).
S. portoricensis *Spreng.* L. – Spreng. Syst. Veg. iii, 444 (1826) ; *S. portoriccensis* Spreng. secundum DC. Prod. v, 625 (1836), secundum DC. autem probabiliter = S. urens *Jacq.*

Sectio II. ACMELLA (Rich.) DC.

S. sp. *Lor.* et *Niederl.* in Inf. Off. Exped. Rio Negr. (Patag.) Entr. ii, Bot. 238 (1881).
S. affinis *H.* et *A.* in Hook. Jour. Bot. iii, 317 (1841).
S. arnicoides *DC.* f. minor *Chod.* in Bull. Herb. Boiss. ser. 2, iii, 725 (1903).
S. arnicoides *DC.* f. nervosa *Chod.* l. c.
S. commutata *Koch* ex Ind. Sem. Hort. Berol. App. 14 (1855).
  S. repens Hort. Par. et Berol. ex Koch l. c.
  Ceratocephalus commutatus (Koch) Ktze. Rev. Gen. Pl. i, 326 (1891).
Eclipta filicaulis *Schumach.* Beskr. Guin. Pl. ii, 164 (1827).
Feaea linearis *Spreng.* L.-Spreng. Syst. Veg. iii, 581 (1826).
  Selloa ? linearis (Spreng.) DC. Prod. v. 612 (1836).
Spilanthes longifolia *DC.* Prod. v, 621 (1836).
S. melampodioides *Gardn.* in Hook. Lond. Jour. Bot. vii, 407 (1848).
S. sphaerocephala *DC.* Prod. v, 621 (1836).

SUBGENUS EXCLUSUM.

Helepta Raf., subgenus *Acmellae* Rich., New Fl. N. Am. i, 52 (1836) = Heliopsis *Pers.* Genus *Helepta* Raf. Neogen. 3 (1825).

SPECIES EXCLUSAE.

Spilanthes arborea (Forst. et Forst. f.) Forst. f. in Comm. Soc. Goett. (Phys.) ix, 67 (1787); DC. Prod. v, 626 (1836), sub. spp. exclus. = Laxmannia arborea *Forst.* et *Forst. f.* Char. Gen. 94, t. 47 (1776).
Spilanthes atriplicifolia L. Syst. Nat. ed. XII, iii, 236 (1768).
  Spilantus atriplifolius R. W. Darw. Fam. Pl. ed. II, ii, 544 (1787) = Isocarpha atriplicifolia (*L.*) *R. Br. Bidens atriplicifolia* L. Amoen. Acad. iv, cent. pl. 2, 329 (Iun. 11, 1756).
Spilanthes atriplicifolia Houtt. ex Miq. Fl. Ind. Bat. ii, 37 (1856) = Dichrocephala latifolia (*Pers.*) *DC.*
Spilanthes bicolor (DC.) Benth. et Hook. f. ex Hemsl. Biol. Centr.-Am. Bot. ii, 192 (Iun. 1881), non Gen. Pl.
  Ceratocephalus bicolor (DC.) Ktze. Rev. Gen. Pl. i, 326 (1891) = Zinnia bicolor (*DC.*) *Hemsl. Mendezia bicolor* DC. Prod. v, 533 (1836).
Pyrethrum Bidens Med. in Act. Acad. vel Hist. et Comment. . . . Theod.-Palat. (Phys.) iii, 241, t. 18 (1775) = Cotula Pyrethraria *L.*

Acmella biflora (L.) Spreng. L.-Spreng. Syst. Veg. iii, 591 (1826) = Wedelia biflora (*L.*) *DC.* *Verbesina biflora* L. Sp. Pl. ed. II, ii, 1272 (1763).

Acmella buphthalmoides (Jacq.) Rich. in Pers. Syn. Pl. ii, 473 (1807) = Heliopsis buphthalmoides (*Jacq.*) *Dun.* *Anthemis buphthalmoides* Jacq. Pl. Rar. Hort. Caes. Schoenbr. ii, 13, t. 151 (1797).

Spilanthus cascata Steud. Nom. Bot. ed. I, 108 (1821) = Verbesina crocata (*Cav.*) *Less.*

Spilanthes cernua (L.) Koehn. in Just Bot. Jahresber. xxii, **2**, 595 (1897) = Spiranthes cernua (*L.*) *Rich.* *Ophrys cernua* L. Sp. Pl. 946 (1753).

Spilanthes crocata (Cav.) Sims in Curt. Bot. Mag. xl, 1627, t. 1627 (1814); DC. Prod. v, 626 (1836), sub spp. exclus. Verbesina crocata (*Cav.*) *Less.* *Bidens crocata* Cav. Ic. Descr. Pl. i, 66, t. 99 (1791).

Spilanthes ecliptoides Gardn. in Hook. Lond. Jour. Bot. vii, 407 (1848).

Ceratocephalus ecliptodes (Gardn.) Ktze. Rev. Gen. Pl. i, 326 (1891) = Jaegeria hirta (*Lag.*) *Less.*

Acmella flavicaulis Raf. New Fl. N. Am. i, 52 (1836) = Heliopsis helianthoides (*L.*) *Sweet.*

Acmella Garcini (Burm. f.) Spreng. L.-Spreng. Syst. Veg. iii, 591 (1826) = Anvillea Garcini (*Burm. f.*) *DC.* *Buphthalmum Garcini* Burm. f. Fl. Ind. t. 60, f. 1 (1768), descr. sub nomine *Anthemidis Garcini* Burm. f. l. c. 183.

Acmella globosa (Ort.) Spreng. L.-Spreng. Syst. Veg. iii, 592 (1826) = Zaluzania globosa (*Ort.*) *Sch. Bip.* *Anthemis globosa* Ort. Nov. Rar. Pl. Hort. Bot. Matrit. Descr. Dec. iv, 47 (1797).

Spilanthes gracilis (Big.) Koehn. in Just Bot. Jahresber. xxii, **2**, 595 (1897) = Spiranthes gracilis (*Big.*) *Beck.* *Neottia gracilis* Big. Fl. Bost. ed. II, 322 (1824).

Spilanthes guatemalensis Vatk. ex J. D. Sm. Enum. Pl. Guat., etc., i, 23 (1889), nom. nud. = Melampodium paludosum *HBK.*

Spilanthus hirta (Lag.) Steud. Nom. Bot. ed. II, ii, 622 (1841).

Acmella hirta Lag. Gen. et Sp. Nov. 31 (1816) = Jaegeria hirta (*Lag.*) *Less.*

Spilanthes Karvinskiana DC. Prod. v, 623 (1836).

Ceratocephalus Karwinskianus (DC.) Ktze. Rev. Gen. Pl. i, 326 (1891) = Jaegeria hirta (*Lag.*) *Less.*

Acmella lanceolata Raf. New Fl. N. Am. i, 52 (1836) = Heliopsis scabra *Dun.*

Spilanthes nitidus Llav. in Llav. et Lex. Nov. Veg. Descr. i, 28 (1824); sp. dub. DC. Prod. v, 626 (1836).
Ceratocephalus nitidus (Llav.) Ktze. Rev. Gen. Pl. i, 326 (1891) = Salmea scandens (*L.*) *DC.*

Acmella nudicaulis Raf. New Fl. N. Am. i, 52 (1836) = Heliopsis helianthoides (*L.*) *Sweet.*

Acmella parvifolia Raf. New Fl. N. Am. i, 52 (1836) = Heliopsis helianthoides (*L.*) *Sweet.*

Spilanthus Pseudo-Acmella (L.) Murr. L.-Murr. Syst. Veg. ed. XIII, 610 (1774); DC. Prod. v, 625 (1836).
Verbesina Pseudo-Acmella L. Sp. Pl. ed. I, ii, 901 (1753); Trim. Hand Bk. Fl. Ceyl. iii, 40 (1895). *Chrysanthemum Maderaspatanum, latifolium, Scabiosae capitulis parvis,* Pluk. Alm. Bot. 99, t. 159, f. 4 (1720).
Ceratocephalus Acmella (L.) Ktze. var. *Pseudacmella* (L.) Ktze. Rev. Gen. Pl. i, 326 (1891). (Species mixta e generibus alienis.)

Spilanthes pseudo-gummifera Forst. ex DC. Prod. v, 626 (1836) = Laxmannia arborea *Forst.* et *Forst. f.*

Spilanthes Romanzoffiana (C. et S.) Koehn. in Just Bot. Jahresber. xxii, **2**, 595 (1897) = Spiranthes Romanzoffiana *C.* et *S.* in Linnaea. iii, 32 (1828).

Spilanthes sessilifolia Hemsl. Biol. Centr.-Am. Bot. ii, 193 (Oct. 1881).
Ceratocephalus sessilifolius (Hemsl.) Ktze. Rev. Gen. Pl. i, 326 (1891) = Jaegeria hirta (*Lag.*) *Less.*

Spilanthes sessilis Poepp. Nov. Gen. et Sp. Pl. iii, 50 (1844).
Ceratocephalus sessilis (Poepp.) Ktze. Rev. Gen. Pl. i, 326 (1891) = Jaegeria hirta (*Lag.*) *Less.*

Acmella spilanthoides Cass. in Dict. Sc. Nat. xxiv, 330 (1882) = Wedelia carnosa *Pers.*

Spilanthes tetrandra Roxb. in Beats. Tracts Relat. St. Helena, App. i, 325 (1816); DC. Prod. v, 626 (1836), sub. spp. exclus. = Laxmannia arborea *Forst.* et *Forst. f.*

Spilanthes tinctoria Lour. Fl. Cochinch. ii, 484 (1790); DC. Prod. v, 626 (1836), sub spp. exclus. = Adenostemma tinctorium (*Lour.*) *Cass.*

Acmella trilobata Spreng. L.-Spreng. Syst. Veg. 591 (1826) = Zaluzania triloba (*Ort.*) *Pers.*

Spilanthes wedelioides H. et A. in Hook. Jour. Bot. iii, 318 (1841).
Ceratocephalus wedeliodes (H. et A.) Ktze. Rev. Gen. Pl. i, 326 (1891) = Eclipta elliptica *DC.*

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A. Fendler (Panama) 166, S. ocymifolia f. radiifera. (Venezuela) 691 et 691B, S. ocymifolia.

K. Fiebrig (Paraguay) 345, S. americana var. stolonifera.

A. Fredholm (Jamaica) 3059, S. uliginosa.

G. Gardner (Brasilia) 4252, S. urens — ser. I, 70, S. Lundii — ser. VIII, 3866, S. ciliata.

C. Gaudichaud-Beaupré (Brasilia) 681, S. oleracea — 682, S. ocymifolia f. radiifera.

G. F. Gaumer (Mexico) 1122, 1257, 1465, 2185, 2502, 3835, S. filipes.

A. Ghiesbreght (Mexico) 557, S. americana.

J. M. Greenman (Mexico) 25 et 134, S. americana.

L. Hahn (Martinica) 126 et 726, S. urens — 1107, S. uliginosa.

E. Hall (Texas) 348, S. americana var. repens.

M. B. Halsted (Mexico) 32, S. americana.

T. Hartweg (Ecuador) 867, S. ocymifolia.

E. Hassler (Paraguay) 925, S. grisea var. micra (specimen typicum) — 1211 et 4659, S. grisea — 1639 et 3370, S. americana var. stolonifera — 4475, S. grisea var. setosa — 7651, S. grisea var. Chodatana (specimen typicum) — 8273, S. grisea var. intermedia — 8274, S. nervosa.

J. W. Helfer (Burma aut Insulae Andaman) 3186, S. Acmella.

A. A. Heller et uxor (Porto Rico) 548, S. iodiscaea — 550a, S. iodiscaea f. leucaena (specimen typicum).

A. Henry (Formosa) 219 et 812, S. Acmella. (China) 12706, S. Acmella — 12260A, S. callimorpha (specimen typicum).

S. E. Henschen (Brasilia) ser. I, 269, S. oleracea.

H. T. Heyde et E. Lux (Guatemala) 3812, S. ocymifolia — 3381, S. americana var. parvula f. lanitecta (specimen typicum) — 4209, S. americana var. parvula — 4501, S. americana.

Edidit R. F. Hohenacker (India) 1017, S. calva.

J. R. Johnston (Venezuela) 102, S. ocymifolia.

F. W. Junghuhn (Java ?) 307, S. calva.

W. Lechler (Chile) 1532, S. urens var. hispidula.

F. C. Lehmann (Guatemala) 1319, S. americana var. parvula f. parvifolia — 1596, S. americana. (Colombia) 3487 et 3599, S. americana — 8010, S. subhirsuta. (Colombia aut Ecuador) 6446, S. americana — 7987, S. ciliata.

F. E. Lloyd (Dominica) 473, S. uliginosa.

P. G. Lorentz (Argentina) 76, S. americana var. stolonifera f. ciliatifolia (specimen typicum).

P. G. Lorentz et G. Hieronymus (Argentina) 903, S. alpestris.

K. K. Mackenzie (Louisiana) 477, S. americana var. repens.

G. Mandon (Bolivia) 63, S. ciliata.

K. F. P. v. Martius (Brasilia) 438, S. Lundii.

G. McCarthy (Carolina Borealis) 5 (Carolina Australis) xxxi, S. americana var. stolonifera.

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C. F. Millspaugh (Mexico) 1494, S. filipes. (Jamaica) 1888, S. uliginosa — 2028, S. urens.

C. Moritz (Venezuela) 1387, S. ciliata.

T. Morong (Paraguay) 89, S. americana var. stolonifera.

S. Mossman (Australia) 509, S. anactina.

Friedr. Mueller (Mexico) 267, 274, 4072, S. ocymifolia.

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E. W. Nelson (Mexico) 1032, S. americana.

E. Otto (Cuba) 94, S. insipida ?

E. Palmer (Mexico) 1565 et 1192, S. ocymifolia.

C. C. Parry et E. Palmer (Mexico) 464, S. americana.

G. S. Perrottet (India) 27, S. calva.

H. Pittier (Costa Rica) 3717 et 10546, S. macrophylla.

C. G. Pringle (Cuba) 75, S. limonica (specimen typicum). (Mexico) 1821, 9546, 9964, S. americana — 2946, 4340, 4341, 11572 S. ocymifolia — 3489, S. disciformis — 3643, S. americana var. parvula — 8637, 9539, 11312, S. phaneractis.

A. F. Regnell (Brasilia) ser. II, 168, S. decumbens var. macropoda.

J. Reverchon (Texas) 161 et 1535, S. americana var. repens.

J. N. Rose, J. H. Painter, J. S. Rose (Mexico) 10055, S. americana.

F. Rugel (Cuba) 6, S. insipida.

H. H. Rusby (Bolivia) 919, S. ciliata.

J. G. Schaffner (Mexico) 338 et 763, S. americana.

C. J. W. Schiede (Mexico) 229, S. americana.

W. Schimper (Abyssinia) 134, S. abyssinica.

H. v. Schlagintweit-Sakuenluenski (India) sub catal. no. 12959, S. oleracea.

H. E. Seaton (Mexico) 73, S. americana — 445, S. americana var. ramosa.

F. Sellow (Brasilia) 848 et 1078, S. ciliata — 1764 et 3522, S. decumbens var. macropoda — 1969, S. decumbens — 2796, S. decumbens var. leptophylla.

J. A. Shafer (Cuba) 420, S. americana.

M. A. Shufeldt (Madagascar) 105, S. mauritiana f. madagascariensis.

P. E. E. Sintenis (Porto Rico) 718 (specimen typicum) et 1149, S. iodiscaea.

H. H. Smith (Colombia) 513, S. ocymifolia f. radiifera (specimen typicum) — 553, S. urens — 591, S. ocymifolia.

H. H. et G. W. Smith (St. Vincent) 96, S. uliginosa.

J. D. Smith (Guatemala) 1604, S. americana — 2125, S. americana var. parvula.

L. C. Smith (Mexico) 131, S. americana.

A. Sodiro (Ecuador) 39/1, S. ocymifolia.

C. Thieme (Honduras) 5309, S. macrophylla.

G. H. K. Thwaites (Ceylonia) 684, S. calva.

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J. Tweedie (Argentina) 223, S. pusilla — 861, S. stenophylla.

W. Tyson (Pondoland) 1057, S. mauritiana.

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L. V. Velasco (Salvador) 8851, S. ocymifolia.

A. E. Wight (Jamaica) 35, S. uliginosa.

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C. Wright (Cuba) 3610, S. insipida.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

CONCERNING THE ADIABATIC DETERMINATION OF THE HEATS OF COMBUSTION OF ORGANIC SUBSTANCES, ESPECIALLY SUGAR AND BENZOL.

By Theodore W. Richards, Lawrence J. Henderson, and
Harry L. Frevert.

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Presented January 9, 1907. Received December 4, 1906.

Introduction.

As a preliminary step to the determination of the heats of combustion of an extended series of organic substances, desired for certain theoretical considerations, measurements were made on two common substances, cane-sugar and benzol. These substances were chosen in order that one might represent solids and the other volatile liquids, and because they may be easily obtained in a high state of purity, as well as because their heats of combustion have been carefully studied by other investigators.

This investigation offered the opportunity for further testing and improving the adiabatic calorimetric method recently proposed and tested by Richards, Henderson, and Forbes,[1] by means of which corrections for accidental loss of heat and for the lag of the thermometer are experimentally eliminated. The method was devised in the hope that its use might increase the accuracy of thermochemical work; and this hope is justified by the present experience. The principle of the method is to cause the temperature of the surroundings of the calorimeter to change in the same direction and at the same rate as the calorimeter itself. This is accomplished by surrounding the calorimeter with vessels in which a suitable warming reaction can take place in a manner fulfilling the above conditions. A reaction easily regulated and well suited to this purpose, namely, the neutralization of an alkali with an acid, was chosen for this purpose.

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[1] These Proceedings, **41**, 3 (1905); Zeit. phys. Chem., **52**, 551 (1905).

The Apparatus.

A vertical section of the apparatus is shown in the accompanying diagram. The large outer vessel (A) and the covering vessel (B), designed for holding the alkaline solution, were made of sheet copper. On account of the corrosive action of the caustic solution to which the vessels are continually exposed, the joints must be very thoroughly soldered, otherwise the corrosion may give rise to annoying leaks. These are of serious consequence if they occur in the cover, because then they may allow the solution to pass into the calorimeter or into the narrow air-space surrounding the calorimeter.

The inner vessel (C) protecting the calorimeter itself from the alkaline surrounding liquid was a heavy nickel-plated copper can well burnished in the interior and firmly adjusted in the outer vessel several inches above the bottom, so as to allow a free circulation of the liquid beneath it.

The calorimeter proper (D) was placed inside of this inner vessel, resting on several bits of cork and separated by an air-space of about two millimeters from its burnished nickel inner surface. This calorimeter was made of pure silver; it weighed 1357 grams and had a capacity of about 4432 milliliters; in operation it was filled with water, completely submerging the combustion bomb which rested upon points bearing on its base. The German-silver stirrer (E) which agitated the water in the silver calorimeter consisted of two perforated rings on upright supporting wires, and was moved up and down at a perfectly regular rate by means of an electric motor with a worm-gear attachment. This stirring arrangement was found to be very satisfactory, as it produced a complete and rapid adjustment of the temperature of the calorimetric system during a combustion, and the comparatively slow motion gave rise to no warming correction during the seven or eight minutes necessary for the actual combustion.

The copper pan (B) used as a cover to both calorimeter and outside jacket, was provided with copper tubes for the stirrers and thermometers projecting below it; its temperature also was changed in the same manner as that of the outer jacket by adding acid to its alkaline contents, so as to follow the effect of the heat of the combustion. The liquid in the cover was stirred by a large oscillating perforated copper ring (F) actuated by the same motor which raised and lowered the stirrer of the calorimeter. It was found unnecessary to follow the change in temperature of the calorimeter as closely in the cover as in the jacket, although had this been necessary it might as easily have been done.

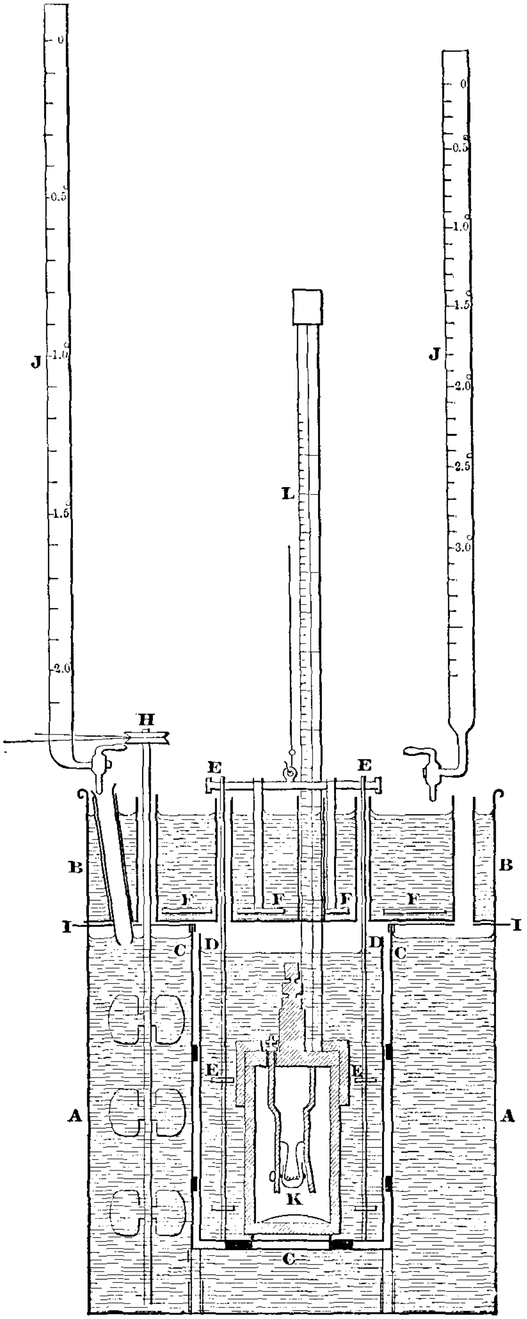

J
0
0.5
1.0
1.5
2.0
J
0
0.5
1.0
1.5
2.0
2.5
3.0
L
H
E
E
B
B
F
F
F
F
I
I
C
D
D
C
E
E
A
A
K
C

FIGURE 1.

Vertical Section.

A. Outer vessel.
B. Covering vessel.
C. Nickel-plated copper can.
D. Silver calorimeter.
E. Stirrer for calorimeter.
F. Stirrer for covering vessel.
H. Stirrer for outer vessel.
I. Copper cover.
J. Burettes.
K. Crucible containing substance to be burned.
L. Thermometer.

On the other hand, it is imperative that the heat generated by the reaction of the acid and the alkali in the outside can should be evenly and very quickly distributed, so that the change in temperature of the outer jacket may follow as closely as possible the change in temperature of the calorimeter. This was accomplished by running the acid into the alkaline solution in the immediate vicinity of a very powerful rotary stirrer (H), which drove the solution downward and at the same time around the vessel. This stirrer was propelled by an electric motor, and kept the liquid very thoroughly agitated. In order to prevent splashing into the calorimeter or into the air-space around the calorimeter, a covering sheet of copper (I) was bent down so as to fit snugly around the inner nickelled copper can (C); this sheet extended to the edge of the outer jacket. This cover effectually prevented injurious splashing, no matter how violent was the agitation of the liquid.

The acid was run both into the outer vessel and into the cover from burettes (J), which were calibrated, not in cubic centimeters, but in tenth degrees of temperature. In other words, the amount of acid necessary to raise each of the outer systems $\frac{1}{10}$° was determined by trial, and the burettes were marked accordingly.

The bomb used in this work was made after the model of those used by Atwater by the firm of Dinsmore and Singleton, Middletown, Connecticut. The interior and top of the bomb were lined heavily with platinum. Every part of the apparatus exposed to the oxygen under pressure during a combustion was of platinum, except a lead gasket which made the joint gas tight when the top of the bomb was screwed down. The presence of a rim of lead caused at first serious difficulty on account of its rapid oxidation by the oxygen under pressure during a combustion. This might introduce a small error of unknown and probably varying magnitude; in some cases the quantity of lead converted to oxide was considerable. Lead gaskets both gold-plated and covered with gold foil were tried without success. The difficulty was finally solved by the use of gold in the following manner. A lead gasket was fitted in position and the top screwed down upon it several times in order to form a depression in the centre

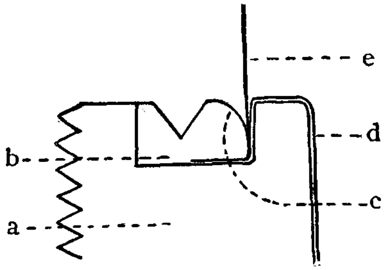
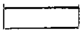


FIGURE 2. Detailed section of edge of bomb, *a*, showing the platinum lining, *d*, and the method of protecting the lead gasket, *b*, with gold foil, *e*. The gasket is partially cut away at *c*.

of the gasket and force out the sides. Some of the lead at the inner edge was then cut away as shown in the diagram. Small strips  of sheet gold, previously annealed, were inserted around this edge so that they might overlap, after which the top was again screwed down several times in order to wedge in the strips of sheet gold. The gold was then bent back over the lead and the top screwed down in order to smooth out the gold lining. A lining of this sort proved entirely satisfactory, though in time a very slight oxidation of the lead occurred, due to the loosening of several of the gold strips. This oxidization was, however, too slight to affect the results, for when a new lining replaced the old, no variation in the results could be detected.

Instead of the shallow dish ordinarily used for holding the substance to be burned, a platinum crucible (K) weighing about 15 grams was substituted. The platinum crucible has probably several advantages over a shallow dish, especially in the combustion of liquids. The high sides of the crucible tend to prevent loss by projection of the material, and the heat at the moment of ignition is probably concentrated, thus securing a more complete combustion.

The thermometer (L) was made by Fuess after the ordinary Beckmann model, with a large bulb containing about 75 grams of mercury and possessing a wide and very freely moving thread of mercury. It had been standardized by the Prussian Physikalisch-Technische Reichsanstalt and was subsequently compared here with a most carefully constructed Baudin thermometer, and with another similarly standardized Beckmann thermometer. All measurements were made over about the same part of the scale, in order to make the results with benzol as closely comparable as possible to the results with sugar.

The oxygen used was similar to that used by Atwater, and was obtained already compressed in steel cylinders from the White Dental Company of Boston. In order to insure a certain standard of purity in the oxygen, samples of each cylinder were analyzed volumetrically for oxygen, and check combustions were frequently performed with sugar. As a rule, this oxygen contained between 2.7 and 3.2 per cent of nitrogen and no chlorine. In work at present in prospect, it is proposed to prepare purer samples of the gas. The effect of the impurity of nitrogen is discussed in detail below.

PURIFICATION OF MATERIAL.

Cane-Sugar, $C_{12}H_{22}O_{11}$. Three samples of sugar, designated A, B, and C, were used in this investigation. The source of each sample was the crystallized sugar or "rock-candy" of commerce. For the first, called Sample A, powdered "rock-candy" was dissolved in boiling distilled water until the solution was almost saturated. The hot solution was filtered through asbestos in a platinum Gooch crucible and allowed to cool in low crystallizing dishes. After inoculating the solution with several minute sugar crystals, the substance began to crystallize slowly in fine granular crystals. Several days afterwards the sugar was separated from the syrup by filtration through a Gooch crucible with suction, being washed with distilled water several times during the process. When the greater part of the solution had been drawn off, the sugar was put into short funnels fitted with platinum cones and subjected to centrifugal action. During the whirling it was several times washed and stirred with distilled water. The sample was desiccated, powdered in an agate mortar, and preserved in a desiccator.

Because this sample, A, might be not completely free from included water, another sample of sugar was crystallized from its solution in a mixture of water and ethyl alcohol which had per gram the same heat of combustion as sugar. In this case, if the sugar included some of the liquid from which it was crystallized, the weighed mother-liquor would give the same heat of combustion as an equal weight of sugar. This preparation was conducted in the following way. A concentrated solution was made from powdered "rock-candy" and a known quantity of hot distilled water; and while the solution was still hot, it was filtered through asbestos in a platinum Gooch crucible. To this solution was added such a weight of absolute alcohol that the mixture of water and alcohol should have the same heat of combustion as sugar. After standing a short time, the sugar began to separate out as fine granular crystals. The dishes with the sugar solution were covered, and at the end of several days the sugar was filtered off and treated in the same manner as sample A, except that it was washed with a mixture of water and alcohol having the same heat of combustion as sugar.

The third sample, C, was prepared at a different time and with different material, but in a manner similar to the latter method.

Benzol, C_6H_6. Two samples of benzol were prepared as follows. For the first, sample A, 500 grams of Merck's benzol, freed from thiophene, were agitated in a flask submerged in a bath of ice and water until about two thirds had crystallized. The crystals were allowed to drain slowly, so that any adhering benzol might be washed off by the liquid formed from the melting crystals. The crystals were then allowed to melt and the above process repeated three more times.

For the second, sample B, the same quantity of Kahlbaum's similar preparation was agitated in a flask submerged in a bath of ice and water until about one third of the benzol had crystallized. These crystals were drained, melted, and then recrystallized, about the first fifth of the crystals being discarded. From the remaining benzol about one third was allowed to crystallize. These crystals were drained, melted, and recrystallized as before, the first crystals being discarded. About one half of the remaining benzol was allowed to crystallize and was retained as the final sample. The freezing-point of this sample was determined in a regular Beckmann freezing-point apparatus, using all of the precautions to prevent supercooling. The freezing-point was found to remain constant until practically all of the sample tested was frozen, thus indicating its purity. Especial care was taken to keep these preparations free from dust and other impurities, both during and after the purification.

The thermochemical results showed that these two samples were essentially the same, as far as the present purpose was concerned.

The Combustion of Sugar.

The details of conducting a combustion of sugar in the calorimetric bomb in the adiabatic calorimeter are easily told.

About 1.5 grams of sugar were accurately weighed in the platinum crucible in which the substance to be burned was placed. The sugar was not compressed into tablets, as has been the custom of previous investigators, but was burned as a powder. This method is the less troublesome and also the safer one, in that it involves less manipulation of the substance; moreover, in a preliminary series of experiments it was found that powdered sugar and sugar tablets gave perfectly concordant results.

The platinum crucible was placed in its support, a platinum ring secured to one of two stout platinum wires projecting downward from the cover of the bomb. These wires formed the terminals of an electric circuit. A spiral of very fine iron wire to serve as the igniter was suspended between them, dipping into the crucible and buried in the sugar. A small amount of water, never more than a milliliter, was sprayed into the bomb, so that the space might be saturated with water-vapor, in order to avoid a correction for the evaporation of part of the water formed by the combustion. The cover was then placed on the bomb, and a gas-tight joint was made by means of the screw-cap fitting over it. Oxygen was run into the bomb until it was under a pressure of 35 atmospheres. This high pressure was always used, in order to be more certain of obtaining complete combustion, unless otherwise indicated.

Meanwhile the rest of the apparatus was being prepared. The dilute alkaline solution in the jacket and cover was brought to a temperature a little below that of the room, and the bomb was placed in the silver calorimeter. Connections were prepared for the later use of an electric current by joining a copper wire to the bomb itself, and another, carefully insulated, to the tip of that one of the two stout platinum wires which was insulated from the bomb. The water for the calorimeter, 3302 grams at 20° C., was now measured in a marked flask, and poured into the silver vessel, submerging the bomb, at nearly the temperature desired. The cover containing dilute alkali was put in position, and its temperature roughly adjusted to that of the calorimeter. The stirrers were set in motion, and the thermometers and burettes clamped in position. The temperatures of the two systems were taken with a 0.01° thermometer; if, after all was ready, a slight difference existed between the temperatures of the calorimeter and the other jackets, the latter were easily adjusted by a little ice, or a little hot water or acid, until the whole apparatus was very nearly the same temperature throughout, that is to say, within a few hundredths of a degree. All measurements were made between 20° and 25° C. Readings every one or two minutes were now taken on the accurate Fuess thermometer, which was always jarred by an electric vibrator before reading, in order to prevent errors due to friction of the mercury thread. As soon as the readings assured constancy of the temperature of the calorimeter, the switch, completing the electric circuit through iron wire in the bomb, was lowered, and the sugar ignited by the combustion of the wire.[2] In a few seconds the temperature of the calorimeter began to rise, and the change in temperature was equalled in the outer system by adding concentrated sulphuric acid from the burettes. For about the first minute the temperature of the outer system, as indicated by the quantity of sulphuric acid used from the burettes, was kept about 0.1° to 0.05° above that indicated by the thermometer in the calorimeter, so as to allow for thermometric lag and give time for the equal distribution of the heat in the jacket. In from four to five minutes the calorimeter reached its maximum temperature, and the thermometer gave constant readings.

The great advantage of this adiabatic method of calorimetry is shown at this point by the constancy of the thermometer at the completion of the combustion. The thermometer usually gave constant readings for a long time after it had reached its maximum, showing indubitably that there was no appreciable heat exchange between the calorimeter and its surroundings. Moreover, there was plenty of time in which to read the thermometer accurately, while the mercury thread was stationary, and no thermometric lag was possible.

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[2] The electrical heat thus introduced is very small in amount, and is wholly eliminated by the comparative method finally used for calculating the results.

After the combustion, the pressure of the gas in the bomb was relieved and the bomb opened. The interior of the bomb and the crucible were always inspected for any caramel or unburned sugar, but in only two cases among the preliminary and rejected experiments was any trace of caramel noted. The remaining gas was repeatedly analyzed after combustions of sugar for carbon monoxide, with negative results. Any unburned iron wire was measured, and a correction subtracted from the known heat of combustion for the original amount used. The interior of the bomb and the crucible were rinsed with distilled water, and these washings, containing the nitric acid formed in the combustion, were titrated with a solution of sodium hydroxide, which was standardized so that one milliliter was equivalent to a known rise in temperature of the calorimetric system. Corrections for the burning of the iron wire and the formation of nitric acid were of course subtracted from the observed rise in temperature.

The data of a typical combustion of cane-sugar follow.

Combustion No. 2.

The weight of sugar (Sample A) taken was 1.5270 grams. The calorimeter was adjusted at 2.55 o'clock.

Time.	Reading of thermometer in calorimeter.
3.03	1.495°
.05	1.496
.06	1.496
.07	1.496
.08	1.496

Temperature of jacket = 1.50°
Temperature of cover = 1.50

The sugar was ignited at 3.08.10 o'clock, and the rapidly rising temperature of the interior was equalled in the jackets and cover.

Time.	Reading of thermometer in calorimeter.
3.10	3.050°
.11	3.108
.12	3.116
.13	3.117
.14	3.117
.15	3.117
.17	3.117
.18	3.117

Temperature of jacket = 3.11°
Temperature of cover = 3.19

The observed rise of temperature was therefore

$$3.117^\circ - 1.496^\circ = 1.621^\circ.$$

15 cm. iron wire were burned, causing a correction of	—0.015°
4.1 milliliters of sodium hydrate solution were used, indicating nitric acid corresponding to a correction of	—0.003°
The correction for graduation of the thermometer was	+0.001°
Total,	—0.017°

Therefore the corrected rise of temperature is

$$1.621° - 0.017° = 1.604°.$$

This would correspond to 1.050° per 1 gram cane-sugar.

Below is given a table containing all the results with sugar, except the few rejected preliminary determinations.

DATA CONCERNING THE COMBUSTION OF SUGAR.

Number.	Sample.	Rise in Temperature per gm. Cane-sugar.	Average.
1	A	° 1.050 ? [*]	° 1.0506
2	A	1.050 [*]	
3	A	1.052 [*]	
4	B	1.050	1.0504
5	B	1.050	
6	B	1.051	
7	B	1.050	
8	B	1.050	
9	B	1.052	
10	B	1.050	
11	C	1.049 [†]	1.0503
12	C	1.052 [†]	
13	C	1.050 [†]	
14	C	1.050 [†]	

Average of 14 determinations, 1.0504°.

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[*] Under a pressure of 25–27 atmospheres of oxygen.

[†] Combustions made by L. J. Henderson. All others were made by H. L. Frevert.

The purity of the sugar is indicated by the essential identity of the results obtained from three different samples.

In Determinations Nos. 7 and 8 the bomb was exhausted before running in the oxygen, and in Determinations Nos. 9 and 10 the amount of nitrogen was increased to about 13 per cent by volume in the compressed atmosphere of the bomb, by pumping in air in the first place, and then supplying the oxygen to 35 atmospheres pressure. The essential identity of these results with the others shows that the presence of even this large amount of inert gas does not affect the heat of combustion of a substance so easily burned as sugar.

In these experiments it appears probable that the greatest experimental error in a combustion lay in the reading of the thermometer. Probably with a scale divided to $\frac{1}{100}$°, even with a good lens, one may make an error of 0.001°. Should this error be made in the same direction in the two extreme readings, the errors would cancel each other. However, should the errors be made in opposite directions, then in a rise of two degrees this error in reading the thermometer would amount to a percentage error of 0.1 per cent. The average result should be much more accurate than this, however. The agreement of the results of cane-sugar is in accord with this consideration of error. In the future, measurements of the more important substances, when other conditions justify a greater refinement, will be made with a platinum resistance thermometer.

The Combustion of Benzol.

The first problem to be solved in the combustion of a volatile liquid was to devise a method by which an accurately weighed quantity of it might be burned. Berthelot[3] determined the heat of combustion of benzol by saturating cellulose with the liquid, which was then ignited in the bomb. This method of procedure is evidently open to the error of a varying loss in weight of benzol by evaporation. Besides, since in this case some of the benzol must burn as vapor, which of course gives a greater heat of combustion than liquid benzol, a correction should be subtracted accounting for the heat of evaporization of that portion of the benzol burned as vapor. These objections account, in part at least, for the irregularity of his results.

Julius Thomsen[4] burned benzol both as vapor mixed with air and as a liquid in his Universal Burner. By these methods he obtained at various times four series of results, one of which was not at all in agreement with the others. Probably, as Stohmann pointed out, the results with the Universal Burner were too high.

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[3] Ann. Chim. et Phys. (5) **23**, 193 (1881).

[4] Thermo. Untersuch., **4**, 59.

Stohmann[5] obtained most of his values for benzol by combustion in a different kind of lamp. Subsequently,[6] however, he sealed benzol in very thin glass bulbs which were placed in the bomb and broken by shaking just before the bomb was placed in the calorimeter. This method involved a necessary correction for the benzol burned as vapor. Stohmann thus secured two results concordant with those previously obtained, but he had much difficulty on account of the incomplete combustion of the benzol. Of a series of determinations, he obtained only these two which were not vitiated by the deposition of soot on the interior of the bomb.

In order to burn completely a weighed quantity of liquid benzol without applying a correction for vaporization, the following procedure was followed in the present investigation. Thin glass bulbs of about 0.7–0.8 milliliters with bent capillary stems were weighed, and filled with benzol by immersing the stems under benzol, alternately cooling and warming the bulbs in cold and hot water. The thin walled capillary stems, filled with benzol, were sealed off near the bulbs in a fine blowpipe flame. In this way bulbs could be obtained either completely filled or with a negligible amount of vapor in the short capillary stem.[7] The bulbs and the detached capillary stems were weighed together under the same conditions as the empty bulbs, and the weights of benzol thus determined.

For combustion one of these bulbs was placed upon the top of about 0.25 gram of carefully weighed pure sugar in the combustion crucible, and the rest of the procedure was exactly the same as in the case of cane-sugar. The great heat of the burning sugar caused the bulb of benzol to burst with the consequent immediate combustion of the benzol. In the preliminary trials small patches of soot due to incomplete combustion of the benzol were noticed on the crucible and on the lining of the bomb. This inadmissible complication was traced to the fact that the stems of the bulbs were from one to two centimeters in length. After sealing new bulbs in such a way that the stems were only three to five millimeters long, no trace of soot ever appeared. Some idea of the temperature in the crucible at the moment of combustion may be gained from the fact that after a combustion the glass of the bulbs was always found at the bottom of the crucible fused into small globules.

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[5] Jour. f. prak. Chem., **33**, 241 (1886).

[6] Ibid., **40**, 77 (1889).

[7] This method of enclosing liquids in thin bulbs in such a way as to be capable of subjection to high pressure was first used by Richards and Stull in their work on Compressibility (Carnegie Inst. Pub., 7, 1903).

A peculiar nitrous odor was in some cases observed in the gas in the bomb after a combustion. In order to discover whether this odor might be due to the formation of small amounts of some of the oxides of nitrogen which were not completely absorbed by the water, the following test was made on the gas in the bomb after combustions No. 12 and No. 25. The gas from the bomb was allowed to bubble slowly through gas-washers made of test tubes containing cold distilled water. This water was then boiled for some minutes to expel carbon dioxide, and was then titrated with a very dilute sodium hydroxide solution and phenolphthalein. The first drop of alkali turned the solution red, so there must have been very little of the acid-forming oxides of nitrogen, nitric or nitrous acids in the gas. Any nitric oxide formed in the bomb would have been immediately converted to nitrogen peroxide in presence of the excess of oxygen, and would have dissolved in the water. The washings of the bomb also were tested for nitrous acid with α-naphthylamine and sulphanilic acid; but negative results only were obtained. In a number of other determinations under varying conditions (Nos. 21, 22, 23, 24, 27), no odor whatever was noticeable in the escaping gas, although the quantitative results were not essentially different from similar ones in which an odor was noticed. Hence it appears that whatever may have caused this odor, it has no appreciable effect on the heat of combustion of benzol; no connection could be found between the presence of the odor and the effect of increasing the per cent of nitrogen present. Nevertheless the matter will receive further attention in the future.

The first trials of this method were from one cause or another unsuccessful, as is usual in such cases; but after eleven such partial failures satisfactory results were regularly obtained. After a series of five consistent values (Nos. 12–16) for benzol had been found, two combustions (No. 17 and No. 18) were made in which the air in the bomb was almost entirely removed before the oxygen was run in. It was hoped that two advantages might be gained by conducting the combustion in the absence of the nitrogen of the air originally contained in the bomb: first, that the cause of the nitrous odor might be removed; second, that the correction for the formation of nitric acid would be decreased. These new experiments gave for the heat of combustion of benzol a value higher than the average previously obtained by over 0.3 per cent, — a greater deviation than could be ascribed to the errors of the method. This interesting observation being new, so far as we could discover, the matter was pursued further. Three more combustions

(Nos. 19, 20, and 21) were made in the usual way with the bomb originally filled with air, and the average value for the heat of combustion of benzol in this series was the same as that obtained in the first series. Again three combustions (Nos. 22, 23, and 28) were made with the air removed from the bomb before the oxygen was run in, and again results were obtained more than 0.3 per cent greater than the results of the combustions in which the bomb was filled with air at the beginning. At the same time, in those cases in which the bomb was exhausted the amount of nitric acid formed during a combustion did not appreciably diminish, hence it was evident that the oxygen in use must contain nitrogen.

After due consideration of these results the logical conclusion seemed to be that the presence of an inert gas such as nitrogen in some way or other interfered with the complete burning of the benzol, and therefore lowered the heat of combustion. In order to test this hypothesis further, more combustions were made in the presence of still greater concentrations of nitrogen, under which conditions the heat of combustion should be still more diminished. To accomplish this, air was forced into the bomb by means of an ordinary bicycle-pump, and the pressure of the air forced in was measured on the pressure-gauge. Oxygen was then run in until the pressure inside the bomb was 35 atmospheres. Since the amount of nitrogen in the oxygen had been determined, it was a simple matter to calculate the approximate amount of nitrogen in the bomb. The results of the combustions made under these conditions agreed with the hypothesis, and while the relation was not a simple ratio, yet the results showed clearly beyond a doubt that the greater the concentration of nitrogen the lower the heat of combustion. This effect of nitrogen in combustion suggests the retarding influence of oxygen on the rate of formation of hydrochloric acid when hydrogen and chlorine are exposed to the light,[8] and numerous similar phenomena, sometimes classified under the general head of negative catalysis.

In presenting these results, a typical determination is given in detail, in order to illustrate further the method of calorimetry; afterwards a table giving all the later results with benzol is recorded.

Determination No. 23,

Weight of benzol (A), 0.5840 gram.

Weight of sugar (B), 0.2073 gram.

Air was exhausted from the bomb before admitting oxygen.

The bomb was immersed in the calorimeter at 11.25 o'clock, and the temperature of jacket and cover were adjusted at 11.30 o'clock.

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[8] Bunsen and Roscoe, Pogg. Ann., 96 (1855); Phil. Trans., **147**, 389 (1857)

Time.	Reading of thermometer.
11.33	2.326°
.35	2.327
.37	2.328
.39	2.330
.41	2.330
.43	2.330
.45	2.330

Temperature of jacket = 2.31°
Temperature of cover = 2.32°

Ignition occurred at 11.45.

Time.	Reading of thermometer.
11.47	4.07°
.48	4.110
.49	4.116
.50	4.117 +
.51	4.118
.52	4.118
.53	4.118
.54	4.118

Temperature of jacket = 4.10°
Temperature of cover = 4.20°

Observed rise of temperature = 4.118° — 2.330° = 1.788°

There was little if any odor in gas remaining in the bomb after combustion. 11.0 centimeters of iron wire were burned, and 4.0 milliliters of the solution of sodium hydrate were used.

From the observed rise of temperature, 1.788°, the following corrections must be subtracted:

For error in graduation of thermometer = —0.001°
For the combustion of 0.2073 gram sugar = —0.218°
" " " " 11 cm. iron wire = —0.011°
" " " " nitrogen to nitric acid = —0.003°
Total, —0.233°

Therefore the combustion of 0.5840 gram of benzol raises the temperature 1.788° — 0.233° = 1.555°, and 1 gram of benzol would raise the temperature 2.662°.

In this result, 2.662°, mistakes of 0.001° in reading the initial and final thermometric height would cause an error of almost 0.004°, if the mistakes happened to be in opposite directions. On comparing a number of determinations, one would therefore expect to find an occasional deviation from the mean as large as this; but the mean should be much more accurate, because in many determinations such accidents tend to balance one another. The study of the results given below confirms these inferences. The following table is arranged according to the amount of nitrogen present in the oxygen at the time of burning; it contains the results of all the combustions except the rejected preliminary ones.

Determination No.	Sample of Benzol.	Per cent Nitrogen.	Rise in Temperature of System for 1 gm. Benzol.	Average and Probable Error.
23	A	2.7	° 2.662	° 2.662 ±0.0006
28	A	3.2	2.661	
17	B	2.7	2.661	
18	B	2.7	2.663	
22	B	2.7	2.663	
12	A	4.9	2.655	2.655 ±0.0007
13	A	4.9	2.658	
14	A	4.9	2.653	
15	A	4.9	2.656	
16	B	4.9	2.651	
19	B	4.9	2.657	
20	B	4.9	2.657	
21	B	4.9	2.650	
26	A	9.7	2.660	2.650 ±0.0023
25	B	9.6	2.649	
24	A	10.1	2.643	
27	B	11.9	2.650	

It is seen at once, from the averages in the right-hand column, that an increasing amount of nitrogen decreases the heat of combustion to an extent far beyond the limit of accidental error. In this respect the results with benzol are very different from those with sugar, which has previously been shown to be unaffected by the presence of much nitrogen. The nature of the gaseous or liquid unburnt residue in the case of benzol has not been determined; solid carbon, at least, was wholly absent.

Those determinations in which the least nitrogen was present undoubtedly represent the closest approach to the true value for the heat of combustion of benzol. The first average of five determinations in the presence of about 3 per cent of nitrogen gives 2.662° as the rise in the calorimetric system for 1 gram of benzol. This, in our opinion, represents a minimum value; the true one may be yet higher. It is planned to determine soon the heat of combustion of benzol in the presence of pure oxygen.

Obviously, data on only two substances will not permit the drawing of certain general conclusions. Nevertheless, it seems reasonably probable that the effect in general of the presence of an inert gas during the combustion of easily burned substances is very slight. On the other hand, other substances like benzol which need much oxygen for their combustion are probably, like it, affected by the presence of an inert gas.

Since the water-equivalent of our bomb and calorimeter has not been determined either by the method of finding the sum of the water-equivalents of the component parts, or by the admirable electrical method of Jaeger and von Steinwehr,[9] absolute values for the heats of combustion of sugar and benzol cannot be computed from these data. Until an accurate value for the absolute heat of combustion of some standard pure substance has been obtained, the present results may be expressed as ratios, taking the rise in temperature of the calorimetric system caused by the combustion of one gram of cane-sugar as the standard of reference. Such ratios can be easily converted to absolute values when a standard has been established, and moreover, the relative values can be used to the same purpose as absolute values in the application of heats of combustion to the primary idea which instigated the present work. The outcome of the present work is thus that a gram of benzol yields at least $\frac{2.662}{1.0504} = 2.5342$ times as much heat on combustion as a gram of sugar.

It is interesting to compare this result with the results of other experimenters. On account of the relation which exists between the weight of water and the weight of metal in the bomb-calorimetric system, and the fact that at temperatures between 0°–30° the specific heat of water decreases, while the specific heats of the metallic parts of the calorimetric system increase with rise in temperature, the water-equivalent of the calorimetric system varies but slightly with change in temperature. In fact, the roughly calculated water-equivalent of our calorimetric system at 20° varied from its water-equivalent at 25° by less than 0.03 per cent. The estimated water-equivalents used in this comparison were, of course, not accurate, but the ratio between them must be very close to the truth. On the assumption that the relation between the weights of the water and the metallic parts in the calorimetric system of other investigators did not differ much from ours, it is evident that but little error will arise in comparing our results obtained at an average temperature of 21.4° with the results of Stohmann and of Fischer at average temperatures of 17° and 15° respectively.

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[9] Verhandl. d. d. physik. Ges., **5**, 50 (1903); Ibid., **5**, 353 (1903); Zeit. phys. Chem., **53**, 153 (1905).

The results of previous investigators, expressed both in gram-calories and kilojoules, are tabulated below.

RESULTS OF OTHER EXPERIMENTERS.

	Cane-Sugar.		Benzol.	
	Calories per Gram at Constant Volume and Pressure.	Kilojoules per Gram at Constant Volume and Pressure at 21°.	Calories per Gram at Constant Volume.	Kilojoules per Gram at Constant Volume at 21°.
Stohmann[10] . .	3955.2	16.533	9977	41.704
Fischer and Wrede[11]	3977.8	16.627	. . .	. . .
Berthelot[12] . .	3961.7	16.560	9949	41.587

Thus our ratio for the heats of combustion of benzol and sugar, 2.534, is about 0.3 per cent higher than 2.527, Stohmann's corresponding ratio. It is interesting to note that had we used the value for benzol obtained when the bomb was filled with air at the beginning, our ratio would have been $\frac{2.655}{1.0504} = 2.528$, which is practically the same as Stohmann's.

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[10] Stohmann and Langbein, Jour. prak. Chem., **45**, 313 (1892); **40**, 77, 81 (1889); Stohmann, Rodatz, and Herzberg, ibid., **33**, 193 (1886).

[11] Sitzb. Berl. Akad. d. Wissen, **19**, **20**, **21**, 687 (1904). Value corrected for error in calibration of bomb, vide, Zeit. phys. Chem., **53**, 164 (1905).

[12] Berthelot and Vielle, Ann. Chim. Phys. (6), **10**, 442 and 458 (1887). Berthelot, Ann. Chim. Phys. (5), **23** ,193 (1881).

The very wide difference between the absolute heats of combustion of sugar as given by Stohmann and Fischer makes a calculation of the absolute value for benzol from the combination of our results with either of theirs a somewhat uncertain matter. If Stohmann's value for sugar be accepted, 41.89 kilojoules is computed as the heat of combustion of a gram of benzol; but if Fischer's be accepted, 42.13 kilojoules must be taken as the true value. No attempt will be made in the present paper to decide this discrepancy, which is probably due either to different standards of temperature or to different standards of heat capacity in the two cases.

In conclusion, it is a pleasure to express our indebtedness to the Rumford Fund of the American Academy of Arts and Sciences for pecuniary support at the commencement of these experiments, and to the Carnegie Institution of Washington for further assistance during their continuation.

SUMMARY.

1. The adiabatic method of calorimetry devised by Richards, Henderson, and Forbes, as applied to the determination of heats of combustion with the calorimetric bomb, was improved and thoroughly tested.

2. The efficiency of this method in eliminating all corrections for heat exchange with the surroundings, and for thermometric lag, was established.

3. A new method was devised for burning an accurately weighed quantity of a liquid in the calorimetric bomb, by sealing it in a thin completely filled bulb, and exploding this by the combustion of a small amount of sugar.

4. The effect of an inert gas, nitrogen, on the heat of combustion of substances was noted for the first time.

5. In the calorimetric combustion of substances needing much oxygen it was shown that the presence of nitrogen may involve a very considerable error.

6. Benzol was found to evolve at least 2.534 times as much heat as an equal weight of sugar on burning.

EXPERIMENTAL DATA — TABLE I.

SUGAR.

Sample.	Number of Combustion.	Weight of Sugar.	Initial Temperature.	Final Temperature.	Observed Rise in Temperature.	Thermometer Correction.	Correction for Iron Burned.[1]	Correction for HNO_3 formed.[2]	Corrected Rise in Temperature.	Rise in Temperature per Gram Sugar.
		gms.	°	°	°	°	°	°	°	°
A	1	1.4985	1.240	2.830 (?)	1.590	+0.001	–0.015	–0.003	1.573	1.050 (?)
A	2	1.5270	1.496	3.117	1.621	+0.001	–0.015	–0.003	1.604	1.050
A	3	1.5019	1.686	3.281	1.595	+0.001	–0.013	–0.003	1.580	1.052
B	4	1.5125	1.421	3.026	1.605	+0.001	–0.015	–0.003	1.588	1.050
B	5	1.5211	1.826	3.441	1.615	0.000	–0.015	–0.003	1.597	1.050
B	6	1.6705	2.916	4.693	1.777	–0.003	–0.014	–0.004	1.756	1.051
B	7	1.4137	2.010	3.511	1.501	+0.001	–0.015	–0.002	1.485	1.050
B	8	1.4957	1.464	3.051	1.587	+0.001	–0.014	–0.003	1.571	1.050
B	9	1.5486	1.323	2.969	1.646	+0.001	–0.012	–0.005	1.630	1.052
B	10	1.5402	1.248	2.883	1.635	–0.001	–0.014	–0.004	1.618	1.050
C	11	1.7317	1.023	2.859	1.836	–0.001	–0.015	–0.003	1.817	1.049
C	12	1.5586	1.573	3.227	1.654	+0.001	–0.013	–0.003	1.639	1.052
C	13	1.6828	2.042	3.826	1.784	0.000	–0.014	–0.003	1.767	1.050
C	14	1.7882	1.773	3.670	1.897	–0.001	–0.015	–0.003	1.878	1.050

[1] 1 centimeter iron wire weighs 0.00194 gm. Hence 1 cm. =⊂⊃= 0.0009° rise, the heat capacity of the system being not far from 3800.

[2] 1 cc. NaOH =⊂⊃= 0.0122 gm. HNO_3. Hence 1 cc. NaOH =⊂⊃= 0.00076° rise.

TABLE II.

BENZOL.

Sample.	No. of Combustion.	Weight of Benzol.	Weight of Sugar.	Initial Temperature.	Final Temperature.	Observed Rise in Temperature.	Thermometer Correction.	Correction for Sugar Burned.	Correction for Iron Burned.	Correction for HNO_3 formed.	Corrected Rise in Temperature.	Per Cent N_2 in Bomb.	Rise in Temp. per gram Benzol.
		gm.	gm.	°	°	°	°	°	°	°	°	%	
A	12	0.4749	0.2058	1.950	3.445	1.495	+0.001	–0.216	–0.015	–0.004	1.261	4.9	2.655
A	13	0.4054	0.2046	2.893	4.208	1.315	–0.004	–0.215	–0.015	–0.003	1.078	4.9	2.658
A	14	0.7319	0.2119	1.697	3.890	2.193	–0.002	–0.222	–0.013	–0.004	1.942	4.9	2.653
A	15	0.7036	0.2879	2.000	4.192	2.192	–0.001	–0.302	–0.015	–0.005	1.869	4.9	2.656
B	16	0.5493	0.2016	1.110	2.796	1.686	–0.001	–0.212	–0.013	–0.004	1.456	4.9	2.651
B	17	0.5561	0.2017	2.150	3.860	1.710	–0.001	–0.212	–0.014	–0.003	1.480	2.7	2.661
B	18	0.7265	0.2053	1.649	3.818	2.169	–0.001	–0.216	–0.013	–0.004	1.935	2.7	2.663
B	19	0.5972	0.2042	1.238	3.058	1.820	+0.001	–0.214	–0.015	–0.005	1.587	4.9	2.657
B	20	0.5284	0.2025	2.700	4.338	1.638	–0.003	–0.213	–0.014	–0.004	1.404	4.9	2.657
B	21	0.6048	0.2015	3.070	4.907	1.837	–0.003	–0.212	–0.015	–0.004	1.603	4.9	2.650
B	22	0.5846	0.2072	2.089	3.880	1.791	–0.001	–0.217	–0.013	–0.003	1.557	2.7	2.663
A	23	0.5840	0.2073	2.330	4.118	1.788	–0.002	–0.218	–0.010	–0.003	1.555	2.7	2.662
A	24	0.6000	0.2034	1.042	2.860	1.818	–0.001	–0.214	–0.014	–0.003	1.586	10.1	2.643
B	25	0.5315	0.2015	1.139	2.776	1.637	–0.001	–0.212	–0.011	–0.005	1.408	9.6	2.649
A	26	0.5420	0.2051	1.910	3.586	1.676	+0.001	–0.215	–0.015	–0.005	1.442	9.7	2.660
B	27	0.6833	0.2065	1.657	3.708	2.051	–0.001	–0.217	–0.015	–0.007	1.811	11.9	2.650
A	28	0.5536	0.2052	1.955	3.661	1.706	0.000	–0.215	–0.015	–0.003	1.473	3.2	2.661

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL
LABORATORY, HARVARD COLLEGE.

ON THE THOMSON EFFECT AND THE TEMPERATURE COEFFICIENT OF THERMAL CONDUCTIVITY IN SOFT IRON BETWEEN 115° AND 204° C.

By Edwin H. Hall, L. L. Campbell, S. B. Serviss,
and E. P. Churchill.

Investigations on Light and Heat made and published, wholly or in part, with Appropriation
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By Edwin H. Hall, L. L. Campbell, S. B. Serviss, and
E. P. Churchill.[1]

Presented May 9, 1906. Received December 7, 1906.

Introduction and Summary of Results.

A paper printed in these Proceedings for May, 1905, by the authors of the present paper gave the following results as obtained from the study of a certain specimen[2] of soft iron :

Composition : iron, 99.93% ; carbon, 0.059%.
Density : about 7.785 at 0° C.
Thermal conductivity, k : 0.1528 at 28.2° C.,
with a temperature coefficient 0.0003 (?) from 28° to 58°,
" " " " 0.0007 (?) " 13° " 87°,
the latter value being the more reliable.
Electric resistance, absolute : 11365 at 0° C.,
with a mean temperature coefficient 0.00519 from 0° to 100°.
"Thermo-electric height" with copper :

$$\text{approximately}\left\{\begin{array}{l} 1028 \times 10^{-8} \text{ volt at } 26.6^\circ \text{ C.,} \\ \ \ 980 \quad \text{" \quad " \quad " } 41.3^\circ \text{ C.,} \\ \ \ 936 \quad \text{" \quad " \quad " } 54.5^\circ \text{ C.,} \\ \ \ 870 \quad \text{" \quad " \quad " } 71.1^\circ \text{ C.,} \end{array}\right.$$

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[1] Mr. Churchill was not engaged in the earlier part of the work described in this paper, but he worked with me during the greater part of July, 1906, in a supplementary investigation. — E. H. H.

[2] It is proverbial that the various properties of any given metal, as set down in physical tables, are usually obtained from different specimens. It has seemed to us worth while to study one particular kind of iron in as many of its different aspects as practicable. The study is still in progress.

Thomson effect coefficient, ν:

$$\text{approximately} \begin{cases} -757 \times 10^{-10} \text{ mean, from } 13^\circ \text{ to } 90^\circ, \\ -715 \quad \text{``} \quad \text{``} \quad \text{``} \quad \text{``} \quad \text{``} \; 51^\circ, \\ -793 \quad \text{``} \quad \text{``} \quad \text{``} \; 51^\circ \text{ ``} \; 90^\circ, \end{cases}$$

[a mean rise of -2×10^{-10} per degree].

We have now to add for the same specimen of iron:

Temperature coefficient of thermal conductivity between 115° and 204° C., referred to the value of the conductivity at 115° = -0.00068, approximately. Electric resistance, absolute: 17260 at 100° and 26140 at 218.2°, with a mean temperature coefficient 0.00661 (on the basis of the 0° value of the resistance) between 100° and 218°.

Thomson effect coefficient, ν ($= \sigma \div T$, where σ is the fraction of a calorie absorbed per second in the Thomson effect by a current of 10 amperes flowing from a cross-section at temperature ($T - \frac{1}{2}$) absolute to a cross-section at temperature ($T + \frac{1}{2}$) absolute), from a combination of our new observations with our old ones, approximately (see pp. 614, 615).

$$-(717 + 2\,(T - 305)) \times 10^{-10},$$

from 32° C. to 182° C. (The rate of change of ν here indicated would make the tangent of the angle of inclination of the iron line on the ordinary thermo-electric diagram about 40 per cent greater at 182° C. than at 32° C.)

Battelli, who worked about twenty years ago with a specimen of iron which he does not describe, except as to its shape and size, got values from which we can deduce the approximate law

$$\nu = -(283 + 0.34\,(T - 326)) \times 10^{-10}$$

from 53° C. to 308° C.

Lecher, who also in his recent paper fails to describe the quality of the iron with which he worked, gets σ as -278×10^{-7}, that is, $\nu = -855 \times 10^{-10}$, at 52° C., and gives a formula which, when allowance is made according to the data of Bède, for the change of specific heat of iron with change of temperature, indicates a considerable increase, about 12 per cent, in the numerical value of ν between 50° and 130°, a nearly constant value from 130° to 160°, a slight numerical fall between 160° and 180°, and an increasing rate of fall with further rise of temperature.

It appears, then, that there is general agreement among those who have worked directly on the Thomson effect in iron that the line for this metal on the thermo-electric diagram should be one of increasing steepness from 50° C. to the neighborhood of 130° C.; though Lecher is in disagreement with others as to the general change of inclination of the line above this temperature.

It is noted that lead, in which the Thomson effect is undoubtedly very small at certain temperatures, has apparently been carefully examined only in the neighborhood of 50° C. (by Le Roux, by Haga, and by Battelli) and 110° C. (by Battelli), and the question is raised whether it is not desirable to examine copper, which is mechanically a much better material to work with than lead, very carefully through the widest practicable range of temperature, with a view to establishing a reference line for the thermo-electric diagram.

Incidentally, it is observed that the thermal conductivity of calcined oxide of magnesium, packed to a density of about 0.17 gram per cu. cm., is considerably less, perhaps 30 per cent less, at 218° C. than at 100° C.

General Arrangement of Apparatus.

The general method followed in our recent work was the same as that which was described in the paper already referred to; but, as many changes in detail were necessitated by working at a higher temperature, it seems desirable to show by means of diagrams the later form of the apparatus.

Figure 1 shows a horizontal section through the middle at the level of the two main iron bars, α and β, each 1 cm. in diameter, which were the subjects of the research. The pots are of cast iron, about 16.5 cm. long, in the direction of the bars, 27 cm. wide, and 25 cm. deep.

Around the main bars are two "guard-rings" (see Figure 2, a vertical cross-section of the apparatus), each consisting of twenty iron bars about 0.6 cm. in diameter, the primary object of which is to lessen lateral outflow of heat from the main bars. The outer guard-ring is a new feature of the apparatus.

Behind the brass nuts *n* on the main bars are, first, washers of brass; next, washers of leather prepared for plumbers' use, to bear a rather high temperature.[3] Mica and a plumber's cement, consisting of silicate

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[3] A trial which we now know to have been too short had persuaded us that this prepared leather would withstand the action of hot naphthalin. When, after a long course of experiments, the main apparatus here described was closely examined, it appeared that the leather washers had been reduced to dust. But the cement had apparently held, so that, so far as we know, there was no leakage at the flanges of the main bars during the experiments on the Thomson effect, though there was such leakage in experiments made later, in July, 1906. See p. 619.

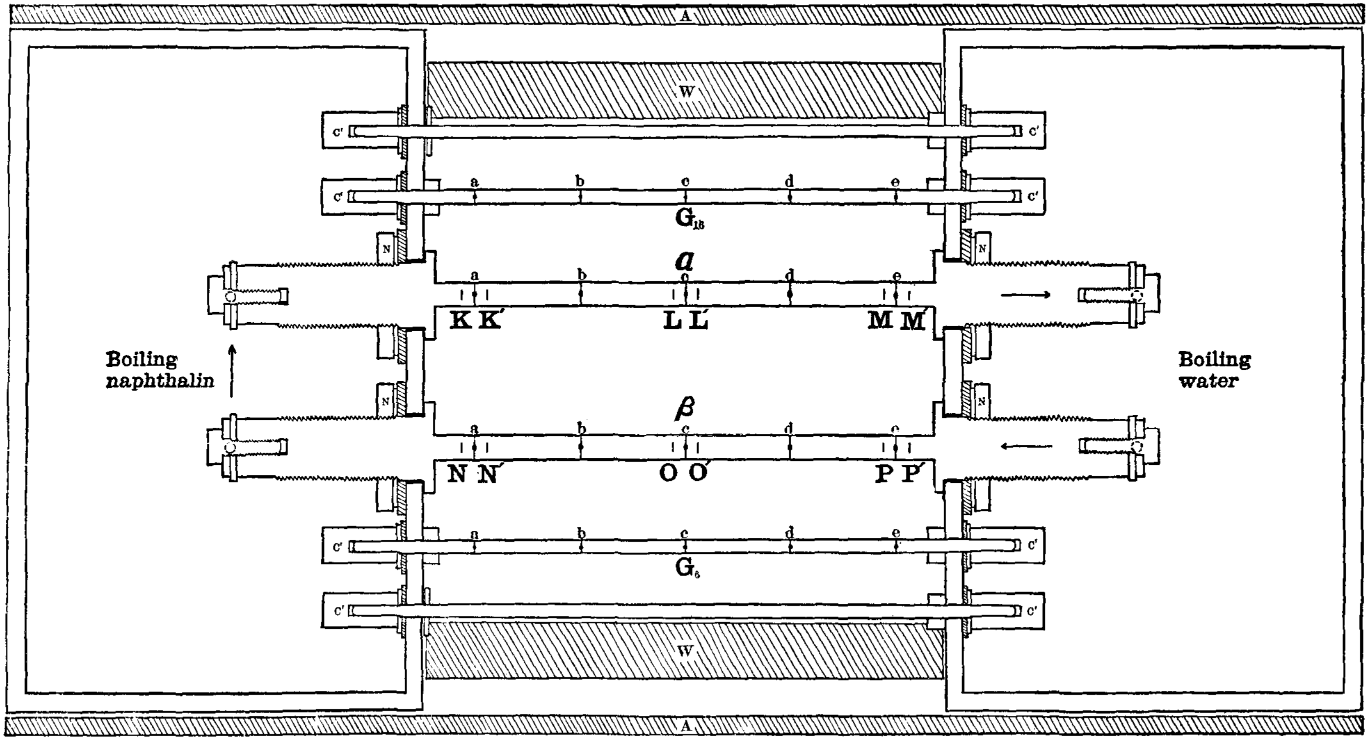
A
W
c'
a
b
c
d
e
G_{18}
α
K K'
L L'
M M'
N
Boiling naphthalin
Boiling water
β
N N'
O O'
P P'
G_6
W
A

Figure 1.

of soda, graphite, and asbestos, with perhaps other ingredients, were used for packing and insulation under the flanges of the main bars and within the cylindrical apertures adjacent.

To prevent leakage at the ends of the numerous guard-ring bars we depended solely on lead washers squeezed hard between the brass cap nuts *c* and the walls of the pots; and to lessen the danger of straining the apparatus and producing rupture of the joints around the main bars at the tightening of these nuts, we used on or in each guard-ring bar just outside the walls of pots, nuts or iron pins, as Figure 1 shows. Although in most cases the lead washers thus used were entirely satisfactory, we shall see later that through one or two joints leakage finally occurred.

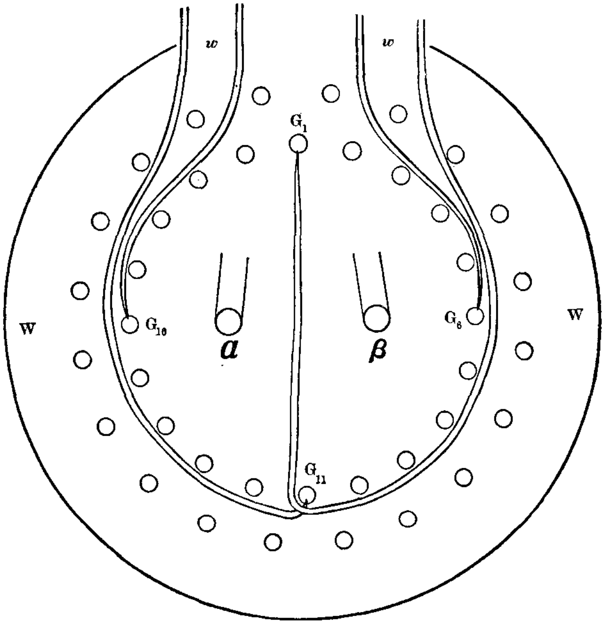

FIGURE 2.

K and K′ are the junctions, 1 cm. apart, of a copper-German-silver thermo-electric couple wound spirally in a groove made in the asphaltum coating of the bar *a*, as shown in Figure 5 of our former paper, which represents a part of the surface of the bar laid out in a plane. L L′, M M′, N N′, O O′, and P P′ are similar spirally placed couples. Each such couple is to indicate the gradient of temperature in its part of the bar on which it lies. Cemented by asphaltum to the bar α, midway between K and K′ is α_a, one junction of a copper-German-silver couple, the other junction of which is kept at a known temperature in a Crafts[4] boiler containing water or naphthalin, which apparatus is not shown in the figure. The object of this couple is to indicate the mean temperature of that part of α which lies between K and K′. The junctions α_b, α_c, α_d, α_e, β_a, β_b, β_c, β_d, and β_e, have each a function similar to that of α_a; for each of them there is a corresponding junction in one or another of four Crafts boilers. The bar G_6 carries five copper-German-silver junctions, 6_a, 6_b, etc., and the bar G_{16} a like equipment, 16_a, 16_b, etc., corresponding in position and function to the junctions α_a, β_a, etc., already noted, on the main bars. In every case the junction indicated in the figure is connected with a mate, which can be kept at a definite temperature in one of the Crafts boilers. Each junction attached to a guard-ring bar is really lodged in a small hole drilled in the bar, insulation being secured by means of thin-walled glass tubing, and fixity of position by means of asphaltum varnish. Bars G_1 and G_{11}, shown in Figure 2, are similarly provided. We have, accordingly, on the main bars and the guard-ring taken together, six *a* junctions, six *b* junctions, etc., each set of six lying approximately, though by no means accurately, in one cross-sectional plane. The effect of displacements from such a plane, though for the most part eliminated in the end by the practice of reversing the gradient of temperature in the apparatus, will require notice in some cases.

In every case the distance from *a* to *b*, from *b* to *c*, etc., was intended to be 4 cm. The distance from pot to pot was very nearly 21 cm.

The copper and German-silver wires, all about 0.02 cm. in diameter, leading up from the various thermo-electric junctions which have been described, were insulated, where insulation seemed necessary, by passing them through short, thin-walled glass tubes. The wires from the main bars ran almost straight up, as Figure 2 indicates, while those from the guard-ring bars were made to follow the contour of the inner guard-ring for a greater or less distance before leaving the packing with which the guard-rings are surrounded. The wires from bar G_1, at the top, were carried down to the bottom of the inner ring, as Figure 2 shows, and then brought up on its outside. The object of these precautions was to make sure that the gradient of temperature in the wires leading from any junction should be very small in the neighborhood of that junction. All the wires, after leaving the packing surrounding the bars, ran nearly straight upward to a wooden platform about 30 cm. above the outer guard-ring. Here all the copper wires were connected with larger copper wires leading off toward the du Bois and Rubens *Panzer-galvanometer* with which all the thermo-electric currents were measured, while all the German-silver wires were continued to the Crafts boilers, about 1 meter distant.

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[4] See a paper *On Thermo-electric Heterogeneity in Certain Alloys, especially German-Silver*, by the authors of the present paper, these Proceedings, March, 1906, for an account of the calibration of thermo-electric couples by use of the Crafts apparatus.

To obstruct the lateral flow of heat from the main bars outward, the outer guard-ring was covered, except at the top, where the wires came through, by a wrapping of magnesia and asbestos about 2.5 cm. thick, and the space within was stuffed rather loosely with wisps or wads of asbestos fibre. Concerning this packing something will be said later.

The space between the square pots was closed on both sides by thick asbestos board, between which and the packing outside the outer guard-ring there was at mid-level an interval about 2 cm. wide.

The space beneath the packed bars was walled in on all four sides by similar asbestos board, which was intended to prevent, to a great degree, circulation of air, and especially of hot gases from the large Bunsen-burner flames used beneath the pots, from passing up past the bars. The space above the bars, up to the wooden platform already mentioned, was boxed in in the same way, though there were, before April 13, two gaps near the top of this enclosure, through which escape of air from the interior was facilitated, it being at first considered unsafe to close in completely the space under the wooden platform.

STUDY OF THE LATERAL FLOW.

The first observations of this kind were made January 26–27, 1906. Both pots contained water boiling at atmospheric pressure, and the outer junctions of all the thermo-electric couples *a*, *b*, *c*, *d*, and *e* connected with the main bars and the guard-ring bars were placed in two of the Crafts boilers containing water boiling at atmospheric pressure.

The time at which boiling began is not recorded in this case. Observations to test the approach of the bars to a constant condition of temperature began about 6 P. M., January 26, and were continued about two hours, the indications being that a fairly stable condition had been attained by 8 P. M. The boiling was continued, however, till past midnight before any of the thermo-electric observations finally used were made.

The final operations were substantially as follows, though not made in the precise order here given : —

1. The couples 1_a, 6_a, 11_a, and 16_a, all the guard-ring couples belonging to the cross-section *a*, were joined in series, and the current sent by this combination through a known resistance was measured. This made it possible to find the difference between the boiling-point of water in the Crafts boilers and the mean temperature of the points *a* on the guard-rings, and so to find this mean temperature itself.

Similar observations were made with respect to the sections *b*, *c*, *d*, and *e* of the guard-rings.

2. The couples α_a, α_e, β_a, β_e, on the main bars, joined in series, were placed in opposition to the couples 1_a, 6_a, 11_a, and 16_a, joined in series, and the current sent by this combination through a known resistance was measured. This gave the means of finding the excess of the mean temperature of the points *a* and *e* on the main bars above the mean temperature of the points *a* on the inner guard-ring.

Similar observations gave the means of finding the excess of the mean temperature of the same points *a* and *e* on the main bars above the mean temperature of the points *e* on the inner guard-ring.

Further observations of a like character gave the excess of the mean temperature of the points *b* and *d* on the main bars above the mean temperature of the points *b* and the points *d*, respectively, on the inner guard-ring.

The excess of the mean temperature of the points *c* on the main bars above the mean temperature of the points *c* on the inner guard-ring was found in like manner.

3. The spiral couples K K′, M M′, N N′, P P′, on the main bars, were joined in series, and the current sent by this combination through a known resistance was measured. This made it possible to find the mean gradient of temperature at the points *a* and *e* on the main bars.

From (1) and (2) above we get the following summary of temperatures and temperature differences: —

	Mean *a* and *e* Temperature.	Mean *b* and *d* Temperature.	Mean *c* Temperature.
Main bars,	100.28° [5]	100.07°	100.06°
Guard-ring,	99.56°	98.55°	98.06°
Differences,	0.72°	1.52°	2.00°

Proceeding with these data, we find by a graphical method that the difference between the mean temperature of the main bars between the cross-sections *a* and *e*, a distance of 16 cm., exceeds the mean temperature of the inner guard-ring bars between the same two planes approximately 1.48°.

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[5] The barometer read about 77 cm. corrected, making the temperature of boiling water about 100.4° at the top of the pot, and perhaps 100.7° at the level of the main bars.

The mean temperature gradient, leading toward the middle, at the points *a* and *e* of the main bars was found, according to the evidence of (3) above, to be approximately 0.066° per cm. The diameter of each bar being 1 cm., and its thermal conductivity at 100° approximately 0.145, according to our observations of last year, we find as the amount of heat led into each bar per second, by conduction past *a* and *e*,

$$2 \times \frac{\pi}{4} \times 0.066 \times 0.145 = 0.0150 \text{ calorie.}$$

This is also, when a stable condition is reached, the amount of heat lost from each bar per second by lateral flow from the parts between *a* and *e*.

As the lateral outflow per second from each bar per 1 degree excess of mean temperature of the bar above the mean temperature of the inner guard-ring, all relating to the region between the cross-sections *a* and *e*, we have, then,

$$0.0150 \div 1.48 = 0.0101 \text{ calorie.}$$

On February 8 and 9 the lateral flow was measured again, all the pots in use now containing naphthalin boiling under atmospheric pressure, which was about 77.3 cm. The boiling temperature of high-grade naphthalin, such as was used in the Crafts boilers, is 218.8° at this pressure. That of the very cheap variety, which was used in the square pots at the ends of the bars, is not so well determined; but there was no need of an accurate knowledge of the temperature in these pots.

Naphthalin was boiling in all of the pots about 7 P. M. on February 8, and continued in that condition until after 6 A. M. of February 9, operations being continued through the night. The first thermo-electric observations recorded were made about 3 A. M., and the last about 6.20 A. M. The following statement will give some indication of the degree of constancy of temperature which was maintained during these observations. Under *Sensitiveness* are given the galvanometer deflections produced by a current of a certain constant strength used in determining the sensitiveness of the instrument; under *Spirals*, the deflections given by the couples K K′, M M′, N N′, P P′, joined in series; under *M against G*, the deflections given by the main bar *a*'s and *e*'s joined in series, in opposition to the guard-ring *a*'s in series.

Time.	Sensitiveness.	Spirals.	*M* against *G*.
2 : 55 A.M	13.18	. . .	. . .
3 : 15	. . .	19.85	. . .
3 : 45	. . .	. . .	12.98
5 : 53	. . .	. . .	13.44
6 : 10	. . .	20.35	. . .
6 : 25	13.08	. . .	. . .

It appears that both the mean gradient at the spirals and the mean difference of temperature between the main bars and the guard-ring bars *increased* a few per cent during the course of the observations, keeping pace with each other pretty closely in this change. This is a sufficient approach to equilibrium for our purposes.

A summary of the results from the observations of February 9, which were in a general way similar to those of January 26 and 27, is given below. The *c* temperature of the main bars was found by direct thermo-electric observation; the other main bar temperatures were estimated from this one by the use of data furnished by the gradient measurements and the *M against G* measurements. The guard-ring temperatures were found by means of the subtraction here shown:

	Mean *a* and *e* Temperature.	Mean *b* and *d* Temperature.	Mean *c* Temperature.
Main bars . .	216.0°	214.7°	214.2°
$M-G$ . . .	3.6°	6.5°	8.1°
Guard-ring .	212.4°	208.2°	206.1°

Plotting an M–G curve, we get, as the excess of the mean temperature of the main bars, between the cross-sections *a* and *e*, over the mean temperature of the corresponding part of the inner guard-ring, 6.32°.

Taking the thermal conductivity of iron to be 0.134 at 216°, an estimate which is in accord with our measurements of the temperature coefficient of thermal conductivity, we get, as the total flow of heat per second in each bar at the junctions *a* and *e*,

$$2 \times \frac{\pi}{4} \times 0.403 \times 0.134 = 0.848 \text{ calorie.}$$

Accordingly we get, as the lateral outflow from each bar per second, in the stable condition, per degree excess of mean temperature of the bar above the mean temperature of the inner guard-ring, all relating to the region between the cross-sections *a* and *e*,

$$0.0848 \div 6.32 = 0.0134 \text{ calorie.}$$

The difference between this estimated outflow, per degree difference of temperature in the neighborhood of 218° C., and the corresponding outflow in the neighborhood of 100° C., 0.0134 and 0.0101, respectively, is too large, apparently, to be readily attributed to mere errors of measurement. On the other hand, one may well doubt whether the coefficient of thermal conductivity of a packing of asbestos fibre really increases by 33 per cent in the rise of temperature from 100° to 218°, though we are unacquainted with any experiments upon this point except our own. We have already stated that the fibre was loosely packed, 1 gram, in, let us say, 15 cu. cm. It is easy to make the density of such packing several times as great as this, and the question has arisen whether there may not have been considerable currents of air through our loosely placed fibre and whether the increase in facility of lateral heat transmission with rise of temperature may not have been due to the part played by such air currents. In this state of doubt one naturally asks whether the coefficient of lateral outflow of heat at either temperature is consistent with the results obtained by previous experimenters on the thermal conductivity of fibrous materials.

Assuming that our lateral outflow is due to conduction proper, we can make from our data a tolerable estimate of the coefficient of thermal conductivity of the packing. Thus, examining Figure 2, we see that no heat should flow across the vertical between G_1 and G_{11}. Accordingly, a gives heat to the half-ring $G_1 G_{16} G_{11}$ only. The length of this arc, along the centres of the guard-ring bars, is approximately 7π cm., 22 cm. It is evident that a, with a given difference of temperature between it and the guard-ring, sends less heat to this arc than it would send to the same length of arc bent to a radius equal to the distance, about 4 cm., from the centre of a to the centre of the nearest guard-ring bar, G_{16}. On the other hand, it is evident that a gives out more heat to the half-ring in the actual case than it would give to the same half-ring, with the same difference of temperature, if it were placed at the centre of curvature of the ring, the radius of which is 7 cm. We can, therefore, make no very large mistake in assuming that, for a given difference of temperature, the lateral outflow of heat from a is equal to that which would occur between a and a guard-ring arc 22 cm. long, having a radius of curvature equal to $\frac{1}{2}(4+7)$ cm., a being at the centre of curvature. A point of some doubt just here is, how large a part of the circumference of a we should consider as giving off heat to the guard-ring in the imagined arrangement. This part should evidently be more than half. A considerable variation in this particular would make no great difference in the estimate of the total outflow from a to the guard-ring. We shall make no serious error if we assume that the outgiving arc of a is the same part of a circumference as the receiving arc, 22 cm. bent to a radius of 5.5 cm. As the radius of a is 0.5 cm., we take 2 cm. as the length of the effective arc on a in the case imagined. In circular measure the length of each arc is 1.27 π.

In the case of steady flow, if q represents the number of calorics transmitted per second from 1 cm. length of a to the corresponding 1 cm. length of guard-ring, and if k stands for the thermal conductivity of the packing, while T stands for temperature, we have for any ring of radius r and width dr,

$$-k \times 1.27 \pi r \frac{dT}{dr} = q, \quad \text{or} \quad dT = -\frac{q}{k \times 4} \cdot \frac{dr}{r}.$$

Integrating between limits we get

$$t_1 - t_2 = \frac{q}{4k} \log_e \frac{r_2}{r_1} = 0.6\, q \div k.$$

At 100° for $t_1 - t_2 = 1$ we have $q = 0.0101 \div 16 = 0.00063$, whence $k_{100} = 0.00038$.

At 218° for $t_1 - t_2 = 1$ we have $q = 0.0134 \div 16 = 0.00084$, whence $k_{218} = 0.00050$.

Of course no great accuracy can be claimed for a result obtained by so many rude approximations; but it seems unlikely that these approximations have introduced an error so great as 25 per cent into the estimated values of k.

If now we seek to compare these values of k for loosely packed asbestos fibre with the values obtained by other investigators, we fail to find any information given in the 1905 edition of *Landolt and Börnstein* concerning the thermal conductivity of this material. If we look at the values of k given for other fibrous materials, we see figuring prominently in tables of physical constants the values obtained many years ago by Professor George Forbes for haircloth, cotton-wool (divided), cotton-wool (pressed), flannel, and coarse linen. These values range from 0.0000402 for haircloth to 0.0000298 for coarse linen. They at first seem to make the values obtained above for asbestos packing rather improbable and tend to strengthen the suspicion of considerable air currents through this packing. But the values found by Forbes are, in all probability, decidedly too small, for they are considerably less than the fairly well-established value, about 0.00005, of the thermal conductivity of "stagnant" air at ordinary temperatures. Rubner[6] found $k = 0.000098$ for cotton-wool packed closely, about 1 gm. to 8 cu. cm., though he found smaller values for wool and silk. Lees and Chorlton[7] found $k = 0.00043$ for asbestos-paper. They give no description of this paper beyond its name and the thickness, about 0.3 cm., of the layer composed of it. The mean temperature of this layer was about 87°. The value 0.00043 lies between the two, 0.00038 and 0.00050, derived above for the effective k of our layer of asbestos fibre; but, as it is altogether likely that the asbestos-paper of Lees and Chorlton made a much more dense layer than the one which we used, and would therefore have a greater conductivity proper, it seems probable that there was more or less convective movement of air through the layer of fibre as we used it.

The coefficient of outflow through this layer at 100° was about 1.7 times as great as that found in our previous work with closely packed cotton-wool at 100°.

In making allowances for the lateral outflow at temperatures intermediate between 100° and 218° we assumed that the coefficient of lateral flow, which may be defined as *the fraction of a calorie lost per second from unit length of a main bar per degree difference of temperature between main bar and guard-ring*, increases at a uniform rate with rise of temperature from 100° to 218°.

It is assumed, furthermore, that this coefficient is practically independent of the gradient of temperature which exists in the bars and in the packing parallel to the length of the bars. This is equivalent to the assumption that the various currents of heat through the packing, whether maintained by conduction proper or by convection, are practically independent of each other. As the currents of air through the packing must be comparatively slow, this assumption is not, perhaps, a violent one, though the doubt attaching to it is sufficient, in view of the large corrections which must be made for lateral flow, to make these corrections the least satisfactory part of our work. But see p. 616.

TEMPERATURE COEFFICIENT OF THERMAL CONDUCTIVITY.

On February 7, and again during the night of March 2–3, 1906, observations were made on the temperature coefficient of thermal conductivity in the main bars. During these observations there was no electric current in these bars and the two were treated as one, in this sense, that no attempt was made to determine the temperatures or the gradients of temperature on the main bars singly, mean values only being looked for. Accordingly, in what follows t_a will indicate the mean temperature of the two main bars in the cross-section *a*, g_a will indicate the mean gradient of temperature of the two bars at the same cross-section, etc. Similarly t'_a will indicate the mean temperature of the guard-ring bars, 1, 6, 11, and 16, in the cross-section *a*, etc.

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[6] Archiv für Hygiene, **24**, 320 (1895).

[7] Phil. Mag., 5th Series, **41**, 502 (1896).

The general method followed in this phase of the investigation was to establish practically fixed conditions of temperature, by keeping the boilers in action several hours before the beginning of observations for record, and then proceed substantially as follows:

1. Data were taken from which the gradients of temperature g_a and g_e could be found.

2. Data were taken from which the difference between the temperature of the main bars and the temperature of the guard-ring bars could be found for each of the five cross-sections, *a*, *b*, *c*, *d*, *e*. These data were obtained, in ways already sufficiently indicated, by means of the thermo-electric couples, α_a, β_a, G_{1a}, G_{6a}, G_{11a}, G_{16a}, etc.

As the various junctions lying nominally in any given cross-section do not strictly so lie, there is opportunity for very considerable errors in the estimation of the temperature differences mentioned in (2) above. It is possible to eliminate such errors almost wholly from the final result by making two complete sets of observations of the kinds indicated in (1) and (2) above, one set with the *a* end of the apparatus, the east end, the hotter and the other set with the *e* end, the west end, the hotter. The first condition existed on February 7, and the second condition on March 2 and 3.

The following table exhibits the main data obtained from the two sets of observations. The temperatures, *t*, given for points on the main bars, were in some cases estimated from data obtained at other times; but the possible inaccuracy of these values cannot be a source of important error. The columns headed $(t-t')$ give the excess of temperature of points on the main bars over nominally corresponding points on the guard-ring bars.

At the foot of each gradient column is given the difference of gradient; at the foot of each *t* column, the range of temperature; at the foot of each t-t' column, the mean value of the column.

The values in the last three columns, which are mean values taken from the preceding columns, are used in finding the temperature coefficient of thermal conductivity. In what immediately follows the subscript h will refer to the high temperature cross-section, which was at one time a and another time e, while the subscript l will refer to the low temperature section, which was at first e and later a.

We must now take account of the fact that a part of the excess of g_h over g_l is due to the lateral escape of heat from the main bars between the h and the l cross-sections. From the data given in the last column of the table above a curve is plotted from which may be found the difference of temperature of the main bars at any section between h and l. Then, with the use of the data already given concerning lateral flows, the amount of heat escaping laterally from each one of the 16 linear centimeters of each bar between the h and l cross-sections is found. The total lateral outflow as thus found, from each bar, is 0.0369, let us say 0.037, calorie per second. Taking first a tentative value of the thermal conductivity at t_h, 203.7°, and replacing this value later by one obtained by use of the data we are now dealing with, proceeding, in other words, by a method of approximation, we find that the gradient of temperature at t_h needed to carry through the h cross-section this amount of heat is 0.346, or 0.35. This we must subtract from g_h in order to find just how much of the latter gradient is necessary to transmit the amount of heat that passes through the l cross-section. But before this subtraction is made we raise the value of g_h from 5.90+, to 5.91, in order to make due allowance for the fact of expansion in the main bars. We thus get $5.91 - 0.35 = 5.56$ as the value of g_h, which is to be compared with 5.26, the value of g_l. Accordingly, if k stands for thermal conductivity, and if f

February 7.				March 2 and 3.				Means.			
	Gradient, g.	t.	$t-t'$.		Gradient, g.	t.	$t-t'$.		Gradient, g.	t.	$t-t'$.
a	5.92°	203.2°	2.44°	e	5.89°	204.1°	5.02°	$a\ e$	5.90+°	203.7°	3.73°
b	. . .	179.4	3.44	d	. . .	179.8	5.10	$b\ d$	. . .	179.6	4.27
c	. . .	156.7	3.71	c	. . .	158.2	4.46	c	. . .	157.4	4.09
d	. . .	135.3	1.72	b	. . .	135.5	2.16	$d\ b$	. . .	135.4	1.94
e	5.30	114.4	–0.32	a	5.22	114.3	–0.39	$e\ a$	5.26	114.4	–0.35
	0.62°	88.8°	2.20°		0.67°	89.8°	3.27°		0.64+°	89.3°	2.73+°

stands for the temperature coefficient of this property between t_l and t_h, we have

$$k_h \times g_h = k_l \times g_l$$

and
$$f = \frac{k_h - k_l}{k_l} \div (t_h - t_l) = \left(\frac{k_h}{k_l} - 1\right) \div (t_h - t_l)$$
$$= \left(\frac{g_l}{g_h} - 1\right) \div (t_h - t_l) = -0.00060+.$$

As the value of this temperature coefficient is a matter of considerable interest, and as such coefficients are notoriously difficult to determine, it seems worth while to inquire how much the two values differ which are obtainable from the data of February 7 and the data of March 2 and 3, taken separately and corrected, as well as possible, for the small displacements of the various junctions from their nominal planes of cross-section.

Measurements, which were capable of no great accuracy, have indicated that the junctions on the guard-ring bars are placed, on the average, about 0.15 cm. farther toward the east end of the apparatus than the corresponding junctions on the main bars. This displacement, with the gradient of temperature ordinarily prevailing in the bars, about 5.5, would make a difference of about 0.15 × 5.5, or 0.83, degree. The values of $t - t'$, corrected accordingly, would be $2.20 + 0.83 = 3.03$ for February 7, and $3.27 - 0.83 = 2.44$ for March 2 and 3. The values of f, worked out from the two sets of data, as thus amended would be very nearly -0.00047 and -0.00073 respectively.

The value of f found in our observations of last year on the same bars of iron, between 13° and 87°, was -0.0007, the value of k at 13° being the basis. The values of f found in this paper are for the interval from 115° to 204°, approximately, and the value of k at 115° is here taken as the basis. If we use the value of k at 13° as our basis now, we get approximately -0.00056, instead of -0.00060, as the value of f between 115° and 204°. The work on thermal conductivity described in the present paper is probably entitled to more confidence than that described in our paper of last year, though the correction for lateral flow was very much greater this year than last year.

On the whole, in view of the acknowledged difficulty of making anything like an accurate determination of f, the general accord of all our results in this phase of our work is reassuring.[8]

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[8] See the Appendix to this paper for an account of further experiments on the temperature coefficient of thermal conductivity in bars α and β.

Jaeger and Disselhorst found (see Beib. zu den Ann. der Phys. for 1901, p. 20)

MEASUREMENT OF THE THOMSON EFFECT.

Our general method of procedure in getting data on the Thomson effect itself, and our method of handling these data, were so fully set forth in the paper[9] already mentioned that, inasmuch as we have made no fundamental changes in these methods, we shall not describe them in detail here; but certain modifications may well be mentioned. In the previous paper occurs the following sentence: "It will be observed that such a day's work includes six complete cycles, or runs, through all the pairs of corresponding couples from M P to K N and back, three runs with the main current in condition F [cold to hot in the *front* bar, β], and three with this current in condition B [cold to hot in the *back* bar, α]." In the work described in the present paper we have usually, though not always, made more cycles during a single heating, as the following practically complete table shows:

Date.	Cycles.			Date.	Cycles.		
	B	F			B	F	
January 31	2	2	4	March 2–3	5	3 2	10
February 5–6	3 3	6	12	April 13–14	9	5 4	18
" 16–17	5 3	4 4	16	" 17–18	4 5	8	17

The main data obtained from the experiments just indicated, and from supplementary observations made at other times on the temperature differences of the main bars, are exhibited in the table on p. 608, in which

Δ_l = the difference of temp. gradient at low temp. spirals.
Δ_m = " " " " " " middle " "
Δ_h = " " " " " high " "

The + sign indicates that the steeper gradient is in the warmer bar, and *vice versa*.

t_l = the mean of the temperatures at the low temp. spirals.
t_m = " " " " " middle " "
t_h = " " " " " high " "
τ_l = " diff. " " " low " "
τ_m = " " " " " middle " "
τ_h = " " " " " high " "

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as the value of this coefficient −0.00065 in one bar of iron and −0.00014 in another bar of iron, between 18° C. and 100° C.

[9] These Proceedings, May, 1905.

EAST END HOT.

Date.	Δ_{l}.	Δ_{m}.	Δ_{h}.	t_{l}.	t_{lm}.	t_{m}.	t_{mh}.	t_{h}.	τ_{l}.	τ_{lm}.	τ_{m}.	τ_{mh}.	τ_{h}.	A.	Sets of Observations.
Jan. 31	+0.0700	+0.0083	−0.0761	114.4°	135.8°	158.0°	179.9°	203.2°	0.20°	0.39°	0.46°	0.44°	0.22°	25.21	4
Feb. 5, 6, 7	+0.0704	+0.0085	−0.0767	115.0	136.4	158.7	180.7	203.7	0.20	0.39	0.46	0.44	0.22	25.34	12
April 17, 18	+0.0662	+0.0073	−0.0740	114.7	136.1	158.7	180.4	203.2	0.20	0.39	0.46	0.44	0.22	25.32	17
General Mean	+0.0682	+0.0079	−0.0752	114.8	136.2	158.6	180.4	203.4	0.20	0.39	0.46	0.44	0.22	25.31	33

WEST END HOT.

Date.	Δ_{l}.	Δ_{m}.	Δ_{h}.	t_{l}.	t_{lm}.	t_{m}.	t_{mh}.	t_{h}.	τ_{l}.	τ_{lm}.	τ_{m}.	τ_{mh}.	h.	A.	Sets of Observations.
Feb. 16, 17	+0.0655	+0.0049	−0.0709	115.1°	136.7°	160.4°	181.3°	205.2°	0.20°	0.41°	0.50°	0.47°	0.21°	25.28	16
March 2, 3	+0.0609	+0.0023	−0.0681	114.7	136.3	159.9	180.8	204.6	0.20	0.41	0.50	0.47	0.21	25.26	10
April 13, 14	+0.0660	+0.0043	−0.0721	114.4	136.4	159.2	180.9	204.2	0.19	0.40	0.48	0.43	0.24	25.29	18
General Mean	+0.0647	+0.0041	−0.0708	114.7	136.5	159.8	181.0	204.6	0.196	0.41	0.49	0.45	0.22	25.28	44

t_{lm} = mean of temp's midway between the l and the m spirals.
t_{mh} = " " " " " " m " h "
τ_{lm} = diff. " " " " " l " m "
τ_{mh} = " " " " " " m " h "
Δ = strength of the main electric current in amperes.

In finding the "general mean" for each column except the last each quantity in the column is taken with a *weight* proportional to the number placed on its level in the last column.

The various t's and τ's in this table were not all independently observed. All those in the first part, *East End Hot*, are derived from observations made February 5 to 7, with allowance for the change which differences of barometric pressure would probably make in the temperature of the main bars, the ends of which are in boiling water or boiling naphthalin.

The t's and τ's for February 16–17 and March 2–3 are derived from observations made February 23–24. The t's and τ's for April 13–14 are derived from observations made April 14, during the same heating that gave the Δ's for these days.

In the derivation of the thermal conductivity values of iron which are needed for use with the data given above, the temperature coefficient of this conductivity is taken as −0.0007 below 115°, and as −0.0006 from this temperature up to 205°. We thus get approximately, k_l being the conductivity at t_l, etc. :

k_l	k_m	k_h
0.1436	0.1396	0.1355

The specific electric resistance of the iron, which also is needed in the evaluation of the Thomson effect, was measured by Mr. Campbell with the aid of the Crafts boilers, in the summer of 1905, and was found to be 17,260 at 100° and 26,140 at 218.2°, with a mean temperature coefficient 0.00661 (on the basis of the 0° value of the resistance) between 100° and 218°.

In the following table q' indicates the first approximation to the "Thomson effect heat"; that is, the difference between the amount of heat generated per second by the electric current between the l and the h cross-sections of either bar and the amount accounted for by Joule's law between the same cross-sections, q' being calculated by the formula (4), p. 25, of our previous paper.[10] The value q'', the second approximation to the quantity desired, obtained from q' by taking account of the difference of thermal conductivity due to the slight excess of temperature of one main bar over that of the other, differs so little from q' that it is not given in the table. Under q is given the final value of the Thomson effect heat, obtained from q'' by taking account of the amount of heat which, between the l and the h cross-sections, escapes by lateral flow from the warmer bar in excess of that which escapes in the same way from the cooler bar. This correction, which is large, usually about 23 per cent of the value of q, will be discussed at some length later. Under σ is given $q \div \frac{A}{10}(t_h - t_l)$; that is, the mean Thomson effect heat per electromagnetic unit of current per degree of the range of temperature considered. Under ν is given $\sigma \div (273 + \frac{1}{2}(t_l + h_h))$, which may be called the mean Thomson effect coefficient between t_l and t_h:

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[10] These Proceedings, May, 1905.

	Date.	$t_h - t_l$.	$q'\times10^5$.	$q\times10^5$.	$\sigma\times10^7$.	$\nu\times10^{10}$.	Wt.	General Mean $\nu\times10^{10}$.	
East end hot	Jan. 31	88°.8	799	1028	−459	−1064	4	−1045	−1019
	Feb 5–7	88.7	801	1029	−459	−1061	12		
	Ap. 17–18	88.5	766	996	−444	−1029	17		
West end hot	Feb. 16–17	90.1	745	985	−433	−999	16	−993	
	Mar. 2–3	89.9	704	945	−417	−964	10		
	Apr. 13–14	89.8	755	986	−434	−1004	18		

The difference, about 5 per cent, between ν_E, the value of ν obtained with the east end the hotter, and ν_W, the value obtained with the west end the hotter, as shown in the table just given, has been the subject of a great deal of cogitation, and a thoroughly satisfactory explanation of it has not been found. It is unlikely that the whole difference was purely accidental in the sense that it would have disappeared in a prolonged continuation of the experiments; for the smallest ν_E is larger than the largest ν_W. It is true that t_l and t_h were determined only once with the east end the hotter, which will now be called the state E; and, moreover, the conditions during this determination were not of the best, so that it may well be doubted whether the true value of $t_h - t_l$ was any greater with state E[11] than with state W. If we assume that the two values were the same, the difference between the mean ν_E and the mean ν_W will be reduced to about 3.7 per cent; but even then the smallest ν_E will be somewhat larger than the largest ν_W.

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[11] A redetermination of this quantity for condition E after the observations of April 17–18 was prevented by a partial breaking down of the apparatus, which mishap will be described later.

There is one other important datum, namely, the mean difference of temperature between the main bars, which was measured at only one heating in condition E, namely during February 5–7. This mean value, found from the observed values at the five cross-sections *a*, *b*, *c*, *d*, *e*, by plotting a curve, was 0.389°, or 0.39°. An error of about 20 per cent in this rather small quantity would account for the whole difference between the ν_E and the ν_W as found. That an error approaching this magnitude was made here is shown to be improbable by the fact that the two determinations of the main bar mean differences of temperature made with condition W gave 0.408°, or 0.41°, and 0.391°, or 0.39°, respectively.

Other possibilities which have been considered in the endeavor to account for the difference in question are the following:

1. A partial short circuit of the main electric current in the west pot when this contained naphthalin. There is no good reason for suspecting such an action. A test made on April 18, with boiling water in the west pot and boiling naphthalin in the east pot, showed that there was no serious leakage of electricity through either liquid.

2. Difference in the sensitiveness of the thermo-electric spirals on the main bars.

A difference of two or three per cent, perhaps more, in the sensitiveness of these spirals is possible. But the observations with these spirals are taken and are combined in such ways that the result obtained from a complete set of data for condition E or condition W is dependent on the mean sensitiveness rather than on the differences of sensitiveness of these spirals. We could account for the difference between ν_E and ν_W by means of variation in these spirals, if we should assume that the two *a* spirals, at the east end of the bars, are about seven per cent more sensitive at 204° and seven per cent less sensitive at 115° than the corresponding two *e* spirals at the west end of the bars. This is very improbable.

3. Dissymmetry in the shape of the bars or in the placing of the spirals upon them.

There is none that can account for the difference which we are considering. The bars were ground to 1 cm. diameter by Brown and Sharpe. The spirals are probably not all of exactly the same length, and there may be slight inaccuracies in their emplacement; but these imperfections, though they might affect slightly the value of ν, would have little or no tendency to make a difference between ν_E and ν_W.

4. Difference in the movements of air within the loose asbestos fibre

surrounding the bars, consequent upon a reversal of the gradient of temperature in the apparatus.

Already, in connection with the discussion of loss of heat by lateral outflow, the question has been raised whether this loss is merely, or practically, a function of lateral differences of temperature. If we had to do here with a true conduction only, there would be little risk in answering this question in the affirmative; but when convection, accompanying rather general movements of the air within the packing, comes into play, as it probably does to some extent, the problem becomes more difficult. But it is not easy to find any plausible reason why this longitudinal temperature gradient should be less effective in carrying off heat from the warmer bar in condition E than in condition W. It is true, that a leakage of naphthalin vapor into the packing space while condition E prevailed might reduce the effectiveness of the convection movements by replacing air by a more sluggish medium; but in the one case in which such leakage was known to occur, the value obtained for ν_E was smaller than usual, indicating, if anything concerning lateral loss, that such loss was greater on this occasion than usual. The general matter of lateral loss will be discussed once more later in this paper. It is the most troublesome part of our experimental problem, and it may be that some inequality of packing of the asbestos fibre is at the bottom of the discrepancy which we are now discussing.

5. Change of quality from one end to the other of the main bars.

Both bars were made from pieces taken from the interior of a very soft bar some 10 cm. in diameter, and any difference of composition or physical state sufficient to make such a difference in the Thomson effect with reversal of temperature gradient is very improbable. It is, perhaps, worth noting that in our work of the year 1905 with these same bars, and with the same spirals upon them, what corresponds to our ν_E of 1906 was, apparently, about two per cent greater than what corresponds to our ν_W of 1906; that is, in both 1905 and 1906 the data obtained with the marked ends (bar α is stamped with a *1* and bar β with a *2* on one end) of the bars the hotter have indicated a slightly greater value of the Thomson effect. This is probably an accidental coincidence.

This long search for an explanation of the discrepancy in question, though it has not been decisive, has served the useful purpose of passing in review a number of possible sources of error. A difference of five per cent in the values of ν_E and ν_W is of no great consequence, if we can feel reasonably sure that the mean of the two is not more than two or three per cent different from the true value of ν.

It must be confessed that this assurance is a little difficult to hold. The close scrutiny which we have given to the matter of lateral flow has left a rather troublesome doubt as to the soundness of our working assumption that this flow is not to any important extent a function of the longitudinal gradient of temperature. Upon this point we must bring to bear whatever further evidence is available.

A satisfactory mode of investigation, if it were not so extremely laborious, would be to repeat the determination of ν with other kinds or states of packing than those here used. Something like this variation of conditions we find in a comparison of our 1905 work with that of 1906, or in the former cotton-wool, closely packed in, was used instead of the rather loosely packed asbestos of 1906, and it has already been stated that the coefficient of lateral outflow, at 100°, per degree difference of temperature between the main bars and the guard-ring bars, was about 1.7 times as great with the asbestos as with the cotton. If we have made any large error in using the assumption now on trial, we should expect a marked discordance between the results obtained from the older and the more recent work. But as the two do not cover the same temperature, one lying between 13° and 90°, while the other lies between 115° and 204°, how can we, without begging the main question which this research deals with, tell what is accordance and what is discordance?

In this difficulty we get some help from the fact that in each case we have taken note of the difference of gradient at the middle cross-section, c, of the main bars, so that it is possible to make from the data of each year's research an estimate of the rate of change of ν with change of temperature, which estimate should not be greatly affected by error in regard to the absolute magnitude of the loss by lateral flow. These estimates are :

From Data of	Range of Temperature.	Inc. of ν per Degree Rise of Temp.	Mean.
1905	32° to 71°	-2.0×10^{-10}	-1.8×10^{-10}
1906	137° to 182°	-1.6×10^{-10}	

If now, taking -1.8×10^{-10} as the rate of change of ν from the mean temperature, about 53°, of the 1905 work to the mean temperature, about 159°, of the 1906 work, we reckon from the value of ν at 53° what its value at 159° should be, we get

$$-757 \times 10^{-10} - (159-53)\, 1.8 \times 10^{-10} = -948 \times 10^{-10}.$$

The value found from the data of 1906 is, as we have seen at page 610, -1019×10^{-10}, about 7 per cent larger than the value as estimated above. As an overestimate of the loss of heat by lateral flow would tend to an increase of the numerical value of ν, the test here applied, if it can be called a test, would seem to indicate that too great rather than too small a correction had been applied for lateral flow in getting the value -1019×10^{-10} for ν. Reducing the applied correction by one third of itself would reduce ν at 159° from -1019×10^{-10} to something very near -948×10^{-10}.

Further, and perhaps stronger, evidence tending to the same effect, that too large an allowance has been made for lateral flow in the work of 1906 thus far described, is now at hand. During July, 1906, a re-measurement of the temperature coefficient of thermal conductivity of bars α and β was made[12] with calcined oxide of magnesium as a packing, through which very little, if any, convection of heat could occur. The resulting value of this coefficient was -0.00076, whereas the value found when the asbestos fibre was used as a packing was -0.00060. An overestimate of the allowance for lateral flow tends to give too small a numerical value for this coefficient. Reducing the applied correction, in the case of asbestos fibre, by one fifth of itself would raise the value found for the coefficient from -0.00060 to -0.00074, which is very close to the value found with use of oxide of magnesium as a packing.

Circumstantial evidence, then, favors the opinion that the allowance which we have made for loss by lateral flow through asbestos fibre packing should be reduced about one fourth part of itself. Perhaps the best we can do under the circumstances is to combine the various estimates of the rate of change of ν with rise of temperature as follows:

	Increase of ν per 1° Rise.	Mean.
From values of ν at 32° and 71° . . . . (1905)	-2.00×10^{-10}	
From values of ν at 137° and 182° . . (1906)	-1.55×10^{-10}	-2.0×10^{-10}
From values of ν at 52° and 159° . . (1905 and 1906)	-2.45×10^{-10}	

Applying this rate of change to -757×10^{-10}, the value of ν at 52°, as found in 1905, we get, as the value of ν at 157°, -971×10^{-10}. This value is about 5 per cent less than -1019×10^{-10}, the value at 159° as found from the measurements of 1906 taken alone. This discrepancy is somewhat disappointing, and it must be admitted that our values of ν may be several per cent in error at any temperature.

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[12] By Mr. E. P. Churchill and myself. See the Appendix to this paper.— E. H. H.

On the other hand, it seems to be well established that the numerical value of ν increases at a rate not very different from that indicated above, 2.0×10^{-10} per degree, in the region between 32° and 182°. As ν is proportional to the tangent of the angle of inclination of the iron line from the horizontal line on the thermo-electric diagram, the statement just made means that in the temperature region considered the iron line should be one of increasing steepness with rise of temperature. Taking the value of ν

$$\text{at } 32^\circ \text{ as } -(757 - 20 \times 2) \times 10^{-10} = -717 \times 10^{-10},$$
$$\text{and at } 182^\circ \text{ as } -(757 + 130 \times 2) \times 10^{-10} = -1017 \times 10^{-10},$$

we find that the tangent of the angle of inclination of the iron line should be about 40 per cent greater at 182° than at 32°.

In our previous paper[13] it was pointed out that Battelli's observations on iron gave data which yield the following values for ν:

-283×10^{-10}	at	53° C.	-331×10^{-10}	at	243° C.
-319	"	" 108°	-369	"	" 308°

The mean rate of increase of ν here indicated would make the value at 182° about 18 per cent greater numerically than the value at 32°.

Battelli's general method was to surround a certain length of each of two similar bars of one metal by a narrow trough containing mercury, separated from the bar by a layer of varnish or other insulating material, establish a gradient of temperature in the bars by means of ice and boiling liquids, pass a strong electric current up the gradient in one bar and down in the other, then estimate the Thomson effect from the difference in the rate of heating of the two baths of mercury. He does not give sufficient details to assure us that he took adequate account of the tendency of the mercury, which was continually stirred, to reduce the gradient of temperature, and so the Thomson effect, in the part of the bar which it surrounded. Whether the low numerical value which Battelli found for the Thomson effect in iron, as compared with that found by us or very recently by Lecher,[14] is due to this peculiarity of his method or to his having used a very different kind of iron from that which we have used, is uncertain, but it is quite possible that his relative values of ν at different temperatures were approximately correct; and the per cent rate of change which his data give for ν with rise of temperature, though less than that which we find, is large enough to establish a strong presumption in favor of representing iron on the thermo-electric diagram by a line of constantly increasing steepness from the region of 50° to the region of 300°.

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[13] These Proceedings, **41**, 36 (1905).
[14] Annalen der Physik, 4, **19**, 859 (1906).

On the other hand, the work of Lecher, to which reference has already been made, gives at 52° the value -855×10^{-10} for ν in iron, a quantity about 13 per cent greater than our value of ν at the same temperature. As to the change of ν with rise of temperature, Lecher's data seem to indicate, when account is taken of the change of specific heat of iron with change of temperature, a considerable increase, about 12 per cent, in the numerical value of ν from 50° to 130°; a nearly constant value from 130° to 160°, a slight numerical fall between 160° and 180°, and an increasing rate of fall with further rise of temperature. Indeed, according to the curve by means of which Lecher gives his results for iron, even the quantity σ, that is, νT, begins to diminish numerically with increase of T in the neighborhood of 210° C., and goes on diminishing more and more rapidly to the upper temperature limit of the observations, 441° C., where the value of ν is scarcely more than one sixth as large as the ν at 210° C.

The method of Lecher, though very different from that of Battelli, resembled it in this particular, that both were calorimetric; that is, neither method depended on the establishment of constant conditions, but each made use of a rate of change of temperature of a quantity of metal. Battelli noted the heating effect in mercury, and he observed the change of temperature during a 30-minute run of the electric current. Lecher noted the rise of temperature in the iron itself during a 30-second run of the current. In getting the curve for iron, to which reference is made above, he seems to have ignored the change of specific heat of iron with change of temperature, though he observes that a correction might be made on account of this change, and apparently refrains from making it on the ground that "the data concerning the specific heat of iron are very uncertain." The common belief appears to be that this quantity increases rather rapidly with rise of temperature, and if account were taken of this change, the course of Lecher's curve for iron and the inferences drawn from it would be seriously affected. Lecher himself recognizes other sources of possible error in his work, and it may well be doubted whether the evidence which he presents entirely offsets the force of Battelli's data, which, as we have already seen, tend to the conclusion that not only νT but ν continues to increase with rise of temperature up to at least the neighborhood of 300° C. It must be admitted that the true course of the iron line, above 200° C., on the thermo-electric diagram is uncertain.

In the case of copper and in the case of silver Lecher does make an allowance for the change of specific heat with change of temperature. He gets with the unit current (10 amperes), in copper, —

$$\sigma = +(3.01 + 0.00662\ t)\ 10^{-6},$$

and in silver, —

$$\sigma = +(7.363 + 0.00887\ t)\ 10^{-6},$$

t being the temperature on the ordinary Centigrade scale. From these equations we get for copper, —

$$\nu = \sigma \div T = +\left(\frac{12000}{T} + 66.2\right)10^{-10},$$

and for silver, —

$$\nu = +\left(\frac{49400}{T} + 88.7\right)10^{-10}.$$

Each of these equations indicates a falling value of ν with rise of temperature. Each indicates for the ordinary thermo-electric diagram a line rising from left to right, with constantly though slowly diminishing steepness. Lecher himself would probably not claim that his observations are sufficiently accurate to establish as a fact the change of curvature of these representative lines. They do, however, make it appear probable that the lines for copper and silver should be drawn nearly straight through a great range of temperature on the thermo-electric diagram, and they suggest these metals as good subjects for further and more careful study. For the very difficulty which every one of experience in the matter finds in the measurement of the Thomson effect in any metal impresses upon us the desirability of establishing some one line on the thermo-electric diagram so accurately that others can be satisfactorily determined from it by "thermo-electric height" measurements, without further recourse to direct observations of the Thomson effect.

To be sure, the lead line, taken as horizontal, has been used in this way, and it may be asked whether it is not still sufficient for the purpose indicated. All observers, so far as we know, agree in declaring the Thomson effect in lead to be very small. Le Roux,[15] who rated this effect in iron as 31 and in copper as 2, found it *sensiblement nul* in lead. At or near 50° C., Haga[16] found the effect in "pure" lead to be negative, as in iron, but *tres faible* compared with that in platinum. Battelli,[17] stimulated by the fact that Haga had found a Thomson effect in lead, undertook to measure this effect. He found it *positive*, and gives data from which we get, —

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[15] Ann. de Chimie et de Physique, 4me série, t. X, 1867, p. 201. Le Roux does not, we believe, describe the quality of the lead which he used.

[16] Ann. de l'Ecole Polytechnique de Delft, t. III, 1887, p. 51. Analysis showed the lead used by Haga to contain, "beside a trace of silver, 0.093 per cent of antimony, 0.004 per cent of iron, and 0.013 per cent of zinc." It is easy to get much purer lead now.

$$\nu \text{ at } 53.0° \text{ C.} = 4.37 \times 10^{-10},$$
$$\nu \text{ “ } 108.4° \text{ C.} = 4.29 \times \text{ “ }.$$

Below 50° C. and above 110° C. we have, apparently, no direct experimental evidence concerning the magnitude of the Thomson effect in lead, and between these two temperatures we are, finding Haga on one side and Battelli on the other, in doubt as to the sign of this effect in the purest lead. It seems probable that the lead line on the thermo-electric diagram should be very nearly horizontal below 50° and above 110°; but it can hardly be said that we have a satisfactory assurance in regard to this fundamental matter. Accordingly it seems desirable to study very carefully through a rather wide range of temperature the Thomson effect in some metal which is easily obtained in a state of great purity. Lead is such a metal, but it is mechanically difficult to work with, especially at moderately high temperatures. Copper seems to be, on the whole, a better material for the most careful direct examination, with a view to the establishment of a reference line on the thermo-electric diagram.

Appendix.

The doubt concerning the amount and character of the lateral escape of heat through the loose asbestos fibre packing was so great that two of us[18] undertook in July, 1906, further experiments bearing on this question. By means of a rough test with a small apparatus we satisfied ourselves that closely packed asbestos would not make a satisfactory wrapping, as the flow of heat through it appeared to be even greater than through the loosely packed asbestos which we had used. Accordingly we looked for other materials capable of bearing a temperature of 218° without injury, which might serve better. Magnesium carbonate, which is used commercially as a bad conductor of heat, was found to lose much weight, doubtless from the giving up of water of composition, at 218° and to regain weight when exposed to the air at ordinary temperature. This, therefore, was rejected. The oxide of magnesium, "light calcined magnesia," on the other hand, was found to bear the heating test well, losing at first a small amount of weight, in drying, probably, but remaining constant in weight at high temperature thereafter.

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[17] Atti della Reale Accademia dei Lincei, série 4, t. **3**, 1887, p. 51. Battelli does not give the analysis of the lead, but states that its density at 0° C. was 11.344.

[18] Mr. Churchill and myself. — E. H. H.

Accordingly we undertook to repack our Thomson effect apparatus, using now the dried magnesium oxide, and to repeat the experiments on lateral flow through the packing as well as those on the temperature coefficient of the thermal conductivity of iron.

As we had a strong apprehension that our boilers would now leak vapor of water or of naphthalin into the packing space, though we had done what we could to stop such leaks as we had discovered, and as we feared that any condensation of vapor in the powdery oxide would be highly objectionable, we put a sort of buffer of asbestos fibre about 0.5 cm. thick against the boiler at each end of the space to be packed, trusting that the upward movement of air through this layer would carry off vapor coming into it. Then, after putting in place the wrapping of magnesia and asbestos which lies outside the outer guard-ring (see Figure 3), we poured the magnesium oxide down into the space between the many bars, striving, by tapping the outer covering and by pressing down the oxide with strips of metal, to get the light powder so well packed that it would not by subsequent settling leave empty spaces under the bars. The density of the powder as thus placed was about 0.17 gram per cubic centimeter.

After this we made, in spite of a considerable amount of leakage of naphthalin, especially toward the last, observations which resulted as follows:

Lateral outflow from each main bar per 1° *difference of temperature between the main bars and the inner guard-ring,*

0.00791 cal. with both pots at 100°, July 17–18
0.00894 " " " " " " " 27
0.00843 "

0.00579 " " " " " 218°, " 21–22
0.00665 " " " " " " " 28
0.00622 "

Temperature coefficient of thermal conductivity in iron between 115° *and* 204°, *with reference to the value at* 115°,

$$f = -\,0.00076,\ \text{July 19 and 24.}$$

It is to be noted that in this packing the thermal conductivity appears to be about 30 per cent greater at 218° than at 100°.

No great confidence should be placed in the numerical accuracy of this indication, but the direction of the change seems to be well established.

The cause of the apparent increase in conductivity of the packing, both at 100° and at 218°, with lapse of time is not known.

The value here found for f, combined with that found for the same coefficient when asbestos packing was used, gives as a mean

$$f = -\tfrac{1}{2}(0.00076 + 0.00060) = -0.00068.$$

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE. E. L. MARK, DIRECTOR. — No. 187.

AN ELECTRIC WAX-CUTTER FOR USE IN RECONSTRUCTIONS.

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AN ELECTRIC WAX-CUTTER FOR USE IN RECONSTRUCTIONS.

By E. L. Mark.

Presented December 12, 1906. Received December 17, 1906.

Any one who has had much experience in making models of microscopic objects by means of wax reconstruction-plates must have felt the disadvantages of the ordinary method of cutting out the plates with a scalpel.

Some years ago the advantages of heating the scalpel in a flame suggested to me the desirability of employing a knife heated to a constant temperature, and observation of the methods employed in making corrections to "line process" plates prepared by the so-called wax method led me to think it might be possible to use for this purpose a small gas jet attached to the knife and connected with the gas supply by a small flexible rubber tube. I did not succeed, however, in producing a device that would work satisfactorily. A little later it occurred to me that a wire heated by an electric current might be kept at a sufficiently constant temperature to answer the purpose. Accordingly in an ordinary bow-saw with large bow the saw blade was replaced by a fine wire, the ends of which were insulated from the frame and connected by means of flexible insulating wire with a 110-volt alternating electric circuit. By introducing into the circuit the proper resistance — in the form of electric lamps arranged in multiple — it was possible to heat to the proper degree the wire selected. By means of this apparatus one could melt a wax plate readily along any predetermined course, provided the wire were slowly moved back and forth as in sawing. But this apparatus was defective, owing to the lengthening of the wire and its consequent laxness when heated. It became obvious at once that for accurate work some sort of spring would have to be introduced into the mechanism to take up the slack of the wire. But a still more serious and unexpected drawback presented itself. Unless the cutting was done very slowly, the melted wax cooled in the saw-carf behind the wire, leaving the edges of the cut almost as firmly joined as before the wire had passed through. This, together with the difficulty of holding conveniently the wax plate during the process of cutting, led to the adoption of a fundamental modification. Instead of holding the wax plate in such a position that the wire-saw could be moved freely, it seemed that a better plan would be to move the wax plate on a horizontal platform against a wire fixed in a vertical position. Some preliminary trials were made with such a device, but it soon became evident that the wire would have to be made white hot if the track through the wax plate were to be made with satisfactory rapidity. The disadvantages of this were obvious; among others, the wire became dangerously lax, and the desired rigidity could not be maintained. The remedy which at once suggested itself, and proved in the end to be practicable, was to give the wire, while keeping it taut by means of a spring, a vertical play of an inch or two, as in a jig-saw. By this means the wire surface brought in contact with the wax was multiplied many times, and a sheet of wax that quickly cooled down a *stationary* wire at the level of contact was easily melted by the moving wire.

A domestic experience in the exchange of sewing machines had acquainted me with the fact that an abandoned — but not worn out — sewing machine was worth about what it would fetch for old iron. I selected one of fairly large proportions, and proceeded to replace the needle with the electrically heated wire. It was at this juncture that the mechanical skill and experience of one of my students — Mr. J. A. Long — proved to be of great value. We together planned the details of the alterations to be made, and he executed the most of them, no professional mechanic being required at any time. The obstacle which it took us longest to overcome was the freezing of the wax in the track of the wire. I at first thought it would be necessary to use a jet of hot air to blow the melted wax away, but found on trial that a cold blast was even better, so far as getting the wax out of the saw-carf was concerned, for it left a sharper cut. After removing the presser-foot, in its place a blowpipe with a fine opening was clamped to the presser-foot bar, and so adjusted that a blast, conducted to it through rubber tubing, was directed downward on the wax plate immediately behind the electric wire. The melted wax was easily removed from the sheet, but there were two serious drawbacks to the arrangement. First, the lower end of the wire where it was attached to the lower end of the needle-bar became loaded with congealed wax, which in time interfered with the working of the machine. Secondly, it was necessary to change continually the position of the wax plate, so that while being melted it should always press

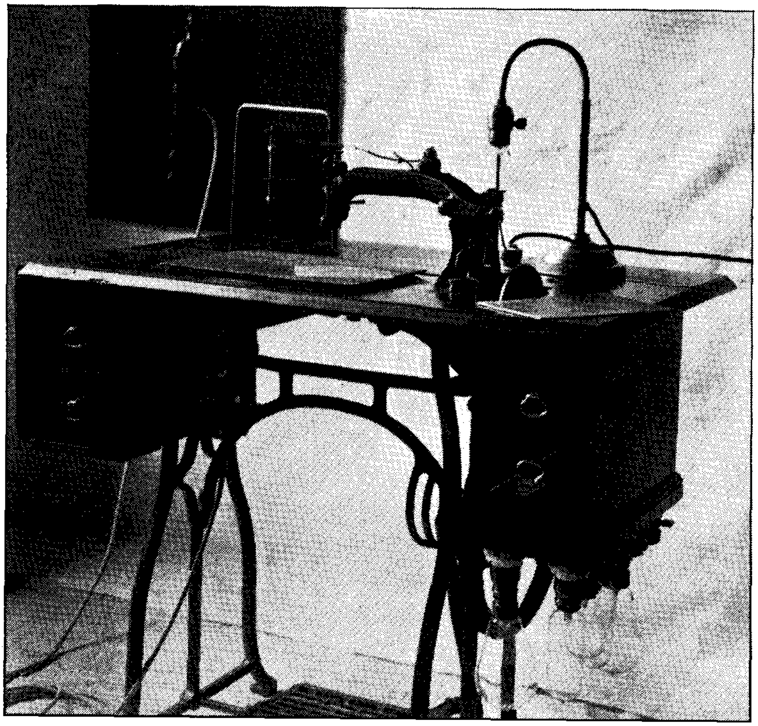

FIGURE 1 (compare Figure 3). Complete machine, showing three of the arms (the two vertical and the upper horizontal) of the "needle-bar," the two horizontal brass bars bound to one of the vertical arms by binding-screws, the short steel rod and (foreshortened) the two bars of indurated fibre supporting the spring and platinum wire. The lower horizontal arm of the needle-bar occupies a "well" which is covered by the horizontal slate platform, raised slightly above the table top. In the background are seen the Richards pump attached to a faucet and the rubber suction tubing leading to the bottle below the water tank. The electric lamps composing the rheostat are seen below the right-hand drawers of the machine.

against the near face of the wire, leaving all the melted wax on the far side of the wire. Changing the position of the blowpipe so that the blast was from below instead of from above, proved to be little, if any, improvement on the first method, for thereby the melted wax, blown upward, fell in drops and spatters on the upper surface of the wax plate, rendering it unfit for accurate reconstruction work, and there was the same disadvantage as before, that the cutting had to be so done as to leave the saw-carf with its melted wax always on the side of the wire away from the operator. These difficulties were finally overcome by substituting suction for blast, and by causing the wire to emerge from the centre of the blowpipe hole, as will be explained directly.

As at present arranged, the apparatus consists of a sewing machine (Wheeler & Wilson), in which only a few changes have been required.

These changes consist in the removal of certain superfluous parts, such as the presser-foot, the bobbin and bobbin-holder, etc., and the substitution for the needle and needle-bar of an arrangement for holding the heated wire.

The needle-bar is replaced by a cylindrical steel bar of precisely the same diameter as the original needle-bar. This new bar may for convenience be called "needle-bar" (B, B, Figure 3). It is thrice bent at right angles, thus giving four regions, or arms, — two vertical and two horizontal. The first of the two vertical arms is inserted, in place of the original needle-bar, into the bearings of the head (H), and fastened by a binding-screw to a block which in turn is connected to the lever (V) by means of the link (L). The upper horizontal arm of the bar is made about as long (20 cm.) as half the diameter of the largest wax plate which it is designed to cut. The bending is such that this arm and the two vertical arms are in one plane; the fourth (lower horizontal) arm is bent out of that plane, its free end being nearer the operator than the corresponding end of the upper horizontal arm. A vertical slit in the free end of the lower arm receives a copper wire, which is held firmly by a binding-screw and terminates in a hook, as seen in Figure 3. The lower end of the electrically heated wire (W), which is platinum, is made into a loop that can be slipped on to this hook. The loop at the upper end of the platinum wire is likewise made to slip on to a hook at the lower end of a brass wire, which is supported indirectly by the first vertical arm of the bent steel rod, or "needle-bar." A part of the brass wire is bent into a spiral spring (S), which serves to keep taut the heated wire (W), the lower end of the brass wire being free to move up and down through a hole in the lower of the two short horizontal square bars of indurated fibre which support it. These two bars are clamped by binding-screws to a short vertical steel rod, which is in turn attached to the first vertical arm of the "needle-bar" by means of two square rods of brass, bored at each end to receive the steel rods, and furnished with set-screws. The whole of the apparatus thus far described will evidently be moved up and down with the oscillating vertical motion of the lever V, and is represented in Figure 3 at about the middle point of its excursion, which in this machine amounts

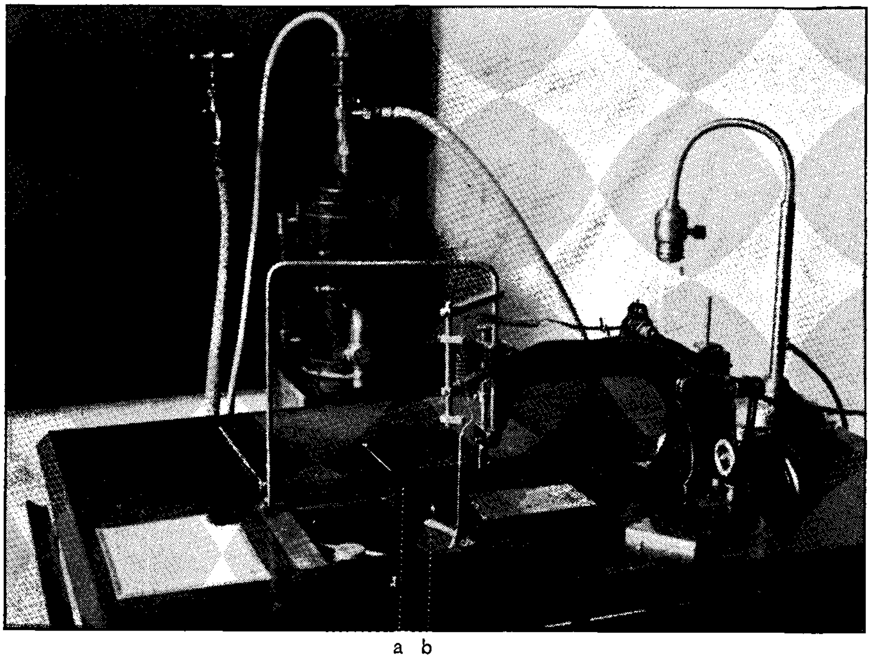

FIGURE 2. The top of the machine seen obliquely from above. The slate platform, showing a central hole for the nozzle of the copper tube and a slot running from it to the front margin of the platform, has been pushed back so as to uncover the "well," in which are seen the round screw cap of the hot-water tank, and immediately beyond it a portion of the lower horizontal arm of the "needle-bar." In addition are seen the two levers (a, b) which are used in raising the far edge of the platform. At the left of the "well" is the removable felt screen, and in the background the Wagner und Muntz pump. The platinum wire (which has been purposely made more conspicuous than it really is) is shown with the supporting brass wire and spring, and likewise the manner in which the latter are supported by passing through the bars of indurated fibre. The portable lamp is seen at the right.

to 35 mm. Supporting the upper wire and spring by two sets of horizontal bars bored to receive the cylindrical rods has the advantage of allowing one to adjust the upper hook vertically over the lower hook with great ease and accuracy. Once clamped in the proper position, these bars need no resetting. The use of indurated fibre, which is a poor conductor, for the shorter horizontal bars, serves to insulate, as well as to support and guide, the upper wire and spring. To prevent lateral vibration of the long, or second, vertical arm of the "needle-bar," a bearing is provided at the level of the table top of the machine, as shown in Figure 2.

The electric circuit used to heat the wire W is established by clamping the wire of one pole to the head (Figure 3, H) by a binding-screw (seen directly below the axle of the lever), and connecting the other with the upper end of the insulated brass wire and spring. Thus, head, lever, and bent "needle-bar" form a part of the circuit. In this circuit is introduced a rheostat (R, Figure 3) consisting of some eight or nine electric lamps of candle-power varying from 4 to 50, arranged in multiple, each with separate key. These are seen in Figure 1, beneath the drawers at the right of the figure. A portable lamp on the table (shown in Figures 1 and 2) is constantly in the circuit, and serves to illuminate the wax plate while cutting. With an alternating current of 110 volts the candle-power required properly to heat the wire (W) — a platinum wire of 26 standard gauge (about 0.4 mm. in diameter) — is between 50 and 100. The rheostat described allows one to adjust the resistance to any voltage ordinarily used in electric lighting.

The removal of the melted wax is effected by suction produced by using a Bunsen pump. Figure 1 shows the machine set up with a pump of the Richards pattern; Figure 2, with a more efficient special pump, made by Wagner und Muntz, Munich. With a pressure of 30 or 35 pounds either pump will effectually withdraw the melted wax. To succeed with this suction apparatus, which is an essential part of the outfit, requires careful attention to certain conditions. The sucking tube must be hot enough to keep the melted wax from immediately hardening, and it must be very close to the under surface of the wax plate at the point where the plate is being cut. The manner in which this has been accomplished is shown in Figure 3. A quarter-inch (6 mm.) copper tube passes through a water reservoir heated with an alcohol lamp, shown indistinctly in Figure 1. The lower end of the copper tube ends in a glass bottle with an air-tight rubber stopper pierced by a bent glass tube (D, Figure 3), to which the suction tube from the pump (Figure 1) is attached. The tube O is for the escape of steam from the water tank. The upper end of the copper tube is drawn out into a bent nozzle (N, Figure 3, seen in vertical and in horizontal section), with a narrow (about 2 mm.) orifice. In the convex side of the terminal bend is cut a longitudinal slot (T), which is slightly wider than the diameter of the platinum wire. The wire, without being detached from its hooks, is slipped into the slot so as to occupy the centre

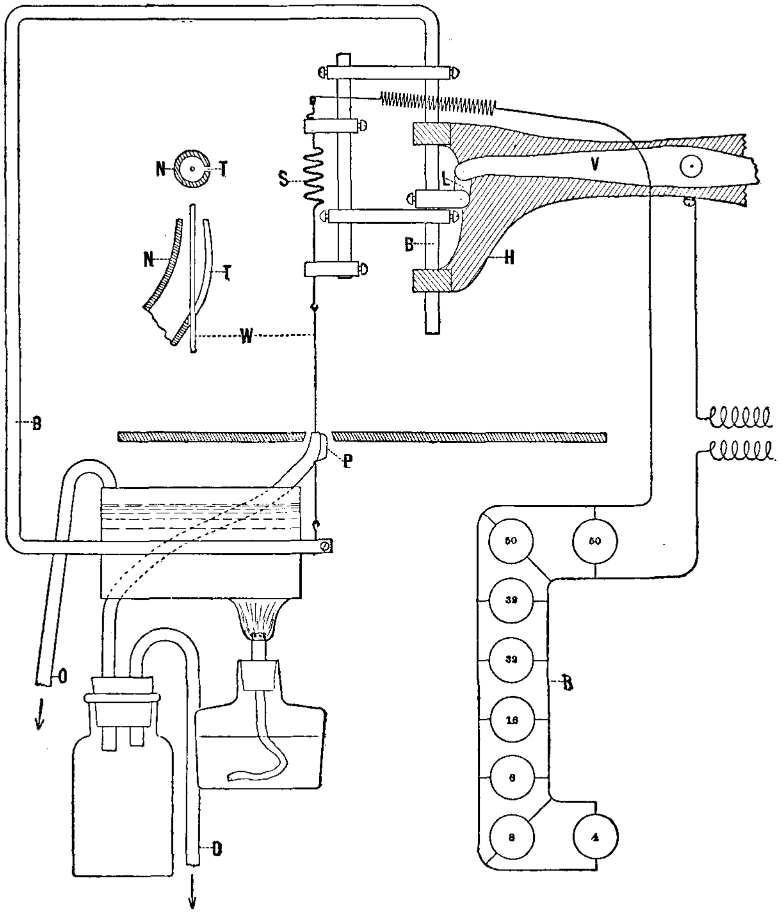

FIGURE 3. Diagrammatic representation of the essential parts of the apparatus. These are all shown in vertical section, except the rheostat, which is shown in horizontal projection. Near the middle of the rectangular area, bounded by the arms of the "needle-bar" (B, B) are shown in vertical section the nozzle (N) with its slot (T) and the platinum wire (W); and immediately above this a horizontal section of the same at the level of the letter N of the vertical section.

B, B. The two vertical arms of the "needle-bar"; D, glass tube terminating in the bottle, which also receives the copper tube; the rubber suction tube is attached to the glass tube at D; H, portion of the "head" of the sewing machine, which receives the "needle-bar"; L, link by which the "needle-bar" is attached to the lever; N, nozzle of the copper tube; O, orifice of tube for the escape of steam from the hot-water tank; P, metal plug to fill the slot in N; R, rheostat; S, brass spring to keep the platinum wire taut when hot; T, slot in one side of copper nozzle; V, lever connected with crank wheel; W, platinum wire.

of the terminal orifice of the nozzle and to emerge below from the bottom of the slot. The slot is then carefully stopped with a thin metal plug P (slight projections from its surface prevent its being forced inward too far), so that air can enter the tube through the terminal orifice only. If this orifice is kept close to the under surface of the wax plate, the melted wax will be completely withdrawn and will run down into the glass bottle; but if the orifice drops only a few millimetres below the under surface of the wax plate, the melted wax will not be withdrawn and will soon congeal, leaving the cut edges firmly reunited. This fact has been taken advantage of to produce at will either a temporary or a permanent cut. As it would be inconvenient to raise and lower the nozzle with its attached water reservoir, the slate platform which supports the wax plate during cutting is made movable in a nearly vertical direction. The front edge (that next the operator) is supported on two round-headed screws, — one seen distinctly near the dotted line a, Figure 2, the other faintly, close to the detached "front plate-slide," further to the right. The height of the front edge of the platform can thus be regulated by turning these screws in or out, and accurate adjustment to the height of the fixed nozzle thus secured. The middle of the far edge of the platform rests on a square block (a, Figure 2) screwed to a long horizontal arm turning on a horizontal pivot at the left. Another horizontal arm (b) turning on a vertical pivot engages the slanting under side of the block, and when moved in a certain direction raises the block some 6 or 8 mm. This second horizontal arm is actuated by levers, not shown in the figures, which are moved by the operator's knee. Thus the far edge of the platform may be quickly raised or lowered at will, so that the middle of the platform, where the heated wire is melting the wax, will also be raised or lowered about half as much as the distant edge.

To prevent the slate platform from becoming heated by the hot-water tank below it, a felt lining is attached to its under surface and a removable screen of the same material is placed over the tank. This is seen at the left in Figure 2 — a square sheet with a square notch cut out of one corner to accommodate the platform-block.

As thus arranged, the wire may be readily heated to the desired temperature, and, by operating the pedal as in sewing, it may be made to make rapid vertical excursions. Since the wire is central to the orifice in the copper tube, the wax plate may be moved in any direction, the melted wax being withdrawn with equal facility, whatever the direction of the cutting. The fine sharp cut, exactly perpendicular to the plane of the wax, which is produced by this machine, seems to meet all the requirements for cutting wax plates rapidly and accurately.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

CONCERNING POSITION ISOMERISM AND HEATS OF COMBUSTION.

BY LAWRENCE J. HENDERSON.

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CONCERNING POSITION ISOMERISM AND HEATS OF COMBUSTION.

By Lawrence J. Henderson.

Presented by T. W. Richards, January 9, 1907. Received December 20, 1906.

Current ideas of valence consist of two distinct conceptions, quantivalence and valence energy, which rest upon two bodies of fact of different sorts. Of these conceptions quantivalence has played by far the greater rôle in the development of organic chemistry and of stereochemistry, though the consideration of valence energy is present or implied in Le Bel's discussion of the asymmetric carbon atom, in von Baeyer's "Spannungstheorie," in Thiele's theory of partial valence, and in Werner's stereochemical theories and recent publications. In Richards's recent discussions of the compressible atom the conception of valence energy has shown itself more pertinent than the conception of quantivalence, and in the theoretical discussion of thermochemical data it has been, and of course is, of the greatest moment.

Conclusions regarding valence energy which are based upon heats of combustion are open to the criticism that from measures of the magnitude of the total-energy change it is sought to determine the magnitude of a quantity which depends perhaps entirely upon the free-energy change. Yet even to-day the determination of heats of combustion remains the one way possible of gaining quantitative data regarding the magnitude of valence energy in organic compounds, and it is probable that such information, properly interpreted, leads to not inaccurate conclusions. On the contrary, these conclusions may be very accurate when differences between the heats of combustion of similar substances are considered, and when such differences in very similar cases are compared, as in this paper, there is good reason to believe that changes in bound energy have been almost wholly eliminated.

Of these two ideas concerning valence, that of valence energy is the less clearly defined. J. Thomsen, it is true, has sought to show that the energy involved in producing a union between any two atoms is the same for all cases of that particular union, excluding double and treble ties, but without regard to the constitution of the rest of the molecule, and with the aid of this theory he has woven the fabric of his complicated thermochemical hypotheses. But, as I have shown in another place,[1] unless together with valence energy other considerable energy relationships exist within the molecule, a possibility as yet unsupported by fact, the energy of the tie between two atoms is variable, and dependent upon the nature of every other atom of the molecule and its position. This logical deduction was established by the consideration of such processes as the substitution of hydrogen by hydroxyl to form primary alcohols or acids as the case may be. The average values of the change in heat of formation of the molecules in these two cases are

$$\mathrm{R\,CH_3 - R\,CH_2\,OH = 40\ Cal.}$$
$$\mathrm{R\,CHO - R\,COOH = 72\ Cal.}$$

If we assume that the valence energy of a tie between two atoms is constant for all cases of the union, and exclude the possibility of other considerable energy relationships within the molecule, it is clear that these two values, 40 Cal. and 72 Cal., should be identical. Therefore the assumption is incorrect.

This conception of varying valence energy is, indeed, old. It was originally stated without experimental basis by Claus,[2] and, again as a theory, was developed by Werner[3] in his original discussion of his stereochemical ideas. More recently[4] the significance of comparative reactions has been carefully considered in this connection, and from that point of view much evidence has been adduced in support of the idea of variable valence energy.

The influence of the nature of those atoms of a molecule which are not directly concerned in a reaction of the molecule upon the heat of a reaction, an influence which must manifest itself in part through variations in the dependent valence energies of the reacting atoms, I have already briefly discussed;[5] with existing data little more can be said, and I postpone the discussion of this aspect of the question till the necessary experimental data are at hand, investigations to this end being now in progress in this laboratory.

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[1] Journal of Physical Chemistry, **9**, 40–56 (1905).

[2] Berichte **14**, 432 (1881).

[3] Vierteljahrsschrift d. Züricher Naturforscher Gesellschaft (1891).

[4] Werner, Berichte, **39**, 1278 (1906). Flürscheim, Jour. f. prakt. Chem., **66**, 321 (1902); **71**, 497 (1905); Berichte, **39**, 497 (1905).

[5] l. c.

On the other hand, the influence of the relative positions of its atoms upon the heat of reaction of a molecule, and consequently upon the valence energies of the reacting atom, is an interesting subject which in some of its aspects may be discussed with the aid of existing data.

The following tables contain the available data which show that the introduction of a foreign group into a molecule influences the heat of a reaction of an atom group of that molecule in a varying degree according to the relative positions of the two groups.

I.

$$CH \longrightarrow C{-}CH_3$$

A

COOH present.[6]

			CAL.
α	$CH_3 \cdot COOH - H \cdot COOH$	207 − 59	= 148
β	$C_2H_5 \cdot COOH - CH_3 \cdot COOH$	364.0 − 206.7	= 157.3
higher	$C_2H_5 \cdot C_nH_{2n} \cdot COOH - CH_3 \cdot C_nH_{2n} \cdot COOH$		= 156.5

B

$CONH_2$ present.[7]

α	$CH_3 \cdot CONH_2 - H \cdot CONH_2$	283 − 135	= 148
β	$C_2H_5 \cdot CONH_2 - CH_3 \cdot CONH_2$	439.8 − 282.7	= 157.1
higher	$C_2H_5 \cdot C_nH_{2n} \cdot CONH_2 - CH_3 \cdot C_nH_{2n} \cdot CONH_2$		= 156.1

C

$CONHC_6H_5$ present.[7]

α	$CH_3 \cdot CONH\,C_6H_5 - H \cdot CONH\,C_6H_5$	1011 − 861	= 150
β	$C_2H_5 \cdot CONH\,C_6H_5 - CH_3 \cdot CONH\,C_6H_5$	1168 − 1011	= 157
higher	$C_2H_5 \cdot C_nH_{2n} \cdot CONH\,C_6H_5 -$ $CH_3 \cdot C_nH_{2n} \cdot CONH\,C_6H_5$		= 156.2

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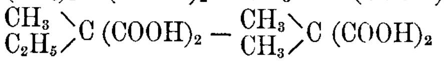
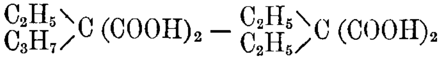
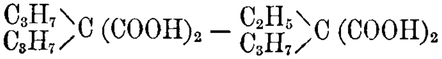
[6] Stohmann, Jour. f. prakt. Chem., **49**, 107 (1894).

[7] Id., **52**, 59 (1895).

D

$CH(COOH)_2$ present.[8]

α $CH_3 \cdot CH(COOH)_2 - H \cdot CH\ (COOH)_2$ 362 — 207 = 155
α $(CH_3)_2\ C\ (COOH)_2 - CH_3 \cdot CH\ (COOH)_2$ 515 — 362 = 153

β $\begin{matrix}CH_3\\C_2H_5\end{matrix}\!\!>C\ (COOH)_2 - \begin{matrix}CH_3\\CH_3\end{matrix}\!\!>C\ (COOH)_2$ 676 — 515 = 461

γ $\begin{matrix}C_2H_5\\C_3H_7\end{matrix}\!\!>C\ (COOH)_2 - \begin{matrix}C_2H_5\\C_2H_5\end{matrix}\!\!>C\ (COOH)_2$ 990 — 833 = 157

γ $\begin{matrix}C_3H_7\\C_3H_7\end{matrix}\!\!>C\ (COOH)_2 - \begin{matrix}C_2H_5\\C_3H_7\end{matrix}\!\!>C\ (COOH)_2$ 1146 — 990 = 156

E

CO present.[9]

α $CH_3 \cdot CO \cdot CH_3 - CH_3 \cdot CHO$ 437 — 282 = 155
α $C_3H_7 \cdot CO \cdot CH_3 - C_3H_7 \cdot CHO$ 754 — 600 = 154
β $CH_3 \cdot CH_2 \cdot CHO - CH_3 \cdot CHO$ 441 — 282 = 159
β $(CH_3)_2 \cdot CH \cdot CHO - CH_3 \cdot CH_2 \cdot CHO$ 600 — 441 = 159

F

COH present.[10]

α $CH_3 \cdot CH_2OH - CH_3OH$ 326 — 171 = 155
α $CH_3 \cdot CHOH \cdot CH_3$[11] $- CH_3 \cdot CH_2OH$ 478 — 322 = 156
α $(CH_3)_3\ COH - (CH_3)_2\ CHOH$ 633 — 478 = 155
β $C_2H_5 \cdot CH_2OH - CH_3 \cdot CH_2OH$ 480 — 322 = 158
β $(CH_3)_2\ CH \cdot CH_2OH - CH_3 \cdot CH_2 \cdot CH_2OH$ 637 — 480 = 157
β $(CH_3)_2\ C_2H_5 \cdot COH - (CH_3)_3\ COH$ 788 — 633 = 155

Averages.

$$C_nH_{2n+1}CH_3 - C_nH_{2n+1}H = 156.2 \text{ Cal.}$$

In the Presence of	α	β	γ	higher.
COOH	148	157.3	156–157	156.5
$CONH_2$	148	157.1	156–157	156.1
$CONH\ C_6H_5$	150	157.0	156–157	156.2
$CH\ (COOH)_2$	154	161	156–157	. . .
CO	154.5	159	. . .	. . .
COH	155.3	156	. . .	. . .

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[8] Id., pp. 114, 116. [9] Thomsen, Thermochemische Untersuchungen.

[10] See Stohmann's tables, Zeits. f. physikal. Chem., **6**, 334 (1890); **10**, 410 (1892).

[11] Louguinine's values are here changed; see Henderson, Jour. of Physical Chemistry, **19**, 43 (1905).

Differences from 156.2 *Cal., the "normal" Value.*

In the Presence of	α	β	γ	higher.
COOH	–8	+1	0	0
$CONH_2$	–8	+1	0	0
$CONH\,C_6H_5$	–6	+1	0	0
$CH\cdot(COOH)_2$	–2	+5	0	. . .
CO	–2	+3	. . .	. . .
COH	–1	0	. . .	. . .

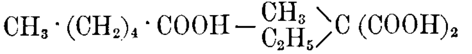
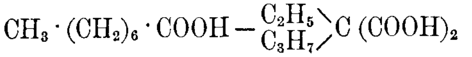
II.

$CH_3 \longrightarrow COOH$

A

COOH present.[12]

α	$CH_3\cdot COOH - COOH\cdot COOH$	207 — 60 = 147
β	$CH_3\cdot CH_2\cdot COOH - COOH\cdot CH_2\cdot COOH$	364 — 207 = 157
γ	$CH_3\cdot(CH_2)_2\cdot COOH - COOH\cdot(CH_2)_2\cdot COOH$	520 — 357 = 163
δ	$CH_3\cdot(CH_2)_3\cdot COOH - COOH\cdot(CH_2)_3\cdot COOH$	677 — 515 = 162
ε	$CH_3\cdot(CH_2)_4\cdot COOH - COOH\cdot(CH_2)_4\cdot COOH$	832 — 669 = 163
ζ	$CH_3\cdot(CH_2)_5\cdot COOH - COOH\cdot(CH_2)_5\cdot COOH$	989 — 829 = 160
η	$CH_3\cdot(CH_2)_6\cdot COOH - COOH\cdot(CH_2)_6\cdot COOH$	1145 — 984 = 161
θ	$CH_3\cdot(CH_2)_7\cdot COOH - COOH\cdot(CH_2)_7\cdot COOH$	1302 — 1141 = 161
ι	$CH_3\cdot(CH_2)_8\cdot COOH - COOH\cdot(CH_2)_8\cdot COOH$	1458 — 1297 = 161

β	$CH_3\cdot CH_2\cdot COOH - COOH\cdot CH_2\cdot COOH$	364 — 207 = 157
β	$CH_3\cdot(CH_2)_2\cdot COOH - CH_3\cdot CH\,(COOH)_2$	520 — 362 = 158
β	$CH_3\cdot(CH_2)_3\cdot COOH - C_2H_5\cdot CH\,(COOH)_2$	677 — 518 = 159
β	$CH_8\cdot(CH_2)_3\cdot COOH - (CH_3)_2\cdot C\,(COOH)_2$	677 — 515 = 162
β	 $CH_3\cdot(CH_2)_4\cdot COOH - {}^{CH_3}_{C_2H_5}{>}C\,(COOH)_2$	832 — 676 = 156
β	$CH_3\cdot(CH_2)_4\cdot COOH - C_3H_7\cdot CH\,(COOH)_2$	832 — 675 = 157
β	$CH_3\cdot(CH_2)_5\cdot COOH - (C_2H_5)_2\cdot C\,(COOH)_2$	989 — 833 = 156
β	 $CH_3\cdot(CH_2)_6\cdot COOH - {}^{C_2H_5}_{C_3H_7}{>}C\,(COOH)_2$	1145 — 990 = 155
β	$CH_3\cdot(CH_2)_7\cdot COOH - (C_3H_7)_2\cdot C\,(COOH)_2$	1302 — 1146 = 156
β	$CH_3\cdot(CH_2)_8\cdot COOH - C_7H_{15}\cdot CH\,(COOH)_2$	1458 — 1303 = 155
β	$CH_3\cdot(CH_2)_9\cdot COOH - C_8H_{17}\cdot CH\,(COOH)_2$	1616 — 1458 = 158

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[12] Stohmann, Jour. f. prakt. Chem., **49**, 107, 114, 116 (1894).

Averages.

$$C_nH_{2n+1}CH_3 - C_nH_{2n+1}COOH = 160 \text{ Cal.}$$

	α	β	γ, δ, ε	higher.
In the presence of COOH	147	157	163	161

Differences from 160 *Cal., the "normal" Value.*

	α	β	γ, δ, ε	higher.
In the presence of COOH	−13	−3	+3	+1

III.

$CH_3 \longrightarrow CONH_2$[13]

A

CONH$_2$ present.

α $CH_3 \cdot CONH_2 - CONH_2 \cdot CONH_2$ $283 - 203 = 80$
β $CH_3 \cdot CH_2 \cdot CONH_2 - CONH_2 \cdot CH_2 \cdot CONH_2$ $440 - 359 = 81$
γ $CH_3 \cdot CH_2 \cdot CH_2 \cdot CONH_2 -$
$CONH_2 \cdot CH_2 \cdot CH_2 \cdot CONH_2$ $596 - 510 = 86$

Summary.

$$C_nH_{2n+1}CH_3 - C_nH_{2n+1} \cdot CONH_2 = 83 \text{ Cal.}$$

	α	β	γ
In the presence of CONH$_2$	80	81	86

Differences from 83 *Cal., the "normal" Value.*

	α	β	γ
In the presence of CONH$_2$	−3	−2	+3

IV.

$CH_3 - CONH\, C_6H_5$[14]

A

COOH present.

α $CH_3 \cdot COOH - C_6H_5 \cdot NH \cdot CO \cdot COOH$ $207 - 863 = - 656$
β $CH_3 \cdot CH_2 \cdot COOH - C_6H_5 \cdot NH \cdot CO \cdot CH_2 \cdot COOH$
$364 - 1013 = - 649$
γ $CH_3 \cdot (CH_2)_2 \cdot COOH - C_6H_5 \cdot NH \cdot CO \cdot (CH_2)_2 \cdot COOH$
$520 - 1166 = - 646$

Summary.

$$C_nH_{2n+1}CH_3 - C_nH_{2n+1}CONH\, C_6H_5 = - 645 \text{ Cal.}$$

---

[13] Id., **52**, 59 (1895); **55**, 263 (1897).

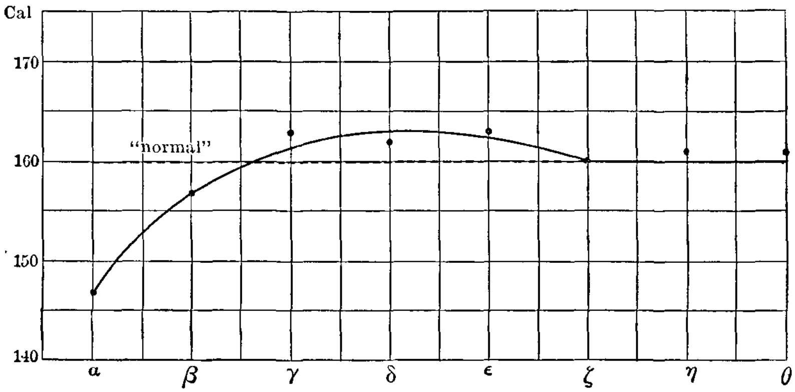
[14] Id.

	α	β	γ
In the presence of COOH	−656	−649	−646

Differences from −645 *Cal., the "normal" Value*

	α	β	γ
In the presence of COOH	−11	−4	−1

It is apparent from a consideration of the above tables that the effect upon the heat of reaction of a molecule exerted by a group which is itself not directly concerned in the reaction varies in a regular way with variation in the position of the group with respect to the reacting portion of the molecule. This is most clearly apparent in the reaction $CH_3 — COOH$ in the presence of the group COOH, probably because here the effect is great and the data abundant and accurate. In this case, when the carboxyl group is α to the reacting group the heat of reaction is diminished by 13 Cal., compared with its "normal" value in the absence of the carboxyl group; when the carboxyl group is in the β position the heat of reaction is diminished by about 3 Cal.; when the carboxyl group is in the γ, δ or ε position it is increased by about 2.5 Cal., returning finally to the "normal" value when the distance between the two groups is greater. The effect may be represented by the accompanying curve.[15]



Curve I.

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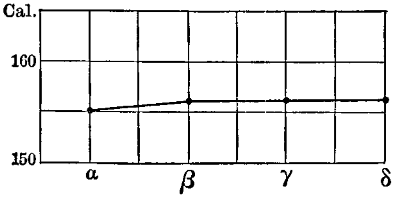
[15] The nature of this curve is apparently inconsistent with the idea developed by Flürscheim (l. c.) and by Biach (Zeits. f. physikalische Chemie, **50**, 43, 1904) that valence energy alternately increases and decreases from one carbon atom to the next, in a chain or ring; the period of alternation being, in the present matter, sometimes one carbon atom, sometimes more, according to the nature of the case.

It is clear in all the above cases that the effect of a group upon the heat of reaction is greatest when it occupies the α position, and that when it is sufficiently distant from the reacting group, this necessary distance varying with the nature of the case, it has no appreciable effect. Evidently, too, in most cases, before the effect disappears it is reversed, as in the γ, δ, and ϵ positions on the above curve and in the β position in those cases where the effects of the groups COOH, $CONH_2$, etc., upon the reaction CH — C CH_3 are considered.

In most cases an atom or atom group does not appreciably influence the heat of a reaction when the reacting group is further away from it than the β or γ position, and it may be noticeable only when it occupies the α position, as in the case represented by the accompanying curve.



CURVE II.

In the case represented by the first curve, however, an effect is still notable when the two groups are in the ϵ position to each other, provided the data are nearly correct, and it is important to consider that the data for this series of compounds are perhaps the best of all Stohmann's admirable determinations. Accordingly it is not impossible that curve I represents at least one general case of variation in heat of reaction with variation in position of an inactive group; at any rate the data here represented is not inconsistent with such a conclusion, and for the most part the curves would be of similar shape.

These regularities in the variation of heats of reaction which have been here considered, as well as those previously pointed out, lead naturally to the conclusion that the valence energy of a tie between two atoms must be capable of *continuous variation*[16] within certain limits, not yet to be defined; otherwise we must assume the existence of large forces of unknown nature within the molecule. *According to this conclusion and the above considerations the valence energies of two ties between like atoms will under like conditions be equal, under unlike conditions different.* Thus in the compound CR_4

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[16] Or, what is less probable, through numerous distinct magnitudes discontinuously.

The final result (40) is the same as was found in formula (20). All the terms in equations (38) to (40) are vector quantities, and are supposed to be added vectorially. It is evident that the e. m. f. impressed at A is unaffected by reflected waves; while the e. m. f. at the distant free end increases by jumps that are numerically smaller and smaller; but the phase relation may be such that the effect on the resultant of any one jump may exceed that of the preceding jump.

In particular cases when the length L of the line is suitably chosen with respect to the frequency and to the line constants, the final voltage at the distant end may greatly exceed the voltage at the sending end, either on the basis of maximum cyclic values, or of effective, i. e. voltmeter, values.

Turning now to the current wave, or magnetic flux wave, when this reaches B for the first time its value is $I_0 \epsilon^{-La}$, a vector quantity, having a certain phase lag La_2 with respect to the current entering simultaneously at A, and also undergoing simple harmonic variation. When a current wave strikes a free end or discontinuity, it is reflected with inversion; i. e., with reversal of sign, or 180° (π radians), change of phase. It starts back, therefore, towards A as $-I_0 \epsilon^{-La}$ amperes, and the total current increment at B is nil. On arriving at A its condition is $-I_0 \epsilon^{-2La}$ amperes. It then passes through the impressed e. m. f. without change and is reflected from the ground. A current reflected from a ground or short circuit undergoes no change of phase. It adds, therefore, a jump of $-2\ I_0 \epsilon^{-2La}$ amperes at A vectorially to the current I_0 impressed there. Whether this will involve an increase or decrease in the magnitude of the current at A will depend upon the relative phases of I_0 and of $-2 I_0 \epsilon^{-2La}$, which in turn will depend on the length of the line L for any given frequency. The current wave then returns to B for the second time, reaching it in the condition $-I_0 \epsilon^{-3La}$. It is again reflected with inversion or starts back to A as $I_0 \epsilon^{-3La}$ amperes, adding nothing to the current at B. It keeps up this battledore and shuttlecock action theoretically forever; but practically until it becomes attenuated to insignificant dimensions. The final resulting current at B is zero, which, of course, must be true at an open end. This is due to the occurrence there of pairs of incident and reflected current waves having equal magnitudes but respectively opposite signs.

The total current strength at A is

I_0 from $t = 0$ to $t = 0.002$
$I_0 - 2\ I_0 \epsilon^{-2La}$ " $t = 0.002$ to $t = 0.004$
$I_0 - 2\ I_0 \epsilon^{-2La} + 2\ I_0 \epsilon^{-4La}$ " $t = 0.004$ to $t = 0.006$

and so on.

phase. The wave after reaching A and going to ground returns back on the circuit towards B, but reversed in sign. The voltage at A is thus not affected by the return of the wave, because although it tends to be increased by $E\epsilon^{-2La}$ volts on the first return it is immediately increased by $-E\epsilon^{-2La}$ volts on the recoil from A, these two impulses being equal and opposite.

The wave reflected with inversion at A now runs back towards B for the second time. When it reaches B it will have run a total distance of $3L$ kilometers from the start, and its magnitude will be $-E\epsilon^{-3La}$ volts. At the open end B, it is reflected without reversal, thus adding a sudden jump of $-2E\epsilon^{-3La}$ to the voltage previously existing at B. Back it comes to A in the condition $-E\epsilon^{-4La}$. It is again reflected from A and ground with reversal, making no change in the voltage impressed on A, but starting out for B with the value $E\epsilon^{-4La}$. It reaches B in the condition $E\epsilon^{-5La}$, adding a sudden jump of $2E\epsilon^{-5La}$ to the voltage existing at that moment, and again sets out for A.

The time of one passage along the line is $\frac{L}{v}$ seconds, where L is the length of the line and v is approximately $\frac{1}{\sqrt{lc}}$ kilometers per second. In overhead aerial wires v is nearly 300,000 kilometers per second or the light speed in air; so that over a circuit 300 kilometers long, the time of flight of a wave would be about $\frac{1}{1000}$ second, and the time of one return trip $\frac{2}{1000}$ second. In one second of time after applying the alternator to A there would have been completed nearly 500 return trips over the circuit.

From time $t = 0$ to $t = 0.001$ second, the voltage at B $= 0$
From time $t = 0.001$ to 0.003 second, the voltage at B $= 2E\epsilon^{-La}$ volts
From time $t = 0.003$ to 0.005 second, the voltage at B $=$

$$2E\epsilon^{-La} - 2E\epsilon^{-3La}$$

From time $t = 0.005$ to 0.007 second, the voltage at B $=$

$$2E\epsilon^{-La} - 2E\epsilon^{-3La} + 2E\epsilon^{-5La}$$

and so on indefinitely, until the succeeding jumps of voltage at B become infinitely small, or the wave has been attenuated away to nothing. The total final voltage at B after the steady state is finally reached is:

$$E_B = 2E\{\epsilon^{-La} - \epsilon^{-3La} + \epsilon^{-5La} - \epsilon^{-7La} + \ldots\ldots \qquad \text{volts} \qquad (38)$$

$$= 2E\epsilon^{-La}\{1 - \epsilon^{-2La} + \epsilon^{-4La} - \epsilon^{-6La} + \ldots\ldots \qquad \text{"} \qquad (39)$$

$$= 2E\frac{\epsilon^{-La}}{1 + \epsilon^{-2La}} = \frac{2E}{\epsilon^{La} + \epsilon^{-La}} = \frac{E}{\cosh La} \qquad \text{"} \qquad (40)$$

At any distance L_1 kilometers from A we should have

$$e = E\,\epsilon^{-L_1\alpha_1}\,\overline{\diagdown \alpha_2} \qquad = E\,\epsilon^{-L_1\alpha} \qquad \text{instantaneous volts} \qquad (34)$$

or
$$\bar{e} = \frac{E}{\sqrt{2}}\,\epsilon^{-L_1\alpha_1}\,\overline{\diagdown \alpha_2} \qquad = \frac{E}{\sqrt{2}}\,\epsilon^{-L_1\alpha} \qquad \text{effective volts} \qquad (35)$$

also
$$i = I_0\,\epsilon^{-L_1\alpha_1}\,\overline{\diagdown \alpha_2} \qquad = I_0\,\epsilon^{-L_1\alpha} \qquad \text{instantaneous amperes} \qquad (36)$$

$$\bar{i} = \frac{I_0}{\sqrt{2}}\,\epsilon^{-L_1\alpha_1}\,\overline{\diagdown \alpha_2} \qquad = \frac{I_0}{\sqrt{2}}\,\epsilon^{-L_1\alpha} \qquad \text{effective amperes} \qquad (37)$$

If, however, the circuit be comparatively short, and be free, or opened, at the distant end ($Z = \infty$ in Figure 1), the wave stream on arriving at B will be reflected back towards A. Let the length of the line be L kilometers, then the voltage when it reaches B will have the value $E\epsilon^{-L\alpha}$ and the current likewise $I_0\,\epsilon^{-L\alpha}$. The effects of reflection are different on voltage and on current, i. e., on moving electrostatic flux and moving magnetic flux, so that each must be considered separately.

Considering first the voltage wave arriving at the distant open end of the line, it rebounds therefrom at light-speed, or is reflected there. The voltage in the reflected wave at B starts back at the same amplitude and phase as when it strikes the free end. It is simply started backwards along the circuit without undergoing any other change. It returns towards A, attenuating as it goes, just as though nothing had happened. At B, however, after the reflection, the voltage is double that which occurred just before reflection; for as soon as the reflected wave starts back, there are the $E\epsilon^{-L\alpha}$ volts arriving at the head of the incoming train, and also $E\epsilon^{-L\alpha}$ volts starting at the head of the returning train. The voltage at B was thus 0 until the wave arrived at B, then it was $E\epsilon^{-L\alpha}$, whereupon reflection immediately occurred and made the voltage jump to $2\,E\epsilon^{-L\alpha}$; where α is a complex number, the real part of $2\,E\epsilon^{-L\alpha}$, being the maximum cyclic value of the voltage at B, and $2\,E\epsilon^{-L\alpha_1}\sin(\omega t - \alpha_2)$ being the voltage at any instant after arrival.

By the time that the wave train gets back to A, the total distance it will have travelled is $2\,L$ kilometers, and the attenuation-coefficient will be $\epsilon^{-2\,L\alpha}$. The arriving voltage will therefore be $E\epsilon^{-2\,L\alpha}$. As soon as the end A of the line is reached, the wave will go direct to ground through the alternator. The e. m. f. of the alternator does not stop the wave, which goes straight through the machine (assumed as impedanceless) without modification. A voltage wave passing through a ground or short-circuit virtually swings round the goal and returns on its path reversed. That is the same, however, as being reflected with change of sign, or with a change of 180 degrees, or π radians, in stands vertical. The shutter must be geared with the shaft O O′, so as to move uniformly and uncover one complete turn of the spiral in the interval of one complete rotation of the shaft. The shadow of any pin P on a horizontal plane at p beneath would then indicate the wave motion at that point from the moment of the wave's first reaching it until reflected waves disturbed the distribution. The spiral of pins thus constructed would just fit into the arrangement of Figure 2, considered as a hollow cylinder or logarithmic trumpet of revolution.

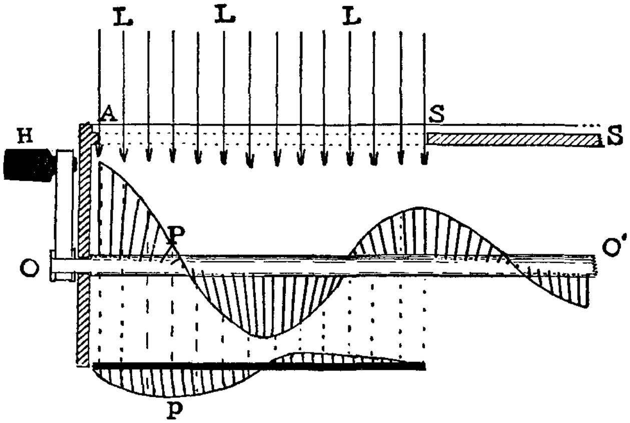

FIGURE 4. — Diagram of model for exhibiting the propagation of the first outgoing wave-train along a uniform line.

In the case of an indefinitely long circuit, as above outlined, the initial current strength at the sending end A would remain the final steady current strength. In other words, there would be nothing to disturb the original simple harmonic voltage and current. At the origin A we should always have:

$$E = E_A \sin \omega t \qquad \text{instantaneous volts} \qquad (30)$$

or

$$\bar{E} = \frac{E}{\sqrt{2}} \qquad \text{effective volts} \qquad (31)$$

also

$$I_0 = I_{0A} \sin \omega t = \frac{E_A}{z_0} \sin \omega t \qquad \text{instantaneous amperes} \qquad (32)$$

or

$$\bar{I}_0 = \frac{I_0}{\sqrt{2}} \qquad \text{effective amperes} \qquad (33)$$

orthogonal projection of a certain radius corresponding thereto in a logarithmic spiral E 1′ 2′ 3′, etc. At successive distances of eight kilometers, the phases at any instant would be the same. Moreover, by rotating the entire spiral system of Figure 3 counter-clockwise about O, with angular velocity ω, the amplitude and phase of the wave stream at any point on the line could be determined, for any moment, by the orthogonal instantaneous projection of the proper radius in the spiral.

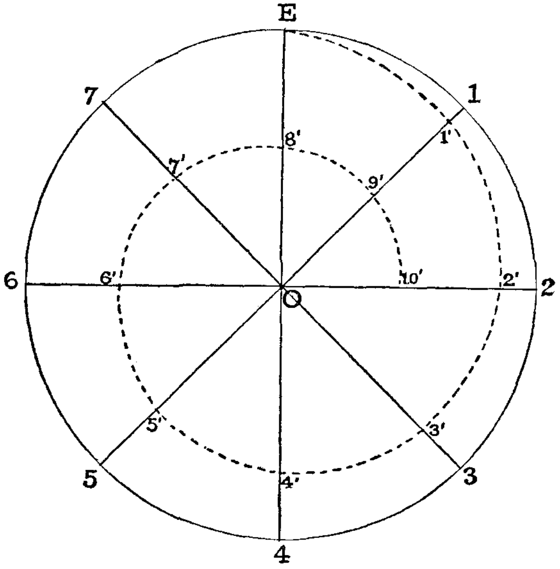

Figure 3. — Diagram of relative magnitudes and phases of outgoing wave over the line of Figure 2, for the first ten kilometers.

By combining Figures 2 and 3, a mechanical model might be prepared to illustrate the first run of an electromagnetic wave over a circuit. Into a cylindrical wooden shaft, of the length O O′, Figure 2, metal pins or wire nails are driven at intervals, corresponding, say, to each quarter kilometer. The radial length of the pin from the axis of the cylinder must conform to the logarithmic curve A B C D, Figure 2, and its angular position about the axis must conform to Figure 3, for the particular line considered, or thirty-two pins to 360°. A shutter S S, Figure 4, movable in a plane parallel to the shaft, like the sliding lid of a box, should start from A and admit vertically-falling light rays L L, on to the spiral, at a moment when the shaft is rotated by the handle H at uniform angular velocity ω, and when the first pin at A

The greater a_2, the shorter the wave-length, or the more waves that can be filled into the diagram.

In the case represented by Figure 2, $a = a_1 + ja_2 = 0.07675 + j\,0.7854$ per kilometer. After running, say thirty kilometers, the wave will have dwindled by the attenuation-coefficient $\epsilon^{-30\,(0.07675 + j\,0.7854)} = \epsilon^{-(2.3026 + j\,23.562)} = \epsilon^{-2.3026} \times \epsilon^{-j\,23.562}$. The first factor is equal to numeric 0.1. The second is equal to a negative angle of 23.562 radians, or $3\frac{3}{4}$ complete rotations or cycles. The waves of current and voltage on arriving at B have thus each dwindled to 10 per cent of their original amplitude, and have also fallen $3\frac{3}{4}$ cycles behind the phase existing at A. The phase of the wave remains the same as it advances, a crest remaining a crest, but the phase at A is constantly advancing at the angular velocity ω radians per second, so that with respect to the phase at A, that of the advancing wave is constantly falling behind.

After advancing another thirty kilometers, or to C, Figure 2, sixty kilometers from A, the voltage and current waves will again shrink to 10 per cent of their amplitudes at B, or to 1 per cent of their original amplitudes, and will have fallen $7\frac{1}{2}$ cycles in phase behind the voltage and current phases at A, respectively, the attenuation-coefficient having become $\epsilon^{-(4.6052 + j47.124)} = 0.01 \diagdown\!\overline{2700^\circ}$. The original phase difference between the voltage and current waves at A is equal to the angle of the vector z_0, the initial sending-end impedance (17). With negligible resistance and leakage conductance, formula (17) shows that the angle of z_0 would approach zero ; so that on such a circuit the voltage and current would be in phase at the start, and also at any point along the line. Whatever the original phase difference might be, according to (17), it would be maintained throughout the first run along the line.

If we represent at O E, Figure 3, a unit vector rotating in a vertical plane about a horizontal axis O, with the angular velocity ω radians per second, its projection on the horizontal plane 6 O 2 will at any instant represent the actual voltage applied to the end A of the line. At a distance of one kilometer from A, the phase of the outgoing wave stream will be O 1, or 45° behind O E. The amplitude will have shrunk to O 1′ or 0.923, an attenuation of 7.7 per cent. The projection of O 1′ on the horizontal plane 6 O 2 will give the amplitude of the wave at the same instant. At a distance of another kilometer beyond, or two kilometers from the origin, the phase of the wave stream will be 90° behind O E, and the amplitude will have again lost 7.7 per cent, or will have fallen to 0.853, 85.3 per cent of the original. By continuing the examination, we should find that at the instant considered, all points along the line would have a wave amplitude determined by the ity by (26) may be reduced, with low frequencies, to 10,000 kilometers per second, or less.

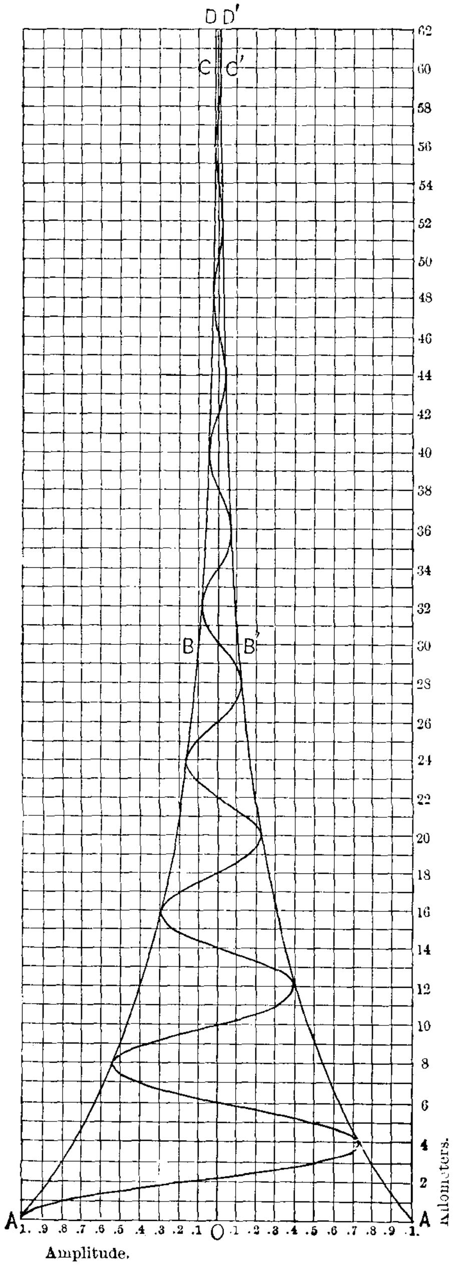

FIGURE 2. — Curve of wave attenuation for the first 62 kilometers of a circuit of attenuation-constant $\alpha = 0.07675 + j0.7854$ per kilometer.

As the electromagnetic wave starting from A at time $t = 0$ runs over the circuit, containing within it both a voltage wave, or electrostatic-flux wave, and a current wave, or magnetic-flux wave, it dwindles or attenuates at the rate of $\epsilon^{-\alpha}$ per kilometer; so that after running L_1 miles, both the voltage and the current will have become weakened by the attenuation-coefficient $\epsilon^{-L_1\alpha}$; or

$$e = E\epsilon^{-L_1\alpha} \quad \text{volts} \quad (28)$$

and

$$i = I_0\epsilon^{-L_1\alpha} \quad \text{amperes} \quad (29)$$

where α is the attenuation-constant of formula (16). This logarithmic attenuation is represented for a particular circuit and frequency in Figure 2, but the same curve, A B C D, may be used to represent any circuit and frequency, by suitably changing the scales of co-ordinates. The length of the horizontal axis is determined by α_1, the real part of the attenuation-constant α in (18), and the wave-length is determined by α_2, the imaginary part. The greater α_1, the shorter the length of circuit that will fit the curve A B C D.

3. When the distant end of the line at B is grounded through a receiving impedance z_r ohms,

$$I_B = \frac{E}{z_0 \sinh L\alpha + z_r \cosh L\alpha} \quad \text{amperes} \quad (23)$$

$$E_B = I_B z_r \quad \text{volts} \quad (24)$$

All of the values of voltage and current in the steady-state formulas (14) to (24) inclusive, may be regarded either as instantaneous voltages and currents, in terms of instantaneous impressed voltage E and current I; or they may be regarded as effective, or square-root-of-mean-square values, in terms of effective impressed voltage E and current I, such as would be indicated by properly calibrated voltmeters and ammeters.

We may now proceed to show how formulas (19) to (24) inclusive may be derived by taking into account the initial outgoing waves of voltage and current, together with superposed reflected waves. The only postulate needed is that when the line is first connected to the alternator at A, the first outgoing current wave has the strength

$$I_0 = \frac{E}{z_0} \quad \text{amperes} \quad (25)$$

and this current strength continues to be delivered to the line at A until such time as current-waves reflected from the distant end modify the current. This proposition is well known.[4]

The outgoing voltage and current waves run along the wire hand in hand, with the velocity :

$$v = \frac{\omega}{\alpha_2} \quad \text{kilometers per second} \quad (26)$$

a velocity which is somewhat less than the speed of long-wave light in the dielectric. When the linear conductor resistance and leakage conductance are very small, this velocity approximates to :

$$v = \frac{1}{\sqrt{cl}} \quad \text{kilometers per second} \quad (27)$$

In the case of overhead aerial copper wires, v approaches 300,000 kilometers per second at high frequencies. At low frequencies the effect of conductor resistance requires the use of formula (26), and may bring down the velocity to 100,000 kilometers per second or less. In paper-insulated, and especially in rubber-insulated cables, the veloc-

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[4] "On the Mechanism of Electric Power Transmission," by A. E. Kennelly. Electrical World, **42**, 673 (Oct. 24, 1903).

"practical" system and use the kilometer (or the mile) as the unit of length:[3]

$$e = E \cosh L_1 a - I z_0 \sinh L_1 a \qquad \text{volts} \qquad (14)$$

$$i = I \cosh L_1 a - \frac{E}{z_0} \sinh L_1 a \qquad \text{amperes} \qquad (15)$$

where L_1 is the distance x measured from A in kilometers,

$$a = \sqrt{(r + jl\omega)(g + jc\omega)} \qquad \frac{1}{\text{kilometer}} \qquad (16)$$

and

$$z_0 = \sqrt{\frac{r + jl\omega}{g + jc\omega}} \qquad \text{ohms} \qquad (17)$$

the vector, or complex quantity, a is commonly called the attenuation-constant of the circuit. It comprises a "real" part a_1 called the real attenuation-constant, and an "imaginary" part ja_2 called the wave-length constant, by the relation

$$a = a_1 + ja_2 \qquad \frac{1}{\text{kilometer}} \qquad (18)$$

The vector impedance z_0 is called the "initial sending-end impedance" of the line, and sometimes the "surge-impedance."

r = the linear resistance	ohms per kilometer.
l = the linear inductance	henrys per kilometer.
g = the linear leakage conductance . .	mhos per kilometer.
c = the linear capacity	farads per kilometer.

Since equations (14) and (15) hold for any moment of time, they must apply to the final steady state. We may easily derive from them the following steady-state formulas:

1. When the distant end of the line at B is free,

$$I_B = 0 \qquad I_A = \frac{E}{z_0 \coth La} \qquad \text{amperes} \qquad (19)$$

$$E_A = E \qquad E_B = \frac{E}{\cosh La} \qquad \text{volts} \qquad (20)$$

2. When the distant end of the line at B is grounded,

$$I_B = \frac{E}{z_0 \sinh La} \qquad I_A = \frac{E}{z_0 \tanh La} \qquad \text{amperes} \qquad (21)$$

$$E_A = E \qquad E_B = 0 \qquad \text{volts} \qquad (22)$$

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[3] "The Distribution of Pressure and Current over Alternating-Current Circuits," by A. E. Kennelly. Harvard Engineering Journal, 1905–1906.

Equation (2) states that the descending gradient of voltage in abvolts per linear centimeter at any point of the line is always equal to the current strength at that point multiplied by the conductor resistance per centimeter plus the time-rate of change of the current into the conductor inductance per centimeter. In other words, the voltage per linear centimeter is equal to the momentary linear Ir drop, plus the linear back e. m. f. of self-induction.

Equation (3) states that the descending gradient of current along the line is always equal to the local voltage multiplied by the linear dielectric conductance, plus the linear capacity into the local time-rate of change of the voltage, i. e. the linear leakage current plus the linear charging current.

Differentiating (2) and (3) with respect to x the length of line, we get

$$e'' = - zi' = yz \cdot e \qquad \frac{\text{abvolts per cm.}}{\text{cm.}} \qquad (6)$$

$$i'' = - ye' = yz \cdot i \qquad \frac{\text{absamperes per cm.}}{\text{cm.}} \qquad (7)$$

By (6) the curvature (or gradient of gradient) of the voltage at any point along the line is always yz times the voltage at that point. By (7) the curvature of the current along the line is also yz times the local current. The complete solutions of the second-order differential equations (6) and (7), for any point distant x from A, are known to be:

$$e = E \cosh x\sqrt{yz} - I\sqrt{\frac{z}{y}} \sinh x\sqrt{yz} \qquad \text{abvolts} \qquad (8)$$

$$i = I \cosh x\sqrt{yz} - E\sqrt{\frac{y}{z}} \sinh x\sqrt{yz} \qquad \text{absamperes} \qquad (9)$$

where E and I are the instantaneous impressed voltage, and current, at the sending end A (Figure 1), respectively.

For an impressed e. m. f. which is sinusoidal, or simply harmonic, in accordance with (1), we have, if $j = \sqrt{-1}$,

$$\frac{dE}{dt} = E_A \omega \cos \omega t = j\omega E \qquad \text{abvolts per second} \qquad (10)$$

and

$$\frac{dI}{dt} = I_A \omega \cos \omega t = j\omega I \qquad \text{absamperes per second} \qquad (11)$$

so that

$$z = r + jl\omega \qquad \text{absohms per cm.} \qquad (12)$$

$$y = g + jc\omega \qquad \text{abmhos per cm.} \qquad (13)$$

y and z are thus plane-vector constants.

We have hitherto kept the equations within the absolute C. G. S. magnetic system of units. We may, however, transfer them to the tions occurring during the initial unsteady state can be easily grasped and followed to the final steady state.[1] It will be readily seen by the aid of these formulas that the final steady state is approached by successive sudden jumps, and not by a uniform continuous advance. The unsteady state is a state of leaps and bounds; but these, although not necessarily or generally of successively diminishing magnitude, diminish in a definitely irregular way, and finally disappear. From a physical point of view, the process is very beautiful, and from a practical engineering standpoint, it is not without importance.

We may first derive the formulas for voltage and current in the circuit of Figure 1 in the usual way, starting with the differential equations of the circuit, and then lead to these already known formulas in the new way.

At any instant t, and at any distance x centimeters from A along the line in Figure 1, we have[2]

$$-e' = zi \qquad \text{abvolts per cm.} \qquad (2)$$

and $$-i' = ye \qquad \text{absamperes per cm.} \qquad (3)$$

where $e =$ the instantaneous e. m. f., abvolts

$e' = \frac{de}{dx}$ the space gradient of e $\frac{\text{abvolts}}{\text{cm.}}$

$i =$ the instantaneous current, absamperes

$i' = \frac{di}{dx}$ the space gradient of i $\frac{\text{absamperes}}{\text{cm.}}$

$r =$ conductor linear resistance, $\frac{\text{absohms}}{\text{cm.}}$

$l =$ conductor linear inductance, $\frac{\text{abhenrys}}{\text{cm.}}$

$g =$ dielectric linear conductance, $\frac{\text{abmhos}}{\text{cm.}}$

$c =$ dielectric linear capacity $\frac{\text{abfarads}}{\text{cm.}}$

$$z = r + l\frac{d}{dt} \qquad \frac{\text{absohms}}{\text{cm.}} \qquad (4)$$

$$y = g + c\frac{d}{dt} \qquad \frac{\text{abmhos}}{\text{cm.}} \qquad (5)$$

$\frac{d}{dt} =$ time-rate of change $\frac{1}{\text{second}}$

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[1] Compare, however, "Alternating Currents," by Bedell and Crehore, 1893, pp. 201–207.

[2] "Electromagnetic Theory," by Oliver Heaviside, **1**, 450 (1893).

THE PROCESS OF BUILDING UP THE VOLTAGE AND CURRENT IN A LONG ALTERNATING-CURRENT CIRCUIT.

BY A. E. KENNELLY.

Presented March 13, 1907. Received March 5, 1907.

LET us consider the simple alternating-current circuit indicated in Figure 1; namely, a long single uniform wire AB, with a ground return circuit. A single-phase sinusoidal alternator of negligible internal impedance and generating an e. m. f.

$$E = E_A \sin \omega t \qquad \text{abvolts} \qquad (1)$$

is connected to the end A at the instant $t = 0$, when E is starting positively from zero at the uniform frequency n cycles per second, or with angular velocity $\omega = 2 \pi n$ radians per second. At B, the distant end of the line, there is connected an impedance Z to ground. If we make $Z = \infty$, we virtually free or disconnect the line at B. If we make $Z = 0$, we virtually ground the line at B. If we make $Z = z_r$, we virtually insert a receiving impedance at B. Any two-wire or multiple-wire metallic circuit is capable of being reduced to an equivalent simple single-wire circuit with ground return, such as is represented in Figure 1.

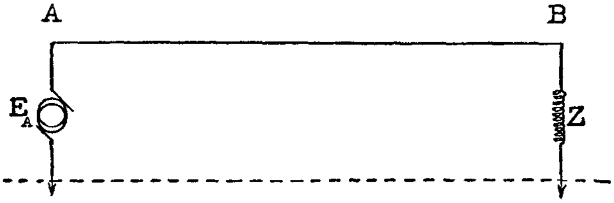

FIGURE 1.—Simple alternating-current circuit.

The current and voltage which exist at any point of the line in Figure 1 can be readily computed, whatever the given terminal impedance at B, after the system has been allowed to operate for a sufficient length of time, usually about one second, during which the initial unsteady state may be regarded from a practical standpoint as developing into the final steady state. It is proposed to discuss in this paper certain formulas, that appear to be new in this application, by which the ac-

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THE PROCESS OF BUILDING UP THE VOLTAGE AND CURRENT IN A LONG ALTERNATING-CURRENT CIRCUIT.

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(3) An effect of *absorption* suffered by the several components of the beam in passing through the substance of the screen.

Experimental results rule out, more or less certainly, the first two effects. Granting these results, a theory of relatively selective absorption is necessary to explain the reduction of penetrating power for one metal by transmission through another. By somewhat indirect experimental evidence, this absorption is shown to be governed by coefficients which are probably constant with varying intensity of the rays.

VI. Conclusions.

The following conclusions are reached in this paper:

(1) The approximate measurement of the energy of Röntgen rays is of the same order of magnitude as the earlier measurements of that energy, and to that extent confirms them.

(2) In the transmission of Röntgen rays through metallic sheets, the effect of the surfaces of the metal is small.

(3) In the transmission of Röntgen rays through metallic sheets, the probability that one sort of ray is transformed into another sort of ray, to an appreciable extent, is small.

(4) The absorption of any particular sort of ray by a metallic sheet is measured by a coefficient which is probably independent of the intensity of that ray.

(5) In one special case, at least, the general effect on absorption by one metal of passing the rays through another metal is not, so far as we can judge, a decrease of absorption, but an increase; and this fact tends to support the theory that the radiation from an ordinary tube is heterogeneous in character, and that the absorption in metallic sheets is more or less selective.

It is hoped to continue this research in the direction suggested by the conclusions of this paper: to test the hypothesis of relatively selective absorption by direct experiment;[21] to obtain, if possible, beams of Röntgen rays which will yield transmission curves of a simpler form than those heretofore found; to study the properties of those beams; and to study the indications of the instrument used in this research in comparison with those of other instruments heretofore used, especially the ionization-electroscope and the photographic plate.

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[21] For a preliminary note on this experiment, see Amer. Jour. Sci., **173**, 91 (1907).

intensity. But to draw conclusions as to the constancy of any or all of the separate coefficients from the constancy of the ratio of transmission is not in strictness possible. It might be that two or more of the coefficients changed together in such a way as to keep the ratio of transmission constant. But the repetition of this coincidence in experiments with different metals and with rays from tubes in very different conditions is exceedingly unlikely, and the constancy of the ratio of transmission with varying intensity is at least very good presumptive evidence that the coefficients characteristic of the absorption of Röntgen rays in metallic sheets are all constant with varying intensity of the rays. It may be of interest to examine the consequences of supposing that the constancy of the ratio of transmission is not a result of the constancy of each of the coefficients. To simplify the argument, let us suppose that the metallic screen is very near the instrument. Let s_1 and s_2 represent respectively the fraction of a certain sort of ray which the metallic screen transmits at the large intensity and at the small intensity. Let p be the fraction of the same sort of ray which the platinum of the instrument transmits at all intensities. Let I_1 and I_2 represent the two intensities of this sort of ray at the instrument. Then the ratio of transmission for this ray alone at the large intensity would be

$$\frac{I_1\, s_1\, (1-p)}{I_1\, (1-p)},$$

or s_1; and at the small intensity,

$$\frac{I_2\, s_2\, (1-p)}{I_2\, (1-p)},$$

or s_2. If the ratios of transmission for the whole beam are found to be the same at the two intensities, the explanation must be either that each s_1 is equal to its corresponding s_2, or that some of the s_1's are larger than their corresponding s_2's while others are smaller. The latter explanation is equivalent to saying that the absorption coefficients for some sorts of rays are increasing functions of the intensity, while others are decreasing functions; and this seems highly improbable.

The argument of this section may be summarized in another form, as follows:

The possible effects of transmission through a metallic screen upon a beam of Röntgen rays are three:

(1) An effect produced upon the beam by transmission across the surfaces of the screen.

(2) An effect of *transformation* suffered by the several components of the beam in passing through the substance of the screen.

beyond our knowledge at present. This consideration will explain many disagreements in the results obtained by observers using different instruments.

It is simplest to suppose that the effect of transmission through a metallic screen upon any one component of a heterogeneous beam is independent of the presence or absence of the other components. In other words, the effect of a given screen upon the transmission of a particular sort of ray is measured by an absorption coefficient which may change with the conditions of the experiment only in so far as those conditions affect the ray in question. An attempt has been made to test the constancy of these coefficients under changing intensity of the rays, by the experiments where the dependence of the ratio of transmission upon the distance between the tube and the screen was investigated. These experiments are given in detail on pages 682, 683.

It seems very probable that in experiments so conducted the variation in the intensity of the rays incident on the screen, involved in moving the tube, very nearly fulfils the condition that the changes in the intensities of all the components of the beam shall be in one ratio. Except for the absorption by the air, this ratio is doubtless fixed for all the different sorts of rays by some function of the distances involved; and from numerous experiments with various instruments[20] it appears that the absorption by air of Röntgen rays in general is negligible for such short distances as these. Of course a real disturbing factor is the changing behavior of the tube, for which correction is made, as well as possible, by alternating and checking readings. Granting that these sources of error have had no appreciable effect, we learn from the observations that the ratios of transmission of the metallic screens examined are independent of the total intensity of the incident radiation, when that intensity changes in such a way that the intensities of all the components change in the same ratio.

If we examine this result in the light of the conclusions already reached in this paper, its practical importance will appear. It is plain that if we knew in advance that all of the absorption coefficients for the metallic screen and the beam in question are constant so far as the intensity is concerned, the constancy of the ratio of transmission would necessarily follow, provided we conclude from the data of Table IV that all of the absorption coefficients of platinum are independent of the

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[20] J. Trowbridge and J. E. Burbank, Amer. Jour. Sci., **157**, 396 (1899); A. M. Mayer, Amer. Jour. Sci., **151**, 467 (1896); C. G. Barkla, Phil. Mag., **7**, 555 (1904).

The experiments of the present research upon the effect of transmission through one metal on penetrating power for another were undertaken for the purpose of confirming or disproving the *necessity* of such a theory of transformation. The results which are summarized on page 688 are in agreement with the general statement which Walter has deduced from his fluoroscopic work, so far as they go; but the result of the experiments with silver and aluminium, given on page 690, is contradictory to that statement, and indicates that relatively selective absorption among metals may exist.

In view of the doubt thus cast upon the *necessity* of a theory of transformation, direct experimental evidence of transformation was sought by means of measurements of transmission through a screen composed of two pieces of different metals. These measurements are described on pages 691, 692. It is to be expected that if transformation occurs in the metals, the effect of the second metal upon rays transformed by the first will not be quantitatively the same as the effect of the first metal upon rays transformed by the second. In other words, the transformation theory leads us to expect that the ratio of transmission of a two-piece screen will depend upon the order of the pieces; and the absence of this dependence, experimentally shown, is evidence of the absence of transformation.

With the possibility of transformation in transmission thus removed, and with experimental evidence [19] showing that any effect of the surfaces of the metal upon transmission is small, the only conceivable action produced upon a beam of rays by transmission through a metallic screen is an absorbing action. To explain the phenomenon observed by Röntgen with different thicknesses of the same metal, we must suppose, with him, that the rays from a tube are heterogeneous, and that different kinds of rays are differently absorbed in any one metal. To explain Walter's apparent reduction of penetrating power by transmission in certain cases, we must suppose that rays which are more penetrating for some metals are less penetrating for others, — that is, that metals show relatively selective absorption, and that the apparent reduction of penetrating power by transmission through silver, etc., is a real reduction of penetrating power with respect to aluminium. In judging of the significance of Walter's fluoroscopic work, as in fact of all work on Röntgen rays, it must be borne in mind that the selectivity of the absorbing media of the instruments themselves must greatly affect the magnitude of their indications, and that an exact interpretation of those indications is the phenomena observed with different thicknesses of one metal are concerned.

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[19] See Tables V–VII and Figure 5.

Another explanation of the phenomena of transmission has recently been proposed by Walter.[17] His theory assumes that the effect of transmission through any substance upon a beam of rays, which may be supposed homogeneous, is essentially a transformation of the character of the beam, — a transformation which in some way renders the rays more penetrating toward all substances. In particular, transmission through a piece of silver, cadmium, or tin is assumed to render the rays especially penetrating toward another piece of either of those metals, and the more so as the atomic weights of the two metals concerned are more nearly equal. These assumptions are a necessary consequence of his experimental results, which are briefly as follows: The penetrating power of the rays direct from the tube was compared with the penetrating power of the rays after transmission through a sheet of silver. The comparison was made by the use of the Benoist-Walter scale of hardness[18] and a photographic plate. It appeared that the rays were rendered less penetrating by transmission through the silver. Transmission through cadmium and through tin showed the same effect to a less extent. The effect of transmission through each of the three metals was also examined by means of scales of hardness in which the central disk of silver was replaced by one of cadmium or by one of tin; in every case it was found that the greatest reduction of penetrating power occurred when the central disk of the scale was of the same metal as the interposed sheet, and that this reduction of penetrating power diminished as the atomic weight of the metal of the disk departed from that of the metal of the interposed sheet. Transmission through sheets of metals other than silver, cadmium, or tin appeared in all cases to increase the penetrating power of the rays as measured by either of the three scales of hardness. A different series of experiments with the fluoroscope indicated that the anomalous reduction of penetrating power by transmission through silver, cadmium, or tin was apparent rather than real, and that in every case the effect of transmission through one metal was to increase the penetrating power of the rays for every metal. This result, if correct, makes untenable an hypothesis of relative selectivity in the absorption of the rays by different metals, which otherwise might explain the experiments with the scales of hardness, and renders necessary some such theory as the one which Walter proposes.

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[17] B. Walter, Ann. d. Phys., **17**, 561 (1905).

[18] B. Walter, loc. cit.

The means of the ratios of transmission for each pair of metals follow :

Copper-aluminium, 0.424	Aluminium-copper, 0.430
Tin-aluminium, 0.567	Aluminium-tin, 0.560
Aluminium-silver, 0.489	Silver-aluminium, 0.489
Copper-tin, 0.336	Tin-copper, 0.331
Tin-silver, 0.368	Silver-tin, 0.365

The thicknesses used were : Copper, 0.0044 cm.
Tin, 0.0012 cm.
Silver, 0.0019 cm.
Aluminium, 0.050 cm.

TABLE XI.

CHECK ON CORRECTNESS OF PROCEDURE IN EXPERIMENT OF TABLE X.

Ratio of Transmission Copper-Silver Screen.	Ratio of Transmission Silver-Copper Screen.
0.47	0.47
0.47	0.47
0.48	0.48
0.47	0.47
	0.48

V. DISCUSSION OF EXPERIMENTAL RESULTS.

The qualitative law which connects the thickness of a screen with the transmission through it, as stated and illustrated on pages 680, 681, was first enunciated by Röntgen.[16] He explained the phenomenon by assuming that the beam emitted by the tube consists of a mixture of different kinds of rays, some of which have more penetrating power than others. If such a beam were passed through a plate of any substance, the less penetrating rays would suffer absorption to a greater extent than the more penetrating ones, and the penetrating power of the emergent beam would be on the whole greater than that of the original beam. This explanation is a satisfactory one so far as in a two-piece screen of silver and copper does not affect the transmission appreciably. This conclusion evidently holds for a considerable range of quality of the incident radiation, as the parallel variations of values in the two columns of ratios in Table X indicate.

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[16] W. C. Röntgen, Ann. d. Phys., **64**, 18 (1898).

TABLE X.

EFFECT OF REVERSING THE ORDER OF THE METALS IN A TWO-PIECE SCREEN.

Throw, cms., no Screen.	Throw, cms., Silver-Copper Screen.	Ratio.	Throw, cms., Copper-Silver Screen.	Ratio.
4.32				
	1.55	0.36		
4.19				
			1.62	0.38
4.43				
	1.66	0.38		
4.28				
			1.79	0.41
4.56				
	1.85	0.40		
4.67				
			1.86	0.39
4.76				
	1.90	0.39		
4.89				
			2.02	0.41
4.89				
	2.05	0.42		
4.88				
			2.04	0.41
4.95				
	2.02	0.41		
4.92				
			2.20	0.44
4.97				
	2.20	0.45		
4.84				
			2.20	0.46
4.76				
	2.21	0.46		
4.89				
			2.28	0.46
4.92				
	2.17	0.45		
4.73				

Similar conclusions were deduced from experiments on other pairs of metals. The four metals, copper, silver, aluminium, and tin, were taken two by two in each of the six possible ways, and in every case it was found that the ratio of transmission for the two-piece screen was independent of the order of the pieces, within the experimental error.

ersed first the silver screen, then the aluminium, and *vice versa*. From the latter measurements it would appear that the effect of transmission through aluminium is actually to "soften" the rays for silver. With no other pair of metals was this anomalous effect observed, and even with this pair it was not always obtained, but depended much on the state of the tube. An experiment performed only a few minutes before the one showing the "softening" effect gave a quite different result, but the change in the condition of the tube between the two experiments was evident to the eye. Direct experiments showed that the secondary radiation diffused from the screens cannot account for the observed effect.

5. *Effect of the Order of the Pieces in a Two-Piece Screen upon Transmission through the Screen.*

In order to throw some light on the nature of the phenomena which are involved in transmission, the following set of experiments was devised:

A screen was made by placing face to face two pieces of different metals, and the deflection obtained from the rays transmitted by this screen was compared with that obtained when the order of the two pieces of metal in the screen was reversed. In practice it was found convenient to prepare two screens, one composed of two metals in one order and the other composed of the same metals in the other order, and to compare the deflections accompanying the interposition of the two screens in succession. In using this method it had to be shown by an auxiliary experiment that there was no difference in the thickness of the two pieces of either metal, which might produce a spurious effect or mask a real one. This test was usually applied by reversing, in each screen, the order of the pieces of metal composing it. In all of the experiments the deflections were standardized in the manner before described, by comparison with deflections obtained when no screen was interposed. In some of the experiments the three-seconds exposure was used, and in others the rays were allowed to run until the greatest attainable deflection was reached. The quality of the rays used in the different experiments varied over a wide range, and sometimes it was allowed to vary considerably during a single experiment, as was the case in the example given in Table X. Copper and silver were the two metals used in this experiment. The copper was 0.0044 cm. thick, and the silver 0.0019 cm. After reversing the order of the plates in each screen, the data summarized in Table XI were obtained.

From these measurements it appears that the order of the pieces

0.0012 cm. When the first screen was of silver, however, and the screen for which the penetrating power was being investigated was of aluminium, the data in Table VIII were obtained. According to these measurements, the effect of transmission through silver on absorption by aluminium is very small. A slight modification of this experiment, performed some days later, gave a more striking result, as shown in Table IX. "Ag.-Al. Screen" indicates that the rays trav-

TABLE IX.

Effect of Transmission through Aluminium on Penetrating Power for Silver.

Throw, cms., no Screen.	Throw, cms., Ag. Screen.	Ratio.	Throw, cms., Al. Screen.	Throw, cms., Al.-Ag. Screen.	Ratio.
2.22	1.43	0.65			
2.18			1.67	1.04	0.63
			1.65		
2.08	1.28	0.65			
1.86			1.30	0.78	0.59
			1.33		
1.57	1.00	0.63			
1.60			1.18	0.73	0.63
			1.14		
1.48	0.96	0.65			
1.46			1.12	0.70	0.62
			1.13		
1.42	0.92	0.65			
1.41			1.06	0.68	0.64
			1.07		
1.35	0.87	0.65			
1.30					
	Means . . .	0.647	. . . .	. . . .	0.622

had passed through a screen of another metal than when they came direct from the tube; in the language of the subject, the rays were "hardened" by passage through the first screen. The following ratios of transmission show this:

Copper alone,	0.587	Copper behind silver,	0.608
Aluminium alone,	0.875	Aluminium behind tin,	0.928
Aluminium alone,	0.85	Aluminium behind copper,	0.91
Copper alone,	0.53	Copper behind tin,	0.65

TABLE VIII.

EFFECT OF TRANSMISSION THROUGH SILVER ON PENETRATING POWER FOR ALUMINIUM.

Throw, cms., no Screen.	Throw, cms., Al. Screen.	Ratio.	Throw, cms., Ag. Screen.	Throw, cms., Ag.-Al. Screen.	Ratio.
2.02	1.69	0.84	1.25	1.05	0.85
2.02			1.21		
1.79	1.54	0.81	1.15	0.96	0.83
1.74			1.15		
1.83	1.59	0.85	1.24	1.07	0.86
1.90			1.23		
1.91	1.71	0.89	1.29	1.13	0.88
1.95			1.28		
2.06	1.76	0.87			
1.98					
	Means	0.852	. . . .		0.855

In these experiments the thicknesses of the various screens were: copper, 0.0044 cm.; silver, 0.0019 cm.; aluminium, 0.027 cm.; tin, Table V contains the data of the experiment with aluminium in full, and in Table VI the mean ratios of transmission for that experiment are collected. Table VII gives the mean ratios of transmission for the experiment with copper. The accuracy of the readings with aluminium was greater than that of the readings with copper, the deflections being larger. It is to be noted, however, in both Table VI and Table VII, that the ratios for solid screens are not strictly comparable with the corresponding ratios for laminated screens, for corresponding screens of the two sets are not quite of the same thickness in most cases. In plotting the curves in Figure 5, this fact is taken into account.

TABLE VII.

SUMMARY OF EXPERIMENT ON EFFECT OF SURFACES OF COPPER SCREEN.

Solid Screens.		Laminated Screens.	
Screen No.	Ratio.	Screen No.	Ratio.
1	0.55	1	0.53
2	0.28	2	0.27
3	0.19	3	0.17

From these results we may conclude that in the case of aluminium and of copper, the effect of the surfaces as such on the transmission of the rays is too small to be detected in measurements of this degree of precision, if it exists at all.

4. *Effect of Transmission through a Screen of One Metal on Penetrating Power for a Screen of Another Metal.*

It has been generally supposed that the transmission of a beam of rays through a screen of any substance renders the beam more penetrating toward another screen of the same or of any other substance than it was originally. In view of the importance of this matter a re-examination of the experimental facts seemed desirable. Accordingly experiments were performed for the purpose of comparing the penetrating power for a given metallic screen of rays direct from the tube with the penetrating power for the same screen of rays which have passed through a screen of some other metal. The ratios of transmission were used as a basis of comparison. With most metals it was found to be true that the ratio of transmission for a given screen was greater when the rays by an automatic device after running three seconds. Though this reduced the size of the deflections, it rendered less rapid the change in condition of the tube. The deflections of the instrument under these conditions appear, from considerations of the motion of the suspended system under the forces involved, to be proportional to the energy received during the exposure, just as in the case where the greatest attainable deflection is used.

A comparison of the mean curve for the solid screens with the curve for the laminated screens should show the effect upon transmission, if any, due to the surfaces of the screen. In Figure 5 the dots and the curves drawn through them are from the data obtained with the solid screens; the crosses are points on the curve for the laminated screens. The screens used in the experiment with aluminium were as follows:

Solid screens: No. 1, 0.027 cm. No. 3, 0.072 cm.
No. 2, 0.050 cm. No. 4, 0.160 cm.

Laminated screens: No. 1, 0.027 cm., 1 layer.
No. 2, 0.054 cm., 2 layers.
No. 3, 0.081 cm., 3 layers.
No. 4, 0.162 cm., 6 layers.

Those used in the experiment with copper were:

Solid screens: No. 1, 0.0044 cm. No. 3, 0.021 cm.
No. 2, 0.014 cm.

Laminated screens: No. 1, 0.0044 cm., 1 layer.
No. 2, 0.013 cm., 3 layers.
No. 3, 0.023 cm., 5 layers.

TABLE VI.

SUMMARY OF TABLE V.

Solid Screens.		Laminated Screens.	
Screen No.	Ratio.	Screen No.	Ratio.
1	0.85	1	0.86
2	0.75	2	0.72
3	0.66	3	0.66
4	0.50	4	0.47

TABLE V.

EFFECT OF SURFACES OF ALUMINIUM SCREEN.

	No. of Screen.	Throw, cms., without Screen.	Throw, cms., with Screen.	Ratio.
Data for First Transmission Curve for Solid Screens.		1.15		
	1		1.06	0.82
		1.43		
	2		1.10	0.72
		1.62		
	3		1.09	0.68
		1.59		
	4		0.73	0.48
		1.46		
	3		1.01	0.67
		1.55		
	2		1.13	0.76
		1.40		
	1		1.14	0.86
		1.24		
Data for Transmission Curve for Laminated Screens.		1.24		
	1		1.02	0.82
		1.23		
	2		0.88	0.73
		1.16		
	3		0.72	0.63
		1.12		
	4		0.51	0.47
		1.05		
	3		0.74	0.68
		1.13		
	2		0.77	0.71
		1.05		
	1		0.89	0.89
		0.95		
Data for Second Transmission Curve for Solid Screens.		0.95		
	1		0.80	0.86
		0.90		
	2		0.69	0.74
		0.95		
	3		0.64	0.67
		0.97		
	4		0.49	0.51
		0.96		
	3		0.59	0.63
		0.92		
	2		0.70	0.76
		0.91		
	1		0.77	0.84
		0.92		

3. *Experiments to show a Possible Effect of the Surfaces of a Screen on Transmission.*

Experiments with two metals, aluminium and copper, were made in order to find evidence of any effect which the *surfaces* of the screen might exert on transmission. The results in both cases were negative. These experiments were conducted in the following way:

A set of screens of different thicknesses was prepared, each screen consisting of a single solid piece of the metal in question. A second set of screens was made, similar to the first except that the screens were laminated. The pieces of metal used were brightly polished on both sides. From the first set of screens a curve was obtained by the method described on page 681. A similar curve was taken for the laminated screens, and lastly, to check the constancy of the tube, a second curve for the solid screens. Readings were taken at intervals of one minute, and in these experiments the rays were not allowed to run until the greatest attainable deflection occurred, but were cut off

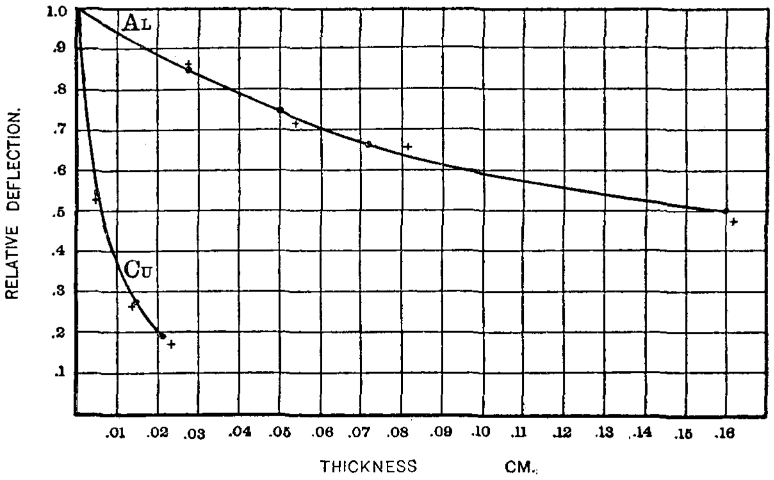

FIGURE 5.

This figure shows the small effect of the *surfaces* of screens upon transmission, in the case of aluminium and of copper. The curves are obtained by plotting as abscissa the thickness of each of a set of one-piece screens of the metal under examination against the corresponding "relative deflection" (that is, the deflection observed when the screen is interposed divided by the deflection observed when the screen is withdrawn) as ordinate. The crosses are points on a curve obtained when laminated screens were substituted for one-piece screens.

was made of the ratio of transmission of the silver screen under these circumstances. The screen was interposed at the same place, relatively to the instrument, as before; hence the motion of the tube involved a considerable variation of the intensity of the radiation incident on the screen. The extremes of intensity were in the ratio of 1 to 8 in this experiment, if the law of inverse squares is applicable. The tube was then returned to its original position, and the first ratio of transmission redetermined as a safeguard. By repeating this process several times, the set of readings in Table III was obtained.

TABLE IV.

Effect of Variation of Intensity upon Transmission by Platinum.

	Screen out. Throw, cms.	Screen in. Throw, cms.	Ratio.	Mean.
Distance, 22 cms.	2.80			
		1.25	0.43	0.39
	3.05			
		1.10	0.36	
	3.10			
		1.20	0.38	
	3.20			
Distance, 42 cms.	0.90			
		0.30	0.33	0.39
	0.90			
		0.40	0.42	
	1.00			
		0.37	0.41	
	0.80			
Distance, 22 cms.	3.00			
		1.12	0.41	0.39
	2.40			
		0.85	0.37	
	2.15			
		0.73	0.39	
	1.60			

The experiment with platinum is of especial interest because of the use made of that metal as the absorbing material of the instrument. Table IV shows the procedure and the result of this experiment.

From the experiments on copper, tin, and aluminium it appeared that for those metals, as for silver and platinum, the ratio of transmission is independent of the intensity of the incident radiation for a considerable range of intensity.

2. *Change in Transmission accompanying Change in Intensity of Incident Radiation.*

The metals used in the following experiments were platinum, 0.0014 cm. thick; copper, 0.0044 cm.; silver, 0.0019 cm.; tin, 0.0012 cm.; aluminium, 0.027 cm. The method was the same in all cases except for differences in detail. The experiment with silver will serve as an example.

TABLE III.

Effect of Variation of Intensity upon Transmission by Silver.

Distance, 42 cms.			Distance, 22 cms.		
Screen out. Throw, cms.	Screen in. Throw, cms.	Ratio.	Screen out. Throw, cms.	Screen in. Throw, cms.	Ratio.
1.62 1.64	1.11	0.68	5.82 5.61	3.98	0.70
1.62 1.60	1.11	0.68	5.78 5.84	3.91	0.67
1.65 1.66	1.11	0.67	5.92 6.12	4.10	0.68
1.77 1.74	1.20	0.68			
	Means	0.678	. . . .		0.683

The tube was placed as far as convenient from the instrument (42 cm. from the platinum of the latter to the centre of the target), and a "ratio of transmission" was obtained by dividing the deflection taken when the screen was in by the mean of two standardizing deflections taken when the screen was out. Then the tube was moved toward the instrument a distance of 20 cm. along a track which had been carefully aligned with the instrument before the latter was jacketed, and a determination drawn, and the deflection produced by the unimpeded rays was observed, for the purpose of making allowance for any small variation in the activity of the tube. The ratio of each deflection obtained with a screen interposed to the mean of these two standardizing deflections was computed, and the relative deflection thus obtained was plotted as ordinate against the thickness of the corresponding screen as abscissa. An experiment of this kind on screens of tin is illustrated by Figure 4. The tinfoil used was not much alloyed by lead, judging from its appearance and from a measurement of its specific gravity. The curve is from data taken January 9 with the tube at 20 cm. from the instrument. A curve taken January 10, with the tube at 25 cm. distance, and another curve taken on the latter day with the tube at 20 cm. distance, coincide with this one so closely as to be indistinguishable from it. Such exact reproduction of results on successive days was possible only when the tube was comparatively new.

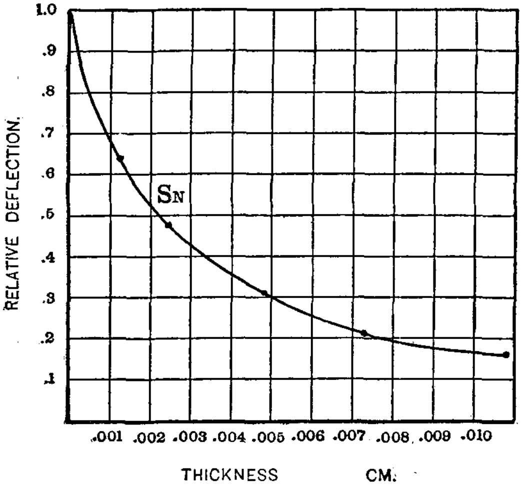

Figure 4.

This curve shows the relation between the deflections of the instrument and the thickness of the tinfoil screens interposed in the path of the rays. The thickness, in centimeters. is plotted as abscissa, and the corresponding "relative deflection" (that is, the deflection observed when the screen is interposed divided by the deflection observed when the screen is withdrawn) is plotted as ordinate. It is to be noticed that each successive equal increment of thickness of the screen is less effective in reducing the deflection than the one preceding it.

any measurement of this quantity is of little significance unless accompanied by complete specification of the conditions under which the rays are produced. But as a confirmation of the qualitative result of the earlier investigations, it serves the purpose for which it was undertaken; and, together with the recently published work of Wien,[12] of Bumstead,[13] of Angerer,[14] and of Carter [15] establishes beyond reasonable doubt the fact that the absorption of Röntgen rays in metals is accompanied by the appearance of a measurable amount of heat. It may be of interest, however, to remark that the result reached above is of the same order of magnitude as those found by the other investigators: Dorn's tube yielded an output of 0.00151 to 0.00168 gm. cal. per sec., Rutherford and McClung found 0.011 gm. cal. per sec., Wien has obtained 0.0015 gm. cal. per sec., Angerer gives a maximum value of 0.013 gm. cal. per sec., and Carter finds 0.00514 gm. cal. per sec. In this connection it may be added that in the latter part of the present research rays were used the energy of which was undoubtedly of the order of 0.01 gm. cal. per sec.

IV. Experiments on Transmission.

1. *Change in Transmission accompanying Change in Thickness of Screen.*

The change in transmission which accompanies a change in the thickness of a metallic screen has been examined by many observers, and has been found in every case to follow the same qualitative law, namely, that each succeeding equal increment of thickness is less effective as an absorbing medium than the one preceding it. Experiments made in the course of the present research confirm this law for all the metals examined, and it seems necessary here only to indicate the mode of procedure followed in these experiments, and to give a specimen of the curves obtained.

A series of screens of the metal to be examined was prepared. Each screen was built up to the desired thickness from thin sheets of the metal, — a method which is justified by experiments described below. The screens were then interposed, one by one, in the path of the rays, and the corresponding deflections of the instrument observed. Just before and just after taking each deflection the screens were with-

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[12] W. Wien, Ann. d. Phys., **18**, 991 (1905).

[13] H. A. Bumstead, Amer. Jour. Sci., **171**, 1 (1906).

[14] E. Angerer, Ann. d. Phys., **21**, 87 (1906).

[15] E. Carter, Ann. d. Phys., **21**, 955 (1906).

or 0.470, and the whole energy of the rays was then $\frac{0.00126}{0.470}$, or 0.00268 gm. cal. per sec. A correction for the absorption in the aluminium window of the instrument, obtained by assuming exponential absorption in aluminium, increased this result to 0.00325 gm. cal. per sec.

TABLE II.

Approximate Measurement of Energy.

	Screen out.		Screen in.	
Zero Reading.	Maximum Reading.	Throw, cms.	Maximum Reading.	Throw, cms.
28.47	29.34	0.87		
28.50			28.76	0.26
28.52	29.39	0.87		
28.49			28.76	0.27
28.52	29.39	0.87		
28.51			28.80	0.29
28.52	29.42	0.90		
28.53			28.78	0.25
28.52	29.40	0.88		
28.53			28.81	0.28
28.51	29.41	0.90		
28.53			28.82	0.29
28.55	29.45	0.90		
28.58			28.86	0.28
28.60	29.53	0.93		
Means	. . . . .	0.890	. . . .	0.274

As a quantitative determination of the energy of Röntgen rays, not too much importance should be attached to this measurement, because the method is subject to obvious inaccuracies, and also because

As a standard of radiation was chosen the "black body"[9] formed by the small opening of a cavity kept at 100° C. by a steam jacket. At 50 cm. distance this standard radiator, 3.14 sq. cm. in area, produced a deflection of 111 mm., the platinum of the instrument being in the normal to the radiating surface; and by using Kurlbaum's[10] measurement of the total net radiation from such a body to its surroundings at the room temperature (0.0156 gm. cal. per sq. cm. per sec., distributed through the half sphere according to the cosine law), it was computed that a small source 1 meter away, radiating energy according to the cosine law, must emit altogether 0.00177 gm. cal. per sec. to produce a deflection of 1 mm. In other words, a deflection of 1 mm. was produced when the platinum was receiving energy at the rate of 5.63×10^{-8} gm. cal. per sec., per sq. cm. of its surface.

With this result, an approximate measurement of the total energy of the Röntgen rays was readily obtained. The tube was placed so that the centre of the target was 20 cm. from the platinum of the instrument. A screen of platinum 0.0026 cm. thick was arranged so that it could be interposed in the path of the rays or withdrawn at pleasure. (A screen 0.0014 cm. thick, like the platinum used in the instrument, would have simplified the calculation, but none was available.) Then readings of the throws produced by the rays with the screen out were taken, alternating with readings with the screen in. These readings are given in Table II. In each case the rays were kept on until the maximum deflection occurred, a period of about 10 seconds. From the mean deflection obtained when the screen was not interposed, 0.890, the total energy of that portion of the rays which was absorbed was found to be

$$2\,\pi \times 20^2 \times 8.90 \times 5.63 \times 10^{-8} \text{ gm. cal. per sec.,}$$

or 0.00126 gm. cal. per sec. To find what fraction of the whole radiation this represented, it was assumed[11] that the fraction of rays transmitted by a piece of platinum is given in terms of its thickness, t, by e^{-kt}. Then k was found from

$$\frac{0.274}{0.890} = e^{-0.0026\,k}$$

to be equal to 453. Hence the fraction of the incident rays which the instrument absorbed was

$$1 - e^{-453 \times 0.0014},$$

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[9] Wien and Lummer, Ann. d. Phys., **56**, 451 (1895).

[10] F. Kurlbaum, Ann. d. Phys., **65**, 746 (1898).

[11] This assumption of an exponential absorption law is without justification, and probably in the present case is far from the truth.

on. The distances used were such that the lamp could be considered as a point source, and hence the constancy of the product

deflection × square of distance

is proof of the proportionality in question. The maximum deflection was used rather than the steady deflection, in this work and in all the work of this paper, because of the drift and other disturbances which rendered inaccurate the determination of the steady deflection. A specimen set of readings is given in Table I. By *d* is indicated the distance from the lamp to the platinum of the instrument, in meters.

TABLE I.

RELATION OF DEFLECTION TO ENERGY ABSORBED.

d.	Zero Reading.	Maximum Reading.	Throw, cms.	d^2 × Throw.
2.90	20.90	21.53	.63	5.30
2.80	20.96	21.64	.68	5.33
2.70	21.01	21.77	.76	5.54
2.60	21.17	21.96	.79	5.34
2.50	21.24	22.11	.87	5.44
2.40	21.36	22.29	.93	5.36
2.30	21.41	22.44	1.03	5.45
2.20	21.50	22.62	1.12	5.42
2.10	21.60	22.83	1.23	5.42
2.00	21.71	23.04	1.33	5.32
1.90	21.80	23.30	1.50	5.42
1.80	21.89	23.57	1.68	5.44
1.70	21.98	23.86	1.88	5.43
1.60	22.03	24.12	2.09	5.35
1.50	22.12	24.52	2.40	5.40
1.40	22.18	24.93	2.75	5.39

A calibration of the instrument in absolute measure was next made.

The distance from the platinum of the instrument to the centre of the target of the tube was 22 cm. in all the experiments described in this paper, with a few exceptions which are noted where they occur. The metallic sheets were interposed in the beam of rays at about 11 cm. from the platinum of the instrument in all cases. Throughout the remainder of this paper these metallic sheets will be referred to as "screens."

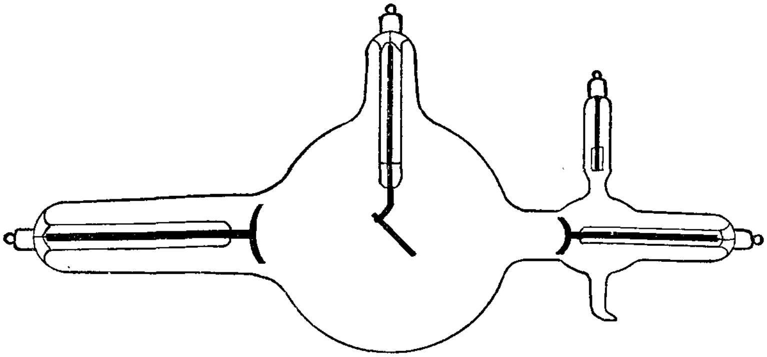

FIGURE 3.

The two electrodes at opposite ends of the bulb were of aluminium, and between them passed the rapidly oscillating discharge from the secondary of the Tesla coil. At the centre of the bulb was fixed the platinum target, upon which the cathode stream from the larger aluminium electrode came to a focus. In the ordinary use of the tube, the target had no metallic connection with the rest of the system. The effect of the constriction in the walls of the tube in front of the smaller aluminium electrode was to choke off more or less the cathode stream emanating from that electrode. The tube was provided with a device for reducing the vacuum.

III. APPROXIMATE MEASUREMENT OF THE ENERGY.

Soon after the instrument was set up, a test was made to ascertain whether its deflections were proportional to the amount of radiant energy which it absorbed per second at its sensitive surface. For the purpose of this test, and of the calibration described below, the platinum of the instrument had been coated on one side with a thin paste of lamp black in kerosene. A small battery-lamp using about 5 watts served as a radiating source, and was placed about 3 meters from the instrument, the aluminium window of the latter being raised so as to expose the platinum to the radiation. After noting the zero, the current was sent through the lamp and the maximum deflection read. Then the lamp was moved nearer and another deflection taken, and so might be independent of transient changes in the external magnetic field, a hollow cylinder of soft iron, with wall 0.6 cm. thick and with the necessary apertures, was slipped over the case. The effects believed to be due to external magnetic disturbances were thereby much diminished. Changes in the room temperature promptly produced a drift of the zero reading. To avoid this drift as much as possible, the instrument was jacketed with cotton wool and hair felt, the aluminium window for the rays being covered only by a thin layer of the latter, which appeared by the use of the fluoroscope to be perfectly transparent. In addition to this, a double-walled house of asbestos, large enough for a man to enter, was built around the instrument, the partition, and the tube. The aperture for observing the mirror was protected by a water-window about 8 cm. thick, and the telescope and scale were placed outside the asbestos house. These precautions were found sufficient for seasons when there was no artificial heat in the building, but during the winter the interior of the house had to be kept at an approximately constant temperature by means of an electric stove and an automatic regulator. Because of the small mass of the suspended system — one or two tenths of a gram — mechanical vibrations, such as those produced by machinery in neighboring buildings or by a high wind, greatly limited the hours available for work with the instrument.

The tube [8] employed as a source of Röntgen rays was one especially designed for use with a Tesla coil. It is illustrated in section in Figure 3. The Tesla coil was operated with power obtained indirectly from the incandescent lighting mains. From those mains the alternating current (110 volts, 60 cycles per second) was led to the primary of a step-up transformer and an adjustable resistance in series with it. The current from the secondary of this transformer charged a condenser of 0.005 mfd. capacity to a difference of potential of from 5,000 to 10,000 volts. The condenser was connected in series with the primary of the Tesla coil and a Cooper-Hewitt interrupter, and discharged itself through that circuit with a frequency of about 7×10^5 oscillations per second. Under these circumstances a spark from the Tesla secondary 33 cm. long between points was obtained; but for ordinary purposes a spark of half that length proved sufficient. By keeping the interrupter at a constant temperature of from 75° to 90° C., it was found possible to maintain a remarkably uniform activity of the tube for considerable periods, especially in the case of new tubes.

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[8] Manufactured by the Macalaster Wiggin Company, Boston.

avoided in setting up and using this instrument. It was placed on a heavy iron pillar rising from the cement floor of a basement room. The outside of the wooden case was covered with tinfoil, and a partition of roofing tin, 2 m. high by 1.3 m. wide, was placed between the instrument and the tube and was put to earth. A hole 1 cm. in diameter was bored in the partition to give passage to the rays. As an indication of the efficacy of these precautions against electrostatic disturbance, it may be stated that when powerful sparks were allowed to pass to the partition from the coil used to drive the tube, the zero reading of the instrument was not in the least affected. It seemed that the magnetic field about the suspended system exerted a considerable directive force upon the latter, notwithstanding the soft-iron block, and direct experiments confirmed this belief. In order, therefore, that not only the sensitiveness of the instrument but its zero reading

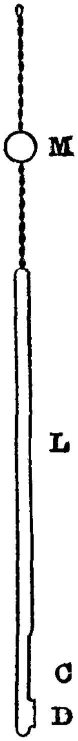

FIGURE 1.

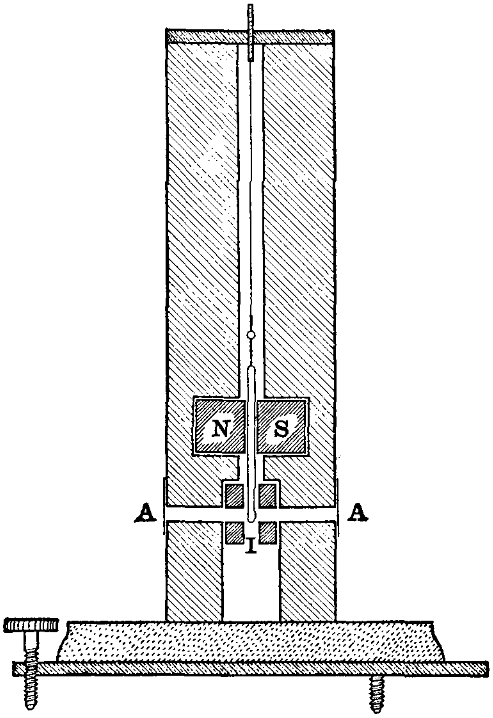

FIGURE 2.

(5) The nature of the effect which occurs within the substance of the sheet.

Most of these subjects had already been investigated by various observers at more or less length, by means of the photographic plate, the fluoroscope, and ionization phenomena; but in view of the semi-qualitative nature of photographic and fluoroscopic work, and considering the difficulty of an exact interpretation of the phenomena of ionization, it seemed desirable to reinvestigate the subjects with a strictly quantitative instrument.

The results obtained confirm the work of earlier investigators in most cases, but in one or two important particulars they are at variance with generally accepted views. It is believed that such measurements as those here published are valuable not only in themselves, but as an indication of the way to a more accurate quantitative examination of the nature of the Röntgen radiation than has yet been made.

II. Apparatus.

Figure 1 illustrates the suspended system of the instrument. It consisted of a loop, L, of copper wire 0.026 cm. in diameter, completed at the lower end by a bit of constantan wire, C, and a thin circular piece of platinum, D. The junctions were made with a soft solder, no acid being used, and all superfluous solder being removed. The wires at the upper end of the loop were twisted together, and carried a light mirror, M. A vertical section through the wooden case of the instrument is shown in Figure 2. The loop of copper was hung by a fine quartz fibre in a magnetic field due to the permanent magnets, N, S. The block of soft iron, I, was drilled horizontally and vertically as represented, and was designed to prevent the magnetic field from exerting a directive force on the suspended system. A copper sheathing, not shown in the section, lined the central cavity of the wooden case throughout its length. The Röntgen rays entered the instrument through a thin aluminium window at A and were partially absorbed in the platinum. The heat thus developed at the lower junction of the copper-constantan couple produced an electric current around the loop, which caused the suspended system to rotate in the magnetic field until brought to rest by the opposing torsion of the fibre. The dimensions of the instrument may be estimated from Figure 2, which is about one third actual size. The thickness of the absorbing platinum was 0.0014 cm., and its diameter 0.38 cm. The thickness of the aluminium window was 0.015 cm. A telescope and scale, the latter 151 cm. from the mirror, were used to read the deflections.

Disturbances of many sorts were encountered and, so far as possible, the means used to detect the heat developed in the metal. Dorn immersed the metal in a body of gas; the rise of temperature of the gas, due chiefly to the heat produced in the metal, caused a corresponding rise of pressure, which was made evident upon a delicate pressure-balance. Rutherford and McClung and Schöps, and Leininger following them, made the metal the sensitive arm of a bolometer. The methods differed also in the correction for the incomplete absorption of the rays. Dorn arranged his apparatus so that the rays had to pass through several layers of the metal, and thus were almost completely absorbed. The others observed the diminution of heating effect which occurred when a piece of metal, of the same kind and thickness as the absorbing sheet, was interposed between the latter and the tube, and hence estimated what fraction of the rays was effective in producing heat.

The method followed throughout the present research was a modification of this general plan. A Boys' radiomicrometer[7] was the instrument used, the thin piece of metal which partially absorbed the rays being placed at one of the junctions of the thermal couple. With this instrument, which is described in detail below, a heating effect was readily perceived; and some measurements were made of the whole energy of the Röntgen rays emitted per second by a tube, which measurements are included in this paper as a confirmation of the results obtained by the earlier investigators.

In the course of this work it became plain that there is at least one possible source of error which may affect, more or less, all measurements of the energy of Röntgen rays based upon observations of the heat developed by the absorption of the rays in thin metallic sheets. The well-known fact that the character of Röntgen radiation is changed by passage through solid substances shows that the methods thus far used to correct for the imperfect absorption of such sheets may yield only a first approximation to the truth. This uncertainty as to the exact effect produced upon a beam of Röntgen rays by passage through metallic sheets suggested the subjects experimentally investigated in this paper. These subjects are:

(1) The effect of varying the thickness of a metallic sheet upon transmission through it.

(2) The effect of varying the intensity of the incident radiation upon transmission through a metallic sheet.

(3) The effect of the surfaces of the sheet upon transmission through it, as distinguished from the effect occurring within its substance.

(4) The effect of transmission through a sheet of one metal upon the penetrating power of the rays for a sheet of another metal.

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[7] C. V. Boys, Phil. Trans., **180**, 159 (1888–1889).

THE TRANSMISSION OF RÖNTGEN RAYS THROUGH METALLIC SHEETS.

By John Mead Adams.

Presented by John Trowbridge, November 14, 1906. Received January 8, 1907.

I. Introduction.

This experimental investigation into the phenomena of the transmission of Röntgen rays through metallic sheets has grown out of a research which was undertaken three years ago for the purpose of confirming the measurements of the energy of Röntgen rays then recently made by Dorn,[1] Rutherford and McClung,[2] Schöps,[3] and others. These measurements had been rendered somewhat doubtful by Leininger's[4] unsuccessful attempt to repeat the work of Rutherford and McClung and of Schöps, and it seemed desirable to obtain new evidence on the question by a method different from those already used.

In general plan the methods of the above-mentioned investigators were similar. Each of them allowed the rays to fall upon thin sheets of metal, and from the heat developed in the metal estimated the energy of the absorbed rays; then, after making a correction for the fact that not all of the rays incident on the metal are absorbed in it, they were able to calculate the rate at which the tube was radiating energy in the form of Röntgen rays, by assuming from the work of Röntgen[5] and of Guillaume[6] that the emission of energy is nearly uniform in all directions from the target. To reduce the results to absolute measure, it was only necessary to send through the absorbing metal a known electric current of such strength that the heating effect thereby produced was equal to that produced by the absorption of the rays; the Joulean heat of the current in the metal was then taken to be equal to the energy of the absorbed rays. The methods differed in

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[1] E. Dorn, Ann. d. Phys., **63**, 160 (1897).

[2] E. Rutherford and R. K. McClung, Proc. Roy. Soc., **67**, 245 (1900).

[3] Schöps, Zeitsch. für Naturwiss., **72**, 145 (1899).

[4] F. Leininger, Phys. Zeitsch., **2**, 692 (1901).

[5] W. C. Röntgen, Ann. d. Phys., **64**, 22 (1898).

[6] Ch.-Ed. Guillaume, Comptes Rendus, **123**, 450 (1896).

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

THE TRANSMISSION OF RÖNTGEN RAYS THROUGH METALLIC SHEETS.

By John Mead Adams.

Investigations on Light and Heat made and published, wholly or in part, with Appropriation from the Rumford Fund.

Similarly for Mars it is

$$\frac{A_1}{g_1};$$

and as the whole mass of the earth's atmosphere over any space $dydz =$ 4.5 that of Mars at a similar point, and $g_1 = .38g$, we have

$$\frac{A}{1} = 4.5\,\frac{A_1}{.38},$$

whence as $A = 30$ inches of barometic pressure, $A_1 = 2.5$ inches.

BOILING POINT ON MARS.

Owing to the less amount of the Martian air and the smaller gravity at the surface of the planet the boiling point of water is greatly reduced, being probably in the neighborhood of one hundred and eleven degrees Fahrenheit. If the whole mass of air be $\frac{1}{4.5}$ of the earth's, while gravity is .38 of ours, the pressure is

$$M_1 g_1 = .09 \text{ of the earth's},$$

whence the boiling point is 44° C. or

$$79 + 32 = 111° \text{ F.}$$

For the same reason sublimation takes place more freely at identical temperatures there. Proportionally, therefore, there would be more water-vapor in the air.

We may summarize the results for Mars :

Mean Temperature	48° F. or 9° C.
Boiling point of water	111° F. or 44° C.
Amount of air per unit surface .	7 in. or 177 mm. ; $\frac{2}{9}$ of the earth's.
Density of air at surface . . .	2.5 in. or 63 mm. ; $\frac{1}{12}$ of " "

The look of the surface entirely corroborates the temperature result of this investigation.

JANUARY 14, 1907.

polar caps. These would combine to give it an albedo of .13. This, however, is illuminated by so much of the light as penetrates the atmosphere only, about three quarters of the whole. Whence the apparent albedo of the surface must be about .10. As the total albedo of the planet is .27, the remaining .17 is the albedo of its air.

Taking the density of the air as proportionate to its brilliancy, which would seem to be something like the fact, since the denser the air the more dust it would buoy up, we have for the Martian air a density about two ninths our own over each square unit of surface.

Now, if the original mass of air on each planet was as its own mass, we should have for the ratio between the Earth and Mars, 9.3 of atmosphere on the former to 1 on the latter. This being distributed as their surfaces, which are in the proportion of 7919^2 to 4220^2, must be divided by 3.5, giving 2.7 times as much air for the earth per unit of surface. The difference between 2.7 and 4.5 found above may perhaps be attributed to the loss of air Mars has since suffered on the supposition of proportionate masses to start with.

AIR DENSITY AT SURFACE OF MARS.

To get the relative density of the air at the surfaces of the two planets these amounts must be divided by the ratio of gravity at the surfaces of the two, that is, by .38.

For the density being proportional to its own increase, if D denote the density at any point, we have

$$dD = -Dg dx,$$

where g denotes the force of gravity at the surface of the earth, and x is reckoned from that surface outward into space, whence

$$D = Ae^{-gx},$$

A being the density at the surface of the planet.

For Mars we have correspondingly

$$D_1 = A_1 e^{-g_1 x}.$$

For the whole mass of air over a space $dydz$ we have, for the Earth,

$$\int_0^\infty D dx = -\frac{A}{g} e^{-gx} \Big|_{x=0}^{x=\infty} = \frac{A}{g}.$$

Flagstaff, 1896, and at Mexico in 1897, from which it appeared that the planet's markings were not obscured by cloud, but seen, as it were, through a veil, and which also showed the correctness of Schiaparelli's deduction that Venus in all probability turned in perpetuity the same face to the sun. That she did so was evident from the long-continued observations at Flagstaff and Mexico. Now such a facing always of one hemisphere sunward would cause convection currents upward in the centre of the disk, and an indraught along its edge, together with an absence of moisture on the sunlit half of the planet. Dry winds of the sort blowing over a perpetual Sahara must be laden with dust, which Very's investigation finds to be the chief cause of reflection in our own air. The high albedo of Venus thus stands accounted for.

Light around Venus.

A sidelight bearing on the albedo of air comes from the prolongation of the crescent of Venus when the planet passes in inferior conjunction before the sun.

It used to be thought that the fine circlet of light that then crowns the disk was due to refraction in the Venusian air. But in 1898 Russell, at Princeton, showed that it is rather reflection from that air than refraction through it which reaches our eyes. Now that such should be the case follows from the planet's albedo, if that albedo be of atmospheric and not of nubial origin. This supports the conclusion reached by the visual observations of Venus at Flagstaff. For refraction means transmission, and if the air of Venus reflects 90 per cent of the incident light, it can refract but 10 per cent at most. The light from it, therefore, must be reflected, not refracted, light in the proportion of nine to one. The albedo, Russell's observations, and the Flagstaff results, thus all concur to the conclusion that Venus is not enveloped in cloud.

Deduction as to Amount of Martian Air.

Another outcome of the consideration of albedoes is a means it gives of approximating to the density of the Martian air. Mars is chiefly Saharan, and dust, therefore, must be largely present in its air. Now from the albedo of various rocks, of forests, and of other superficies, we may calculate the relative quotas in the whole albedo of Mars, of its surface and its air. Five eighths of its surface is desert, and therefore of an albedo of about .16, as its hue shows three eighths of a blue-green, the color of vegetation, with an albedo of about .7, while one sixth is more or less permanently of a glistening white in the which gives not far from what we had before, since it lowers the resulting temperature for Mars by only about 4° F. or 2° C.

ALBEDO AND AIR.

Some interesting conclusions follow on the investigation of planetary albedo.

If we classify the various planets according to their atmospheric envelopes, we shall discover a significance in their several albedoes. Three classes stand forth distinct: 1. those possessing no air; 2. those with air but wholly or in part cloudless; 3. those with a cloud covering. Into these classes the planets fall in the manner below, while the albedoes they respectively present are placed alongside of them.

		Albedo.
I. Airless Bodies.		
Mercury		.17
Moon		.17
II. Air-enveloped Bodies.		
Venus, Cloudless	Medium air	.92
Earth, 50 % Cloudless		.77
Mars, Cloudless, thin air		.27

	Albedo.	
III. Cloud-canopied Bodies.		
Jupiter	.75	
Saturn	.88	or .78 by Struve's latest measures.
Uranus	.73	
Neptune	.63	

The albedo of cloud is .72. Whence it is clear that cloud cannot account for the albedo of Venus, but that it accords with the albedo of the four major planets. That an air-envelope increases the albedo of a planet is witnessed first by the greater brilliancy per unit of disk of the earth, Venus, and Mars as compared with the airless bodies, Mercury and the Moon, and secondly, by the relative specific brightness of Venus and Mars, together with what has above been found as to that of the earth. It appears that the denser the air surrounding the planet the more dazzling the aspect the planet presents. This is undoubtedly due not to the gases themselves, but to the solid or liquid particles the gases support in the shape of dust, ice-particles, or drops of water.

This testimony of the albedo that Venus is not cloud-covered but atmosphere-hid is corroborative of the observations made by me at

For the earth in its entirety $M = o$ and $N = o$, since what is lost by convection in one place is gained in another.

Applying this same formula to the case of Mars we have similarly —

$$\alpha_1 = .40 \text{ approximately,}$$

$$A_1 = \frac{1^2}{1.524^2}(1 - .17 \times .32) = \text{whole spectrum} - \text{albedo of its air} \times \text{visible portion}$$

$$= \frac{.946}{1.524^2}$$

$$\beta_1 = \alpha_1 \text{ approximately.}$$

$$\nu_1 = 1 - .13 = .87$$

Whence for the earth under a clear sky

$$T^4 = \frac{A\,(1 + \nu - \nu\alpha)}{\gamma\,(1 + \nu - \beta\nu)},$$

and similarly for Mars, substituting its values for A, α, and β. Since in both $\alpha = \beta$ and $\gamma_1 = \gamma$ approximately, we have T_1 for Mars, which gives

$$\frac{T_1^4}{T^4} = \frac{A_1}{A}.$$

But the earth is .50 cloud-covered, and the transmission of cloud being not more than .20 (the value he takes), we have finally

$$\frac{T_1^4}{T} = \frac{A_1}{A}\frac{.99}{.60},$$

whence

$$T_1 = .974\ T,$$

and T being 519.4° Abs. on the Fahrenheit,

$$T_1 = 505.7°, \quad \text{that is,} \quad 46.3° \text{ F. or } 8° \text{ C.}$$

a result substantially the same as we have deduced.

Had we assumed β to be .70 and to be in like proportion to α for Mars, we should have had

$$T^4 = 1.140\frac{A}{\gamma}$$

and

$$T_1^4 = 1.101\frac{A_1}{\gamma_1},$$

the radiation must be in the same ratio. Whence, the loss by radiation in twenty-four hours on Mars so far as it depends on the heat received is

$$\begin{aligned} e_1 &= 1.1\,e \\ &= \ .51, \end{aligned}$$

or by the more approximate calculation in the paragraph above, it still

$$= .51$$

Substituting these values in our equation (page 660), we find x, the mean temperature of Mars,

$$\begin{aligned} &= \ 8^\circ.7 \text{ C.} \\ \text{or } &= 47^\circ.7 \text{ F.}, \end{aligned}$$

taking into account the heat radiated away as well as the heat received and gauging the temperature by the heat retained; by the net, instead of the gross, amount of the radiant energy received.

If we assume clouds to transmit less heat than 20 per cent, we diminish y and increase $(1 - .35\,e)$, so that the ultimate result is not greatly altered.

If we take Arrhenius' formula for the temperature T of the earth's surface as affected by the air-envelope, we have as determined in his paper on the effect of carbon dioxide in the air:

$$T^4 = \frac{\alpha A + M + (1 - \alpha)\,A\,(1 + \nu) + N\left(1 + \frac{1}{\nu}\right)}{\gamma\,(1 + \nu - \beta\nu)},$$

where α = atmospheric absorption for solar heat,
β = atmospheric absorption for earth-surface heat,
A = Solar Constant, less loss by selective reflection by the air,
M = heat conveyed to the air from other points,
N = heat conveyed to the surface from other points,
$\nu = 1$ — albedo of the surface,
γ = radiation constant.

The values for these quantities found bolometrically for a clear sky are $\alpha = .50$,
$A = 1 - .79 \times .32 = .747$ = whole spectrum — albedo of the air × visible portion,
$\beta = \alpha$ approximately,
$\nu = 1 - .11 = .89$

under a cloudy one, about 7° F. or 4° C. Taking the average day temperature from these data at 292°Abs. on the centigrade scale or 19°C., and considering an average day sky and a clear night, we have the transmission or loss

$$\tfrac{1}{2}\,(.35 + .50)\,e \text{ or } .425\,e\,;$$

while for an average day and a cloudy night it is

$$\tfrac{1}{2}\,(.35 + .20)\,e \text{ or } .275\,e$$

We form the following equation to determine e:

$$\frac{292^\circ - 10^\circ}{292^\circ - 4^\circ} = \frac{\sqrt[4]{y\,(1 - .425\,e)}}{\sqrt[4]{y\,(1 - .275\,e)}}$$

whence $$e = .47$$

Since the radiation by day is greater by about 1.15 than by night

being as $$\frac{292^4}{282^4},$$

we have more approximately

$$\tfrac{1}{2}\,(.40 + .50)\,e \text{ or } .45\,e$$

for a clear night and average day and

$$\tfrac{1}{2}\,(.40 + .20)\,e \text{ or } .30\,e$$

for a cloudy night under the same conditions.

This gives, $$e = .4634,$$

or substantially what it was before. It changes the final result for the mean temperature of Mars by less than two tenths of a degree.

Determination of e_1.

Since in the mean the planet radiates as much heat as it receives and

$$\frac{y_1}{y} = 1.10$$

Heat received and Heat retained.

Such, then, would be the mean annual temperature of the planet, were the heat retained as well there as here. I am far from saying that such is the temperature. For the retention is not the same on the two planets, being, on account of its denser air, much better on the earth. But that such is the amount received is enough to suggest very different ideas as to the climatic warmth from those hitherto entertained.

Temperature deduced from Heat retained.

To obtain some idea of the heat retained and of the temperature in consequence we may proceed in this way :

Let y = the radiant energy received at the surface of the earth.
y_1 = that similarly received on Mars.
e = the relative emissivity or the coefficient of radiation from the surface of the earth, giving the ratio of the loss in twenty-four hours to the amount received in the same time, due to factors other than the transmissibility of the air, which is separately considered.
e_1 = the same coefficient for Mars.

Clouds transmit approximately 20 per cent of the heat reaching them ; a clear sky at sea-level, 50 per cent. Consequently as the sky is half the time cloudy the mean transmission of its air-envelope for the earth is

$$.35\,e$$

For Mars it is

$$.60\,e_1$$

To get, then, the mean temperature of the planet in degrees, x, from the heat retained, which is the daily mean receipt less the mean loss, we have the following equation, the mean temperature of the earth being [519.4°F. Abs.] 288°C. above absolute zero :

$$\frac{x}{288.5} = \frac{\sqrt[4]{y_1\,(1 - .60\,e_1)}}{\sqrt[4]{y\,\;(1 - .35\,e)}}$$

Determination of e.

To find e we have the data that the fall in temperature toward morning on the earth under a clear night sky is about 18° F. or 10° C. ;

HEAT RECEIVED BY EARTH AND MARS.

We will now apply the argument from the albedo.

HEAT RECEIVED AT THE SURFACES OF MARS AND THE EARTH.

	Per cent of Whole Energy.	Per cent of Heat received to Whole Energy.	
		Mars.	Earth.
Visual spectrum . . .	32	73	23
Infra-red	65	60	50
Total		64	41.5

The ultra-violet rays slightly increase the depletion by selective dispersion for both planets, and probably the more for Mars.

INSOLATION.

But this is not all. The above deduction applies only to such sky as is clear. Now the earth is cloud-covered to the extent of 50 per cent of its surface on the average; Mars, except for about six Martian weeks, at the time of the melting of the polar cap and over an area extending some fifteen degrees from the pole, stands perpetually unveiled. The surface thus fog-enveloped is 0.034 of its hemisphere, and the time 0.23 per cent of the half year, whence the total ratio of cloud to clear the whole year through over the whole surface is less than 1 per cent.

The albedo of cloud being .72, its transmission, including absorption re-given out, cannot exceed .28, and may be taken as 20.[5] Consequently the effective heat received on this score by the earth is about as $20 \times 50 = 60$ per cent, and for Mars 99 per cent, giving the ratio that of .60 to .99.

Taking now Stefan's law that the radiation of a body is as the fourth power of its temperature, and remembering that, since the two planets maintain their respective mean annual temperatures, they must radiate as much heat as they receive, we have the following equation from which to find the mean annual temperature of Mars, x, in which $459.4^\circ + 60^\circ$ or 519.4°F. on the absolute scale denotes the mean annual temperature of the earth:

$$x : 519.4^\circ :: \sqrt[4]{1^2 \times .64 \times .99} : \sqrt[4]{1.524^2 \times .415 \times .60}$$

or

$$x = 519.4^\circ \tfrac{892}{872}$$

giving

$$x = 531.4^\circ \text{ Abs.} = 72^\circ \text{ F. or } 22^\circ \text{ C.}$$

---

[5] This agrees with Arrhenius' estimate of the heat transmissibility of cloud.

The greater bulk of the depletion in this part of the spectrum comes from the absorption by water-vapor, water itself, or ice and carbon dioxide. At the earth's surface the transmission in consequence is about 50 per cent; at Camp Whitney it was about 59 per cent. We might, therefore, suppose it still greater through the air of Mars, which is very thin, and if we did so we should find a still larger fraction of solar heat to be received by the planet's surface; so that such a supposition would actually increase the cogency of the present argument. But the very thinness of the air joined to the lesser gravity at the surface of the planet would lower the boiling point of water, as investigation shows (see later in the paper) to something like 110° F. The sublimation at lower temperatures would be correspondingly increased. Consequently the amount of water-vapor in the Martian air must on that score be relatively greater than in our own.

DEPLETION BY CARBON DIOXIDE.

Carbon dioxide, because of its greater specific gravity, would also be in relatively greater amount, so far as this cause is considered. For the planet would part, *caeteris paribus*, with its lighter gases the quickest. Whence, as regards both water-vapor and carbon dioxide we have reason to think them in relatively greater quantity than in our own air at corresponding barometric pressure. We may therefore assume provisionally that the absorption due this cause is what it is with us at Camp Whitney, or about 40 per cent of the whole, leaving 60 per cent of the heat transmitted.

It is distinctly to be noted not only that this estimate lowers the determination of the heat received at the surface of Mars, but that what is thus lost in reception goes to make the retention of the heat received all the greater.

ALBEDOES OF THE PLANETS.

The albedoes of the several planets, according to the determinations latest obtained, those by Müller at Potsdam, together with that found above for the earth and for the moon by Zöllner, stand thus:

Mercury	.17	Jupiter	.75	(using Struve's latest diametral measures, .78)
Venus	.92	Saturn	.88	
Earth	.75	Uranus	.73	
Moon	.17 (Zöllner)	Neptune	.63	
Mars	.27			

The albedo of various rocks and of the ocean is as follows :

White quartzite . .	.25	Dark slate .	.09
Clay shale	.16	Ocean . .	.075

For forest we may perhaps take .07
and snow according to purity .50 — .78

The percentages of distribution of surfaces being about

Ocean .	72 per cent	Steppes & Desert .	10 per cent
Forest .	10 per cent	Polar Caps . . .	6 per cent,

we deduce 11 for the albedo of the surface. But this being illuminated by only 25 per cent of the light outside the air gives about 3 for its quota to the planet's illumination. When finally the earth's whole albedo to one viewing it from space becomes $.74 + .3 = .77$ albedo of the earth for a clear sky.

As the earth's is about 50 per cent cloud-covered (see the researches of Teisserinc de Bord on Nebulosity) and the albedo of cloud is .72, we get .75 for the mean albedo of the earth.

VALUE OF LOSS OF LIGHT A MINIMAL ONE.

That the value above found for the percentage transmission of solar radiation to the earth's surface is a maximal rather than a minimal amount, and the albedo a minimal rather than a maximal one, is hinted by the fact that the higher the observer ascends above the surface the greater his estimate of the solar constant becomes. Thus Langley in his memoir on the Mt. Whitney expedition says :

"In accordance with the results of previous observers, then, and of our own with other instruments, *we find a larger value for the Solar Constant as we deduce it from observations through a smaller air mass.*" The italics are his.[4]

DEPLETION BY WATER-VAPOR ON MARS.

We are now in position to estimate the heat actually received respectively at the surfaces of Mars and the earth. The visual part of the spectrum containing 32 per cent of the incident solar radiation gives us its quota directly from the albedo, since the heat received = 1 albedo. The infra-red portion containing 65 per cent of the whole depends upon the character of the air and of what it holds in suspension.

---

[4] Researches on Solar Heat, p. 68.

Transmission of Solar Radiation in the Visible Spectrum.

	Calories received from the Whole Spectrum.	Visible Portion transmitted.
Outside the atmosphere	3.127	1.000
Camp Whitney	1.896	.664
Lone Pine	1.533	.482
Sea-Level	1.180	.210

The relative loss in the regions I, $\lambda = 0.393\ \mu$ to $\lambda = 0.76\ \mu$, and II, $\lambda = 0.76\ \mu$ to $\lambda = 1.2\ \mu$, between the several stations is as follows:

	I.	II.
Outside to Camp Whitney . .	0.105	0.029
Camp Whitney to Lone Pine . .	0.055	0.010
Lone Pine to Sea-Level . . .	0.086	0.027

Light received from the Day Sky.

To these transmissions must be added that part of the solar radiation which is lost by reflection and diffraction in the atmosphere before reaching the ground, but is reflected again upon it, causing the brightness of the day sky. This amount is sufficient to obliterate the stars. Compared with direct sunlight, its ratio as determined by Langley[3] is

	Sun.	Sky.
Illumination	80	19

or 24 per cent of the sun's light.

We must therefore increase the energy transmitted by 24 per cent of itself. This gives finally :

	Transmission.		Portions reflected into Space.
Outside	1.000	1000	0
Sea-Level	.21	26	74

Albedo of the Earth.

Now the fraction of the incident energy in the visible spectrum is that by which we see the body and is called its albedo. The albedo of our air, then, comes out .74. To get the whole albedo of the earth we must add to it the albedo of the surface.

---

[3] Professional Papers of the Signal Service, Vol. 15.

This value of 1.180 is one which is probably about the average of clear days in our latitude, the day in question being registered by Crova as "very clear."

From these several data we find the following values for the solar radiation received at the respective posts, in calories in one column, in percentage of that entering the atmosphere in another.

SOLAR RADIATION.

	Bar.	Calories.	Percentage.
Outside the atmosphere .	0.	3.127	1.000
Camp Whitney	500 mm.	1.896	.606
Lone Pine	663 "	1.533	.490
Montpellier	761 "	1.180	.377

The loss in the visible spectrum is almost wholly from selective or general reflection and from diffraction, that in the invisible one from selective absorption. The absorptive loss by bands in the former is only about 1 per cent of the whole, and the loss by reflection in the latter probably not over 7 per cent of its depletion.

In view of the fact that the absorption is known to take place high up in the air, Very adopted the Alleghany amount for Lone Pine, the difference being insensible; but when it comes to Camp Whitney it is clear from the above that 9 per cent of it is got rid of, between $\lambda = 1.2\ \mu$ and $= 10\ \mu$ by rising the 11,700 ft. from sea-level.

DEPLETION IN VISIBLE RAYS.

We may now find the depletion in the visible part of the spectrum which is not in general the same as that for the invisible part, decreasing relatively with the altitude and reversely increasing as the air envelope becomes thicker. It does this at a greater rate than the increase of the air mass, because the particles suspended in the air, dust, water globules, and ice augment more rapidly than the air mass as one approaches the ground.

Drawing the curve for transmission at the sea-level on the same principles as those for outside the atmosphere at Camp Whitney and at Lone Pine, and then measuring the amounts of transmission of each within the limits of the visual rays, from $\lambda = 0.393\ \mu$ the K line to $\lambda = 0.76\ \mu$ the A band, we get the following table:

giving for the

Visible portion,	32 per cent,
Invisible "	68 "

of the whole.

Loss of Heat in Traverse of the Air.

Turning, now, from the question of the initial heat for different parts of the spectrum at the time the solar radiation enters the air, we come next to consider the loss the several rays sustain in their traverse of it.

From Very's curves for the radiation at the confines of the atmosphere at Camp Whitney and at Lone Pine, 18 $\lambda = 1.2\ \mu$, we get the amount transmitted at these two stations, employing planimetric measurement as before, and introducing with him the absorption in the red and infra-red from the Alleghany measures, which he considers the same at Lone Pine.

From Very's measures we have, calling the whole heat at the confines of the atmosphere unity, —

Transmission.

	$\lambda = 0.2\ \mu$–$1.2\ \mu$.	$\lambda = 1.2\ \mu$–$15\ \mu$.
Outside	50.	50.
Camp Whitney . . .	31.3	
Lone Pine	24.3	25.1

To get that for sea-level we shall take Crova's actinometric measures at Montpellier (height 40 m.), made on August 13, 1888, at $12^h\ 30^m$, under a barometer of 761 mm. Simultaneously with these, other self-registering ones were taken by him on Mt. Ventoux (height 2000 m.). The respective calories he obtained were, —

	Montpellier.	Mt. Ventoux.
Aug. 13, $12^h\ 30^m$, 1888.	0.975 calories, bar. 761.1 mm.	1.360 calories, bar. 613.5 mm.

We shall reduce these to the same scale as the Lone Pine results, made with the pyrheliometer and used by Very, to wit:

	Lone Pine.
Aug. 11, 12, 14, 12^h–$12^h\ 30^m$, 1881.	1.533 calories, bar. 663 mm.

giving for

Montpellier.	Mt. Ventoux.
1.180 calories.	1.643 calories.

Violle, Crova, Hansky, and others to the determination of this quantity at given places, and so to a conclusion as to the amount of heat outside our air, or the Solar Constant. Langley's great contribution to the subject was the pointing out that the several wave-lengths of the different rays were not of homogeneous action or modification, and that to an exact determination of the Solar Constant it is necessary to consider the action of each separately, and then to sum them together. To this end he invented his spectro-bolometer.

By means of this instrument Langley mapped the solar radiation to an extension of the heat spectrum unsuspected before. He then carried it up Mt. Whitney in California, and discovered two important facts: one, that the loss in the visible part of the spectrum was much greater, not only actually, but relatively to the rest, than had been supposed; and the other, that the greater the altitude at which the observations were made, the larger the value obtained for the Solar Constant. Both of these are pertinent to our present inquiry.

With a rock-salt prism, instead of a glass one, he next extended still farther the limits of the heat spectrum toward the red, the effect of the solar radiation proving not negligible as far as $\lambda = 15\ \mu$.

In 1901 Professor Very, who had been his assistant earlier, published an important memoir on the Solar Constant, based upon these bolometric observations, but with a value for it got from spectral curves derived from simultaneous actinometric and bolometric determinations at Camp Whitney and Lone Pine, and extended from them outside the atmosphere by taking both air and dust effects into account in selectively reflecting and diffracting the energy waves. The air effect is proportionate to the air mass, but the dust effect increases in greater ratio as one nears the surface of the ground. The formulæ he used were adaptations of those by Rayleigh for accounting for the selective reflection and diffraction of small particles.[2]

ENERGY OF VISIBLE AND INVISIBLE SPECTRUM.

Planimetrical measurement of the area enclosed by the curve deduced for outside our atmosphere gives the following results:

DISTRIBUTION OF HEAT IN THE SPECTRUM.

	Wave-lengths.	Percentage.
Invisible,	$\lambda = 0.2\ \mu$–$0.393\ \mu$	2.5
Visible,	$\lambda = 0.393\ \mu$–$0.76\ \mu$	32.
Invisible,	$\lambda = 0.76\ \mu$–$15\ \mu$	65.5
		100.

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[2] U. S. Department of Agriculture, Weather Bureau, No. 254.

Distance but One of Several Factors.

In these and similar determinations, the only thing considered is the distance from the sun, as if the surfaces of the planets were immaterial points in space, and the whole heat arriving there went to warm the bodies. But such is far from being the fact. Not only are the surfaces material, but any air the bodies possess as a covering is material too. Now, postponing for the moment consideration of the blanketing effect of air, the actual amount of heat received at the surfaces in consequence of their constitution or of their air-envelope, is not at all what mere distance would lead one to infer.

Division of Radiant Energy.

So soon as a radiant ray strikes matter it suffers division of its energy. Part of it is reflected, part absorbed, and part transmitted. What is reflected is sent off again into space, performing no work in the way of heating the body. Now the amount reflected is not the same in all cases, depending for its proportion upon the character of the matter the ray strikes.

If the surface of a planet be itself exposed unblanketed by air, the absorbed and transmitted portions go to heat the planet, directly or indirectly.

If the planet be surrounded by air, the portion transmitted by this air, plus what is radiated or reflected from it to the solid surface, must first be considered. Then, upon this quota as a basis, must secondly be determined how much the surface in its turn reflects. The balance alone goes to warm the ground or ocean.

Light and Heat.

Radiant energy is light, heat, or actinism, merely according to the effect we take note of. If our eyes were sensitive equally to all wave-lengths, we could gauge the amount of heat received by a body by the amount of light it reflected, — that is, by its intrinsic brightness, or albedo. For this percentage deducted from unity would leave the percentage of heat received. This procedure may still be applied, provided account be also taken of the heat depletion suffered by the invisible rays. Two problems, then, confront us.

We must find the albedoes of the several planets in order to compare one with another in its reception of heat, and we must find the relation borne by the visible and invisible rays to the subject. The latter problem may best be attacked first.

Actinometers and pyrheliometers are instruments for measuring *in toto* the heat received from the sun; and they have been used by

TEMPERATURE OF MARS.

A DETERMINATION OF THE SOLAR HEAT RECEIVED.

By Percival Lowell.

Presented December 12, 1906. Received January 15, 1907.

Heat hitherto deduced from Distance only.

Up to the present time the chief obstacle to crediting Mars with the possibility of life has lain in accounting for sufficient heat on the surface of the planet. So far the determination of this heat has been limited to a consideration of distance from the sun. Thus Professor Young, who feels the difficulty acutely, says in his "General Astronomy":[1] "We know that on account of the planet's distance from the Sun the intensity of solar radiation upon its surface must be less than here in the ratio of 1^2 to 1524^2." For the resulting temperature he seems to have assumed either with Newton, that a body radiates heat in direct proportion to its temperature, which would give for the mean temperature of Mars 223.6° Abs. (—236°F.), or Dulong and Petit's law, which would make it 363° Abs., or —96° F.; for he entertains the possibility that the polar caps may be composed of solid carbonic acid, which freezes only at —109° F.

A better determination has recently been made by Moulton by taking Stefan's law of radiation, that of the fourth power of the temperature. Stefan's formula is not only the best experimentally to-day, but has since its enunciation been deduced from theoretic considerations by both Boltzmann and Galitzine. On this basis the mean temperature comes out —33° F., the reasoning being this: If a body remains at the same temperature, it must radiate as much heat as it receives. Consequently the temperature is as the fourth root of the amount received. Absolute zero is minus 459° F. The mean temperature of the earth is usually taken at 60° F. Therefore, to determine the mean temperature of Mars we have, calling x its temperature on the absolute scale, the following equation:

$$x : 518 \text{ Abs.} :: \sqrt[4]{4} : \sqrt[4]{9},$$

which gives —33° F. for the mean temperature of the planet.

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[1] Ed. 1898, p. 363.

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TEMPERATURE OF MARS.

A DETERMINATION OF THE SOLAR HEAT RECEIVED.

By Percival Lowell.

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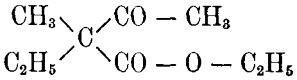
from the Rumford Fund.

the four valence energies of the central carbon atom will be equal, in the compound $CR'R''_3$ three will be equal and one different from these three, and in the compound $CR'R''R'''R''''$ all four will be different, even when the central carbon atom is tied to like atoms by all four of its valences as in methylethylacetacetic ester



because every one of the four carbon atoms which are tied to the central carbon atom is different in position from every other and consequently subjected to different influences from at least some of the atoms of the molecule.

It cannot be too strongly emphasized that this conclusion is a *logical* deduction from established experimental data with the aid of but two postulates, the current theory of chemical constitution and the current theory that the interatomic forces within the molecule are exclusively (or almost exclusively) valence energies.

It remains to be pointed out that through the consideration of many cases similar to those here studied, one may hope to obtain accurate quantitative information regarding the phenomena of orientation of substituting groups[17] and the countless other cases familiar to every organic chemist of the influencing of reaction by substituted groups, and thus to replace the existing empirical rules for such cases by accurately formulated principles.

SUMMARY.

In development of the conception of varying valence energy it is shown that the effect of an inactive atom group upon a reaction of another atom group of the same molecule, measured by the heat of reaction, varies in a regular way according to the relative positions of the two groups.

Curves are presented representing the nature of this variation in one case of special importance because of the magnitude of the effect and the accuracy of the data, and in a case where the effect is slight.

It is shown that, in accordance with this conclusion and the previous considerations, the valence energies of two ties between like atoms are under like conditions equal, under unlike conditions different.

The importance of these considerations for the understanding of the influencing of reactions by substituted groups is pointed out.

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[17] See for instance Flurscheim, l. c.

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RUMFORD PREMIUM.

In conformity with the terms of the gift of Benjamin, Count Rumford, granting a certain fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of said fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a premium to the author of any important discovery or useful improvement in light or in heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference being always given to such discoveries as shall, in the opinion of the Academy, tend most to promote the good of mankind; and to add to such medals, as a further premium for such discovery and improvement, if the Academy see fit so to do, a sum of money not exceeding three hundred dollars.

STANDING VOTES.

1. Communications of which notice has been given to the Secretary shall take precedence of those not so notified.

2. Associate Fellows, Foreign Honorary Members, and Resident Fellows, who have paid all fees and dues chargeable to them, are entitled to receive one copy of each volume or article printed by the Academy on application to the Librarian personally or by written order within two years of the date of publication. Exceptions to this rule may be made in special cases by vote of the Academy.

3. The Committee of Publication shall fix from time to time the price at which the publications of the Academy may be sold. But members may be supplied at half this price with volumes which they are not entitled to receive free, and which are needed to complete their sets.

4. Two hundred extra copies of each paper accepted for publication in the Memoirs or Proceedings of the Academy shall be placed at the disposal of the author, free of charge.

5. Resident Fellows may borrow and have out from the Library six volumes at any one time, and may retain the same for three months, and no longer.

6. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

7. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy, except by special leave of the Librarian.

8. Books, publications, or apparatus shall be procured from the income of the Rumford Fund only on the certificate of the Rumford Committee that they, in their opinion, will best facilitate and encourage the making of discoveries and improvements which may merit the Rumford Premium; and the approval of a bill incurred for such purposes by the Chairman shall be accepted by the Treasurer as proof that such certificate has been given.

9. A meeting for receiving and discussing scientific communications may be held on the second Wednesday of each month not appointed for stated meetings, excepting July, August, and September.

enactment a majority of two-thirds of the members present, and at least eighteen affirmative votes.

2. Standing votes may be passed, amended, or rescinded at a stated meeting, or a special meeting called for the purpose by a majority of two-thirds of the members present. They may be suspended by a unanimous vote.

CHAPTER XII.

OF LITERARY PERFORMANCES.

1. The Academy will not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its publications.

candidate for election, unless he shall have received the approval of at least five members of the Council present at a meeting. All nominations thus approved shall be read to the Academy at any meeting, and shall then stand on the nomination list until the next stated meeting, and until the balloting. No person shall be elected a Resident Fellow, unless he shall have been resident in this Commonwealth one year next preceding his election. If any person elected a Resident Fellow shall neglect for one year to pay his admission fee, his election shall be void; and if any Resident Fellow shall neglect to pay his annual assessments for two years, provided that his attention shall have been called to this article, he shall be deemed to have abandoned his Fellowship; but it shall be in the power of the Treasurer, with the consent of the Council, to dispense (*sub silentio*) with the payment both of the admission fee and of the assessments, whenever in any special instance he shall think it advisable so to do.

3. The nomination and election of Associate Fellows shall take place in the manner prescribed in reference to Resident Fellows.

4. The nomination and election of Foreign Honorary Members shall take place in the manner prescribed for Resident Fellows, except that the nomination papers shall be signed by at least seven members of the Council before being presented to the Academy.

5. Three-fourths of the ballots cast must be affirmative, and the number of affirmative ballots must amount to eleven to effect an election of Fellows or Foreign Honorary Members.

6. If, in the opinion of a majority of the entire Council, any Fellow — Resident or Associate — shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his Fellowship; and provided that a majority of two-thirds of the Fellows at a stated meeting, consisting of not less than fifty Fellows, shall adopt this recommendation, his name shall be stricken off the roll of Fellows.

CHAPTER XI.

OF AMENDMENTS OF THE STATUTES.

1. All proposed alterations of the Statutes, or additions to them, shall be referred to a committee, and, on their report at a subsequent stated meeting or a special meeting called for the purpose, shall require for

7. The Librarian shall have custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

CHAPTER IX.

OF MEETINGS.

1. There shall be annually four stated meetings of the Academy; namely, on the second Wednesday in May (the Annual Meeting), on the second Wednesday in October, on the second Wednesday in January, and on the second Wednesday in March. At these meetings, only, or at meetings adjourned from these and regularly notified, or at special meetings called for the purpose, shall appropriations of money be made, or alterations of the statutes or standing votes of the Academy be effected.

Special meetings shall be called by the Recording Secretary at the request of the President or of a Vice-President or of five Fellows. Notifications of the special meetings shall contain a statement of the purpose for which the meeting is called.

2. Fifteen Resident Fellows shall constitute a quorum for the transaction of business at a stated or special meeting. Seven Fellows shall be sufficient to constitute a meeting for scientific communications and discussions.

3. The Recording Secretary shall notify the meetings of the Academy to each Resident Fellow; and he may cause the meetings to be advertised, whenever he deems such further notice to be needful.

CHAPTER X.

OF THE ELECTION OF FELLOWS AND HONORARY MEMBERS.

1. Elections shall be made by ballot, and only at stated meetings.

2. Candidates for election as Resident Fellows must be proposed by two Resident Fellows of the section to which the proposal is made, in a recommendation signed by them; and this recommendation shall be transmitted to the Corresponding Secretary, and by him referred to the Council. No person recommended shall be reported by the Council as a

Committee, and shall report the same at the expiration of his term of office or whenever called on so to do by the Academy or Council.

3. The Treasurer shall keep separate accounts of the income and appropriation of the Rumford Fund and of other special funds, and report the same annually.

4. The Treasurer may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, the Treasurer shall be responsible; or the Treasurer may employ any Trust Company, doing business in Boston, as agent to perform his duties, the compensation of such Assistant Treasurer or agent to be paid from the funds of the Academy.

CHAPTER VIII.

OF THE LIBRARIAN AND LIBRARY.

1. It shall be the duty of the Librarian to take charge of the books, to keep a correct catalogue of them, to provide for the delivery of books from the Library, and to appoint such agents for these purposes as he may think necessary. He shall make an annual report on the condition of the Library.

2. The Librarian, in conjunction with the Committee on the Library, shall have authority to expend such sums as may be appropriated, either from the General, Rumford, or other special Funds of the Academy, for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

3. To all books in the Library procured from the income of the Rumford Fund, or other special funds, the Librarian shall cause a stamp or label to be affixed, expressing the fact that they were so procured.

4. Every person who takes a book from the Library shall give a receipt for the same to the Librarian or his assistant.

5. Every book shall be returned in good order, regard being had to the necessary wear of the book with good usage. If any book shall be lost or injured, the person to whom it stands charged shall replace it by a new volume or set, if it belongs to a set, or pay the current price of the volume or set to the Librarian; and thereupon the remainder of the set, if the volume belonged to a set, shall be delivered to the person so paying for the same.

6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

meeting he shall present the letters which have been addressed to the Academy since the last meeting. Under the direction of the Council, he shall keep a list of the Resident Fellows, Associate Fellows, and Foreign Honorary Members, arranged in their Classes and in Sections in respect to the special sciences in which they are severally proficient; and he shall act as secretary to the Council.

2. The Recording Secretary shall have charge of the Charter and Statute-book, journals, and all literary papers belonging to the Academy. He shall record the proceedings of the Academy at its meetings; and after each meeting is duly opened, he shall read the record of the preceding meeting. He shall notify the meetings of the Academy, apprise officers and committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy. He shall post up in the Hall a list of the persons nominated for election into the Academy; and when any individual is chosen, he shall insert in the record the names of the Fellows by whom he was nominated.

3. The two Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them calculated to promote its interests.

4. Every person taking any books, papers, or documents belonging to the Academy and in the custody of the Recording Secretary, shall give a receipt for the same to the Recording Secretary.

CHAPTER VII.

Of the Treasurer.

1. The Treasurer shall give such security for the trust reposed in him as the Academy shall require.

2. He shall receive all moneys due or payable to the Academy and all bequests and donations made to the Academy. He shall pay all bills due by the Academy, when approved by the proper officers (except those of the Treasurer's office, which may be paid without such approval). He shall sign all leases of real estate in the name of the Academy. All transfers of stocks, bonds, and other securities belonging to the Academy shall be made by the Treasurer with the written consent of one member of the Committee of Finance. He shall keep an account of all receipts and expenditures, shall submit his accounts annually to the Auditing

and proper execution of the trust. All bills incurred on account of the C. M. Warren Fund, within the limits of the appropriations made by the Academy, shall be approved by the Chairman of the C. M. Warren Committee.

5. The Committee on Publication, to consist of three Fellows, one from each class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Proceedings and Memoirs shall be entrusted. This Committee shall report to the Council in March of each year on the appropriations needed for the coming year. All bills incurred on account of publications, within the limits of the appropriations made by the Academy, shall be approved by the Chairman of the Committee on Publication.

6. The Committee on the Library, to consist of the Librarian *ex officio*, and three other Fellows, one from each class, who shall examine the Library and make an annual report on its condition and management. This Committee, through the Librarian, shall report to the Council in March of each year, on the appropriations needed for the Library for the coming year. All bills incurred on account of the Library, within the limits of the appropriations made by the Academy, shall be approved by the Librarian.

7. The President and Recording Secretary shall be a Committee on the general expenditures of the Academy. This Committee shall report to the Council in March of each year on the appropriations needed for the general expenditures for the coming year, and either member of the Committee may approve bills incurred on this account within the limits of the appropriations made by the Academy.

8. An auditing Committee, to consist of two Fellows, for auditing the accounts of the Treasurer, with power to employ an expert and to approve his bill.

9. In the absence of the Chairman of any Committee, bills may be approved by a member of the Committee designated by the Chairman for the purpose.

CHAPTER VI.

OF THE SECRETARIES.

1. The Corresponding Secretary shall conduct the correspondence of the Academy, recording or making an entry of all letters written in its name, and preserving on file all letters which are received; and at each

CHAPTER IV.

OF THE PRESIDENT.

1. It shall be the duty of the President, and, in his absence, of the senior Vice-President present, or next officer in order as above enumerated, to preside at the meetings of the Academy; to direct the Recording Secretary to call special meetings; and to execute or to see to the execution of the Statutes of the Academy. Length of continuous membership in the Academy shall determine the seniority of the Vice-Presidents.

2. The President, or, in his absence, the next officer as above enumerated, shall nominate members to serve on the different committees of the Academy which are not chosen by ballot.

3. Any deed or writing to which the common seal is to be affixed shall be signed and sealed by the President, when thereto authorized by the Academy.

CHAPTER V.

OF STANDING COMMITTEES.

1. At the Annual Meeting there shall be chosen the following Standing Committees, to serve for the year ensuing, viz.: —

2. The Committee on Finance to consist of three Fellows to be chosen by ballot, who shall have, through the Treasurer, full control and management of the funds and trusts of the Academy, with the power of investing and of changing the investment of the same at their discretion.

3. The Rumford Committee, to consist of seven Fellows to be chosen by ballot, who shall consider and report to the Academy on all applications and claims for the Rumford premium. They shall also report to the Council in March of each year on all appropriations of the income of the Rumford Fund needed for the coming year, and shall generally see to the due and proper execution of the trust. All bills incurred on account of the Rumford Fund, within the limits of the appropriation made by the Academy, shall be approved by the Chairman of the Rumford Committee.

4. The C. M. Warren Committee, to consist of seven Fellows to be chosen by ballot, who shall consider and report to the Council in March of each year on all applications for appropriations from the income of the C. M. Warren Fund for the coming year, and shall generally see to the due

ing Secretary, and the Treasurer, proposing the appropriations for their work during the year beginning the following May. The Treasurer at the same meeting shall report on the income which will probably be received on account of the various Funds during the same year.

At the Annual Meeting, the Council shall submit to the Academy, for its action, a report recommending the appropriations which in the opinion of the Council should be made for the various purposes of the Academy.

4. If any office shall become vacant during the year, the vacancy shall be filled by a new election, at the next stated meeting, or at a meeting called for this purpose.

CHAPTER III.

OF NOMINATIONS OF OFFICERS.

1. At the stated meeting in March, the President shall appoint a Nominating Committee of three Resident Fellows, one for each Class.

2. It shall be the duty of this Nominating Committee to prepare a list of candidates for the offices of President, Vice-Presidents, Corresponding Secretary, Recording Secretary, Treasurer, Librarian, Councillors, and the Standing Committees which are chosen by ballot; and to cause this list to be sent by mail to all the Resident Fellows of the Academy not later than four weeks before the Annual Meeting.

3. Independent nominations for any office, signed by at least five Resident Fellows, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call for the Annual Meeting, which shall then be issued not later than one week before that meeting.

4. The Recording Secretary shall prepare for use, in voting at the Annual Meeting, a ballot containing the names of all persons nominated for office under the conditions given above.

5. When an office is to be filled at any other time than at the Annual Meeting, the President shall appoint a Nominating Committee in accordance with the provisions of Section 1, which shall announce its nomination in the manner prescribed in Section 2 at least two weeks before the time of election. Independent nominations, signed by at least five Resident Fellows and received by the Recording Secretary not later than one week before the meeting for election, shall be inserted in the call for that meeting.

the Commonwealth. Each Resident Fellow shall pay an admission fee of ten dollars and such annual assessment, not exceeding ten dollars, as shall be voted by the Academy at each annual meeting. Resident Fellows only may vote at the meetings of the Academy.

3. The number of Associate Fellows shall not exceed one hundred, of whom there shall not be more than forty in either of the three classes of the Academy. Associate Fellows shall be chosen from persons residing outside of the Commonwealth of Massachusetts. They shall not be liable to the payment of any fees or annual dues, but on removing within the Commonwealth they may be transferred by the Council to resident fellowship as vacancies there occur.

4. The number of Foreign Honorary Members shall not exceed seventy-five; and they shall be chosen from among persons most eminent in foreign countries for their discoveries and attainments in either of the three departments of knowledge above enumerated. There shall not be more than thirty Foreign Members in either of these departments.

CHAPTER II.

OF OFFICERS.

1. There shall be a President, three Vice-Presidents, one for each Class, a Corresponding Secretary, a Recording Secretary, a Treasurer, and a Librarian, which officers shall be annually elected, by ballot, at the annual meeting, on the second Wednesday in May.

2. There shall be nine Councillors, chosen from the Resident Fellows. At each annual meeting, three Councillors shall be chosen, by ballot, one from each Class, to serve for three years; but the same Fellow shall not be eligible for two successive terms. The nine Councillors, with the President, the three Vice-Presidents, the two Secretaries, the Treasurer, and the Librarian, shall constitute the Council. Five members shall constitute a quorum. It shall be the duty of this Council to exercise a discreet supervision over all nominations and elections. With the consent of the Fellow interested, they shall have power to make transfers between the several sections of the same Class, reporting their action to the Academy.

3. The Council shall at its March Meeting receive reports from the Rumford Committee, the C. M. Warren Committee, the Committee on Publication, the Committee on the Library, the President and Record-

STATUTES AND STANDING VOTES.

STATUTES.

Adopted May 30, 1854: *amended September* 8, 1857, *November* 12, 1862, *May* 24, 1864, *November* 9, 1870, *May* 27, 1873, *January* 26, 1876, *June* 16, 1886, *October* 8, 1890, *January* 11, *and May* 10, 1893, *May* 9, *and October* 10, 1894, *March* 13, *April* 10, *and May* 8, 1895, *May* 8, 1901, *January* 8, 1902, *May* 10, 1905, *February* 14 *and March* 14, 1906.

CHAPTER I.

OF FELLOWS AND FOREIGN HONORARY MEMBERS.

1. The Academy consists of Resident Fellows, Associate Fellows, and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, viz.: Class I. The Mathematical and Physical Sciences; — Class II. The Natural and Physiological Sciences; — Class III. The Moral and Political Sciences. Each Class is divided into four Sections, viz.: Class I., Section 1. Mathematics and Astronomy; — Section 2. Physics; — Section 3. Chemistry; — Section 4. Technology and Engineering. Class II., Section 1. Geology, Mineralogy, and Physics of the Globe; — Section 2. Botany; Section 3. Zoölogy and Physiology; — Section 4. Medicine and Surgery. Class III., Section 1. Theology, Philosophy, and Jurisprudence; — Section 2. Philology and Archæology; — Section 3. Political Economy and History; — Section 4. Literature and the Fine Arts.

2. The number of Resident Fellows residing in the Commonwealth of Massachusetts shall not exceed two hundred, of whom there shall not be more than eighty in any one of the three classes. Only residents in the Commonwealth of Massachusetts shall be eligible to election as Resident Fellows, but resident fellowship may be retained after removal from

SECTION III. — 5.

Zoölogy and Physiology.

Ludimar Hermann,	Königsberg.
H. Kronecker,	Bern.
E. Ray Lankester,	London.
Elias Metschnikoff,	Paris.
M. Gustav Retzius,	Stockholm.

SECTION IV. — 7.

Medicine and Surgery.

Emil von Behring,	Marburg.
Sir T. L. Brunton,	London.
A. Celli,	Rome.
Sir V. A. H. Horsley,	London.
R. Koch,	Berlin.
Lord Lister,	London.
F. v. Recklinghausen,	Strassburg.

CLASS III. — *Moral and Political Sciences.* — 23.

SECTION I. — 5.

Philosophy and Jurisprudence.

A. J. Balfour,	Prestonkirk.
Heinrich Brunner,	Berlin.
A. V. Dicey,	Oxford.
F. W. Maitland,	Cambridge.
Sir Frederick Pollock, Bart.,	London.

SECTION II. — 7.

Philology and Archæology.

Ingram Bywater,	Oxford.
F. Delitzsch,	Berlin.
Hermann Diels,	Berlin.
W. Dörpfeld,	Athens.
Sir John Evans,	Berkhampsted.
H. Jackson,	Cambridge.
G. C. C. Maspero,	Paris.

SECTION III. — 5.

Political Economy and History.

James Bryce,	London.
Adolf Harnack,	Berlin.
Sir G. O. Trevelyan, Bart.,	London.
John Morley,	London.
Pasquale Villari,	Florence.

SECTION IV. — 6.

Literature and the Fine Arts.

E. de Amicis,	Florence.
Gaston Boissier,	Paris.
Georg Brandes,	Copenhagen.
S. H. Butcher,	London.
Jean Léon Gérôme,	Paris.
Rudyard Kipling,	Burwash.

FOREIGN HONORARY MEMBERS.—68.

(Number limited to seventy-five. Elected as vacancies occur.)

CLASS I.—*Mathematical and Physical Sciences.*—22.

SECTION I.—7.

Mathematics and Astronomy.

Arthur Auwers,	Berlin.
George H. Darwin,	Cambridge.
Sir William Huggins,	London.
Felix Klein,	Göttingen.
Émile Picard,	Paris.
H. Poincaré,	Paris.
H. C. Vogel,	Potsdam.

SECTION II.—5.

Physics.

Oliver Heaviside,	Newton Abbot.
F. Kohlrausch,	Marburg.
Joseph Larmor,	Cambridge.
Lord Rayleigh,	Witham.
Joseph J. Thomson,	Cambridge.

SECTION III.—5.

Chemistry.

Adolf Ritter von Baeyer,	Munich.
J. H. van't Hoff,	Berlin.
Wilhelm Ostwald,	Leipsic.
Sir H. E. Roscoe,	London.
Julius Thomsen,	Copenhagen.

SECTION IV.—5.

Technology and Engineering.

Sir Benjamin Baker,	London.
Lord Kelvin,	Largs.
Maurice Lévy,	Paris.
H. Müller-Breslau,	Berlin.
W. Cawthorne Unwin,	London.

CLASS II.—*Natural and Physiological Sciences.*—23.

SECTION I.—5.

Geology, Mineralogy, and Physics of the Globe.

Sir Archibald Geikie,	London.
Julius Hann,	Vienna.
Albert Heim,	Zurich.
Sir John Murray,	Edinburgh.
Henry C. Sorby,	Sheffield.

SECTION II.—6.

Botany.

E. Bornet,	Paris.
A. Engler,	Berlin.
Sir Joseph D. Hooker,	Sunningdale.
W. Pfeffer,	Leipsic.
H. Graf zu Solms-Laubach,	Strassburg.
Eduard Strasburger,	Bonn.

Section II.—6.

Botany.

L. H. Bailey,	Ithaca, N. Y.
D. H. Campbell,	Palo Alto, Cal.
J. M. Coulter,	Chicago.
C. G. Pringle,	Charlotte, Vt.
John D. Smith,	Baltimore.
W. Trelease,	St. Louis.

Section III.—9.

Zoölogy and Physiology.

Joel A. Allen,	New York.
W. K. Brooks,	Lake Roland, Md.
C. B. Davenport,	Cold Spring Harbor, N. Y.
F. P. Mall,	Baltimore.
S. Weir Mitchell,	Philadelphia.
H. F. Osborn,	New York.
A. E. Verrill,	New Haven.
C. O. Whitman,	Chicago.
E. B. Wilson,	New York.

Section IV.—8.

Medicine and Surgery.

John S. Billings,	New York.
W. S. Halsted,	Baltimore.
Abraham Jacobi,	New York.
W. W. Keen,	Philadelphia.
William Osler,	Baltimore.
T. Mitchell Prudden,	New York.
Wm. H. Welch,	Baltimore.
H. C. Wood,	Philadelphia.

Class III.—*Moral and Political Sciences.*—27.

Section I.—6.

Philosophy and Jurisprudence.

Joseph H. Choate,	New York.
Melville W. Fuller,	Washington.
William W. Howe,	New Orleans.
Charles S. Peirce,	Milford, Pa.
G. W. Pepper,	Philadelphia.
T. R. Pynchon,	Hartford, Conn.

Section II.—7.

Philology and Archæology.

Timothy Dwight,	New Haven.
B. L. Gildersleeve,	Baltimore.
D. C. Gilman,	Baltimore.
T. R. Lounsbury,	New Haven.
Rufus B. Richardson,	New York.
Thomas D. Seymour,	New Haven.
A. D. White,	Ithaca, N.Y.

Section III.—7.

Political Economy and History.

Henry Adams,	Washington.
G. P. Fisher,	New Haven.
Arthur T. Hadley,	New Haven.
Henry C. Lea,	Philadelphia.
Alfred T. Mahan,	New York.
H. Morse Stephens,	Ithaca.
W. G. Sumner,	New Haven.

Section IV.—7.

Literature and the Fine Arts.

James B. Angell,	Ann Arbor, Mich.
H. H. Furness,	Wallingford, Pa.
R. S. Greenough,	Florence.
Herbert Putnam,	Washington.
Augustus St. Gaudens,	Windsor, Vt.
John S. Sargent,	London.
E. C. Stedman,	Bronxville, N. Y.

ASSOCIATE FELLOWS. — 97.

(Number limited to one hundred. Elected as vacancies occur.)

Class I. — *Mathematical and Physical Sciences.* — 37.

Section I. — 14.

Mathematics and Astronomy.

Edward E. Barnard,	Williams Bay, Wis.
S. W. Burnham,	Williams Bay, Wis.
George Davidson,	San Francisco.
Fabian Franklin,	Baltimore.
Asaph Hall,	S. Norfolk, Conn.
George W. Hill,	W. Nyack, N.Y.
E. S. Holden,	New York.
Emory McClintock,	Morristown, N.J.
E. H. Moore,	Chicago.
Simon Newcomb,	Washington.
Charles L. Poor,	New York.
George M. Searle,	Washington.
J. N. Stockwell,	Cleveland, O.
Chas. A. Young,	Hanover, N. H.

Section II. — 6.

Physics.

Carl Barus,	Providence, R.I.
G. E. Hale,	Williams Bay, Wis.
T. C. Mendenhall,	Worcester.
A. A. Michelson,	Chicago.
E. L. Nichols,	Ithaca, N. Y.
M. I. Pupin,	New York.

Section III. — 9.

Chemistry.

Wolcott Gibbs,	Newport, R.I.
Frank A. Gooch,	New Haven.
Eugene W. Hilgard,	Berkeley, Cal.
S. W. Johnson,	New Haven.
J. W. Mallet,	Charlottesville, Va.
E. W. Morley,	Cleveland, O.
Charles E. Munroe,	Washington.
J. M. Ordway,	New Orleans.
Ira Remsen,	Baltimore.

Section IV. — 8.

Technology and Engineering.

Henry L. Abbot,	Cambridge.
Cyrus B. Comstock,	New York. [Va.
W. P. Craighill,	Charlestown, W.
John Fritz,	Bethlehem, Pa.
James D. Hague,	New York.
F. R. Hutton,	New York.
William Sellers,	Edge Moor, Del.
Robt. S. Woodward,	Washington.

Class II. — *Natural and Physiological Sciences.* — 32.

Section I. — 10.

Geology, Mineralogy, and Physics of the Globe.

Cleveland Abbe,	Washington.
George J. Brush,	New Haven.
T. C. Chamberlin,	Chicago.
Edward S. Dana,	New Haven.
Walter G. Davis,	Cordova, Arg.
Samuel F. Emmons,	Washington.
G. K. Gilbert,	Washington.
R. Pumpelly,	Newport, R.I.
Israel C. Russell,	Ann Arbor.
Charles D. Walcott,	Washington.

"Description of Bermuda Hydroids." By Edgar Davidson Congdon. Presented by E. L. Mark.

"The Transmission of Röntgen Rays through Metallic Sheets." By John Mead Adams. Presented by John Trowbridge.

Nine hundred and sixty-ninth Meeting.

December 12, 1906.

The Academy met at its House.

The President in the chair.

There were present twenty-two Fellows.

The Corresponding Secretary read a letter from Mr. Henry H. Edes, presenting to the Academy a photographic copy of the certificate of Membership in the Academy, which was given in 1781, to Benjamin Franklin.

On motion of the Corresponding Secretary, it was

Voted, That the thanks of the Academy be tendered to Mr. Edes for his gift.

The Chair announced the following deaths: —

Samuel Cabot, Resident Fellow in Class I, Section 3; F. Brunetière, Foreign Honorary Member in Class III, Section 4.

A proposition from the Council for the establishment of an emeritus list of Resident Fellows was read; this proposition, requiring a change in the statutes, was referred to a committee consisting of Charles P. Bowditch, A. Lawrence Rotch, William Watson.

The following recommendation from the Council was read by the Corresponding Secretary: —

That an appropriation of five hundred dollars ($500) be made from the income of the Publication Fund, for the use of the Publishing Committee for the remainder of the year.

The following communication was given: —

"The Surface Temperature of Mars (from a New Determination of the Solar Heat received by it)." By Professor Percival Lowell.

The following papers were read by title: —

"An Electric Wax-Cutter for Use in Reconstruction." By E. L. Mark.

"On the Magnetic Behavior of the Finely Divided Core of an Electro-magnet or of a Transformer while a Steady Current is being established in the Exciting Coil." By B. O. Peirce.

"The Sensation of Loudness." By W. C. Sabine.

Nine hundred and seventieth Meeting.

JANUARY 9, 1907. — STATED MEETING.

The Academy met at its House.

The PRESIDENT in the chair.

There were present twenty-five Fellows.

On the recommendation of the Council, at the request of the Chairman of the Publication Committee, and with the approval of the Treasurer, it was

Voted, To appropriate the sum of five hundred dollars ($500) from the income of the Publication Fund for the use of the Publication Committee for the remainder of the year.

On motion of the Chairman of the Rumford Committee, it was

Voted, To appropriate the sum of three hundred and seventy-five dollars ($375) from the income of the Rumford Fund for the use of the Rumford Committee in the furtherance of research.

The report of the Committee on Amendments to the Statutes was read and accepted, and, in accordance therewith, it was

Voted, To amend the Statutes as follows: —

Chapter I, Section 2, line 1. After the word "Fellows" add "residing in the Commonwealth of Massachusetts."

This amendment was passed by a vote of three-fourths of the members present, there being eighteen (18) affirmative votes.

The following communications were given: —

"The Meteorology of the North and South Polar Zones." By R. DeC. Ward.

"The Corpuscular Theory as applied to Electric Conduction in Gases." By Theodore Lyman.

The following papers were read by title: —

"On Linear Differential Equations of the Parabolic Type." By M. M. Smith. Presented by Maxime Bôcher.

"The Spermatogenesis of the Myriapods. V. On the Spermatocytes of Lithobius." By Maulsby W. Blackman. Presented by E. L. Mark.

"Concerning the Adiabatic Determination of the Heats of Combustion of Organic Compounds, especially Sugar and Benzol." By T. W. Richards, L. J. Henderson, and H. L. Frevert.

"Concerning Position Isomerism and Heats of Combustion." By L. J. Henderson. Presented by T. W. Richards.

"The Determination of Small Amounts of Antimony by the Berzelius-Marsh Process." By C. R. Sanger and James A. Gibson.

"The Determination of Arsenic in the Urine." By C. R. Sanger and Otis F. Black.

"The Determination of Arsenic by the Guthzeit Method." By C. R. Sanger and Otis F. Black.

Nine hundred and seventy-first Meeting.

FEBRUARY 13, 1907.

The Academy met at the Walker Building of the Massachusetts Institute of Technology.

The PRESIDENT in the chair.

There were present twenty-five Fellows and eighteen guests.

The following letters were read: From the Société Imperiale russe de Géographie, on the occasion of the eightieth anniversary of M. Pierre de Séménov; from E. van Everdingen, announcing his appointment as Director of the Netherlands Meteorological Institute; from the Académie de Macon, enclosing the medal of its centennial; from the Seventh International Zoölogical Congress, inviting the Academy to be represented by delegates at its meeting in Boston, August 19 to 23, 1907.

The President announced that the Rumford Medal had been awarded to Professor Ernest Fox Nichols for his researches on radiation, particularly on the pressure due to radiation, the heat of the stars, and the infra-red spectrum. He then called upon Professor Charles R. Cross, Chairman of the Rumford Committee, to state the grounds on which the award was made.

Professor Cross gave a brief account of Professor Nichols' researches, after which the President, in the name of the Academy, presented the medal to Professor Nichols, who made an appropriate reply expressing his appreciation of the high honor which had been conferred upon him.

Professor Nichols then showed experiments demonstrating the pressure of rays of light.

Remarks were made on this subject by Professor Webster, who concluded with the motion that a vote of thanks be given to Professor Nichols for his admirable experimental demonstrations.

The following papers were presented by title: —

"The Kathode-luminescence of Fluorite." By Harry W. Morse. Presented by John Trowbridge.

"On the Self and Mutual Inductance of Cylindrical Solenoids." By A. G. Webster.

Contributions from the Gray Herbarium of Harvard University. New Series. — No. XXXIV. I. "New Species of Senecis and Schoenocaulon from Mexico." By J. M. Greenman. II. "New or otherwise noteworthy Spermatophytes, chiefly from Mexico." By B. L. Robinson. III. "New Plants from Guatamala collected by C. C. Deam." By B. L. Robinson and H. H. Bartlett. IV. "Diagnoses of New Spermatophytes from Mexico." By M. L. Fernald. Presented by B. L. Robinson.

Nine hundred and seventy-second Meeting.

March 13, 1907. — Stated Meeting.

The Academy met at its House.

The President in the chair.

There were present twenty-one Fellows.

Letters were read from Henry S. Pritchett, Henry G. Denny, David W. Cheever, and Jeremiah Smith, resigning Fellowship; from the Michigan Agricultural College, inviting the Academy to be represented by delegates at its fiftieth anniversary, May

28–31, 1907; from the Smithsonian Institution, informing the Academy of the election of Charles Doolittle Walcott as its Secretary; from G. Capellini, inviting the Academy to participate in the celebration of the three hundredth anniversary of the death of Ulisse Aldrovandi, June 12, 1907, at Bologna; also, printed communications regarding the adoption of an international language.

On motion of the Recording Secretary, it was unanimously

Voted, To invite Dr. William Everett, Professor Amos E. Dolbear, and Mr. Henry G. Denny to attend the meetings of the Academy, and to make use of the library.

The Chair announced the following death: —

Dimitri Mendeleeff, Foreign Honorary Member in Class I, Section 3.

The following gentlemen were elected Members of the Academy: —

William Zebina Ripley, of Newton, as Resident Fellow in Class III, Section 3 (Political Economy and History).

Edward Henry Hall, of Cambridge, as Resident Fellow in Class III, Section 4 (Literature and the Fine Arts).

William Coolidge Lane, of Cambridge, as Resident Fellow in Class III, Section 4 (Literature and the Fine Arts).

Magnus Gustav Retzius, of Stockholm, as Foreign Honorary Member in Class II, Section 3 (Zoölogy and Physiology), in place of the late Albert von Kölliker.

Hermann Diels, of Berlin, as Foreign Honorary Member in Class III, Section 2 (Philology and Archaeology), in place of the late J. W. A. Kirchhoff.

The President appointed the following Councillors to act as Nominating Committee: —

William T. Sedgwick, of Class II.
Elihu Thomson, of Class I.
Andrew McF. Davis, of Class III.

The following Communications were given: —

Professor S. P. Sharples, "Cobalt and Silver Deposits of Cobalt, Canada."

Professor D. W. Johnson, "The Volcanic Necks of the Mt. Taylor Region, New Mexico" (by invitation).

The following papers were read by title: —

"The Process of building up the Voltage and Current in a Long Alternating-Current Circuit." By A. E. Kennelly.

"High Electromotive Force with its Application to Spectrum Analysis." By John Trowbridge.

Nine hundred and seventy-third Meeting.

APRIL 10, 1907.

The PRESIDENT in the chair.

There were present sixteen Fellows.

The Corresponding Secretary read the following letters: From Edward H. Hall, William C. Lane, and William Z. Ripley, acknowledging election as Resident Fellows; from Hermann Diels, acknowledging election as Foreign Honorary Member; from Amos E. Dolbear and Henry G. Denny, thanking the Academy for its invitation to attend the meetings and to use the library; from the Geological Society of London inviting the Academy to appoint a delegate to attend its centennial celebration, September 26, 27, and 28, 1907; from the Comité Géologique de la Russie, announcing the death of M. Nicholas Sokolow; from Columbia University in the City of New York, announcing the Loubat prizes to be awarded in 1908.

The Chair announced the following death: —

Marcellin Berthelot, Foreign Honorary Member in Class I, Section 3.

The following delegates were appointed by the chair: —

Professor Ephraim Emerton (Professor Angelo Celli of Rome, alternate), to attend the celebration at Bologna of the three hundredth anniversary of the death of Ulisse Aldrovandi, June 12, 1907; President James Burrill Angell, to attend the celebration at Lansing, Michigan, of the fiftieth anniversary of the Michigan Agricultural College, May 28 to 31, 1907; Professor Edward Laurens Mark, to attend the Seventh International Zoölogical Congress at Boston, August 19 to 23, 1907.

The following communications were given: —

"On the Physiological Basis of Illumination." By Dr. Louis Bell.

"Meteorological Observations in 1906 above the Tropical and Equatorial Atlantic." By Professor A. Lawrence Rotch.

The following paper was presented by title: —

"The Demagnetizing Factors for Cylindrical Magnets of Different Dimensions." By C. L. B. Shuddemagen. Presented by B. O. Peirce.

Nine hundred and seventy-fourth Meeting.

MAY 8, 1907. — ANNUAL MEETING.

The PRESIDENT in the chair.

There were present twenty-two Fellows.

Letters were read from E. F. Sawyer and J. S. Kingsley, resigning Fellowship; from Gustav Retzius, acknowledging election as Foreign Honorary Member; from the New York Academy of Sciences, inviting the Academy to be represented at the celebration of the two hundredth anniversary of the birth of Carl von Linné, and also to contribute an official document appreciative of the work of Linné.

On motion of the Recording Secretary it was

Voted, That the appointment of Delegates to the celebrations of the Geological Society of London and the New York Academy of Sciences be intrusted to the President.

The Chair announced the death of Sir Michael Foster, Foreign Honorary Member in Class II, Section 3.

The annual report of the Council was read.[1]

The annual report of the Treasurer was read, of which the following is an abstract: —

GENERAL FUND.

	Receipts.	
Investments	$1,683.01	
Assessments	1,880.00	
Admission fees	40.00	
Rent of offices	1,200.00	$4,803.01
	Expenditures.	
General expenses	$3,603.47	
Library	1,199.54	$4,803.01

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[1] See p. 761.

RUMFORD FUND.

Receipts.

Balance, April 30, 1906	$ 451.23	
Investments, etc.	2,614.75	$3,065.98

Expenditures.

Research	$1,490.00	
Publication	788.54	
Library	149.35	
Charged to reduce book value of stocks	451.23	$2,879.12
Balance, April 30, 1907		186.86
		$3,605.98

C. M. WARREN FUND.

Receipts.

Balance, April 30, 1906	$ 940.06	
Investments	347.74	$1,287.80

Expenditures.

Research	$ 450.00	
Vault rent	4.00	
Premium on bonds charged off	50.00	
Accrued interest on bonds bought	20.83	$ 524.83
Balance, April 30, 1907		762.97
		$1,287.80

PUBLICATION FUND.

Receipts.

Balance, April 30, 1906	$ 961.68	
Investments	2,854.50	
Sale of publications	130.35	
Authors	179.23	$4,125.76

Expenditures.

Publication	$3,275.42	
Charged to reduce book value of investments	625.00	
Vault rent	12.50	$3,912.92
Balance, April 30, 1907		212.84
		$4,125.76

The following reports were also presented: —

REPORT OF THE LIBRARIAN.

During the past year the Assistant Librarian and Miss Wyman, as cataloguer, have been employed in verifying and continuing the type-written catalogue, begun by Dr. Holden, and also in classifying the books according to the classification adapted by him to this library. The subjects, mathematics, astronomy, physics, light, heat, electricity, chemistry, the useful arts (including engineering), natural history, geology, mineralogy, and about one-half of meteorology, are entirely catalogued and classified. This includes all serial publications relating to the subjects mentioned. There remains to be done, paleontology, botany, zoölogy, agriculture, medicine, the moral and political sciences, and the books and serial publications on general science. As Miss Wyman can only give two or three hours a day during the coming year, the catalogue will probably not be completed for some months, but the thoroughness with which the work has been done compensates for the length of time taken.

It is very much to be desired that each member of the Academy should present to the library a copy of any scientific work of which he is the author. The books on the various subjects were, as a rule, well chosen in the past, and form what might be called the nucleus to a valuable library, but some years ago, through lack of funds, it was decided to purchase no more books from the General Fund; therefore donations are much needed.

The accessions during the year have been as follows: —

	Vols.	Parts of Vols.	Pams.	Maps.	Total.
By gift and exchange . . .	144	2396	85	8	2633
By purchase — General Fund	3	690			693
By purchase — Rumford Fund	1	336			337
Total	148	3422	85	8	3663

61 books have been borrowed from the library by 18 persons, including 10 Fellows. The library has been consulted many times when no books were taken.

All the books borrowed during the year have been returned for the annual examination. Of the books reported as still out a year ago, all have been returned except 5 (taken out by one person).

The expenses charged to the library are as follows: Miscellaneous, $400 (which includes $206.25 for cataloguing); Binding, $417.65, General, and $44.70, Rumford, Funds; Subscriptions, $381.89, General, and $104.65, Rumford, Funds; making a total of $799.54 for the General, and $149.35 for the Rumford, Funds as the cost of binding and subscriptions.

A. Lawrence Rotch, *Librarian.*

May 8, 1907.

Report of the Rumford Committee.

During the past year the Committee has made the following grants in aid of research, from the amount placed at its disposal by the Academy for that purpose.

Oct. 10, 1906. Professor Arthur A. Noyes, of the Massachusetts Institute of Technology, for the construction of a calorimeter for the determination of heats of reaction at high temperatures . . .	$300
Professor R. W. Wood, of the Johns Hopkins University, for the purchase of quartz lamps (additional appropriation) .	200
Nov. 14, 1906. Professor Norton A. Kent, of Boston University, for the continuation of work on spectral lines (additional appropriation)	75
Professor L. R. Ingersoll, of the University of Wisconsin, for an investigation of the Kerr Effect in the infra red rays . . .	200
Professor Frederick E. Kester, of the Ohio State University, for a research on the thermal properties of gases flowing through porous plugs (additional appropriation)	315
March 13, 1907. Dr. Harry W. Morse, of Harvard University, for his research on fluorescence (additional appropriation) .	400

Since the last Annual Meeting papers as follows have been published at the expense of the Rumford Fund. Two additional papers are now in press.

"Dispersion in Electric Double Refraction." H. L. Blackwell, May, 1906.

"The Thermal Conductivity of Lead." F. L. Bishop, May, 1906.

"Fluorescence and Magnetic Rotation Spectra of Sodium Vapor, and their Analysis." R. W. Wood, November, 1906.

"Expansion and Compressibility of Ether and of Alcohol in the Neighborhood of their Boiling Points." A. W. Smith, January, 1907.

"Concerning the Adiabatic Determination of the Heats of Combustion of Organic Substances, especially Sugar and Benzol." T. W. Richards and Messrs. Henderson and Frevert. March, 1907.

"On the Thomson Effect and the Temperature Coefficient of Thermal Conductivity in Soft Iron between 115° and 204° C." E. H. Hall and Messrs. Campbell, Serviss, and Churchill. March, 1907.

"Concerning Position Isomerism and Heats of Combustion." L. J. Henderson. March, 1907.

"Temperature of Mars. A Determination of the Solar Heat Received." P. Lowell. March, 1907.

"The Transmission of Röntgen Rays through Metallic Sheets." J. M. Adams. April, 1907.

Reports of progress in their respective researches have been received from Messrs. A. L. Clark, J. A. Dunne, E. B. Frost, G. E. Hale, W. J. Humphreys, L. R. Ingersoll, N. A. Kent, F. E. Kester, A. B. Lamb, C. E. Mendenhall, R. S. Minor, H. W. Morse, A. A. Noyes, J. A. Parkhurst, T. W. Richards, F. A. Saunders, C. B. Thwing, J. Trowbridge, R. W. Wood.

At its meeting of January 9, 1907, the Committee voted for the first time, and at its meeting of February 13, 1907, for the second time, to recommend to the Academy that the Rumford Premium be awarded to Edward Goodrich Acheson for the Application of Heat in the Electric Furnace to the Industrial Production of Carborundum, Graphite, and other New and Useful Substances.

Charles R. Cross, *Chairman.*

May 8, 1907.

Professor Cross continued as follows: —

Mr. President and Gentlemen of the Academy, — In presenting the claims of Mr. Acheson for the Rumford Premium, in view of the character of the work upon which the recommendation of the Committee is based, instead of making an independent statement of my own, I will read a report upon that work which was presented to the Committee by one of its members, than whom there is none more competent to pronounce an opinion upon any subject concerning applied electricity. I hardly need to say that I refer to Professor Elihu Thomson.

The report is as follows:

"The early experimenters with the carbon arc from voltaic batteries found that the temperature developed transcended that of all other sources of artificial heat, and numerous substances subjected thereto were found to undergo profound changes. It may suffice to quote from Silliman's Physics, a text-book in common use some forty years ago, page 589, paragraph 886 of the edition of 1870, 'Heat of the Voltaic Arch Deflagration': 'Where the positive electrode is fashioned into a small crucible of carbon, gold, silver, platinum, mercury, and other substances are speedily fused, deflagrated, or volatilized, with various colored lights. The fusion of platinum (like wax in a candle) before the voltaic arch is significant of its intense heat, and still more, the volatilization and fusion of carbon, a result first announced by Professor Silliman in 1822 and since confirmed by Despretz, who by the union of the heat of six hundred carbon couples arranged in numerous parallel series, and conjoined with the jet of an oxyhydrogen blowpipe, and the heat of the midday sun, focalized by a powerful burning glass, succeeded in volatilizing the diamond, fusing magnesia and silica, and softening anthracite. The diamond is also softened, and converted into a black spongy mass resembling coke, or, more nearly, the black diamond found in the Brazilian mines.'

"The fusion of carbon referred to is evidently a mistake, but the softening of anthracite probably resulted in its conversion into graphite.

"Upon the advent of the dynamo during the decade of 1870 to 1880, the ability to obtain and maintain electric arcs with comparative ease made it possible to apply electric heat to larger crucibles and to note again the facile conversion of impure varieties of carbon into graphite. The first attempts to apply electric heating on a fairly large scale in obtaining new products was probably made by Cowles in 1886, using a mixture of carbon, aluminum oxide, and copper, whereby there was formed aluminum bronze. The operation involved the direct reduction of alumina by carbon, but if this were attempted without the presence of the alloying metal copper, a carbide of aluminum was formed and no metal. The operation was rather irregular and uncertain, but stands as a worthy effort to apply electricity to smelting. The heat generated in the mass of materials undergoing the operation was due to resistance of innumerable contacts throughout the granular mass, to the resistance of the materials themselves, and possibly to incipient arcing between particles.

"Mr. Edward G. Acheson, who had assisted in the Edison Menlo Park laboratory in 1880 and 1881, and who up to 1889 and later was connected with electric work as engineer, superintendent, or electrician, began to work with an electric furnace in 1891. In attempting to 'impregnate clay with carbon under the influence of the high heat obtainable with the electric current' he noted the formation of a few bright specks. After having separated out one of these specks, he drew it across a pane of glass, which it scratched and cut. This led to further experiments, and a small vial of the new product was taken to New York under the name 'carborundum,' as Mr. Acheson was laboring under the mistake that it contained carbon and corundum. It was tried in cutting diamond, with the result that the diamond cutter bought the material at thirty cents per carat. This was the first sale of the new abrasive. On finding that it was the silica in the clay which had reacted with the carbon, sand was substituted for the clay before used. Mr. Acheson then planned a furnace substantially the same as that now used in the manufacture at Niagara, — a central core of carbon pieces resting on the floor of the furnace and connecting heavy carbon terminals or electrodes, around which core was packed the mixed materials to be heated, which in the case of the production of carborundum are sand, coke-powder, and a little common salt.

"Meeting with troubles and trials, the usual lot of pioneers, and having his efforts at last crowned with success, the carborundum manufacture has become a large industry. In 1894 a steam generating plant of less than 150 horse power was used, and the cost of production greatly restricted the applications of the product. Only one half the product found a market. It was then that, in spite of the opposition of the directors of his company, Mr. Acheson insisted on removing to Niagara, so as to manufacture more cheaply with large amounts of relatively cheap electric energy. The wisdom of this insistence soon became manifest. One thousand horse power was consumed in a single carborundum furnace. In 1904 the Carborundum Company had a plant of five thousand electrical horse power, and produced over seven millions of pounds of carborundum or silicon carbide.

"Later on Mr. Acheson found that when carborundum was very highly heated in the furnace in which it was formed the silicon was evaporated from the crystals and graphite left as a pseudomorph. This observation led the way to experiments and methods which formed the basis of another large industry carried on by the National Acheson Graphite Company at Niagara. Its product is plumbago, a graphite made by heating to a high temperature impure carbon, such as anthracite coal. The silica and clay in the ash apparently provide material for the chemical changes which result in the conversion, and the impurities are themselves finally vaporized out of the mass to a large extent. That impure carbon was convertible into graphite at high temperatures was known. Rods of carbon were so converted by the writer in 1882. The carbon electrodes of arc lights, especially large arcs, were noted to have been after burning so converted for a small distance from the ends at the arc, and this was particularly noticeable at the positive craters of large arcs, which, of course, during burning attained a temperature limited only by the sublimation of the carbon itself. It is, however, due to Mr. Acheson that the production of artificial graphite has become a great commercial success. Rods, bars, plates of carbon converted into Acheson graphite, are now extensively used in electro-chemical and electro-metallurgical industries, almost to the complete exclusion of the ordinary moulded carbon formerly used.

The graphite is used as a filler for dry batteries, and is being applied in place of natural graphite in stove polish, lubrication, foundry facings, and as a pigment for paints. It is easily reducible to a very fine powder. The graphite is nearly pure, containing about one-half per cent of impurities when used for electro-chemical electrodes, and about three per cent when used for paint. A good quality of natural graphite may be eighty-five per cent pure, and much of it on the market is as low as forty-five per cent. The solid bars of artificial graphite have a relatively high electric conductivity, estimated at about four times that of ordinary amorphous carbon rods. They disintegrate in electro-chemical work at a much lower rate than the former electrodes, for which they have become the substitute. In some cases the life of the electrode is increased twelve times. During 1905 the Acheson Graphite Company produced about two and a half millions of pounds of graphite electrodes and nearly two millions of pounds of bulk graphite, made principally from anthracite coal.

"Two other electric furnace products have been added more recently as the outcome of Mr. Acheson's experiments. In attempting to reduce the element silicon by electrically heating an intimate mixture of fine graphite and pure silica, the amount of carbon being only that sufficient to remove the oxygen, Mr. Acheson noted not only the reduction of the silicon in part, but the production of a quantity of a greenish gray fluffy substance which, from the circumstances of its production, he recognized as highly refractory. The manufacture of silicon itself was turned over to the carborundum company in 1901, since which time this manufacture has been extended and perfected so that the element silicon, once a rare curiosity of the chemical laboratory, seen only in small crystals and in samples of small amount, has become a commercial product of low price.

"The greenish gray fluffy mass referred to when analyzed gave the formula Si_2C_2O, and was named siloxicon. It is amorphous and said to be inert to acid and basic slag, insoluble in fused iron, and packs or binds itself when pressed and then heated to about 2500 degrees F. Briquettes can thus be made, resisting the highest furnace temperatures. Attention is now being given to this substance as a material for crucibles, muffles, fire bricks, fire-resisting linings, etc.

"Mr. Acheson has contributed several papers to technical journals and societies, an incomplete list being as follows: —

"'The Influence of the Condenser on Disruptive Discharges.' Electrical World, July 7, 1888.

"'Disruptive Discharges and their Relations to Underground Cables.' Before the National Electric Light Association, August 29, 1888.

"'Lightning Arresters and the Photographic Study of Self-Induction.' Before the American Institute of Electrical Engineers, January 8, 1889.

"'Carborundum: its History, Manufacture, and Uses.' Before the Franklin Institute, June 21, 1893.

"'Graphite: its Foundation and Manufacture.' Before the Franklin Institute, March 15, 1899.

"'Egyptianized Clay.' Before the American Ceramic Society, in 1903.

"'Discovery and Invention.' Before the Society of Mining Engineers, Massachusetts Institute of Technology, March 9, 1906.

"He has received some forty United States patents, chiefly relating to carborundum, graphite, and other products of the electric furnace.

"In conclusion, it may be said that the labors of Mr. Acheson have, through the agency of electric heat, in furnaces especially designed for its utilization, and by methods devised by him, provided a new abrasive only inferior to the diamond in hardness and toughness, much harder than emery or carborundum, of which it largely takes the place, whether used in the form of graded sizes of grains or in slabs, or in the construction of grinding wheels. His work has also developed a source of nearly pure graphite aside from the usual mining of this substance, and which promises to become the principal supply of such material. It has supplied compact graphite in any form desired, and of high endurance and purity, for electrodes in electro-chemical industry. There has resulted from the work of Mr. Acheson a commercial production of the element silicon, before known in small amounts as a curiosity of the chemical cabinet. This element promises to fill a need for electric-resistance materials, and for chemical ware unattackable by strong acids even when hot, hydrofluoric among the number It can be cast in any form in ordinary moulds, and stands rapid changes of temperature when in the form of crucibles or dishes not too thick. It oxidizes but slightly in the air even at high temperatures. Lastly, the work of Mr. Acheson provides a new material, — siloxicon, — the properties of which promise to render it a very valuable addition to our resources in high temperature furnace processes. He is to be commended for his persistency, ingenuity, and sagacity in applying the heating effect of electrical energy to technical problems of the highest importance."

I can add to this only that the committee is heartily unanimous in the opinion that the labors of Mr. Acheson fully entitle him to the honor of the award.

It was then

Voted, That the Rumford Premium be awarded to Mr. Edward Goodrich Acheson for the "Application of Heat in the Electric Furnace to the Industrial Production of Carborundum, Graphite, and other new and useful Substances."

REPORT OF THE C. M. WARREN COMMITTEE.

The C. M. Warren Committee beg leave to report that during the last year they have made the following grants from the C. M. Warren Fund: —

Professor Walter L. Jennings, of the Worcester Polytechnic Institute for a research on the acetyl derivatives of rosaniline and pararosaniline	$100
Dr. J. Bishop Tingle, of Johns Hopkins University, for research on the hydrazine derivatives of the camphor oxalic acids	50
Mr. Richard C. Tolman, Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology, for the construction of apparatus to be used in the measurement of the difference in electrical potential between the inner and outer ends of rotating tubes containing salt solutions, this apparatus to be the property of the American Academy of Arts and Sciences, and to be returned to the C. M. Warren Committee on the completion of Mr. Tolman's investigations	300

Acknowledgments of these grants and satisfactory reports of progress have been received.

LEONARD P. KINNICUTT, *Chairman*.

May 8, 1907.

REPORT OF THE PUBLICATION COMMITTEE.

During the last year there were published of the Proceedings three numbers of Volume XLI (33–35), and twenty-six numbers of Volume XLII; also one biographical notice; in all, 854 + v pages and eleven plates. Seven numbers of Volume XLII (13, 17, 21, 22, 24, 25, and 26) were paid for from the income of the Rumford Fund.

Of the Memoirs, one number (Volume XIII, No. 4, pp. 149–179, plates IX–XXIV) has been published.

Under the ruling that balances of appropriations unexpended at the end of the year revert to the general funds of the Academy, and that the proceeds from sales of publications are not available for the purposes of publication except by special vote of the Academy to that effect, the funds available for publication during the year just closed have been limited to the appropriation of $2600 voted at the annual meeting, May 9, 1906, and an additional appropriation of $500 voted January 9, 1907; in all, $3100. However, several authors bore a portion of the expense of their papers. Sometimes it was convenient to have the author pay a lump sum to the Academy rather than send him a separate bill. The money so received aggregated $179.23, and was turned over to the Treasurer for credit to the General Publication Fund. Against this total sum of $3279.23 bills aggregating $3279.22 have been approved, thus leaving an unexpended balance of one cent.

There are now three papers in press for the Proceedings, the estimated unpaid expense of which is $282, and unpaid bills amounting to $27.25.

Bills for publications on account of the Rumford Fund amounting to $887.01 have been forwarded to the chairman of the Rumford Committee for approval.

One number of the Memoirs, estimated to cost for the text $145, and one number of the Proceedings, estimated to cost $86, are in press as Rumford papers.

EDWARD L. MARK, *Chairman.*

May 8, 1907.

FINANCIAL REPORT OF THE COUNCIL.

The income of the Academy for the year 1907–8, as estimated by the Treasurer, is as follows:

GENERAL FUND	Investments	$1608	
	Assessments	1800	
	Rent of offices	1200	$4608
PUBLICATION FUND	Appleton Fund investments	$ 556	
	Centennial Fund investments	2178	$2734
RUMFORD FUND	Investments		$2547
WARREN FUND	Investments		$584

The following appropriations are recommended:

GENERAL FUND.

House expenses	$1200	
Library expenses	1600	
Books, periodicals, and binding	900	
Expenses of meetings	250	
Treasurer's office	150	$4100

RUMFORD FUND.

Research	$1000	
Periodicals and binding	150	
Books and binding	50	
Publication	700	
To be used at discretion of Committee	264	
Unexpended balance of 1906–7 appropriation	186	$2350

C. M. WARREN FUND.

Research	$500

PUBLICATION FUND.

Publication	$2400

In accordance with the recommendations in the foregoing report it was

Voted, To appropriate for the purposes named the following sums: —

From the income of the General Fund	$4100
From the income of the Rumford Fund	2350
From the income of the C. M. Warren Fund	500
From the income of the Publication Fund	2400

On the motion of the Treasurer, it was

Voted, That the annual assessment for the ensuing year be ten dollars ($10).

The annual election resulted in the choice of the following officers and committees: —

WILLIAM W. GOODWIN, *President.*
JOHN TROWBRIDGE, *Vice-President for Class I.*
HENRY P. WALCOTT, *Vice-President for Class II.*
JOHN C. GRAY, *Vice-President for Class III.*
EDWIN H. HALL, *Corresponding Secretary.*
WILLIAM WATSON, *Recording Secretary.*
CHARLES P. BOWDITCH, *Treasurer.*
A. LAWRENCE ROTCH, *Librarian.*

Councillors for Three Years.

HENRY P. TALBOT, of Class I.
JOHN E. WOLFF, of Class II.
GEORGE L. KITTREDGE, of Class III.

Finance Committee.

WILLIAM W. GOODWIN,
ELIOT C. CLARKE,
FRANCIS BARTLETT.

Rumford Committee.

CHARLES R. CROSS, ARTHUR G. WEBSTER,
EDWARD C. PICKERING, ELIHU THOMSON,
THEODORE W. RICHARDS, ERASMUS D. LEAVITT,
LOUIS BELL.

C. M. Warren Committee.

LEONARD P. KINNICUTT, CHARLES R. SANGER,
ROBERT H. RICHARDS, ARTHUR A. NOYES,
THEODORE W. RICHARDS, HENRY P. TALBOT,
GEORGE D. MOORE.

The following standing committees were appointed by the Chair: —

Publication Committee.

WALLACE C. SABINE, of Class I, EDWARD L. MARK, of Class II,
CRAWFORD H. TOY, of Class III.

Library Committee.

HARRY M. GOODWIN, of Class I, SAMUEL HENSHAW, of Class II,
HENRY W. HAYNES, of Class III.

Auditing Committee.

A. LAWRENCE LOWELL, FREDERICK J. STIMSON.

House Committee.

WILLIAM R. WARE, A. LAWRENCE ROTCH,
MORRIS H. MORGAN.

The following gentlemen were elected members of the Academy: —

Charles Ladd Norton, of Boston, to be a Resident Fellow in Class I, Section 2 (Physics).

George Washington Pierce, of Cambridge, to be a Resident Fellow in Class I, Section 2 (Physics).

Gregory Paul Baxter, of Cambridge, to be a Resident Fellow in Class I, Section 3 (Chemistry).

The following communications were given:

"Variable Stars in the Globular Clusters." By Professor S. I. Bailey.

"The Eastern Escarpment of the Mexican Plateau." By Professor W. M. Davis.

The following paper was presented by title:

Contribution from the Research Laboratory of Physical Chemistry. — No. 15. "On the Density, Conductivity, and Viscosity of Fused Electrolytes." By H. M. Goodwin and R. D. Mailey.

AMERICAN ACADEMY OF ARTS AND SCIENCES.

REPORT OF THE COUNCIL. — PRESENTED MAY 8, 1907.

BIOGRAPHICAL NOTICE.

EDWARD ATKINSON . . . . THOMAS WENTWORTH HIGGINSON.

REPORT OF THE COUNCIL.

The Academy has lost nine members by death since the report of the Council at the Annual meeting, May 9, 1906: three Resident Fellows, Edward J. Young, Bennett H. Nash, Samuel Cabot; six Foreign Honorary Members, J. W. A. Kirchhoff, Ludwig Boltzmann, F. Brunetière, D. Mendeleeff, M. Berthelot, Sir Michael Foster.

Six Resident Fellows have resigned; one has abandoned Fellowship.

New members have been elected as follows: Resident Fellows, three; Foreign Honorary Members, two.

The roll of the Academy now includes 189 Resident Fellows, 98 Associate Fellows, and 68 Foreign Honorary Members.

EDWARD ATKINSON

Edward Atkinson, a member of the American Academy of Arts and Sciences since March 12, 1879, was born in Brookline, Mass., on February 10, 1827, and died in Boston on December 11, 1905. He was descended on his father's side from the patriot minuteman, Lieutenant Amos Atkinson, and on the maternal side from Stephen Greenleaf, a well known fighter of Indians in the colonial period; thus honestly inheriting on both sides that combative spirit which marked his life in good causes. Owing to the business reverses of his father he was prevented from receiving, as his elder brother William Parsons Atkinson had received, a Harvard College education, a training which was also extended to all of Edward Atkinson's sons at a later day. At fifteen he entered the employment of Read and Chadwick, Commission Merchants, Boston, in the capacity of office boy; but he rapidly rose to the position of bookkeeper, and subsequently became connected with several cotton manufacturing companies in Lewiston, Maine, and elsewhere. He was for many years the treasurer of a number of such corporations, and in 1878 became President of the Boston Manufacturers Mutual Insurance Company. This business was in a somewhat chaotic state when he took hold of it, but he remained in this position until his death, having during this time organized, enlarged, and perfected the mutual insurance of industrial concerns. In 1855 he married Miss Mary Caroline Heath of Brookline, who survives him with seven children, — Mrs. Ernest Winsor, E. W. Atkinson, Charles H. Atkinson, Wm. Atkinson, Robert W. Atkinson, Miss C. P. Atkinson, and Mrs. R. G. Wadsworth.

This gives the mere outline of a life of extraordinary activity and usefulness which well merits a further delineation in detail. Mr. Atkinson's interest in public life began with a vote for Horace Mann in 1848. Twenty years after, speaking at Salem, he described himself as never having been anything else than a Republican; but he was one of those who supported Cleveland for President in 1884 and whose general affinities were with the Democratic party. He opposed with especial vigor what is often called "the imperial policy," which followed the Cuban war, and he conducted a periodical of his own from time to time, making the most elaborate single battery which the war-party had to encounter.

He was from an early period of life a profuse and vigorous pamphleteer, his first pamphlet being published during the Civil War and entitled "Cheap Cotton by Free Labor," this publication leading to his acquaintance with David R. Wells and Charles Nordhoff, thenceforth his life-long friends. His early pamphlets were on the cotton question in different forms (1863–76); he wrote on blockade running (1865); on the Pacific railway (1871); and on mutual fire insurance (1885), this last being based on personal experience as the head of a mutual company. He was also, during his whole life, in print and otherwise, a strong and effective fighter for sound currency.

A large part of his attention from 1889 onward was occupied by experiments in cooking and diet, culminating in an invention of his own called "The Aladdin Oven." This led him into investigations as to the cost of nutrition in different countries, on which subject he also wrote pamphlets. He soon was led on into experiments so daring that he claimed to have proved it possible to cook personally, in open air, a five-course dinner for ten persons, and gave illustrations of this at outdoor entertainments. He claimed that good nutrition could be had for $1 per week, and that a family of five, by moderate management, could be comfortably supported for $180 per year ("Boston Herald, Oct. 8, 1891). These surprising figures unfortunately created among the laboring class a good deal of sharp criticism, culminating in the mistaken inquiry, why he did not feed his own family at $180 a year, if it was so easy? I can only say for one, that if the meals at that price were like a dinner of which I partook at his own house with an invited party, and at which I went through the promised five courses after seeing them all prepared in the garden, I think that his standard of poverty came very near to luxury.

Mingled with these things in later years was introduced another valuable department of instruction. He was more and more called upon to give addresses, especially on manufactures, before Southern audiences, and there was no disposition to criticize him for his anti-slavery record. There could hardly be found another man whose knowledge of manufacturing and of insurance combined made him so fit a man to give counsel in the new business impulse showing itself at the South. He wrote much (1877) on cotton goods, called for an international cotton expedition, giving an address at Atlanta, Ga., which was printed in Boston in 1881.

Looking now at Atkinson's career with the eyes of a literary man, it seems clear to me that no college training could possibly have added to his power of accumulating knowledge or his wealth in the expression of it. But the academic tradition might have added to these general statements in each case some simple address or essay which would bring out clearly to the minds of an untrained audience the essential points of each single theme. Almost everything he left is the talk of a specially trained man to a limited audience, also well trained, — at least in the particular department to which he addresses himself. The men to whom he talks may not know how to read or write, but they are all practically versed in the subjects of which he treats. He talks as a miner to miners, a farmer to farmers, a cook to cooks; but among all of his papers which I have examined that in which he appears to the greatest advantage to the general reader is his "Address before the Alumni of Andover Theological Seminary" on June 9, 1886. Here he speaks as one representing a wholly different pursuit from that of his auditors; a layman to clergymen, or those aiming to become so. He says to them frankly at the outset, "I have often thought [at church] that if a member of the congregation could sometimes occupy the pulpit while the minister took his place in the pew, it might be a benefit to both. The duty has been assigned to me to-day to trace out the connection between morality and a true system of political or industrial economy."

He goes on to remind them that the book which is said to rank next to the Bible toward the benefit of the human race is Adam Smith's "Wealth of Nations," and that the same Adam Smith wrote a book on moral philosophy which is now but little read. He therefore takes the former of Smith's books, not the latter, as his theme, and thus proceeds:

"I wonder how many among your number ever recall the fact that it has been the richest manufacturers who have clothed the naked at the least cost to them; that it is the great bonanza farmer who now feeds the hungry at the lowest price; that Vanderbilt achieved his great fortune by reducing the cost of moving a barrel of flour a thousand miles, — from three dollars and fifty cents to less than seventy cents. This was the great work assigned to him, whether he knew it or not. His fortune was but an incident, — the main object, doubtless, to himself, but a trifling incident compared to what he saved others."[1]

He then goes on to show that whatever may be the tricks or wrongs of commerce, they lie on the surface, and that every great success is based upon very simple facts. "The great manufacturer," he says, "who guides the operations of a factory of a hundred thousand spindles, in which fifteen hundred men, women, and children earn their daily bread, himself works on a narrow margin of one fourth of a cent on each yard of cloth. If he shall not have applied truth to every branch of construction and of the operation of that factory, it will fail and become worthless; and then with toilsome labor a hundred and fifty thousand women might try to clothe themselves and you, who are now clothed by the service of fifteen hundred only.

"Such is the disparity in the use of time, brought into beneficent action by modern manufacturing processes.

"The banker who deals in credit by millions upon millions must possess truth of insight, truth of judgment, truth of character. Probity and integrity constitute his capital, for the very reason that the little margin which he seeks to gain for his own service is but the smallest fraction of a per cent upon each transaction. I supervise directly or indirectly the insurance upon four hundred million dollars' worth of factory property. The products of these factories, machine-shops, and other works must be worth six hundred million dollars a year. It is n't worth fifty cents on each hundred dollars to guarantee their notes or obligations, while ninety-nine and one half per cent of all the sales they make will be promptly paid when due."[2]

He elsewhere turns from viewing the factory system with business eyes alone to the consideration of it from the point of view of the laborer. There is no want of sympathy, we soon find, in this man of inventions and statistics. He thus goes on:

"The very manner in which this great seething, toiling, crowded mass of laboring men and women bear the hardships of life leads one to faith in humanity and itself gives confidence in the future. If it were not that there is a Divine order even in the hardships which seem so severe, and that even the least religious, in the technical sense, have faith in each other, the anarchist and nihilist might be a cause of dread.

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[1] Address to Alumni of Andover, p. 1.
[2] Ibid., p. 10.

"As I walk through the great factories which are insured in the company of which I am president, trying to find out what more can be done to save them from destruction by fire, I wonder if I myself should not strike, just for the sake of variety, if I were a mule-spinner, obliged to bend over the machine, mending the ends of the thread, while I walked ten or fifteen miles a day without raising my eyes to the great light above. I wonder how men and women bear the monotony of the workshop and of the factory, in which the division of labor is carried to its utmost, and in which they must work year in and year out, only on some small part of a fabric or an implement, never becoming capable of making the whole fabric or of constructing the whole machine."[3]

We thus find him quite ready to turn his varied knowledge and his executive power towards schemes for the relief of the operative, schemes of which he left many.

Mr. Atkinson, a year or two later (1890), wrote a similarly popularized statement of social science for an address on "Religion and Life" before the American Unitarian Association. In his usual matter-of-fact way he had prepared himself by inquiring at the headquarters of different religious denominations for a printed creed of each. He first bought an Episcopal creed at the Old Corner Bookstore for two cents, an Orthodox creed at the Congregational Building for the same amount, then a Methodist two-cent creed also, a Baptist creed for five cents, and a Presbyterian one for ten, Unitarian and Universalist creeds being furnished him for nothing; and then he proceeds to give some extracts whose bigotry makes one shudder, and not wonder much that he expressed sympathy mainly with the Catholics and the Jews, rather than with the severer schools among Protestants. And it is already to be noticed how much the tendency of liberal thought, during the last twenty years, has been in the direction whither his sympathies went.

As time went on he underwent the test which awaits all Northern public men visiting the Southern States, but not met by all in so simple and straightforward a way as he. Those who doubt the capacity of the mass of men in our former slave States to listen to plainness of speech should turn with interest to Atkinson's plain talk to the leading men of Atlanta, Ga., in October, 1880. He says, almost at the beginning: "Now, gentlemen of the South, I am going to use free speech for a purpose and to speak some plain words of truth and soberness to you. . . . I speak then to you here and now as a Republican of Republicans, as an Abolitionist of early time, a Free Soiler of later date, and a Republican of to-day." And the record is that he was received with applause. He goes on to say as frankly: "When slavery ended, not only were blacks made free from the bondage imposed by others, but whites as well were redeemed by the bondage they had imposed upon themselves. . . . When you study the past system of slave labor with the present system of free labor, irrespective of all personal considerations, you will be mad down to the soles of your boots to think that you ever tolerated it; and when you have come to this wholesome condition of mind you will wonder how the devil you could have been so slow in seeing it." [Laughter.]

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[3] Address to Workingmen in Providence, April 11, 1886, p. 19.

Then he suddenly drops down to the solid fact and says: "Are you not asking Northern men to come here, and do you not seek Northern capital? If you suppose either will come here unless every man can say what he pleases, as I do now, you are mistaken." Then he goes on with his speech, rather long as he was apt to make them, but addressing a community much more leisurely than that which he had left at home; filling their minds with statistics, directions, and methods, till at last recurring to the question of caste and color he closes fearlessly: "As you convert the darkness and oppression and slavery to liberty and justice, so shall you be judged by men, and by Him who created all the nations of the South."

After tracing the course and training of an eminent American at home, it is often interesting to follow him into the new experiences of the foreign traveller. In that very amusing book, "Notes from a Diary," by Grant Duff (now Sir Mountstuart Elphinstone Grant Duff), the author writes that he came unexpectedly upon a breakfast (June, 1887), the guests being "Atkinson, the New England Free Trader, Colonel Hay, and Frederic Harrison, all of whom were well brought out by our host and talked admirably." I quote some extracts from the talk:

"Mr. Atkinson said that quite the best after-dinner speech he had ever heard was from Mr. Samuel Longfellow, brother of the poet. An excellent speech had been made by Mr. Longworth, and the proceedings should have closed, when Mr. Longfellow was very tactlessly asked to address the meeting, which he did in the words: 'It is, I think, well known that worth makes the man, but want of it the fellow,' and sat down." After this mild beginning we have records of good talk.

"Other subjects," Grant Duff says, "were the hostility of the Socialists in London to the Positivists and to the Trades' Unions; the great American fortunes and their causes, the rapid melting away of some of them, the hindrance which they are to political success; and servants in the United States, of whom Atkinson spoke relatively, Colonel Hay absolutely, well, saying that he usually kept his from six to eight years. . . .

"Atkinson said that all the young thought and ability in America is in favor of free trade, but that free trade has not begun to make any way politically. Harrison remarked that he was unwillingly, but ever more and more, being driven to believe that the residuum was almost entirely composed of people who would not work. Atkinson took the same view, observing that during the war much was said about the misery of the working women of Boston. He offered admirable terms if they would only go a little way into the country to work in his factory. Forty were at last got together to have the conditions explained — ten agreed to go next morning, of whom one arrived at the station, and she would not go alone!"

On another occasion we read in the "Diary": —

"We talked of Father Taylor, and he [Atkinson] told us that the great orator once began a sermon by leaning over the pulpit, with his arms folded, and saying, 'You people ought to be very good, if you 're not, for you live in Paradise already.'

"The conversation, in which Sir Louis Mallet took part, turned to Mill's economical heresies, especially that which relates to the fostering of infant industries. Atkinson drew a striking picture of the highly primitive economic condition of the South before the war, and said that now factories of all kinds are springing up throughout the country in spite of the keen competition of the North. He cited a piece of advice given to his brother by Theodore Parker, 'Never try to lecture down to your audience.' This maxim is in strict accordance with an opinion expressed by Hugh Miller, whom, having to address on the other side of the Firth just the same sort of people as those amongst whom he lived at Cromarty, I took as my guide in this matter during the long period in which I was connected with the Elgin Burghs."

"Atkinson went on to relate that at the time of Mr. Hayes's election to the presidency there was great danger of an outbreak, and he sat in council with General Taylor and Abraham Hewitt, doing his best to prevent it. At length he exclaimed: 'Now I think we may fairly say that the war is over. Here are we three acting together for a common object, and who are we? You, Mr. Hewitt, are the leader of the Democratic party in New York; I am an old Abolitionist who subscribed to furnish John Brown and his companions with rifles; you, General Taylor, are the last Confederate officer who surrendered an army, and you surrendered it not because you were willing to do so, but, as you yourself admit, because you could n't help it.'"

The publication which will perhaps be much consulted in coming years as the best periodical organ of that party in the nation which was most opposed to the Philippine war will doubtless be the work issued by Mr. Atkinson on his own responsibility and by his own editing, from June 3, 1899, to September, 1900, under the name of "The Anti-Imperialist." It makes a solid volume of about 400 octavo pages, and was conducted wholly on Atkinson's own responsibility, financially and otherwise, though a large part of the expense was paid him by volunteers to the extent of $5,657.87 or more, covering an outlay of $5,870.62, this amount being largely received in sums of one dollar, obtained under what is known as the chain method. For this amount were printed more than 100,000 copies of a series of pamphlets, of which the first two were withdrawn from the mail as seditious under President McKinley's administration. A more complete triumph of personal independence was perhaps never seen in our literature, and it is easy to recognize the triumph it achieved for a high-minded and courageous as well as constitutionally self-willed man. The periodical exerted an influence which lasts to this day, although the rapidity of political change has now thrown it into the background for all except the systematic student of history. It seemed to Mr. Atkinson, at any rate, his crowning work.

The books published by Edward Atkinson were the following: "The Distribution of Profits," 1885; "The Industrial Progress of the Nation," 1889; "The Margin of Profit," 1890; "Taxation and Work," 1892; "Facts and Figures the Basis of Economic Science," 1894. This last was printed at the Riverside Press, the others being issued by Putnam & Co., New York. He wrote also the following papers in leading periodicals: "Is Cotton our King?" (Continental Monthly, March, 1862); "Revenue Reform" (Atlantic, October, 1871); "An American View of American Competition" (Fortnightly, London, March, 1879); "The Unlearned Professions" (Atlantic, June, 1880); "What makes the Rate of Interest" (Forum, 1880); "Elementary Instruction in the Mechanics Arts" (Century, May, 1881); "Leguminous Plants suggested for Ensilage" (Agricultural, 1882); "Economy in Domestic Cookery" (American Architect, May, 1887); "Must Humanity Starve at Last?" "How can Wages be Increased?" "The Struggle for Subsistence," "The Price of Life" (all in Forum for 1888); "How Society Reforms Itself," and "The Problem of Poverty" (both in

Forum for 1889); "A Single Tax on Land" (Century, 1890); and many others. When the amount of useful labor performed by the men of this generation comes to be reviewed a century hence, it is doubtful whether a more substantial and varied list will be found credited to the memory of any one in America than that which attaches to the memory of Edward Atkinson.

THOMAS WENTWORTH HIGGINSON.

American Academy of Arts and Sciences

OFFICERS AND COMMITTEES FOR 1907-08.

PRESIDENT.

William W. Goodwin.

VICE-PRESIDENT.

Class I.	Class II.	Class III.
John Trowbridge,	Henry P. Walcott,	John C. Gray.

CORRESPONDING SECRETARY.

Edwin H. Hall.

RECORDING SECRETARY.

William Watson.

TREASURER.

Charles P. Bowditch.

LIBRARIAN.

A. Lawrence Rotch.

COUNCILLORS.

Class I.	Class II.	Class III.
Wallace C. Sabine,	William G. Farlow, *Terms expire* 1908.	Edward Robinson.
Ira N. Hollis,	James C. White, *Terms expire* 1909.	William R. Ware.
Henry P. Talbot,	John E. Wolff, *Terms expire* 1910.	George L. Kittredge.

COMMITTEE OF FINANCE.

William W. Goodwin, Eliot C. Clarke, Francis Bartlett.

RUMFORD COMMITTEE.

Charles R. Cross, *Chairman*,
Erasmus D. Leavitt, Edward C. Pickering, Elihu Thomson.
Arthur G. Webster, Theodore W. Richards, Louis Bell.

C. M. WARREN COMMITTEE.

Leonard P. Kinnicutt, *Chairman*,
Robert H. Richards, Charles R. Sanger, Arthur A. Noyes.
Henry P. Talbot, Theodore W. Richards, George D. Moore.

COMMITTEE OF PUBLICATION.

Edward L. Mark, of Class II, *Chairman*,
Wallace C. Sabine, of Class I, Crawford H. Toy, of Class III.

COMMITTEE ON THE LIBRARY.

A. Lawrence Rotch, *Chairman*,
Harry M. Goodwin, of Class I, Samuel Henshaw, of Class II,
Henry W. Haynes, of Class III.

AUDITING COMMITTEE.

A. Lawrence Lowell, Frederick J. Stimson.

HOUSE COMMITTEE.

William R. Ware, *Chairman*.
A. Lawrence Rotch, Morris H. Morgan.

LIST

OF THE

FELLOWS AND FOREIGN HONORARY MEMBERS.

(Corrected to June 1, 1907.)

RESIDENT FELLOWS.—192.

(Number limited to two hundred.)

CLASS I.—*Mathematical and Physical Sciences.*—78.

SECTION I.—15.

Mathematics and Astronomy.

Solon I. Bailey,	Cambridge.
Maxime Bôcher,	Cambridge.
William E. Byerly,	Cambridge.
Seth C. Chandler,	Wellesley Hills.
Gustavus Hay,	Boston.
Percival Lowell,	Boston.
Edward C. Pickering,	Cambridge.
William H. Pickering,	Cambridge.
John Ritchie, Jr.,	Roxbury.
Arthur Searle,	Cambridge.
William E. Story,	Worcester.
Henry Taber,	Worcester.
Harry W. Tyler,	Boston.
O. C. Wendell,	Cambridge.
P. S. Yendell,	Dorchester.

SECTION II.—26.

Physics.

A. Graham Bell,	Washington, D.C.
Louis Bell,	Boston.
Clarence J. Blake,	Boston.
Francis Blake,	Weston.
George A. Campbell,	Boston.
Harry E. Clifford,	Newton.
Charles R. Cross,	Brookline.
A. W. Duff,	Worcester.
H. M. Goodwin,	Roxbury.
Edwin H. Hall,	Cambridge.
Hammond V. Hayes,	Cambridge.
William L. Hooper,	Somerville.
William W. Jacques,	Newton.
Frank A. Laws,	Boston.
Henry Lefavour,	Boston.
Theodore Lyman,	Brookline.
Charles L. Norton,	Boston.
Benjamin O. Peirce,	Cambridge.
George W. Pierce,	Cambridge.
A. Lawrence Rotch,	Boston.
Wallace C. Sabine,	Boston.
John S. Stone,	Boston.
Elihu Thomson,	Swampscott.
John Trowbridge,	Cambridge.
A. G. Webster,	Worcester.
Robert W. Willson,	Cambridge.

SECTION III.—19.

Chemistry.

Gregory Paul Baxter,	Cambridge.
Arthur M. Comey,	Cambridge.
James M. Crafts,	Boston.
Charles W. Eliot,	Cambridge.
Charles L. Jackson,	Cambridge.
Walter L. Jennings,	Worcester.

Leonard P. Kinnicutt,	Worcester.
Charles F. Mabery,	Cleveland, O.
Arthur Michael,	Boston.
George D. Moore,	Worcester.
John U. Nef,	Chicago, Ill.
Arthur A. Noyes,	Boston.
Robert H. Richards,	Jamaica Plain.
Theodore W. Richards,	Cambridge.
Charles R. Sanger,	Cambridge.
Stephen P. Sharples,	Cambridge.
Francis H. Storer,	Boston.
Henry P. Talbot,	Newton.
Charles H. Wing,	Ledger, N. C.

SECTION IV. — 18.

Technology and Engineering.

Comfort A. Adams,	Cambridge.
Alfred E. Burton,	Boston.
Eliot C. Clarke,	Boston.
Heinrich O. Hofman,	Jamaica Plain.
Ira N. Hollis,	Cambridge.
L. J. Johnson,	Cambridge.
Arthur E. Kennelly,	Cambridge.
Gaetano Lanza,	Boston.
E. D. Leavitt,	Cambridge.
William R. Livermore,	New York.
Hiram F. Mills,	Lowell.
Cecil H. Peabody,	Brookline.
Andrew H. Russell,	Manila.
Albert Sauveur,	Cambridge.
Peter Schwamb,	Arlington.
H. L. Smyth,	Cambridge.
George F. Swain,	Boston.
William Watson,	Boston.

CLASS II. — *Natural and Physiological Sciences.* — 62.

SECTION I. — 14.

Geology, Mineralogy, and Physics of the Globe.

H. H. Clayton,	Milton.
Algernon Coolidge,	Boston.
William O. Crosby,	Jamaica Plain.
William M. Davis,	Cambridge.
Benj. K. Emerson,	Amherst.
O. W. Huntington,	Newport, R. I.
Robert T. Jackson,	Cambridge.
T. A. Jaggar, Jr.,	Cambridge.
William H. Niles,	Cambridge.
Charles Palache,	Cambridge.
John E. Pillsbury,	Boston.
Robert DeC. Ward,	Cambridge.
John E. Wolff,	Cambridge.
J. B. Woodworth,	Cambridge.

SECTION II. — 12.

Botany.

F. S. Collins,	Malden.
Geo. E. Davenport,	Medford.
William G. Farlow,	Cambridge.
Charles E. Faxon,	Jamaica Plain.
Merritt L. Fernald,	Cambridge.
George L. Goodale,	Cambridge.
John G. Jack,	Jamaica Plain.
Edward C. Jeffrey,	Cambridge.
B. L. Robinson,	Cambridge.
Charles S. Sargent,	Brookline.
Arthur B. Seymour,	Cambridge.
Roland Thaxter,	Cambridge.

SECTION III. — 23.

Zoölogy and Physiology.

Alexander Agassiz,	Cambridge.
Robert Amory,	Boston.

Henry P. Bowditch,	Jamaica Plain.
William Brewster,	Cambridge.
Louis Cabot,	Brookline.
Walter B. Cannon,	Cambridge.
William E. Castle,	Cambridge.
Samuel F. Clarke,	Williamstown.
W. T. Councilman,	Boston.
Harold C. Ernst,	Jamaica Plain.
Edward G. Gardiner,	Boston.
Samuel Henshaw,	Cambridge.
Theodore Hough,	Boston.
Edward L. Mark,	Cambridge.
Charles S. Minot,	Milton.
Edward S. Morse,	Salem.
George H. Parker,	Cambridge.
William T. Porter,	Boston.
James J. Putnam,	Boston.
Samuel H. Scudder,	Cambridge.
William T. Sedgwick,	Boston.
James C. White,	Boston.
William M. Woodworth,	Cambridge.

Section IV. — 13.

Medicine and Surgery.

Edward H. Bradford,	Boston.
Arthur T. Cabot,	Boston.
Thomas Dwight,	Nahant.
Reginald H. Fitz,	Boston.
Charles F. Folsom,	Boston.
Frederick I. Knight,	Boston.
Samuel J. Mixter,	Boston.
W. L. Richardson,	Boston.
Theobald Smith,	Jamaica Plain.
O. F. Wadsworth,	Boston.
Henry P. Walcott,	Cambridge.
John C. Warren,	Boston.
Francis H. Williams,	Boston.

Class III. — *Moral and Political Sciences.* — 52.

Section I. — 9.

Philosophy and Jurisprudence.

James B. Ames,	Cambridge.
Joseph H. Beal, Jr.,	Cambridge.
John C. Gray,	Boston.
Francis C. Lowell,	Boston.
Hugo Münsterberg,	Cambridge.
Josiah Royce,	Cambridge.
Frederic J. Stimson,	Dedham.
Edward H. Strobel,	Cambridge.
Samuel Williston,	Belmont.

Section II. — 19.

Philology and Archæology.

Charles P. Bowditch,	Jamaica Plain.
Lucien Carr,	Cambridge.
Franklin Carter,	Williamstown.
J. W. Fewkes,	Washington.
William W. Goodwin,	Cambridge.
Henry W. Haynes,	Boston.
Albert A. Howard,	Cambridge.
Charles R. Lanman,	Cambridge.
David G. Lyon,	Cambridge.
George F. Moore,	Cambridge.
Morris H. Morgan,	Cambridge.
Frederick W. Putnam,	Cambridge.
Edward Robinson,	New York.
Edward S. Sheldon,	Cambridge.
Herbert Weir Smyth,	Cambridge.
F. B. Stephenson,	Boston.
Crawford H. Toy,	Cambridge.
John W. White,	Cambridge.
John H. Wright,	Cambridge.

SECTION III. — 12.

Political Economy and History.

Charles F. Adams,	Lincoln.
Thomas N. Carver,	Cambridge.
Andrew McF. Davis,	Cambridge.
Ephraim Emerton,	Cambridge.
A. C. Goodell,	Salem.
Charles Gross,	Cambridge.
Henry C. Lodge,	Nahant.
A. Lawrence Lowell,	Boston.
James F. Rhodes,	Boston.
William Z. Ripley,	Newton.
Charles C. Smith,	Boston.
F. W. Taussig,	Cambridge.

SECTION IV. — 12.

Literature and the Fine Arts.

Francis Bartlett,	Boston.
Arlo Bates,	Boston.
Kuno Francke,	Cambridge.
Edward H. Hall,	Cambridge.
T. W. Higginson,	Cambridge.
George L. Kittredge,	Cambridge.
William C. Lane,	Cambridge.
Charles Eliot Norton,	Cambridge.
Denman W. Ross,	Cambridge.
William R. Ware,	Milton.
Herbert L. Warren,	Cambridge.
Barrett Wendell,	Boston.

Section 2; J. W. A. Kirchhoff, Foreign Honorary Member in Class III, Section 2.

Professor G. F. Swain was appointed by the President to represent the Academy at the dedication of the building of the engineering department of the University of Pennsylvania.

At the request of the President, Professor Ware gave some information in regard to the ventilation of the Meeting-room, with its estimated cost. After some discussion, on motion of the Recording Secretary, the whole subject was referred to the President, with full power.

The following communications were given: —

"An Ascent of Orizaba, Mexico," by Professor John E. Wolff; "On Some Elements of Conservatism in Ancient Greece," by Professor Herbert Weir Smyth.

The following paper was presented by title: —

Contributions from the Gray Herbarium of Harvard University. New Series. — No. XXXIII. "Revision of the Genus Spilanthes." By Albert Hanford Moore. Presented by B. L. Robinson.

Nine hundred and sixty-eighth Meeting.

NOVEMBER 14, 1906.

The Academy met at its House.

The PRESIDENT in the chair.

There were present twenty-two Fellows.

The Corresponding Secretary read the following letters: From the Délégation pour l'Adoption d'une Langue Auxiliaire Internationale, offering to members of the Academy the resources of the Delegation for the propaganda in the United States; from the Board of Managers for Massachusetts, of the Jamestown Exposition, asking for the loan of objects of historical interest for display at the Exposition.

The following communications were given: —

"The Ancient Coniferous Flora of New England," by Professor E. C. Jeffrey; "The Ancient Water-levels of the Champlain and Hudson Valleys," by Professor J. B. Woodworth.

The following papers were read by title: —

RECORDS OF MEETINGS.

Nine hundred and sixty-seventh Meeting.

OCTOBER 10, 1906. — STATED MEETING.

The Academy met at its House.

The PRESIDENT in the chair.

There were present twenty-three Fellows.

The Corresponding Secretary read the following communications: From the University of Pennsylvania, inviting the Academy to send a representative to the dedication of the new building of its engineering department; from the Curator of the Reale Universita Napoli, offering for sale collections of specimens of the materials ejected in the recent eruption of Vesuvius; from the American Philosophical Society, presenting the medal struck by order of the Congress of the United States in honor of the two hundredth anniversary of the birth of Benjamin Franklin; one from the same Society acknowledging the address presented by the Academy on the occasion of the above celebration; from the Director of the Observatorio Astronomico de la Plata, announcing the annexation of the Observatorio to the Universidad Nacional; from the family of Ludwig Boltzmann, announcing his death, and also the same announcement from the University of Vienna; from the Museo Nacional, Mexico, announcing the death of Manuel Urbina, chief of the department of natural history.

The following deaths were announced by the Chair: —

Edward J. Young, Resident Fellow in Class III, Section 2; Bennett H. Nash, Resident Fellow in Class III, Section 2; Ludwig Boltzmann, Foreign Honorary Member in Class I,

Proceedings of the American Academy of Arts and Sciences.

VOL. XLII. NO. 29. — AUGUST, 1907.

RECORDS OF MEETINGS, 1906-1907.

REPORT OF THE COUNCIL: BIOGRAPHICAL NOTICE.

EDWARD ATKINSON. BY THOMAS WENTWORTH HIGGINSON.

OFFICERS AND COMMITTEES FOR 1907-1908.

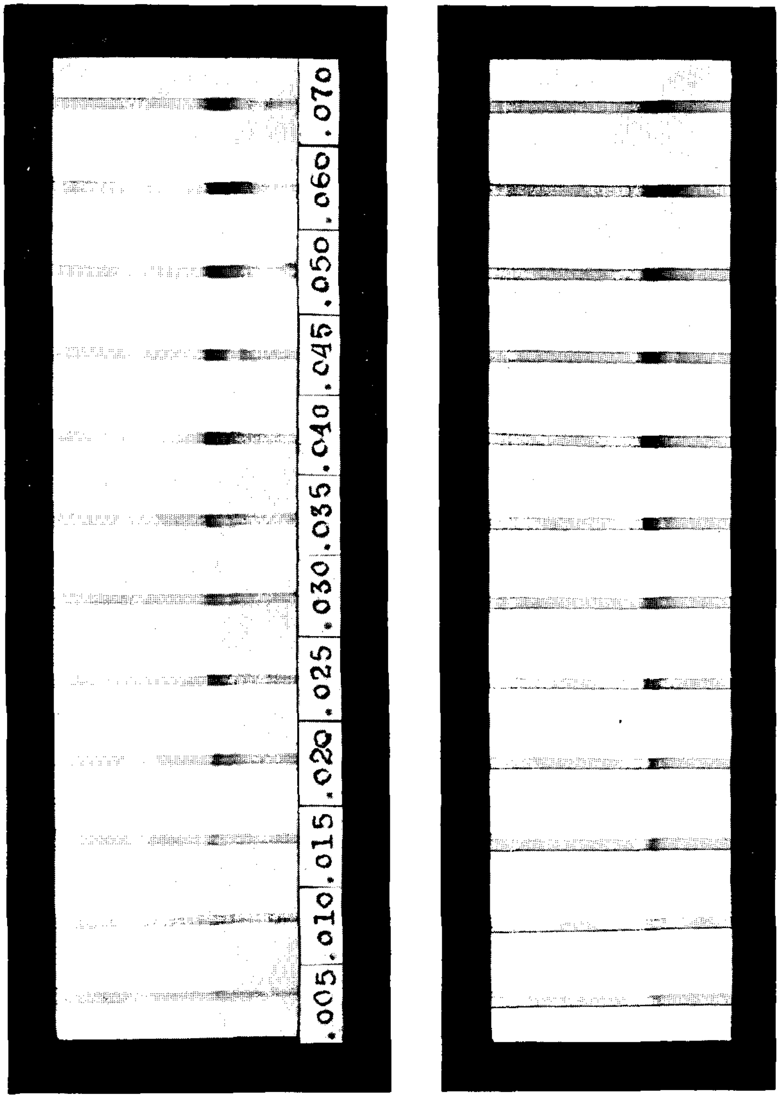
LIST OF THE FELLOWS AND FOREIGN HONORARY MEMBERS.

STATUTES AND STANDING VOTES.

RUMFORD PREMIUM.

INDEX.

(TITLE PAGE AND TABLE OF CONTENTS).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STANDARD ANTIMONY MIRRORS IN MILLIGRAMS OF Sb_2O_3.

As to the delicacy of the process in the detection of minimal amounts of antimony, a mirror corresponding to 0.005 mg. can be detected and identified with certainty. With 0.001 mg. a faint deposit is seen. Yet the differentiation of amounts under 0.01 mg. presents some difficulty. Possibly the solution of the problem of how to sensitize the zinc properly in the Marsh process as applied to arsenic will enable one to detect smaller amounts of antimony with certainty. The question of the delicacy of the process has not so much concerned us, however, as its practicability. Its application to the determination of small amounts must of course be worked out for each particular problem. This does not come, however, within the scope of the present investigation, but certain applications of the method are under consideration in this laboratory.

Harvard University, Cambridge, Mass.,
January 1, 1907.

with the standards and the amount of antimony read off. From the amount in the aliquot portion, the amount in solution was calculated.

Very often the mirror obtained from a given portion of the solution will be found too small or two large for estimation. In this case one can readily determine the proper amount of solution to be used.

The following table shows the results of these analyses :

TABLE II.

No. of Analysis.	Standard Solution used.	C.c. of Solution taken.	Sb_2O_3 taken.	Total Weight Diluted Solution.	Weight Diluted Solution taken for Analysis.	Mirror Reading.	Sb_2O_3 found.	Sb_2O_3 found, Mean.	Per cent Sb_2O_3 found.
			mg.	gm.	gm.	mg.	mg.	mg.	
2	II	5	0.05	24.49	*a*) 5.56 *b*) 13.93	0.014 0.027	0.06 0.05	0.055	110
1	II	10	0.10	25.36	*a*) 11.64 *b*) 13.72	0.038 0.042	0.08 0.08	0.080	80
6	II	30	0.30	29.95	*a*) 3.14 *b*) 2.40	0.035 0.025	0.33 0.31	0.320	107
7	I	0.5	0.50	24.48	*a*) 1.91 *b*) 1.62	0 043 0.038	0.55 0.57	0.560	112
3	I	1.0	1.00	23.94	*a*) 0.83 *b*) 1.58	0.038 0.048	1.10 0.73	0.915	92
4	I	1.5	1.50	26.22	*a*) 0.72 *b*) 0.79	0.040 0.045	1.46 1.49	1.475	98
5	I	2.0	2.00	23.21	*a*) 0.45 *b*) 0.68	0.038 0.048	1.96 1.64	1.800	90
Average percentage									98

The results are as good as can be expected, considering the small amounts of antimony under estimation and the difficulty of reading the mirrors more accurately than within 0.002 or 0.003 mgr. The demand on the method made by these analyses is much more severe than would occur in actual practice. It is safe to say that the method will give a close approximation to the amount of antimony present in a given product, within the limits for which it is intended. It will serve also as a means of quickly, though roughly, estimating amounts which would require the ordinary analytical methods for an exact determination.

through the upper and lower sides of the frame. As the antimony mirrors are affected by moist air, and in the course of time by light, we take the precaution to seal the tubes and to keep the set of standards in the dark. The deterioration of the standards with time is not a serious objection to the method, since a fresh set can readily be made when desired.[16]

The amounts of antimony used by us in the set of standards shown in the plate are, in milligrams of antimonious oxide (Sb_2O_3) as follows: 0.005; 0.01; 0.015; 0.02; 0.025; 0.03; 0.035; 0.04; 0.045; 0.05; 0.06; 0.07. Above 0.07 mgr. it is of no advantage to make a mirror, as the recognition of the differences in the higher mirrors is difficult. These mirrors may be used as standards when viewed by reflected light, as shown in the upper set of the plate, but it is much better to use transmitted light. This is best arranged by mounting the frame in a box, similar in form to the fluoroscope, which may be of wood or metal. This box, about 25 cm. long and open at each end, is larger at the bottom than at the top, and carries a rabbet at the bottom through which the frame can be slipped. It is provided at the top with means of shading the eyes against all light except that which comes through the set of mirrors. If the box is held against a white surface, the mirrors gain greatly in sharpness, and smaller differences may be in this way detected when comparing mirrors obtained in an analysis. The lower part of the plate shows the set of standards when viewed by this device.

Analysis of Solutions containing Antimony.

To test the availability of the standard mirrors, a series of seven analyses was made of solutions in which the amounts of antimony were unknown to the analyst. The solutions were weighed to the second decimal place in a side-neck test tube of about 30 c.c. capacity. After the apparatus had been running for about ten minutes, a few drops of the solution were added to the reduction flask. If no mirror appeared within ten minutes, a larger portion of the solution was added, and if this again gave no mirror, the addition was continued. After the appearance of the mirror, the run was continued for thirty or forty minutes, until there was no probability of further deposit. By reweighing the test tube after the addition of the portion or portions which produced the mirror, the amount of solution taken was determined. The mirror obtained from this amount was then compared

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[16] The suggestion of Panzer (Chem. Centralbl., **74** (1), 821 (1903), to seal standard arsenic mirrors with phosphorus pentoxide could probably be applied with advantage to the set of antimony mirrors.

The glass tubing varies from 5 or 6 mm., inside diameter, to 7 or 8 mm. outside. The inside of the tube should be thoroughly cleaned before use. For this purpose a bundle of tubes is entirely immersed for some time in concentrated sulphuric acid to which sodic chromate has been added. The tubes are then washed, dried, and stored away from dust. In drawing out the capillary, care should be taken to draw to the same outside diameter and as nearly as possible to the same length. If one starts with tubes of as nearly equal size as possible, and observes these conditions, a capillary of nearly constant bore is obtained. It is on this uniformity of bore that the gradation of the standards depends. One should therefore at least draw the tubes to the same diameter, since this condition is easily governed and is the chief factor in determining the bore. We have used for this purpose a Brown and Sharpe wire gauge, and have drawn the tubing to gauge No. 13, which corresponds to 1.8 mm., or 0.072 inch.

In beginning the run, three to five grams of zinc are placed in the reduction flask, which is then attached at one end to the stopcock of the constant generator, at the other to the drying tube. To the latter is then attached the ignition tube, with the collar over the capillary. 20 c.c. dilute hydrochloric acid are added through the funnel tube, and the apparatus may be tested for tightness by closing the end of the capillary with the finger while adding the acid. The hydrogen is then turned on from the generator, and, when the air is expelled, is lighted at the end of the capillary and turned down to a height of about one millimeter. It is important that this height be maintained as nearly as possible throughout the run and that the flame should burn steadily, since an irregular flow of hydrogen results in an uneven deposit of antimony. After the apparatus has been in action for five or ten minutes, the measured amount of antimony is added to the reduction flask through the funnel, which is then rinsed into the flask with a little water. No air is introduced into the apparatus if the funnel is sufficiently small compared to the tube in which it is set, and if the lower end of the tube is constricted.

The deposit of antimony makes its appearance in five to ten minutes, and is completely deposited in thirty, though we have usually waited forty to fifty minutes, to avoid the possibility of loss. We have satisfied ourselves that all the antimony is deposited from the hydride at this point, since a further heating along the tube gives no mirror.

The tubes containing the mirrors are sealed at each end and mounted in a frame, as shown in the plate. This frame is 185 mm. by 70 mm. outside, 135 by 35 mm. inside, and made of blackened wood 6 mm. in thickness. The tubes are fastened in holes passing amount which is inappreciable in our work on antimony.[14] By use of hydrochloric acid and purification of the hydrogen from the generator, there is no necessity for purification of the hydrogen from the reduction flask, provided sulphur compounds reducible to hydrogen sulphide are absent. Whenever the hydrogen contains hydrogen sulphide, the deposit in the heated tube is more or less reddish-yellow in color, due presumably to the presence of antimonious sulphide.

The fused sticks of calcic chloride used for drying (Merck) dissolved clear in water, and the solution showed an alkalinity of not over 0.3 per cent. We have seen no reason to believe that hydride of antimony is held back by this preparation, nor has there been any loss of antimony observed from an unduly moist condition of the chloride. We have, however, kept the chloride as active as possible by refilling the rear half of the tube with fresh chloride after every half dozen runs, as explained, or when the accumulation of moisture becomes noticeable. When not in use, the tubes are kept stoppered.

The selection of hard glass tubing is a matter of the highest importance for the success of the method. We used at first a German glass, source not known to us, which had been used in arsenic work without apparent disadvantage. This glass had a slight brown color after long ignition. Antimony mirrors deposited on this, under the conditions about to be described, were often white and not clearly defined. We next tried a sample of American glass, but this gave the brown color on ignition still more, and the deposits of antimony were entirely white. Next a Jena glass, which on ignition gave the well-known opaque appearance, and on which the mirrors were also entirely white. Finally we resorted to a glass of Kavalier, which did not give any color or opacity on long heating. The deposits of antimony on the capillaries drawn from this glass were satisfactory, and it was used in the preparation of the standard mirrors.[15]

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[14] This would mean not over 0.00004 mg. of arsenic in the amount of hydrochloric acid used, which is beyond the limit of the delicacy of the process as applied to arsenic.

[15] A cursory qualitative examination of the different samples of tubing showed no marked points of difference except that the fourth tubing contained no barium, while the others did. The amount of barium in the others was proportional to the degree of change in appearance of the mirror produced on the samples. Lack of time prevents an investigation on this point, and an opinion as to the influence of the barium, if any, would be mere conjecture; as, for example, whether the barium oxide could act catalytically in causing an oxidation of the antimony, since it seems probable that the white deposit is due to an oxide of antimony. In the absence of any definite knowledge on the matter one can only determine the availability of a sample of glass by actual trial.

pass in opposite directions through the connectors, in which they can be clamped by the screws. The end hooks are thus capable of being raised or lowered or of being moved laterally, so that the combination of the three hooks will support any tube and capillary.

We found it best to protect the capillary from direct contact with the flame, and at the same time to secure a more uniform heating by enclosing it in a brass tube or collar which is slipped over the capillary and rests on the two anterior hooks of the support. This collar is 5 cm. long, with an outer diameter of 6 mm. and an inner of 4 mm. It is heated towards the anterior end by the tip of a flame 5 to 6 mm. high from a burner with a good air supply and protected by a conical chimney. We assume that under these conditions the capillary is heated, through a space of 3 cm., at about 500°.

The Preparation of Standard Mirrors.

A standard solution of antimony was made by dissolving 2.3068 gr. of recrystallized tartar emetic in water and making up to one liter. This solution (I) contains 1 mg. of antimonious oxide in each cubic centimeter. Of solution I, 10 c.c. were diluted to a liter, giving a solution (II) containing 0.01 mg. per c.c. The tartrate offers the most convenient and accurate solution, and we satisfied ourselves that the presence of the tartrate ions had no effect whatever on the deposition of the mirror. The strength of solution I was also checked by analysis.

The zinc used in the reduction flask was the same as that used in the generator, but in smaller pieces, averaging perhaps 1 cm. in their longest dimension. The weight used was from three to five grams. This zinc, which is obtained of the New Jersey Zinc Company of New York, contains not over 0.019 per cent of lead and not more than 0.013 per cent of iron; hence the evolution of hydrogen by its contact with dilute sulphuric acid is slow. Platinum or other sensitizing agents, either in form of foil or as a deposit on the zinc, cannot be used, as we have proved by trial, since they show a tendency to hold back antimony even greater than in the case of arsenic.[13] We had recourse, therefore, to hydrochloric acid, in a dilution of one to ten, of which we use exactly 20 c.c. for each run. The hydrochloric acid, which is obtained of Messrs. Baker and Adamson, of Easton, Pa., contains no antimony, and only about 0.02 mg. of arsenious oxide per liter, an the same time. The generator which we have found most convenient is one proposed by Richards, and is similar to the forms described by him some years ago.[11] Any form of constant generator will answer which provides for withdrawal of the spent acid without disconnection. The zinc in the generator is conveniently sensitized, according to the suggestion of Gooch,[12] by brief treatment with a solution of cupric sulphate and subsequent washing. The acid is sulphuric, at a dilution of one to eight. The hydrogen from our generator shows no arsenic or antimony when run for hours at a time.

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[13] The discussion as to the effect of other metals on the evolution of arsenic hydride has been revived in recent years through the endeavors to increase the delicacy of the Marsh test for arsenic. This question, as applied to arsenic and antimony, will be taken up by one of us in a future paper.

As the hydrogen with which the antimony hydride is heated must contain no hydrogen sulphide, as hereafter shown, the hydrogen from the generator is passed through a ten per cent solution of cupric sulphate contained in an Allihn or other suitable washing bottle. From this, the hydrogen, which needs no further purification, passes to a Y-tube with two glass stopcocks. To these stopcocks are attached the reduction flasks, which are wide-mouth bottles of 60 to 75 c.c. capacity. These are fitted with a pure rubber stopper with three holes. Through one hole passes, to the bottom of the bottle, a right-angle tube connected with the stopcock; through the second a tube passing nearly to the bottom of the bottle and extending 3 to 5 cm. above the stopper. The upper end of this tube is open, the lower somewhat constricted. In the tube is placed a small funnel, blown from narrow tubing, through which the solution to be reduced is added. The third hole in the stopper carries the right-angle delivery tube for the hydrogen which passes just below the stopper. On its upper end is a rubber stopper over which is placed a 15 cm. (total length) tube filled with calcic chloride in fused sticks. We have found it convenient to half fill the tube with calcic chloride, and then to introduce a second, smaller tube filled with the chloride. By this arrangement the rear portion of the chloride, which soon becomes moist, may be frequently and conveniently renewed without disturbing the rest.

To the calcic chloride tube is connected the hard glass reduction tube drawn out to a straight capillary and ending in a capillary point. This tube is supported throughout its length by three adjustable brass hooks (No. 6 gauge, 4.1 mm.). The middle hook, which has a shank 10 cm. long, is fastened to a stand by the ordinary double clamp. On the top of the shank and about two thirds the distance from the clamped end are soldered two ordinary screw connectors at right angles to the shank. The other hooks, which are somewhat longer than the first, are bent at right angles in the plane of the table and their shanks

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[11] Amer. Chem. Jour., **20**, 189 (1898).

[12] Amer. Jour. Sci., (3) **48**, 292 (1894).

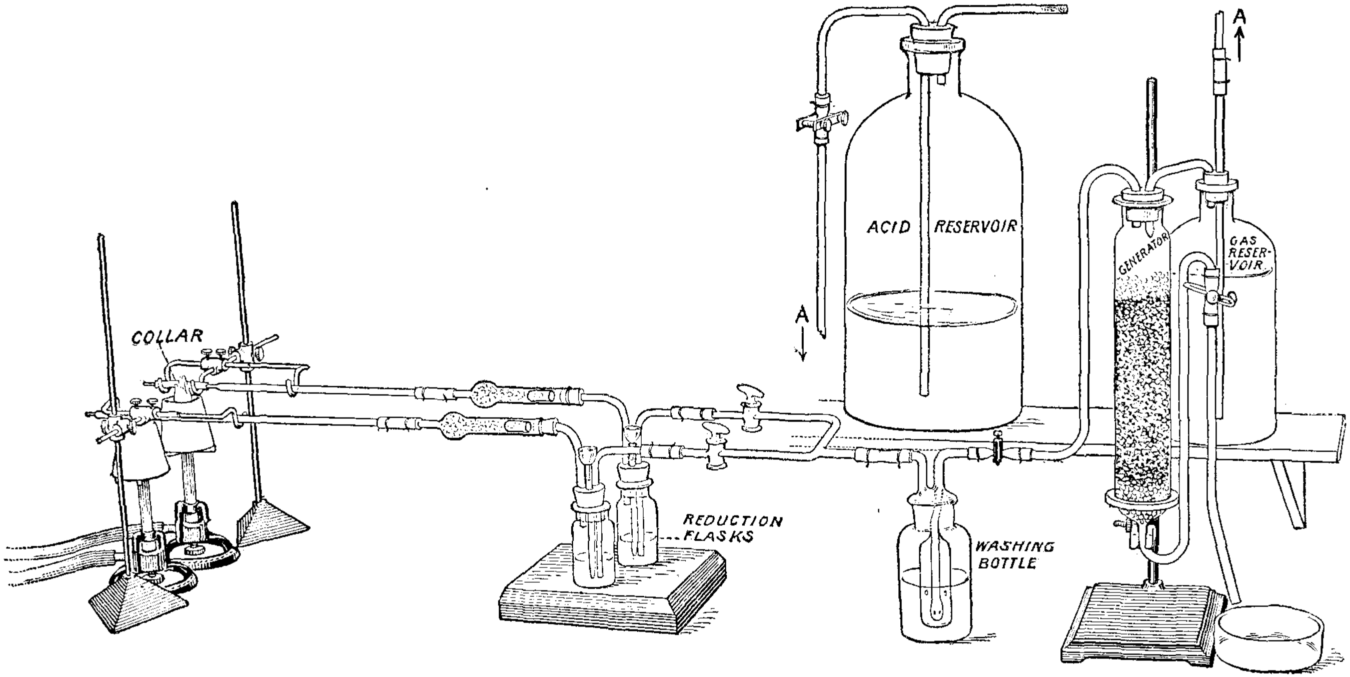
COLLAR
REDUCTION
FLASKS
ACID
RESERVOIR
A
WASHING
BOTTLE
GENERATOR
GAS
RESER-
VOIR
A

FIGURE 2.

Apparatus for the Determination of Small Amounts of Antimony.

it as a means of determining the temperatures by observation of the electromotive forces produced.

The study, as far as it concerned the separation of arsenic and antimony, is not completed, hence the results are reserved for another publication. Sufficient data were obtained to show that there was a possibility of separating arsenic and antimony by the difference in decomposition points of their diluted hydrides, and the investigation will be continued in this laboratory. Concerning the arsenic hydride, it will be sufficient to say that an amount of diluted hydride corresponding to 0.04 mg. of arsenious oxide is completely decomposed at 340° if the length of capillary heated is 3.5 cm.; at 410° if the length is 2 cm.; and at about 450° if the length is 1 cm. Conversely, no arsenic is deposited from this amount at 330° with a heating length of 1 cm.; none at about 300° if the length is 2 cm.; and none at about 250° if the length is 3.5 cm.

We were unable to get as definite results on the decomposition of antimony hydride, since the difficulty in deposition of the antimony mirror on the glass tubing used, which was afterwards solved as explained later, prevented the results from being uniform. We think, however, that we are safe in saying that amounts of hydride from 0.1 mg. antimonious oxide and under are entirely decomposed at a temperature of 300° and a heating length of 2 cm.

Having shown that practically all the antimony may be evolved as hydride from the small amounts to be used in our work, and that the hydride, diluted with hydrogen, can be entirely decomposed by heating the capillary through which the gases pass at a temperature which is easily controlled, we then proceeded to develop a method for estimating minimal amounts of antimony.

The Apparatus.

The apparatus[10] is essentially the same as that proposed by Sanger for the estimation of small amounts of arsenic. Slight modifications of this have been introduced by subsequent workers, many of whom have overlooked the original article. As shown in the figure, the parts comprise a constant hydrogen generator, two reduction flasks, and two heating tubes, so that duplicate determinations may be carried out at the hydrogen is very great compared with that of the hydride, and the decomposition would not take place except at a relatively high temperature.

---

[10] This apparatus, which is shown in Figure 2, is essentially the same as that described, but not illustrated, in the paper of Sanger (1891) above referred to. An apparatus of practically the same principle is shown on page 226 of Volume II of the Final Report of the Royal Commission on Arsenical Poisoning (Appendix 22, Report to the Commission by McGowan and Finlow on the methods employed in testing for arsenic); London, Eyre and Spottiswoode, 1903.

We have, therefore, no data as to the decomposition point of antimony hydride in such dilution, since investigators of this point, for example Brunn,[9] have dealt with mixtures containing much greater amounts of antimony. Brunn gives the decomposition point of antimony hydride as 150°; of arsenic hydride as 230°; but the amount of each in the mixture is not stated. Furthermore, in our work the differences in the amounts of antimony are relatively so slight in comparison with the large amount of diluting hydrogen that the temperatures required for decomposition should not vary within wide limits. As the amount of antimony to be estimated in mirror form should not exceed 0.1 mg. of antimonious oxide, the question to be considered is the lowest temperature at which the hydride from this amount, mixed with a relatively very large volume of hydrogen, would be decomposed when passed through the heated tube. At the same time, with a view to the possibility of separating small amounts of arsenic and antimony by the difference in decomposition points of the diluted hydrides, we included in our investigation the decomposition point of diluted arsenic hydride. While it is well known that antimony hydride, in any dilution, decomposes at a lower temperature than arsenic hydride at equal dilution, there were no exact data to guide us.

At the beginning of the study, the tube was heated just behind the drawn-out portion, or, as we shall call it, the capillary. It was very soon found that even with small portions of antimony there was a tendency to deposition in the wide part of the tube back of the heated portion, since the larger surface offers greater chance for deposition of the rapidly condensing antimony. It was then found necessary to heat the capillary itself, which had the effect of concentrating the deposit at the desired place just in front of the heated space.

For determining the decomposition temperature, we used the thermoelectric method of Le Chatelier, employing for the purpose a couple of which the wires were platinum and an alloy of platinum with ten per cent of rhodium. The couple was standardized by determining the electromotive force (in micro-volts) produced by heating the junction in the vapor of boiling naphthaline (218°), diphenylamine (302°), and sulphur (445°). With these three boiling points as ordinates and the corresponding electromotive forces as abscissae, a perfectly straight line was obtained. As the temperature to be measured did not exceed 700° or fall below 200°, it was justifiable to extend this line and use of antimony, when introduced into the Marsh apparatus, can be converted to the hydride and collected as a metallic mirror. This conclusion is reached by a consideration of the averages, for each series, of the ratios of antimony precipitated on the zinc to that deposited in the heated tube. These ratios vary from 7.7 in Series A to 0.3 in Series D, and the rate of progression is not the result of chance. ·Plotting these results, we have the following curve (Figure 1), in which the ordinates are the amount of antimony taken and the abscissae the ratios of antimony precipitated on the zinc to that deposited in the heated tube.

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[9] Ibid., **22**, 3202 (1889).

From inspection of this curve, it seems to us reasonable to suppose that amounts of antimony under a milligram when introduced into the reduction flask would be practically converted to hydride, and we have proceeded on this assumption in the development of the method which follows, since in that method the amount of antimony usually taken for a determination in no case exceeds one tenth of a milligram and is usually considerably under that amount.

With the exception of an abstract of a paper by Rieckher,[6] we have found no previous study of the ratio of antimony deposited on the zinc to that evolved as hydride. In this abstract the ratio is said to be between 92 to 8 and 96 to 4, i. e., from 11.5 to 24, but the data for this conclusion are not given. As the original paper of Rieckher is not accessible to us, we can only assume that the concentration of the antimony ions must have been much greater than in our experiments.

II. The Temperature and Cross-Section of the Deposition Tube.

The preparation of pure, gaseous antimony hydride has been accomplished in recent years by Stock and Doht[7] and by Stock and Guttmann.[8] The pure gas has been shown by these authors to have in many respects quite different properties from those of the various mixtures of hydrogen and hydride of antimony which have been in the hands of so many investigators. As to the property which concerns us in this investigation, the decomposition of the hydride by heat, Stock and Guttmann have shown that the pure gas decomposes slowly at ordinary temperature, but very readily in the presence of catalytic agents, notably antimony itself. In the case, however, of the mixture of hydrogen and antimony hydride which comes from a Marsh apparatus containing minimal amounts of antimony, the concentration of

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[6] Neues Jahrbuch. d. Pharm., **28**, 10 (1867 ?); ref., Jahresber. 1867, 255.

[7] Ber., **34**, 2339 (1901); **35**, 2270 (1902).

[8] Ibid., **37**, 885 and 901 (1904).

SERIES D.

1.	9.4	7.8	1.8	1.4	0.2
2.	10.4	8.7	1.0	1.9	0.1
3.	9.5	4.9	2.2	4.7	0.4
4.	9.5	5.7	2.0	2.7	0.3
5.	9.6	8.7	1.4	3.2	0.2
6.	9.6	7.8	2.0	3.3	0.3

Average of Series D . . . **0.3**

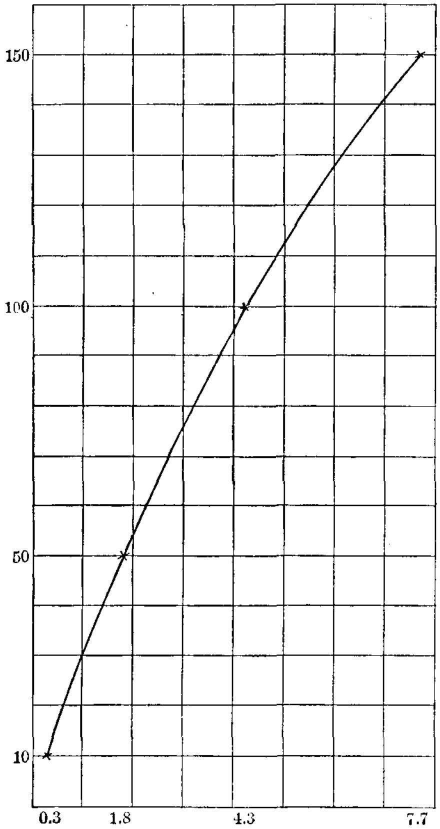

FIGURE 1.

A study of the above table will show that there is apparently a considerable amount of unreduced antimony in solution, and that all the antimony taken is not accounted for, except in Series D, in which it is less than the amount apparently found. Though the impurities in the zinc were probably too slight to influence the reaction in any way, yet the amount of zinc, which was purposely small in order to avoid the introduction of large quantities into the solutions to be titrated, was perhaps insufficient for reduction. It is not unfair to suppose that a more complete reduction would have given the same ratios between the precipitated antimony and that evolved as hydride. As to the inequality between the amounts of antimony taken and found, we think that this may be explained by the errors in the titration of such small amounts.

In spite of these criticisms of the above results, it seems to us that the study points to the conclusion that practically all of a small amount phurous acid, and the solution, neutralized with sodic hydroxide, was titrated with iodine after addition of acid sodic carbonate.

In the filtrate from the metallic antimony and zinc, the unreduced antimony in solution was determined also by titration with iodine.

Four series of determinations were made, using weighed amounts of the tartar emetic solution equivalent respectively to about 150, 100, 50, and 10 milligrams of antimony. The results are given in the following table. The analyses under columns *c* and *d* were made, in Series A and B, with an approximately decinormal solution of iodine, standardized, through sodic thiosulphate and potassic bichromate, against pure iron. In Series C and D an approximately centinormal solution of iodine was used, standardized against a determinate solution of thiosulphate made from the decinormal.

TABLE I.

No.	*a*	*b*	*c*	*d*	*e*
	mg. Sb. taken.	mg. Sb. dep. in tube.	mg. Sb. ppted on zinc.	mg. Sb. not reduced.	Ratio c/b.
			SERIES A.		
1.	147.8	9.6	106.8	14.3	11.1
2.	156.1	13.8	117.7	7.6	8.5
3.	149.2	19.2	105.0	9.6	5.5
4.	152.0	19.6	90.0	26.9	4.6
5.	151.0	12.9	136.7	12.3	10.6
6.	152.9	16.9	100.1	19.7	5.9
			Average of Series A . . .		**7.7**
			SERIES B.		
1.	101.5	17.3	71.9	7.1	4.2
2.	100.9	21.6	lost	lost	—
3.	100.9	20.0	65.3	5.5	3.3
4.	100.7	12.8	73.5	11.2	5.7
5.	100.3	13.5	lost	4.7	—
6.	100.5	15.7	64.4	7.4	4.1
			Average of Series B . . .		**4.3**
			SERIES C.		
1.	49.7	13.3	28.3	3.6	2.1
2.	"	14.9	22.7	3.6	1.5
3.	"	11.7	27.4	6.3	2.3
4.	"	lost	24.9	2.9	—
5.	"	16.9	21.8	3.9	1.3
			Average of Series C . . .		**1.8**

in the wide part of the tube; hence a series of standard mirrors under these conditions is impossible. We have found, however, that both of these obstacles are surmountable, and that small amounts of antimony in solution can be reduced with practical completeness to the hydride, which, when suitably heated, deposits the antimony in such a form as to admit of estimation with reasonable accuracy by comparison with standard mirrors.

In arriving at this result two studies were made: first, as to the influence of the concentration of the antimony ions on the deposition of the antimony upon the zinc; second, as to the influence of the temperature and cross section of the heated tube on the formation of the mirror.

I. The Evolution of Antimony Hydride in the Reduction Flask.

A solution of pure recrystallized tartar emetic was prepared of such a strength that 10 grams contained 0.0996 gram of metallic antimony (average of five determinations of the antimony as the pentasulphide, also confirmed by the volumetric method given below). A definite amount of this solution was introduced in small portions into the reduction flask (see Figure 2), which contained 0.5 gram of zinc and 20 cubic centimeters of dilute sulphuric acid (1 to 12). The evolved hydride was carried through a hot tube by a current of hydrogen according to the method described below, except that the tube was heated, as in the determination of arsenic, at its wide portion. After the deposition of the antimony in the heated tube had evidently reached its maximum, we determined the amount of the deposit, as well as the amount precipitated on the zinc and the residue still in solution. The volume of liquid in the reduction flask was approximately the same in all cases.

To find the amount of antimony deposited in the heated tube, the portion of the tube containing the deposit was weighed, the mirror dissolved in hydrochloric acid and potassic chlorate, and the tube, after washing with alcohol and ether, reweighed.

The metallic antimony and zinc left in the reduction flask were filtered, washed, and dissolved in hydrochloric acid with the aid of a small amount of potassic chlorate. The antimony in this solution was determined by the iodometric method of Gooch and Gruener.[5] The solution, free from chlorine, was reduced by boiling with potassic iodide in excess of sulphuric and tartaric acids. Any residual iodine was bleached by careful addition of approximately hundredth-normal sul-

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[5] Amer. Journ. Sci., **42**, 213 (1891).

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

THE DETERMINATION OF SMALL AMOUNTS OF ANTIMONY BY THE BERZELIUS-MARSH PROCESS.

By Charles Robert Sanger and James Andrew Gibson.

Presented January 9, 1907. Received March 11, 1907.

The Berzelius-Marsh method has been successfully applied to the quantitative determination of arsenic. On the one hand, the process which depends upon the weighing of the "mirror," first suggested by Berzelius himself,[1] has been carefully worked out by Gautier,[2] Chittenden and Donaldson,[3] and several others, and is applicable to cases in which the amount of arsenic is too small for precipitation as sulphide or in which the separation of the latter would be inconvenient. On the other hand, for cases in which the mirror of arsenic is too small for weighing, or in the rapid estimation of very small amounts, methods similar to that which was first described by one of us[4] have found wide application.

The extension of the gravimetric Berzelius-Marsh method to the determination of antimony is impossible for the reason that, under ordinary circumstances, a large part of the antimony is deposited on the zinc in the reduction flask and does not leave the solution as the hydride. In attempting to apply the method of Sanger[4] to the determination of small amounts of antimony, we were met not only by this difficulty, but also by the well-known irregularity in the deposition of the antimony in the heated tube, the temperature at which the antimony hydride is decomposed being so low that much of the deposit is formed

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[1] Berzelius, Jahresbericht, **17**, 191 (1837). The decomposition of the arsenic hydride by heating the tube was also suggested by Liebig, Ann., **23**, 217 (1837), but the idea of weighing the arsenic seems to have originated with Berzelius.

[2] Bull. Soc. Chim. (2) **24**, 250 (1876).

[3] Am. Chem. Journ., **2**, 235 (1881); Chem. News., **43**, 21 (1881).

[4] These Proceedings, **26**, 24 (1891); Amer. Chem. Journ., **13**, 431 (1891). Abstracted in Jour. Chem. Soc., **62**, 382 (1892); Jour. Soc. Chem. Ind., **11**, 370 (1892); Chem. Centralbl., **63**, 335 (1892); Ber., **25**, 47, r, (1892); Zeitschr. Anal. Chem., **38**, 137 and 377 (1899).

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By Charles Robert Sanger and James Andrew Gibson.

With a Plate.

NOTICE

The concluding Part of Vol. 42 Proceedings, containing Title-page and Index, will be issued shortly, probably in July.

pedance at a point, sets up a pair of secondary waves; namely, those that go on through the impedance, and those that go back reflected. The final steady state is the harmonic variation of a vector sum at each point, due to all the primary, secondary, tertiary, etc., waves, summed to infinity. In many cases this vector sum may be very simply expressed in hyperbolic functions; but if these functions are not made use of, there is a fearful and wonderful summation.

the current $I_0 \epsilon^{-La}$ arriving at B after the first passage, is partly reflected with inversion as from a discontinuity, and the rest of it passes through the impedance z_r to ground without further reflection. Let m be the fraction of the current that goes through z_r. This coefficient m may be called the transmission-coefficient of the impedance z_r. We have

$$m = \frac{2\, z_0}{z_0 + z_r} \tag{53}$$

a vector. Consequently the remaining portion of current $1 - m$ is reflected with inversion, as from a discontinuity, and may be called the reflection-coefficient of the impedance z_r.

$$1 - m = \frac{z_r - z_0}{z_0 + z_r} \tag{54}$$

This portion of current retreats with its sign reversed.

The current which flows through the receiving impedance z_r on first passage is $m\ I_0\ \epsilon^{-La}$ amperes, and a current $-\ (1 - m)\ I_0\ \epsilon^{-La} = (m - 1)\ I_0\ \epsilon^{-La}$ returns to the sending end. It reaches that end in the condition $(m - 1)\ I_0\ \epsilon^{-2La}$ and is reflected from the ground back to B without inversion. On reaching B the second time, it will have dwindled to $(m - 1)\ I_0\ \epsilon^{-3La}$ amperes and subdivides. The portion $m\ (m - 1)\ I_0\ \epsilon^{-3La}$ passes directly to ground through z_r, and the remainder is reflected homewards as $(m - 1)^2\ I_0\ \epsilon^{-3La}$. The final current strength built up in the receiving impedance z_r will be :

$$\begin{aligned} I_B &= m\ I_0\, \epsilon^{-La} + m\ (m - 1)\ I_0\, \epsilon^{-3La} + m\ (m - 1)^2\ I_0\ \epsilon^{-5La} + \cdots \quad (55) \\ &= m I_0 \epsilon^{-La} \{ 1 + (m - 1) \epsilon^{-2La} + (m - 1)^2 \epsilon^{-4La} + (m - 1)^3 \epsilon^{-6La} + \cdots \\ &= \frac{m I_0\ \epsilon^{-La}}{1 - (m - 1)\ \epsilon^{-2La}} = \frac{m I_0}{\epsilon^{La} - (m - 1)\ \epsilon^{-La}} = \frac{I_0}{\frac{\epsilon^{La}}{m} - \left(\frac{m - 1}{m}\right) \epsilon^{-La}} \\ &= \frac{I_0}{\left(\frac{2 - m}{m}\right) \cosh La + \sinh La} = \frac{I_0}{\frac{z_r}{z_0} \cosh La + \sinh La} \qquad \text{amperes} \\ &= \frac{E}{z_0 \sinh\ La\ +\ z_r \cosh\ La} \end{aligned} \tag{56}$$

The final result of (56) is thus identical with the value of I_B in (23). The receiving-end-impedance with the impedance z_r at B is

$$Z_l = z_0 \sinh La + z_r \cosh La \qquad \text{ohms} \tag{57}$$

Following the above line of reasoning, we can readily understand that every discontinuity in a circuit, such as a leak or a lump of im-

After an indefinitely great lapse of time, the current at A is

$$I_A = I_0 - 2\,I_0\,\epsilon^{-2La} + 2\,I_0\,\epsilon^{-4La} - 2\,I_0\,\epsilon^{-6La} + \ldots\ldots \qquad \text{amperes} \tag{41}$$

$$= I_0\{1 - 2\,\epsilon^{-2La} + 2\,\epsilon^{-4La} - 2\,\epsilon^{-6La} + \ldots\ldots \qquad \text{''} \tag{42}$$

$$= I_0\{1 - 2\,\epsilon^{-2La}\,(1 - \epsilon^{-2La} + \epsilon^{-4La} - \epsilon^{-6La} + \ldots\ldots \qquad \text{''} \tag{43}$$

$$= I_0\left\{1 - \frac{2\,\epsilon^{-2La}}{1 + \epsilon^{-2La}}\right\} = I_0\left\{1 - \frac{2}{\epsilon^{2La} + 1}\right\} = I_0\left\{\frac{\epsilon^{La} - \epsilon^{-La}}{\epsilon^{La} + \epsilon^{-La}}\right\} \tag{44}$$

$$= \frac{I_0}{\coth La} = \frac{E}{z_0 \coth La} \qquad \text{amperes} \tag{45}$$

The final result (45) thus agrees with the result of (19).

If now the line be grounded directly at B, the final voltage at B is nil, because each voltage wave is now reflected from that end with inversion. The voltage at A also remains steady at E because the voltage waves are reflected from the ground at A with inversion.

The current arriving at B the first time is $I_0\,\epsilon^{-La}$. It is reflected directly from the ground there, thus producing at B a total increment of $2\,I_0\,\epsilon^{-La}$ amperes. On arriving back at A its value is $I_0\,\epsilon_{-2L}{}^{a}$. Being reflected from the ground at A without inversion, it adds a total increment of $2\,I_0\,\epsilon^{-2La}$ to I_0 the vector current at A. When it reaches B the second time, its value is $I_0\,\epsilon^{-3La}$, and it adds there an increment of $2\,I_0\,\epsilon^{-3La}$. Reversal does not recur at either end of the line. The final current strength at B after a sufficient lapse of time is

$$I_B = 2\,I_0\,\epsilon^{-La} + 2\,I_0\,\epsilon^{-3La} + 2\,I\,\epsilon^{-5La} + \ldots\ldots \qquad \text{amperes} \tag{46}$$

$$= 2\,I_0\,\epsilon^{-La}\,(1 + \epsilon^{-2La} + \epsilon^{-4La} + \epsilon^{-6La} + \ldots\ldots \qquad \text{''} \tag{47}$$

$$= \frac{2\,I_0\,\epsilon^{-La}}{1 - \epsilon^{-2La}} = \frac{I_0\,2}{\epsilon^{La} - \epsilon^{-La}} = \frac{I_0}{\sinh La} = \frac{E}{z_0 \sinh La} \qquad \text{''} \tag{48}$$

The result (48) thus agrees with I_B in (21) or the final receiving-end-impedance of the grounded circuit is $z_0 \sinh La$ ohms.

Similarly, the final current strength at A will be the vector sum of the initial outgoing current plus all the subsequently added reflected currents, or

$$I_A = I_0 + 2\,I_0\,\epsilon^{-2La} + 2\,I_0\,\epsilon^{-4La} + 2\,I_0\,\epsilon^{-6La} + \ldots\ldots \qquad \text{amperes} \tag{49}$$

$$= I_0\{1 + 2\,\epsilon^{-2La}\,(1 + \epsilon^{-2La} + \epsilon^{-4La} + \ldots\ldots \qquad \text{''} \tag{50}$$

$$= I_0\left\{1 + \frac{2\,\epsilon^{-2La}}{1 - \epsilon^{-2La}}\right\} = I_0\left(\frac{1 + \epsilon^{-2La}}{1 - \epsilon^{-2La}}\right) = I_0 \coth La \qquad \text{''} \tag{51}$$

$$= \frac{E}{z_0 \tanh La} \qquad \text{''} \tag{52}$$

The result in (52) thus agrees with the value for I_A in (21).

If now the line be grounded through an impedance z_r ohms at B,

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Western Reserve University. *See* Laboratory of Histology and Embryology, Western Reserve University.
Wolff, J. E., An Ascent of Orizaba, Mexico, 738.
Wood, R. W., Fluorescence and Magnetic Rotation Spectra of Sodium Vapor, and their Analysis, 233–260.
Woodworth, J. B., The Ancient Water-levels of the Champlain and Hudson Valleys, 738.

Young, Edward J., Death of, 737.

Zoölogical Laboratory of the Museum of Comparative Zoölogy at Harvard College, E. L. Mark, Director, Contributions from, 101, 215, 333, 627.
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PLATE II.

Figure 15. *Entomophthora* "*x*." Young zygospore, showing stage in the formation of the endospore (× 750).

Figure 16. *Entomophthora* "*x*." Mature zygospore, three months after formation; fixed in hot sublimate-acetic (× 750).

Figure 17. *E. Geometralis.* Septate conidiophore (× 500); fixed in Flemmings.

Figure 18. *E. Geometralis.* Mature conidium (× 1000).

Figure 19. *E. Geometralis.* Large, elongated nucleus in the conidiophore (× 1000).

Figures 20–22. *E. echinospora.* Zygospore-formation (× 750).

Figure 20. Young zygospore, showing method of formation.

Figure 21. Zygospore, after the formation of the cross-wall.

Figure 22. Mature zygospore, three months after formation.

PLATE III.

Empusa Grylli.

FIGURE 23. Young hyphal body (× 750).

FIGURE 24. Upper portion of conidiophore, showing the formation of the conidium (× 750).

FIGURE 25. Mature conidium (× 1000).

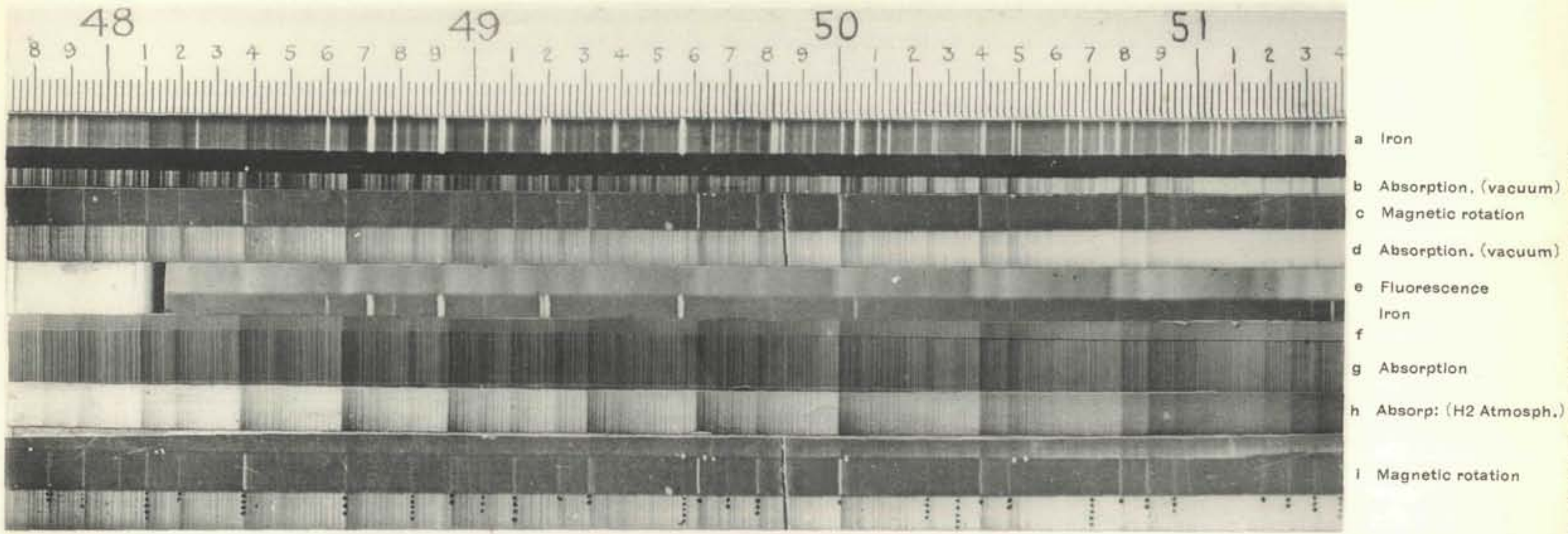
FIGURE 26. Young azygospore, showing a stage in the passage of the contents of the hyphal body into the young spore (× 750).

FIGURE 27. Young azygospore, before the formation of the cross-wall, showing the multi-nucleate condition and the attached empty hyphal body (× 750).

FIGURE 28. Mature azygospore, just after formation (× 750).

FIGURE 29. Mature azygospore, three months after formation, showing that no changes in the nuclei have taken place (× 750).

FIGURE 30. Encysted hyphal body; poor fixation due to the thick and resistant wall (× 750).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49
50
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a Iron
b Absorption. (vacuum)
c Magnetic rotation
d Absorption. (vacuum)
e Fluorescence
Iron
f
g Absorption
h Absorp: (H2 Atmosph.)
i Magnetic rotation

Fig. 1.

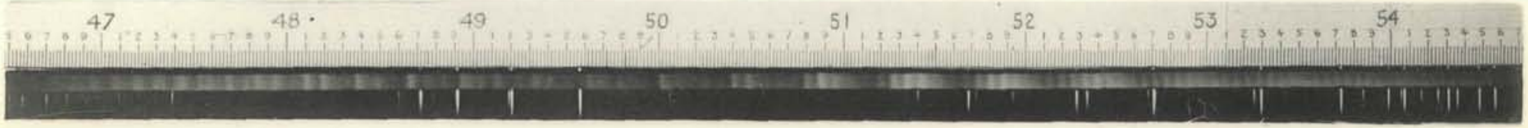
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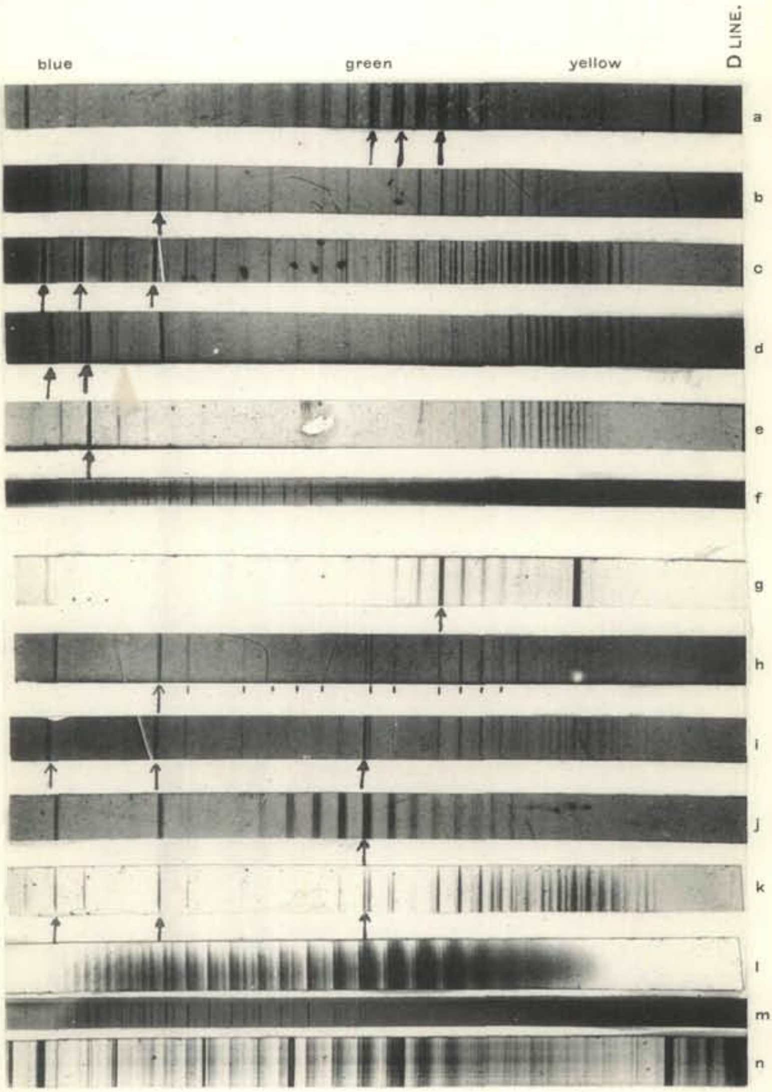
Fig. 2.

Fluorescence of Na. Fe Comparison.



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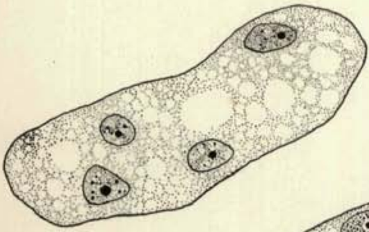

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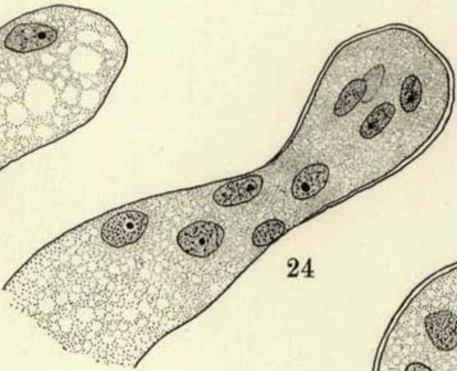

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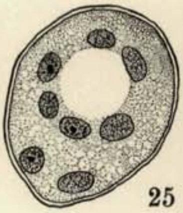

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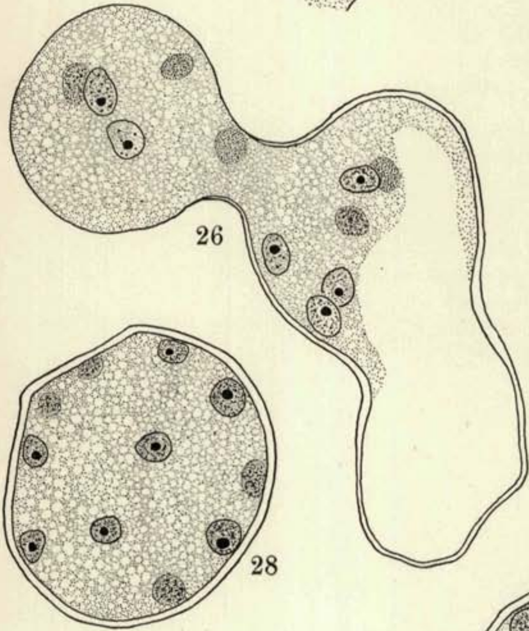

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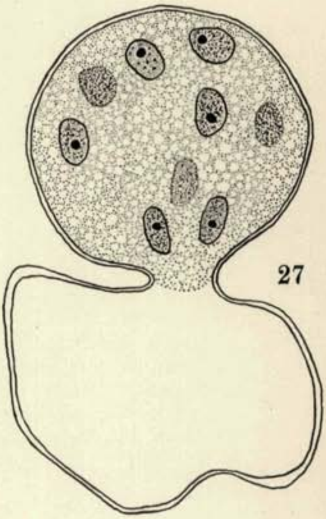

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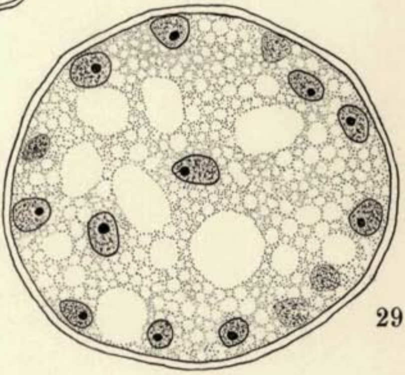

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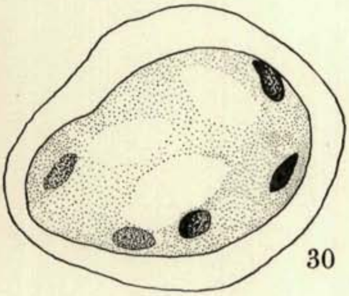

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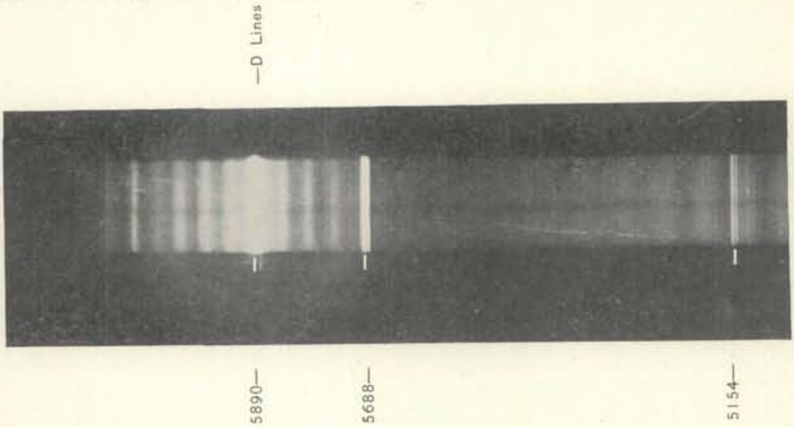

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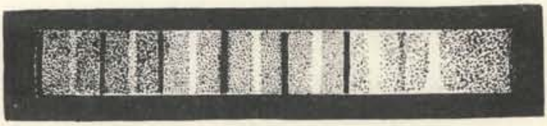

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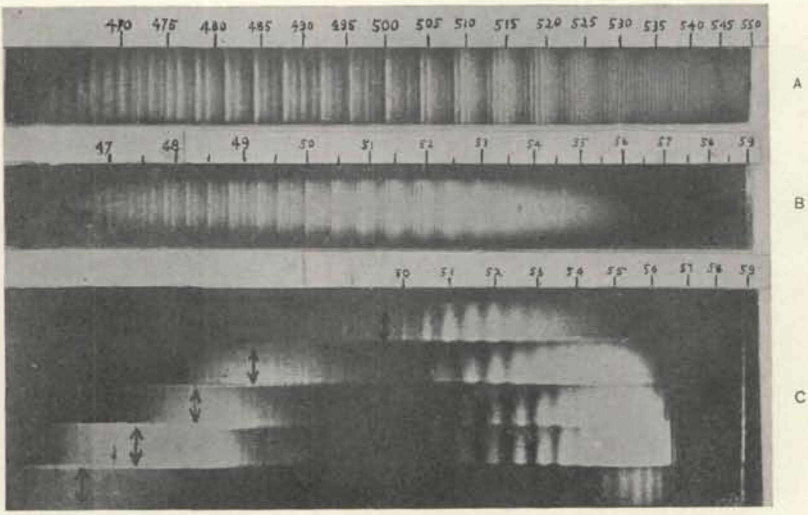

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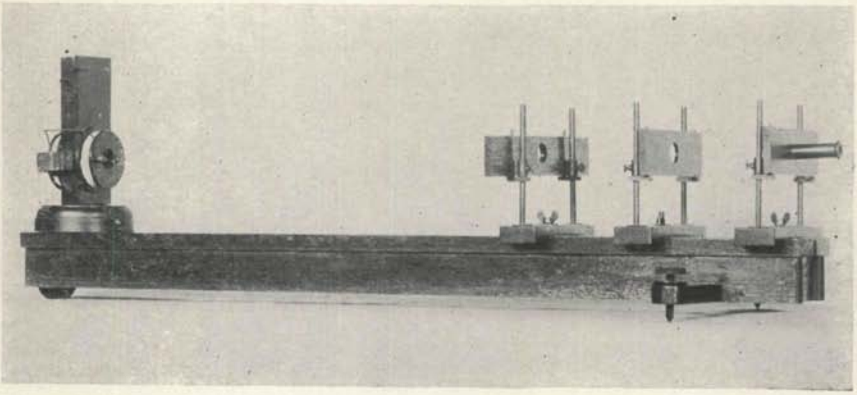

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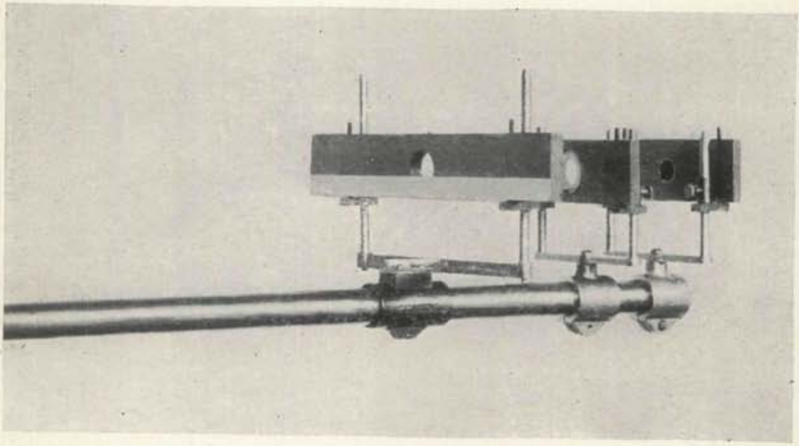

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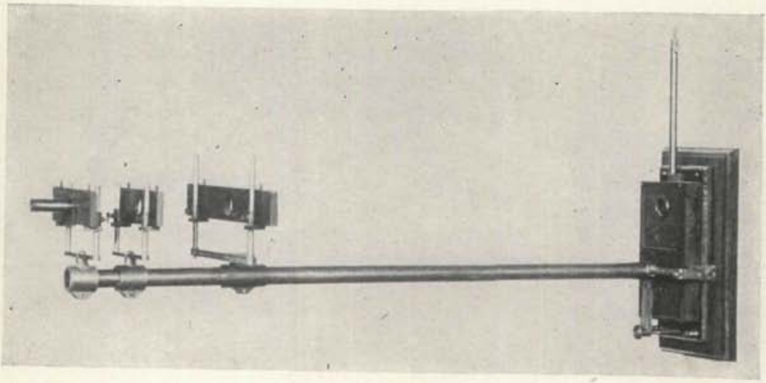

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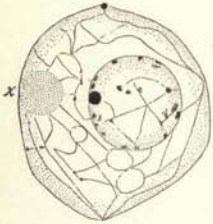

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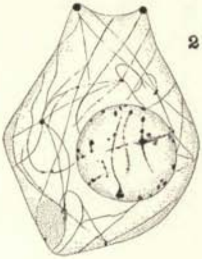

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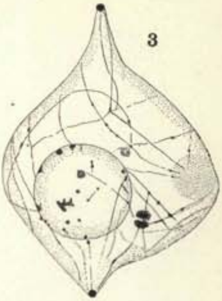

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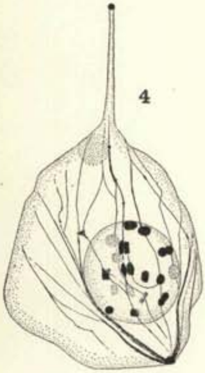

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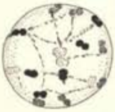

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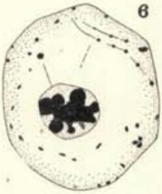

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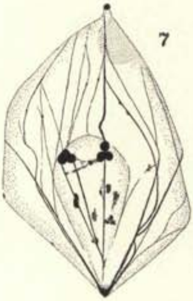

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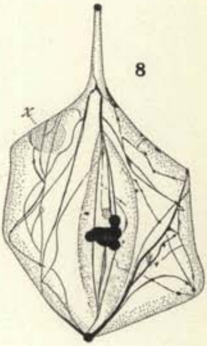

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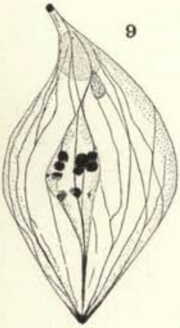

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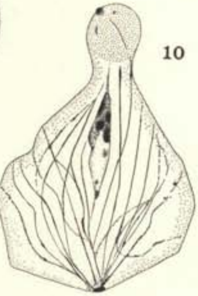

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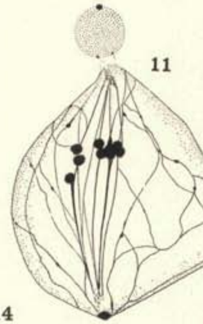

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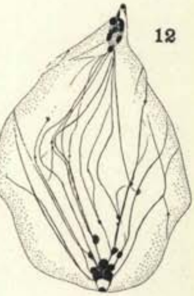

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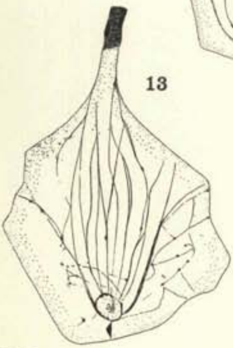

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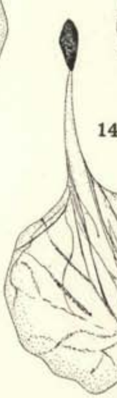

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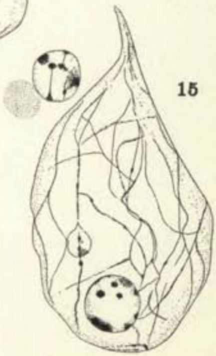

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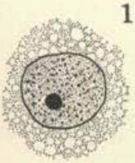

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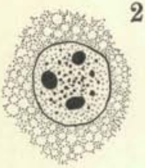

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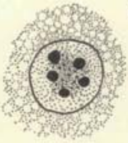

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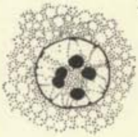

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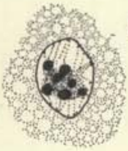

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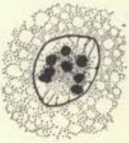

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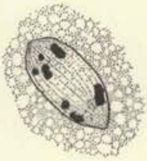

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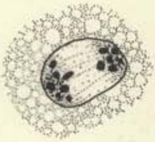

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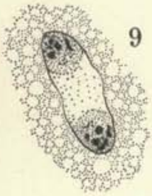

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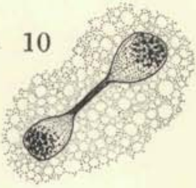

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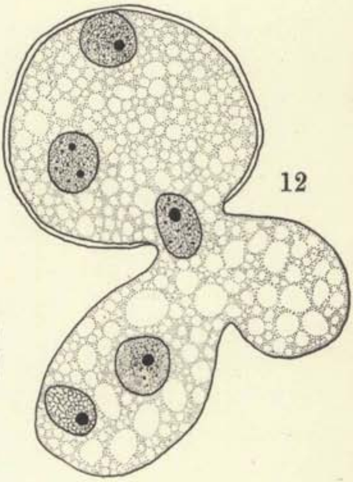

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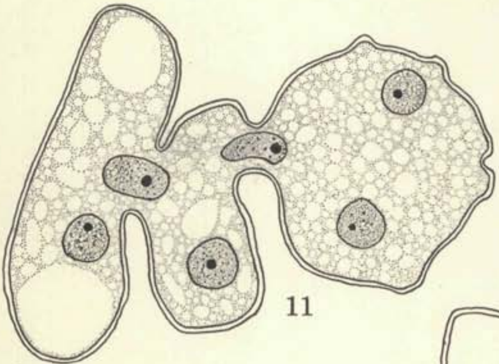

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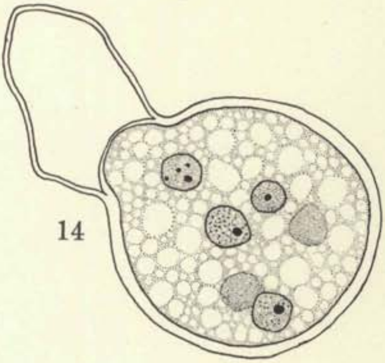

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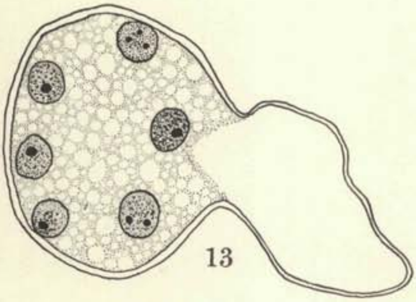

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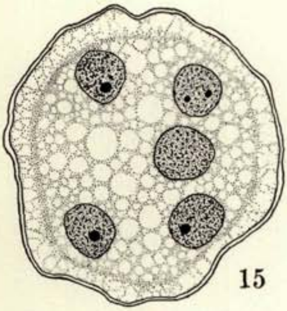

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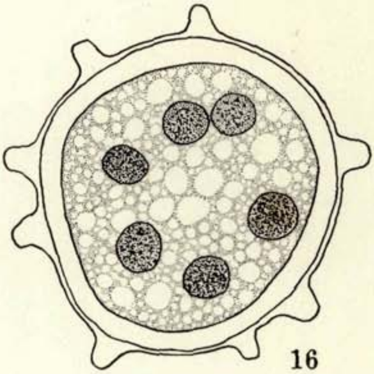

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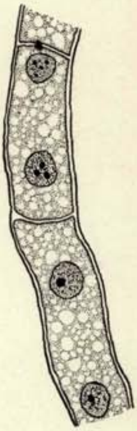

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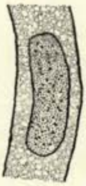

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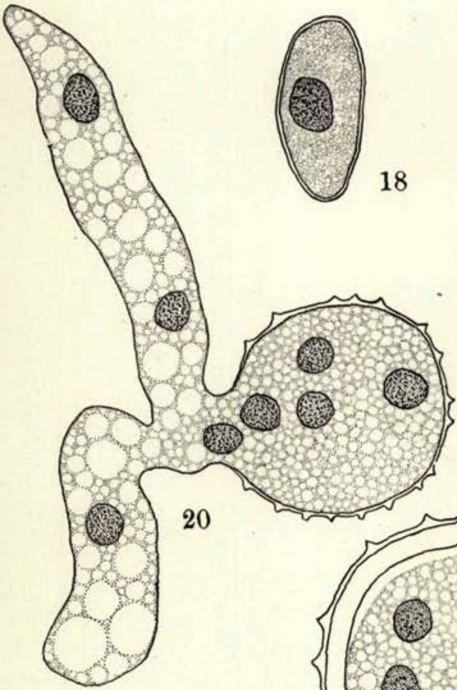

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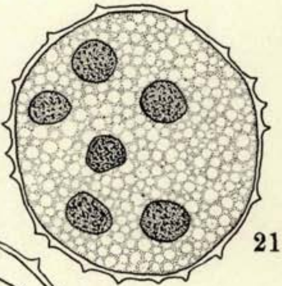

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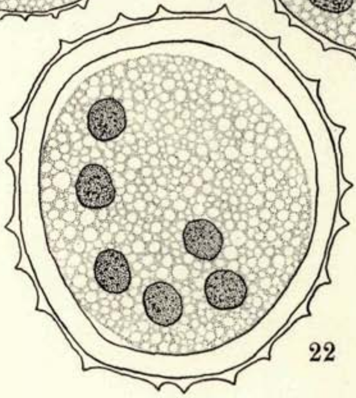

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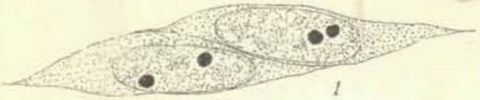
BLACKMAN.— SPERMATOCYTES OF LITHOBIUS.

PLATE 1.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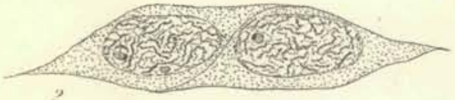
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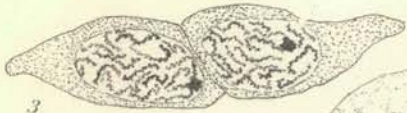
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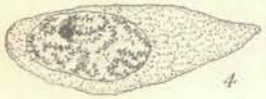
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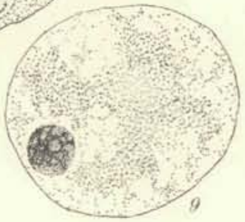
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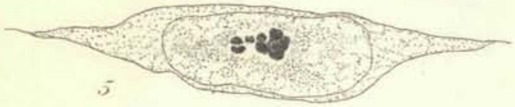
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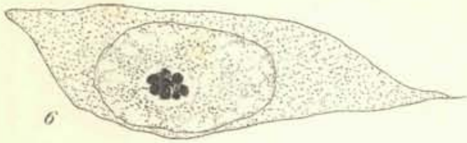
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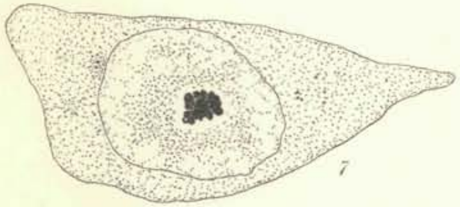
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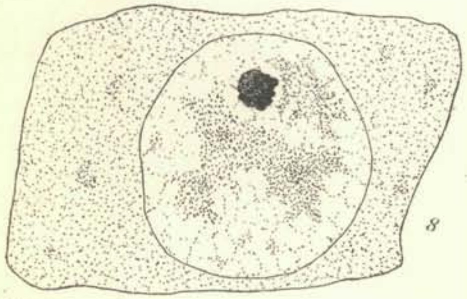
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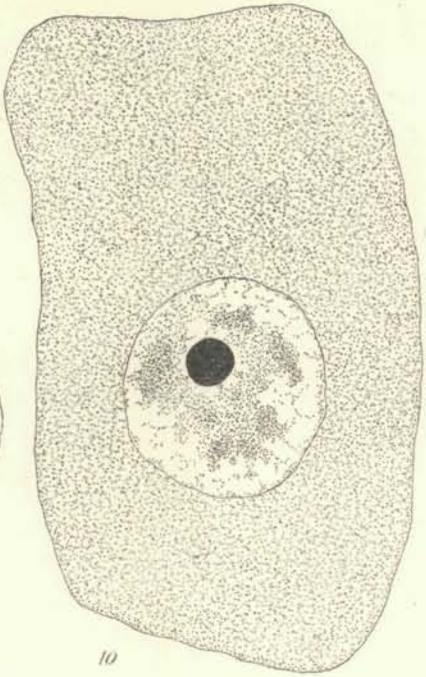
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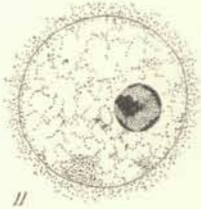
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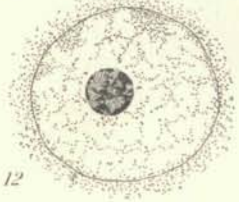
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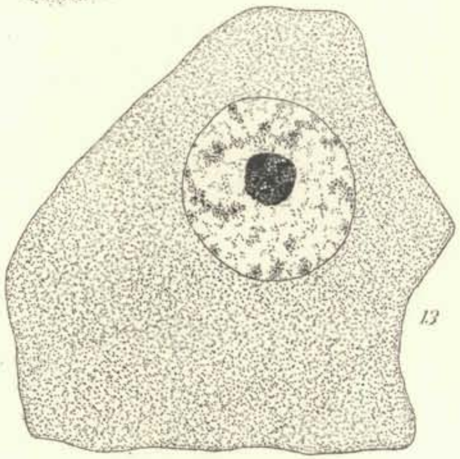
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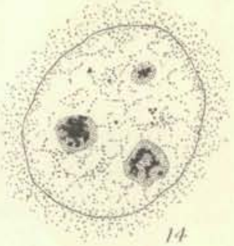
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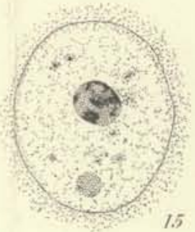
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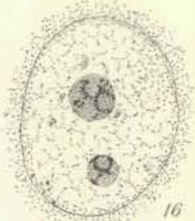
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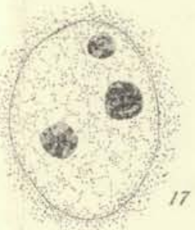
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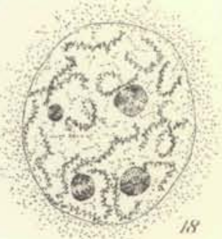
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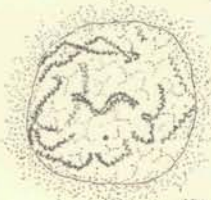
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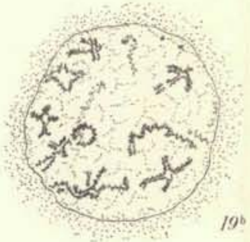
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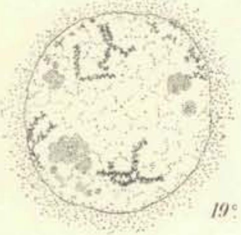
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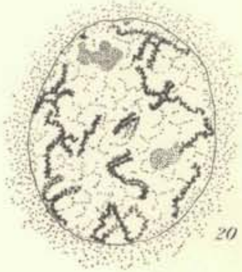
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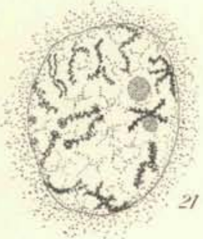
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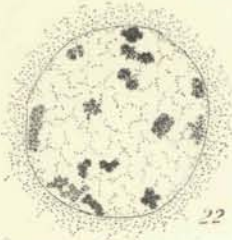
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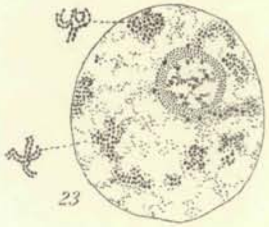
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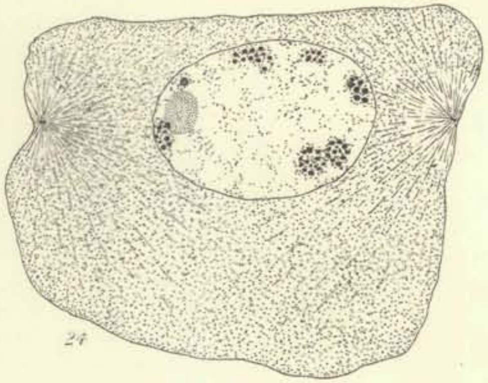
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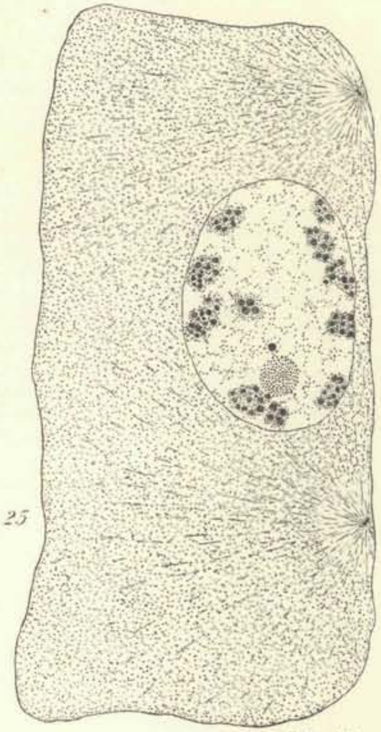
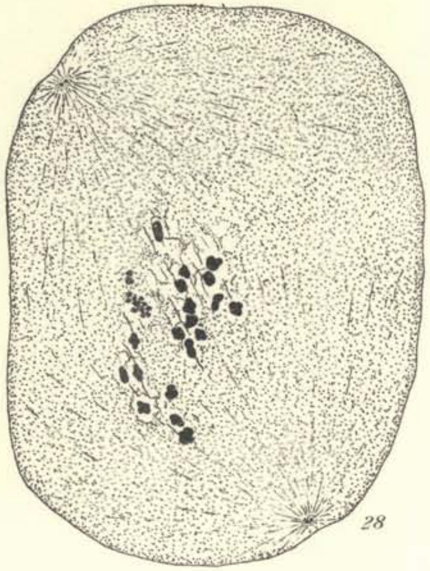

25

M.W.B. del.

B. Meisel lith.

BLACKMAN.—SPERMATOCYTES OF LITHOBIUS.

PLATE 2.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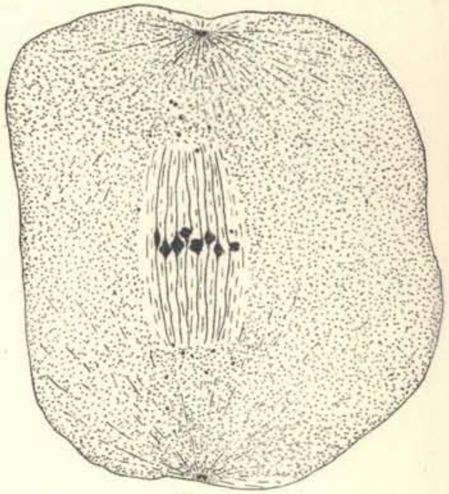
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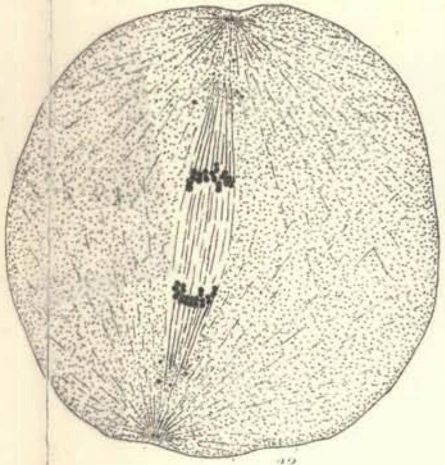
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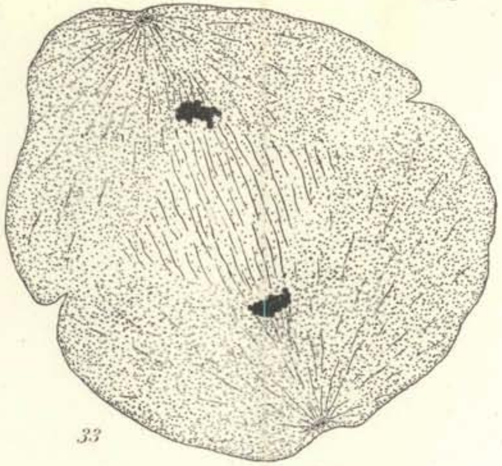
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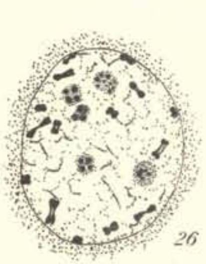
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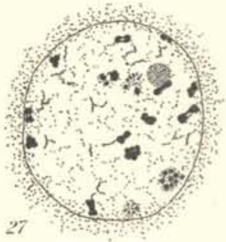
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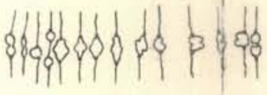
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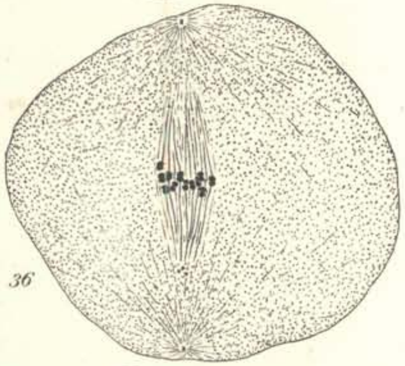
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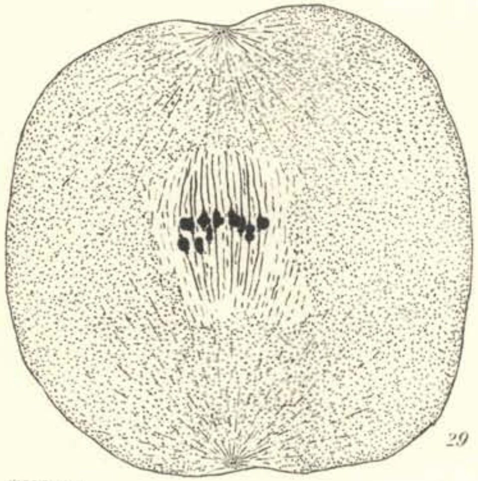
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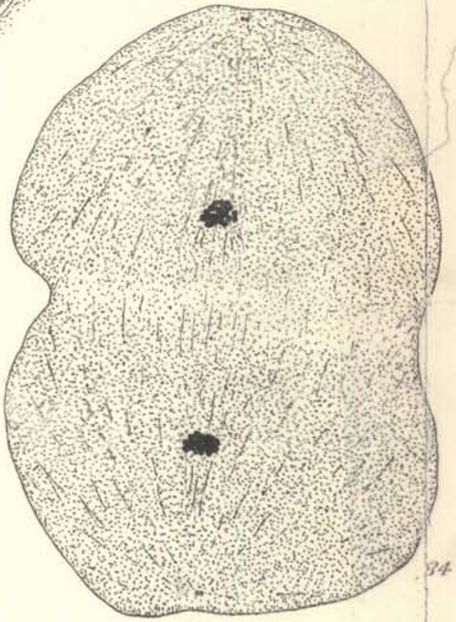
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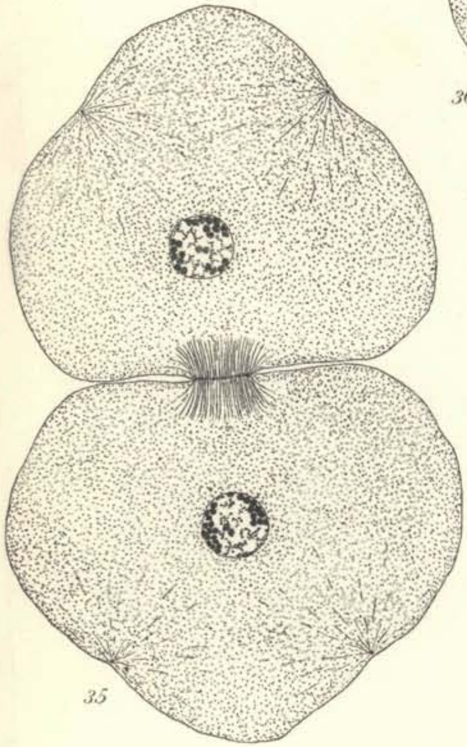
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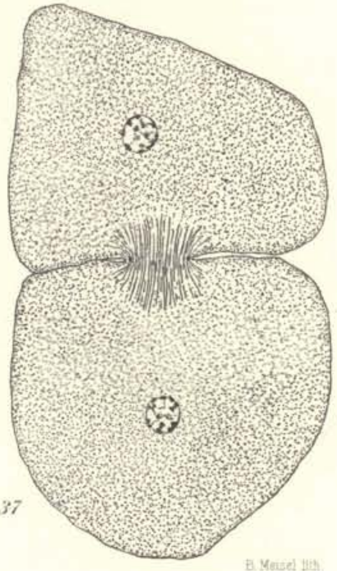
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M.W.B. del.

B. Meisel lith.

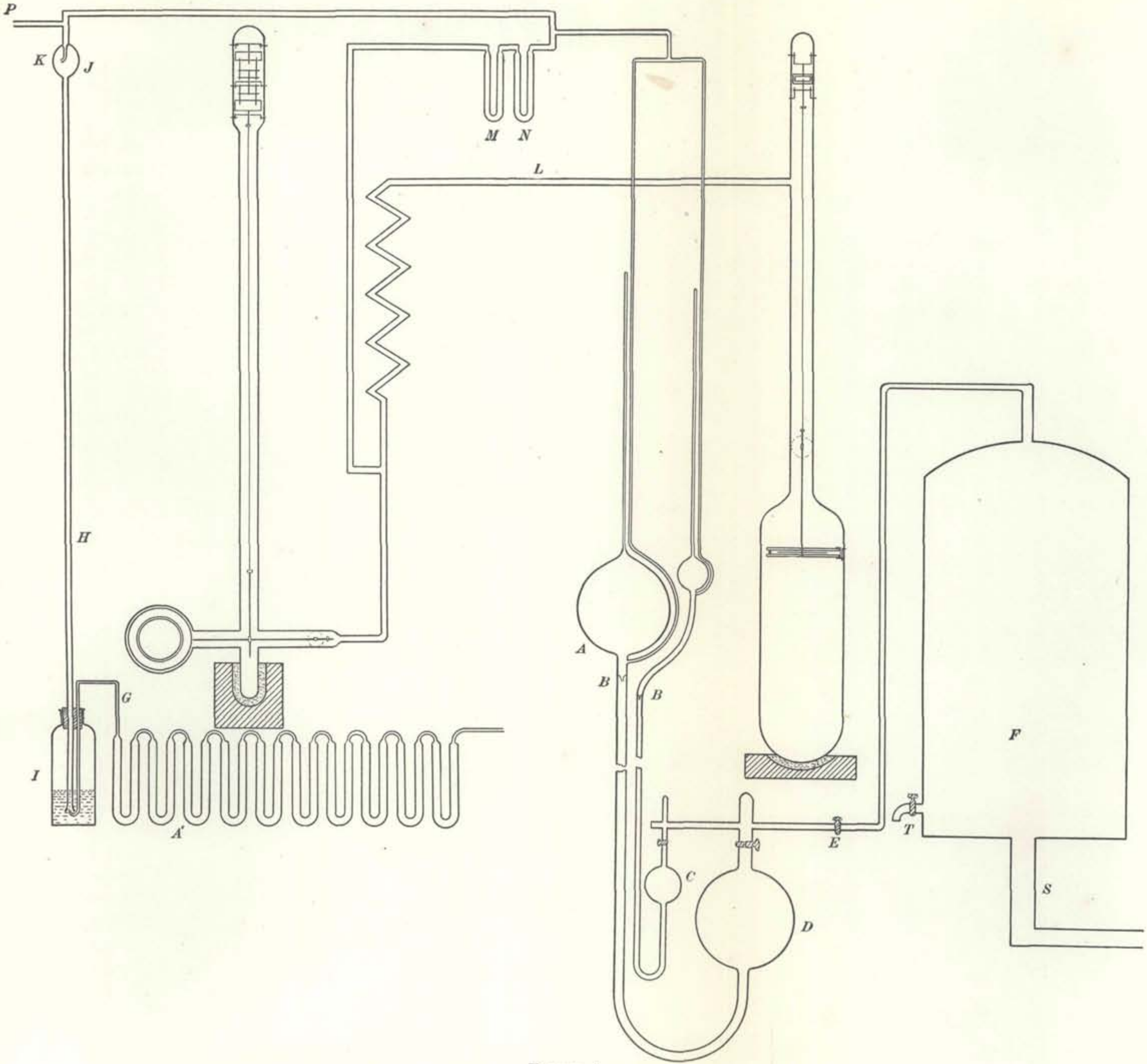
P
K
J
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FIGURE 5.

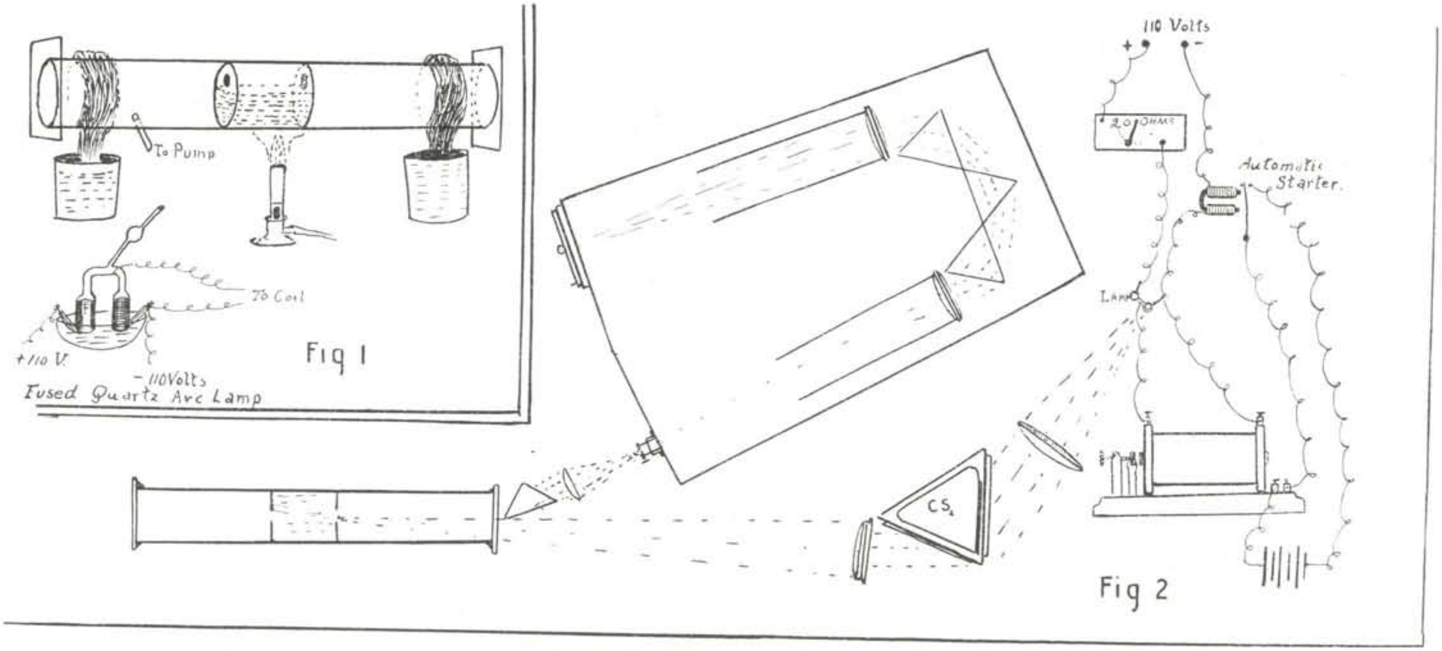

To Pump
B
To Coil
+110 V.
- 110Volts
Fused Quartz Arc Lamp
110 Volts
+
-
20 OHMS
Automatic Starter.
Lamp
CS₂

Fig 1

Fig 2

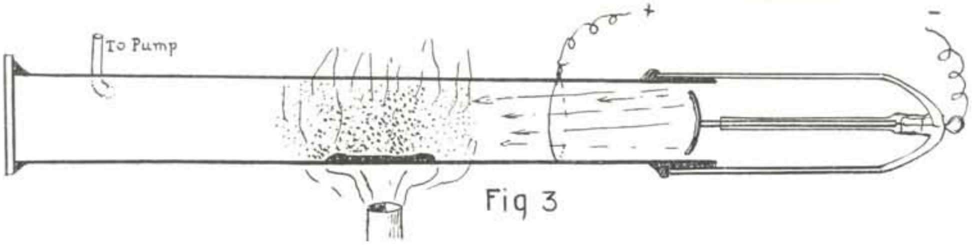
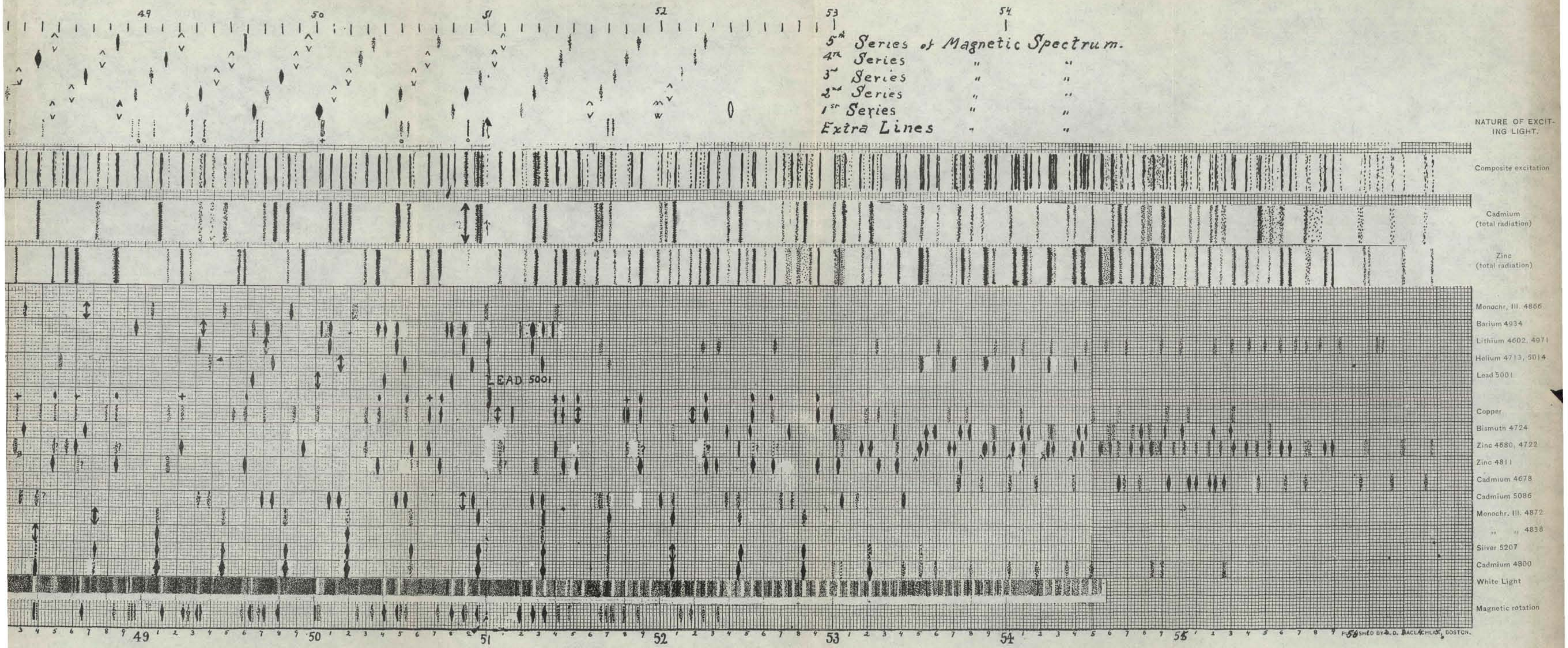


Fig 3


49
50
51
52
53
54
5th Series of Magnetic Spectrum.
4th Series " "
3rd Series " "
2nd Series " "
1st Series " "
Extra Lines " "
LEAD 5001
NATURE OF EXCITING LIGHT.
Composite excitation
Cadmium (total radiation)
Zinc (total radiation)
Monochr. Ill. 4866
Barium 4934
Lithium 4602, 4971
Helium 4713, 5014
Lead 5001
Copper
Bismuth 4724
Zinc 4680, 4722
Zinc 4811
Cadmium 4678
Cadmium 5086
Monochr. Ill. 4872
" " 4838
Silver 5207
Cadmium 4800
White Light
Magnetic rotation
49
50
51
52
53
54
55
56
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